

# Knapsack Problem

# Economy stares at crisis as rupee suffers worst one-day fall of 142p

By Ashish Joshi  
New Delhi

The Indian rupee fell to a record low of 64.78 against the US dollar on Monday, as investors fled the currency in the wake of a global financial crisis. The rupee's fall was the steepest in its history, with a one-day drop of 142 paise.



By Ashish Joshi  
New Delhi

The rupee's fall was the steepest in its history, with a one-day drop of 142 paise. The fall was driven by a combination of factors, including a global financial crisis and a loss of confidence in the Indian economy.

# Army intel sees foreign hand in Darjeeling stir

By Ashish Joshi  
New Delhi



The Indian Army's intelligence reports suggest that there is a foreign hand in the recent unrest in Darjeeling. The reports indicate that the situation is being manipulated by external forces.

# Train crushes 7 pilgrims in Bihar, road sets it ablaze

By Ashish Joshi  
New Delhi

A train accident in Bihar, India, resulted in the deaths of seven pilgrims. The train was carrying a large number of people, and the accident was caused by a road setting the train on fire.

# Train crushes 7 pilgrims in Bihar, road sets it ablaze

The accident occurred on a busy road, and the train was carrying a large number of people. The road was set on fire, and the train was crushed by the flames.



# Key Congress ally says he will not join

By Ashish Joshi  
New Delhi

A key Congress ally has announced that he will not join the party. The ally is a prominent figure in the Congress party, and his decision has caused a significant impact on the party's leadership.

# Mamata criticizes UPA's Bangla policy again

By Ashish Joshi  
New Delhi

Mamata Banerjee has once again criticized the UPA's policy towards Bangladesh. She has accused the government of being inconsistent and not taking a clear stance on the issue.



# Governor's friendliness to show the way

By Ashish Joshi  
New Delhi

The Governor's friendliness towards the state government is seen as a positive sign for the future of the state. It is hoped that this will lead to a more stable and prosperous state.

# Good for India's image

The recent events in India are seen as a positive sign for the country's image. It is hoped that this will lead to a more stable and prosperous India.



# Now, let's all unite and fight

By Ashish Joshi  
New Delhi

The author calls for a united front against the current challenges facing the country. It is hoped that this will lead to a more stable and prosperous India.

# 'Hello, I'm Arnie,' Kandam's star call-up

By Ashish Joshi  
New Delhi



The star call-up for Kandam is seen as a positive sign for the industry. It is hoped that this will lead to a more stable and prosperous industry.

**NEW CHARGE, JUST ONE!**

For the first time, the new charge is available in a single package.

**DOUBLE BENEFIT!!**

1. Double the benefit of the new charge.

2. Double the benefit of the new charge.

Item	Price
1. Double the benefit of the new charge	1000
2. Double the benefit of the new charge	1000
3. Double the benefit of the new charge	1000
4. Double the benefit of the new charge	1000
5. Double the benefit of the new charge	1000
6. Double the benefit of the new charge	1000
7. Double the benefit of the new charge	1000
8. Double the benefit of the new charge	1000
9. Double the benefit of the new charge	1000
10. Double the benefit of the new charge	1000

- An editor has 5 news items. The table shows the importance of each item (between 1 and 10) and number of lines for each item.

- | <u>news</u> | <u>lines</u> | <u>importance</u> |
|-------------|--------------|-------------------|
| 1           | 120          | 5                 |
| 2           | 150          | 5                 |
| 3           | 200          | 4                 |
| 4           | 150          | 8                 |
| 5           | 140          | 3                 |

- The problem is to edit the report, by deleting lines or rewriting, so that overall importance is maximized, limiting the total lines to 600 (Capacity is 600).
- Let  $x_i$  be the amount of news used (number of lines used/number of lines), and let  $p_i$  be the importance of that news, and let the capacity be  $C$ .
- The importance of the resulting draft would be  $\sum x_i p_i$ .
- The problem is to maximize this sum subject to the overall news capacity  $C$ .

- This is an example of class of problems known as **Knapsack Problem**.
- Given  $n$  objects and a knapsack of capacity  $C$ ,
- where each object has weight  $w_i$ , and earns profit  $p_i$ ,
- find values of  $x_i$  that maximizes the total profit  $\sum x_i p_i$ ,
- subject to the constraint that  $\sum x_i w_i < C$ .
- The constraint guarantees that the capacity of the knapsack is not exceeded.

