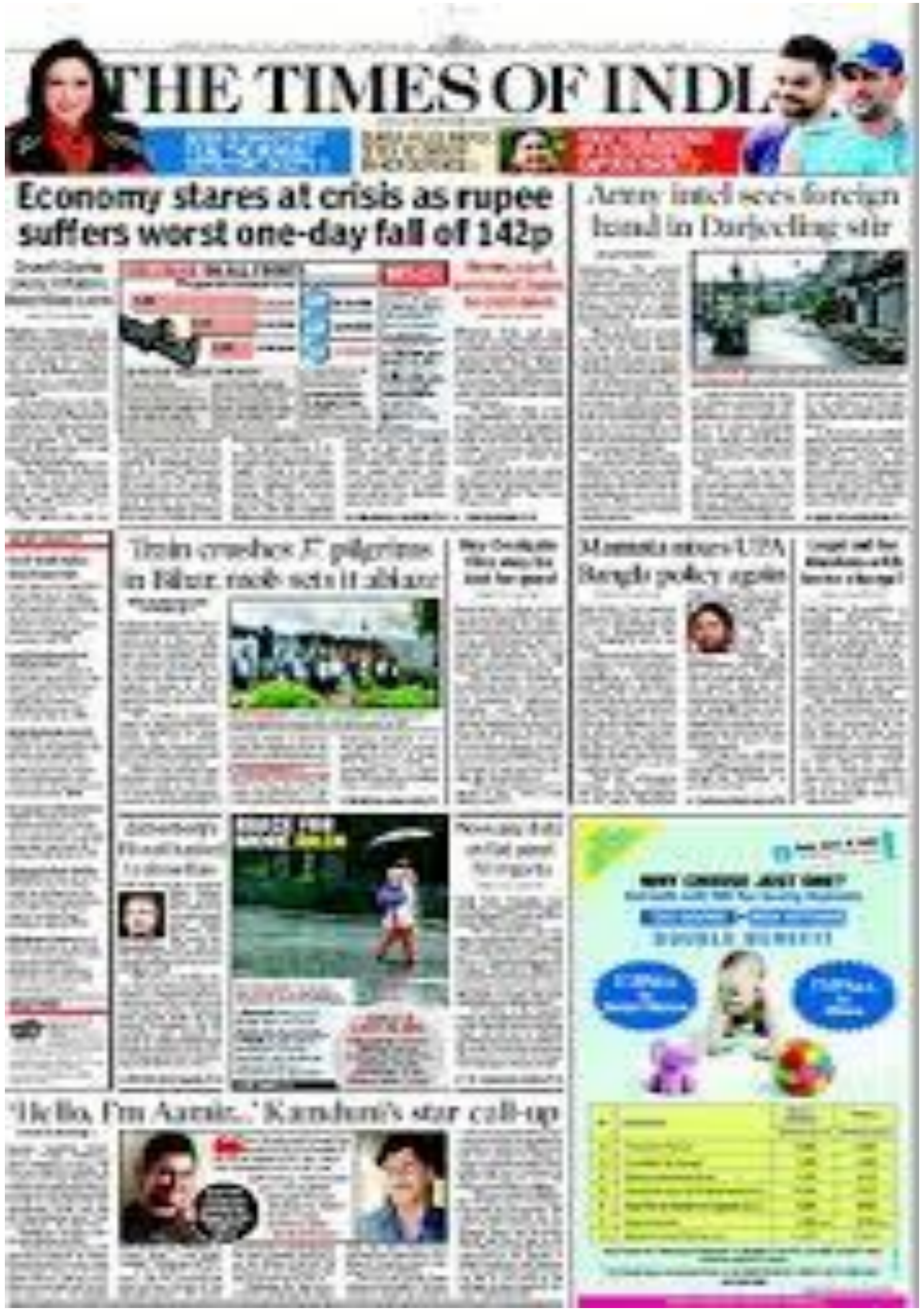
Knapsack Problem


THE TIMES OF IND

Economy stares at crisis as rupee suffers worst one-day fall of 142p

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WHY CHOOSE JUST ONE?

DOUBLE BENEFIT

- An editor has 5 news items. The table shows the importance of each item (between 1 and 10) and number of lines for each item.

news	lines	importance
1	120	5
2	150	5
3	200	4
4	150	8
5	140	3

- The problem is to edit the report, by deleting lines or rewriting, so that overall importance is maximized, limiting the total lines to 600 (Capacity is 600).
- Let x_i be the amount of news used (number of lines used/number of lines), and let p_i be the importance of that news, and let the capacity be C.
- The importance of the resulting draft would be $\sum x_i p_i$.
- The problem is to maximize this sum subject to the overall news capacity C.

- This is an example of class of problems known as Knapsack Problem.

- Given n objets and a knapsack of capacity C,
- where each object has weight w_i ,and earns profit p_i ,
- find values of x_i that maximizes the total profit $\sum x_i p_i$,
- subject to the constraint that $\sum x_i w_i < C$.

