

MAT315 Combinatorial Enumeration
Monsoon 2024

Instructor: Manjil Saikia

TA: Kanak Dhotre

MATHEMATICAL AND PHYSICAL SCIENCES DIVISION, SCHOOL OF ARTS AND
SCIENCES, AHMEDABAD UNIVERSITY, AHMEDABAD 380009, GUJARAT, INDIA
Email address: `manjil.saikia@ahduni.edu.in`

2020 *Mathematics Subject Classification.* Primary 05-01
Key words and phrases. Combinatorics, Discrete Mathematics.

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Preface

These are the lecture notes of MAT315 Combinatorial Enumeration, offered in Monsoon 2024 semester at Ahmedabad University, India. The notes were written down by the TA for the course, Kanak Dhotre and is as close to the classroom teaching as possible.

There are several very good textbooks in combinatorics, however the material that I wish to cover in this course is not available in a single source to my liking. So, I decided to make my own notes for this iteration, as well as for any future iterations of this course.

The background required to enroll for the course is very minimal, in fact, it is not even mandatory for a student to have done a first course in Discrete Mathematics. So, we introduce several basic concepts along the way and if there is a scope for some digression then we will take it. The course is supplemented by some homework assignments, some of the problems in the text were set in those assignments, some even appeared in the examinations.

I am thankful to Kanak Dhotre for typing these notes. Any errors that remain are mine. If there are any errors, comments, or corrections, please write to me via email.

Manjil Saikia

CHAPTER 1

What is Combinatorics?

1.1. Counting Principles

1.2. The Pigeon-hole Principle

1.3. The Principle of Inclusion-Exclusion

CHAPTER 2

The Art of Bijections

2.1. Binomial Coefficients

2.2. Catalan Numbers

CHAPTER 3

Generating Functions

3.1. Ordinary Generating Functions

3.2. Exponential Generating Functions

CHAPTER 4

Partitions

4.1. Set Partitions

4.2. Integer Partitions

CHAPTER 5

Lattice Path Combinatorics

5.1. Dyck Paths Revisited

5.2. Motzkin and Schröder Paths

5.3. Non-intersecting lattice paths

5.4. q -Counting of lattice paths

CHAPTER 6

A Combinatorial Miscellany

6.1. Domino Tilings

6.2. Permutations

6.3. Symmetric Functions

6.4. Graphs and Trees

Bibliography

- [A] T. Aoki, *Calcul exponentiel des opérateurs microdifférentiels d'ordre infini*. I, Ann. Inst. Fourier (Grenoble) **33** (1983), 227–250.
- [B] R. Brown, *On a conjecture of Dirichlet*, Amer. Math. Soc., Providence, RI, 1993.
- [D] R. A. DeVore, *Approximation of functions*, Proc. Sympos. Appl. Math., vol. 36, Amer. Math. Soc., Providence, RI, 1986, pp. 34–56.