# Knapsack Problem.

- Given  $n$  objects and a knapsack of capacity  $C$ , where each object has weight  $w_i$  and earns profit  $p_i$ , find values of  $x_i$  that maximizes the total profit  $\sum x_i p_i$ , subject to the constraint that  $\sum x_i w_i < C$ .
- A **greedy algorithm** would examine the objects with maximum possible profit to decide how much of each object be selected.
- Since we want to maximize the profit and minimize the weight, we should maximize the **ratio** of profit to weight.
- Greedy algorithm selects the objects in terms of decreasing order of this ratio.
- If Capacity of Knapsack is not exceeded, It selects all objects.
- If it is going to exceed, then take whatever portion of last object can be taken to fill the knapsack.

- We now calculate profit to weight ratio for the news items.

<u>news</u>	<u>lines</u>	<u>importance</u>	<u>ratio p/w</u>
1	120	5	$5/120 = .0417$
2	150	5	.0333
3	200	4	.0200
4	150	8	.0533
5	140	3	.0214

We select the items in order of 4,1,2, 5, 3

Knapsack weight =  $150+120+150+140 = 560$ .

Now we cannot take all of item 3,

so we take only  $40/200$  (that is 0.2) of item 3.

- We now calculate profit to weight ratio for the news items.

<u>news</u>	<u>lines</u>	<u>importance</u>	<u>ratio p/w</u>	<u>profit</u>
1	120	5	$5/120 = .0417$	5
2	150	5	.0333	5
3	200	4	.0200	$0.2 \times 4 = 0.8$
4	150	8	.0533	8
5	140	3	.0214	3

We select the items in decreasing order of 4,1,2, 5, 3

Knapsack weight of items 4,1,2, 5 =  $150+120+150+140 = 560$ .

Now we cannot take all of item 3,

so we take only  $40/200$  (that is 0.2) of item 3.

Total Profit = 21.8

- Prob 2

- 3 Sugar bottles with weight and profit given below. Fill up sugar in bottle of size 20.

<u>items</u>	<u>Weight</u>	<u>Profit</u>	<u>ratio p/w</u>
1	14	24	1.71
2	18	24	1.11
3	10	16	1.6

We select the items in order of 1, 3, 2

Knapsack Capacity = 20.

Knapsack Capacity after item 1 =  $20 - 14 = 6$ .

we can take only 6/10 of item 3.

Knapsack Capacity after item 3 = 0, (No space for item 2)

NET PROFIT:  $24 + 16 (0.6) = 24 + 9.6 = 33.6$



# Complexity of Knapsack

- All profits need to be sorted.
- Sorting step takes  $O(n \log n)$
- Looping through all the items takes  $O(n)$  time.
- So overall complexity is  $O(n \log n)$ .
  
- A **brute force** approach would involve trying out all possible combinations of weights.
- For  $n$  items, there can be  $2^n$  subsets of weights,  $O(2^n)$  is practically impossible to solve if  $n$  is large.
- If  $n$  is 20, it is close to 1 million steps.

- These type of problems are known as **Continuous Knapsack problems**.
  - The greedy approach generates the optimal (best ) solution .
  - However, there is another type of Knapsack problems for which solution may not be optimal.
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- <https://www.javatpoint.com/fractional-knapsack-problem>
  - [https://www.tutorialspoint.com/design\\_and\\_analysis\\_of\\_algorithms/design\\_and\\_analysis\\_of\\_algorithms\\_fractional\\_knapsack.htm](https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_fractional_knapsack.htm)

# The 0/1 Knapsack Problem

- The 0/1 Knapsack problem is slightly different.
- Here you cannot take portion of an item.
- You either take the whole item, or leave it.

# Thief in a museum

## What should he take?

- The burglar wishes to carry away the most valuable items subject to the weight constraint.
- No point taking fraction of an object, so he must make a decision to take the object entirely or leave it



- Prob 3 (0/1 Knapsack Problem)

- A thief wants to maximize his profit by stealing items from a museum. He has got a bag which can hold 60 Kg of goods.

<u>items</u>	<u>Price</u>	<u>weight</u>	<u>ratio p/w</u>
1	1000	10	100
2	2800	40	70
3	1200	20	60

The Greedy approach selects the items in order of 1, 2, 3

Knapsack Capacity = 60.

Knapsack Capacity after item 1 =  $60 - 10 = 50$ .

Knapsack Capacity after item 2 =  $50 - 40 = 10$ .

Profit after selling the items :  $1000 + 2800 = 3800$ .

However, this is not the Optimal solution (best solution).

The best solution is to take items 2 and 3, which gives a profit of 4000.

- Thus the Greedy approach may sometimes fail for 0/1 Knapsack problems.

# Job Scheduling Problem



- The goal is to schedule the jobs such that task completion profit is maximized.
- The job needs to be completed before deadline for the job.
- A sequence of jobs is feasible if all the jobs end by their deadline.
- A set of jobs is called a feasible set, if at least one sequence is possible.
- The sequence associated with maximum profit is called the optimal sequence.