- The constraint guarantees that the capacity of the knapsack is not exceeded.

Knapsack Problem.

- Given n objets and a knapsack of capacity C, where each object has weight w_i . and earns profit p_i , find values of x_i that maximizes the total profit $\sum x_i p_i$, subject to the constraint that $\sum x_i w_i < C$.
- A greedy algorithm would examine the objects with maximum possible profit to decide how much of each object be selected.

- Since we want to maximize the profit and minimize the weight, we should maximize the ratio of profit to weight.
- Greedy algorithm selects the objects in terms of decreasing order of this ratio.
- If Capacity of Knapsack is not exceeded, It selects all objects.
- If it is going to exceed, then take whatever portion of last object can be taken to fill the knapsack.

- We now calculate profit to weight ratio for the news items.

news	lines	importance	ratio p/w
1	120	5	5/120 = .0417
2	150	5	.0333
3	200	4	.0200
4	150	8	.0533
5	140	3	.0214

We select the items in order of 4,1,2, 5, 3

Knapsack weight = 150+120+150+140 = 560.

Now we cannot take all of item 3,

so we take only 40/200 (that is 0.2) of item 3.

- We now calculate profit to weight ratio for the news items.

news	lines	importance	ratio p/w	profit
1	120	5	5/120 = .0417	5
2	150	5	.0333	5
3	200	4	.0200	0.2x4= 0.8
4	150	8	.0533	8
5	140	3	.0214	3

We select the items in decreasing order of 4,1,2, 5, 3

Knapsack weight of items 4,1,2, 5 = 150+120+150+140 = 560.

Now we cannot take all of item 3,

so we take only 40/200 (that is 0.2) of item 3.

Total Profit = 21.8

- Prob 2
- 3 Sugar bottles wiith weight and profit given below. Fill up sugar in bottle of size 20.

items	Weight	Profit	ratio p/w
1	14	24	1.71
2	18	24	1.11
3	10	16	1.6

We select the items in order of 1, 3, 2

Knapsack Capacity = 20.

Knapsack Capacity after item 1 = 20 – 14 = 6.

we can take only 6/10 of item 3.

Knapsack Capacity after item 3 = 0, (No space for item 2)

NET PROFIT: 24 + 16 (0.6) = 24 + 9.6 = 33.6

Complexity of Knapsack

- All profits need to be sorted.
- Sorting step takes O(n log n)
- Looping through all the items takes O(n) time.
- So overall complexity is O(nlog n).

- A brute force approach would involve trying out all possible combinations of weights.
- For n items, there can be 2^n subsets of weights, $O(2^n)$ is practically impossible to solve if n is large.
- If n is 20, it is close to 1 million steps.

- These type of problems are known as Continuous Knapsack problems.
- The greedy approach generates the optimal (best) solution .
- However, there is another type of Knapsack problems for which solution may not be optimal.

- https://www.javatpoint.com/fractional-knapsack-problem
- https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_fractional_knapsack.htm

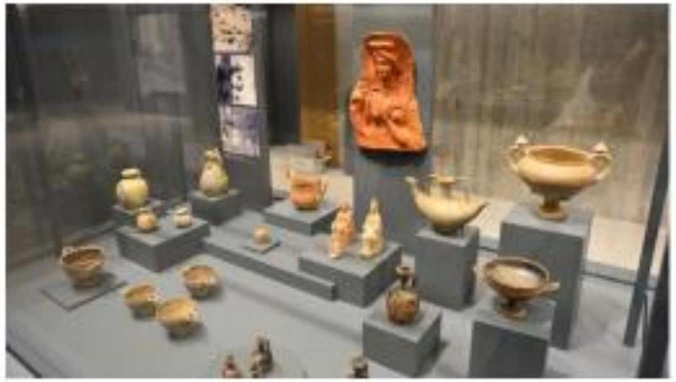
The 0/1 Knapsack Problem

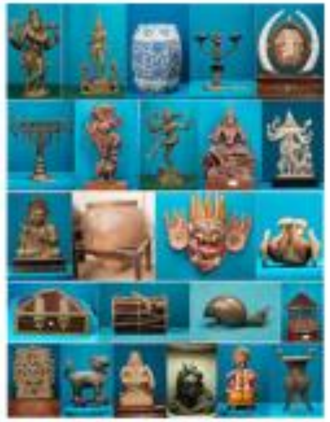
- The 0/1 Knapsack problem is slightly different.
- Here you cannot take portion of an item.
- You either take the whole item, or leave it.

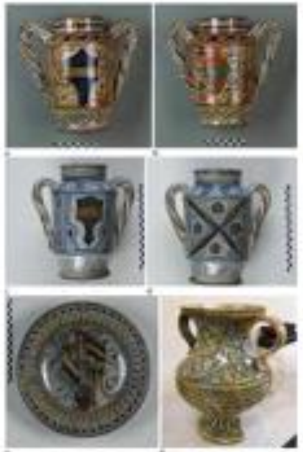
Thief in a museum

What should he take?

