

QUESTION 2:

- Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviations of $\sigma_1 = 0.015$ and $\sigma_2 = 0.018$. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

- Input Data :

- Machine 1 16.03 16.04 16.50 16.05 16.02 16.10 15.96 15.98 16.02 15.99
- Machine 2 16.02 15.97 15.96 16.01 15.99 16.03 16.04 16.02 16.01 16.00

a) State the hypotheses that should be tested in this experiment.

- The hypotheses that should be tested in this experiment are:
 - Null Hypothesis: $\mu_1 - \mu_2 = 0$ (Both machines fill to the same net volume)
 - Alternative Hypothesis: $\mu_1 - \mu_2 \neq 0$ (Both machines do not fill to the same net volume)

where μ_1 and μ_2 are the population means of the net volume filled by machine 1 and machine 2, respectively.

b) Test these hypotheses using $\alpha = 0.05$ and state whether we accept or reject them?

In [22]:

```
import numpy as np
from scipy import stats
import math

Machine_1 = [16.03, 16.04, 16.50, 16.05, 16.02, 16.10, 15.96, 15.98, 16.02, 15.99]
Machine_2 = [16.02, 15.97, 15.96, 16.01, 15.99, 16.03, 16.04, 16.02, 16.01, 16.00]

n_1 = len(Machine_1)
n_2 = len(Machine_2)

dof=n_1 + n_2 - 2 # Degree of freedom

print("Degree of Freedom:",dof)

mean_1 = sum(Machine_1)/n_1
mean_2 = sum(Machine_2)/n_2

# Given standard deviations
std_1 = 0.015
std_2 = 0.018

# Performing z-test since std are given

# Calculate z value
z_calculated = abs((mean_1 - mean_2) / math.sqrt(std_1**2/n_1 + std_2**2/n_2)) # Taking absolute value of z

# Calculate critical value of t
alpha = 0.05
z_critical = np.abs(np.round(stats.norm.ppf(alpha/2), 5))

# Determine p-value
p_value = 2 * stats.t.cdf(-np.abs(z_calculated), dof)

print("\nSample 1 mean:", mean_1)
print("Sample 2 mean:", mean_2)
print("Sample 1 standard deviation:", std_1)
print("Sample 2 standard deviation:", std_2)
print("\nCalculated z-value:", z_calculated)
print("Calculated z-value for dof=18 and two tailed alpha=0.05:", z_critical)
print("p-value:", p_value)

if(z_calculated<z_critical and p_value>alpha):
    print("\nAccept Null Hypothesis")
else:
    print("\nReject Null Hypothesis")
```

Degree of Freedom: 18

Sample 1 mean: 16.069000000000003

Sample 2 mean: 16.005000000000003

Sample 1 standard deviation: 0.015

Sample 2 standard deviation: 0.018

Calculated z-value: 8.637614188575112

Calculated z-value for dof=18 and two tailed alpha=0.05: 1.95996

p-value: 8.085405851143598e-08

Reject Null Hypothesis

Comments:

- Since the $z_{\text{calculated}}$ value is much higher than z_{critical} value we reject the Null Hypothesis.

c) Find a 95 percent confidence interval on the difference in mean fill volume for the two machines.

In [23]:

```
SE=math.sqrt(std_1**2/n_1 + std_2**2/n_2)
CI_lower = (mean_1 - mean_2) - z_critical * SE
CI_upper = (mean_1 - mean_2) + z_critical * SE

print(z_critical * SE)

print("\n95% Confidence Interval :")
print("Lower limit of Confidence Interval:",CI_lower)
print("Upper limit of Confidence Interval:",CI_upper)
```

0.014522232327291832

95% Confidence Interval :

Lower limit of Confidence Interval: 0.04947776767270823

Upper limit of Confidence Interval: 0.07852223232729189