- You are given a set of  $n$  jobs.
- Each job takes **one** unit of time to complete, and *only one job can be scheduled at a time.*
- Suppose the unit is one day.
- Each job has a deadline. If deadline is 2, it means job can be delivered after 2 days.
- We earn the profit associated with a job, if and only if the job is completed by its deadline.
- Find the job scheduling of the given jobs that ensure maximum profit.

- Consider a problem of scheduling 4 jobs on 5<sup>th</sup> Feb.

- | <u>Job</u> | <u>Deadline</u> | <u>Profit</u> |
|------------|-----------------|---------------|
| 1          | 2               | 60            |
| 2          | 1               | 30            |
| 3          | 2               | 40            |
| 4          | 1               | 80            |

- Consider job Sequence {1,2}. Job 1 is to be delivered on 7<sup>th</sup> Feb and Job 2 is to be delivered on 6<sup>th</sup> Feb.
- Job 1 can be scheduled on 5<sup>th</sup> Feb. This would be delivered on 6<sup>th</sup> Feb.
- Job 2 can be scheduled only after job 1 is delivered. But job 2 cannot be scheduled as it is already 6<sup>th</sup> Feb.
- So sequence {1, 2} is not feasible.
- For same reason sequences {1,4}, {2,4}, {1,2,3} ,. . . . .are not feasible

- | <u>Job</u> | <u>Deadline</u> | <u>Profit</u> |
|------------|-----------------|---------------|
| 1          | 2               | 60            |
| 2          | 1               | 30            |
| 3          | 2               | 40            |
| 4          | 1               | 80            |

- Consider now the **job Sequence {2,1}**. Job 2 is to be delivered on 6<sup>th</sup> Feb and Job 1 is to be delivered on 7<sup>th</sup> Feb.
- Job 2 can be scheduled on 5<sup>th</sup> Feb. This would be delivered on 6<sup>th</sup> Feb.
- Job 1 can be scheduled after that ( on 6<sup>th</sup> Feb), and this can be delivered on 7<sup>th</sup> Feb in time.
- So both jobs will be delivered in time. We say that sequence {2, 1} is feasible.

- Here are the feasible sequences and corresponding profits.

• Job sequence	<u>Total Profit</u>
2,1	90 (60 + 30)
1,3	100
2,3	70
4,1	140
4,3	120

- Max profit is associated with Sequence {4,1}.
- Optimal order is Sequence {4,1}.
- Note that Sequence {1,4} is not feasible, as deadlines are violated.

# Greedy approach to Scheduling Algorithm

- First, sort the jobs in the decreasing order of their profits.
- Then find the highest deadline among all deadlines.
- Next, we need to assign time slots to individual job ids.
- First, check if the maximum possible time slot for the job, i.e., its deadline, is assigned to another job or not.
  - If it is not filled yet, assign the slot to the current job id.
  - Otherwise, search for any empty time slot less than the deadline of the current job.
  - If such a slot is found, assign it to the current job id, and move to the next job id.

- Consider a problem of scheduling 5 jobs.

- | <u>Job</u> | <u>Deadline</u> | <u>Profit</u> |
|------------|-----------------|---------------|
| 1          | 2               | 100           |
| 2          | 1               | 19            |
| 3          | 2               | 30            |
| 4          | 1               | 25            |
| 5          | 3               | 15            |

- Sort the jobs in decreasing order of profit.

- | <u>Job</u> | <u>Deadline</u> | <u>Profit</u> |
|------------|-----------------|---------------|
| 1          | 2               | 100           |
| 3          | 2               | 30            |
| 4          | 1               | 25            |
| 2          | 1               | 19            |
| 5          | 3               | 15            |

- Consider first the job with highest profit (job 1)
- Next highest profitable job is job 3. In 2 days, job 1 and job 3 can be delivered.
- However, job 4 and job 2 cannot be scheduled as they don't have enough deadline.
- But job 5 can be scheduled, as its deadline permits job to be scheduled on 3<sup>rd</sup> day.
- So { 1, 3, 5} is an optimal solution. Note {3, 1, 5} is also an optimal solution.



# complexity analysis

Main computation of scheduling problem is the sorting step

- This takes  $O(n \log n)$  time.
- All other steps take constant time.
- Therefore complexity of scheduling problem is  $O(n \log n)$ .

- read up more about this topic:
- [https://www.tutorialspoint.com/design\\_and\\_analysis\\_of\\_algorithms/  
design\\_and\\_analysis\\_of\\_algorithms\\_job\\_sequencing\\_with\\_deadline.  
htm](https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_job_sequencing_with_deadline.htm)