- The burglar wishes to carry away the most valuable items subject to the weight constraint.
- No point taking fraction of an object, so he must make a decision to take the object entirely or leave it







- Prob 3 (0/1 Knapsack Problem)
- A thief wants to maximize his profit by stealing items from a museum. He has got a bag which can hold 60 Kg of goods.

items	Price	weight	ratio p/w
1	1000	10	100
2	2800	40	70
3	1200	20	60

The Greedy approach selects the items in order of 1, 2, 3

Knapsack Capacity = 60.

Knapsack Capacity after item 1 = 60 – 10= 50.

Knapsack Capacity after item 2 = 50 – 40= 10.

Profit after selling the items : 1000+2800 = 3800.

However, this is not the Optimal solution (best solution).

The best solution is to take items 2 and 3, which gives a profit of 4000.

- Thus the Greedy approach may sometimes fail for 0/1 Knapsack problems.

Job Scheduling Problem

- The goal is to schedule the jobs such that task completion profit is maximized.
- The job needs to be completed before deadline for the job.

- A sequence of jobs is feasible if all the jobs end by their deadline.
- A set of jobs is called a feasible set, if at least one sequence is possible.
- The sequence associated with maximum profit is called the optimal sequence.

- You are given a set of n jobs.
- Each job takes **one** unit of time to complete, and *only one job can be scheduled at a time.*
- Suppose the unit is one day.
- Each job has a deadline. If deadline is 2, it means job can be delivered after 2 days.
- We earn the profit associated with a job, if and only if the job is completed by its deadline.
- Find the job scheduling of the given jobs that ensure maximum profit.

- Consider a problem of scheduling 4 jobs on 5th Feb.

Job	Deadline	Profit
1	2	60
2	1	30
3	2	40
4	1	80

- Consider job Sequence {1,2}. Job 1 is to be delivered on 7th Feb and Job 2 is to be delivered on 6th Feb.
- Job 1 can be scheduled on 5th Feb. This would be delivered on 6th Feb.
- Job 2 can be scheduled only after job 1 is delivered. But job 2 cannot be scheduled as it is already 6th Feb.
- So sequence {1, 2} is not feasible.
- For same reason sequences {1,4}, {2,4}, {1,2,3} ,. . . . . . . .are not feasible

Job	Deadline	Profit
1	2	60
2	1	30
3	2	40
4	1	80

- Consider now the job Sequence {2,1}. Job 2 is to be delivered on 6th Feb and Job 1 is to be delivered on 7th Feb.
- Job 2 can be scheduled on 5th Feb. This would be delivered on 6th Feb.
- Job 1 can be scheduled after that (on 6th Feb), and this can be delivered on 7th Feb in time.
- So both jobs will be delivered in time. We say that sequence {2, 1} is feasible.

- Here are the feasible sequences and corresponding profits.
- Job sequence **Total Profit**

Job sequence	Total Profit	
2,1	90	(60 + 30)
1,3	100	
2,3	70	
4,1	140	
4,3	120	

- Max profit is associated with Sequence {4,1}.
- Optimal order is Sequence {4,1}.
- Note that Sequence {1,4} is not feasible, as deadlines are violated.

Greedy approach to Scheduling Algorithm

- First, sort the jobs in the decreasing order of their profits.
- Then find the highest deadline among all deadlines.
- Next, we need to assign time slots to individual job ids.
- First, check if the maximum possible time slot for the job, i.e., its deadline, is assigned to another job or not.
  - If it is not filled yet, assign the slot to the current job id.
  - Otherwise, search for any empty time slot less than the deadline of the current job.
  - If such a slot is found, assign it to the current job id, and move to the next job id.

- Consider a problem of scheduling 5 jobs.
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Job	Deadline	Profit
1	2	100
2	1	19
3	2	30
4	1	25
5	3	15

- Sort the jobs in decreasing order of profit.
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Job	Deadline	Profit
1	2	100
3	2	30
4	1	25
2	1	19
5	3	15

- Consider first the job with highest profit (job 1)
- Next highest profitable job is job 3. In 2 days, job 1 and job 3 can be delivered.
- However, job 4 and job 2 cannot be scheduled as they don't have enough deadline.
- But job 5 can be scheduled, as its deadline permits job to be scheduled on 3^{rd} day.
- So { 1, 3, 5} is an optimal solution. Note {3, 1, 5} is also an optimal solution.

complexity analysis

Main computation of scheduling problem is the sorting step

- This takes O(n log n) time.
- All other steps take constant time.
- Therefore complexity of scheduling problem is O(n log n).

- read up more about this topic:
- https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_job_sequencing_with_deadline.htm