Aerodynamic optimization design of airfoil based on CST parameterization method

Article *in* Xibei Gongye Daxue Xuebao/Journal of Northwestern Polytechnical University · October 2013

4 authors, including:

Wenping Song
Northwestern Polytechnical University

Zhong-Hua Han
Northwestern Polytechnical University

Jianhua Xu
Northwestern Polytechnical University

Aerodynamic Optimization Design of Airfoils with Adaptive Design Space Based on CST Parameterization Method

Yue-Peng Bu[1], Wen-Ping Song[2*], Zhong-Hua Han[3], Jian-Hua Xu[4], Jun Liu[5]

National Key Laboratory of Science and Technology on Aerodynamic Design and Research, Northwestern Polyt echnical University, Xi'an, 710072, China

Abstract

An important step of optimization is to confirm a design space, however, for an unknown and complicated problem, there isn't any clue for the optimal solution. So the design space should be as large as possible in order to obtain a global optimal solution. Whereas, larger design space may lead to more time consuming and computational expense. In this paper, two adaptive design space methods based on CST[1] (Class function / Shape function Transformation) are proposed to find optimal solutions excluding the influences of initial design space and improve the efficiency of optimization. Surrogate-based optimization[2] is adopted in this paper ,the Kriging models are refined by infilling samples to converge to the optimum. The aerodynamic force are solved by Reynolds-Averaged Navier-Stokes equations combined with S-A(Spalart-Allmaras) turbulence model. RAE2822 airfoil is utilized as the baseline. Single- or multi-point optimizations for drag minimization of transonic airfoil are studied by the above strategies. The results show that compared with fixed design space, both of the adaptive design space methods improve the efficiency of optimization, method 2 leads to better optimization result than method 1.

Keywords: aerodynamic optimization design, airfoil, adaptive design space, Class function / Shape function Transformation.

Introduction

Design space for the optimization design has a very important impact, sufficient optimization design space enables satisfactory results, but with the increase of the design space, optimization time also increases, and in the design space may appear strange airfoil geometry or airfoil geometry cross circumstances (especially near the trailing edge). So under the premise of sufficient design space and optimization quality assurance, To study how to improve optimization design efficiency is particularly important

Airfoil parameterization method has also a very important impact on final optimization result. It has significant influence on the optimization design efficiency and result. A variety of parameterization methods are used for optimization, such as analytic function superposition method (Hicks-Henne parametric methods, etc.), spline fitting method (including B-spline method and cubic spline interpolation method, etc.). In recent years, Kulfan put forward a parameterization method which based on class function / shape function transformation, It calls CST parameterization method, It has a clear geometrical meaning , a few parameters and better accuracy. Kulfan noted that the method can be used not only to coordinate airfoil parameterized, but also for other geometric features such as the curvature and the parameters of the arc. It has a good prospect for aerodynamic optimization in two-dimensional airfoil and three-dimensional wing etc. So CST parameterization method has also been widely studied and applied.

**Wen-Ping Song Email Address: wpsong@nwpu.edu.cn*
Foundation item: National Natural Science Foundation of China (Grant No. 11302177)

Unlike Hicks-Henne parameterization method which based on linear superposition functions, CST parameterization method can directly fit airfoil, design space can widely change, this ensures optimization design has sufficient design space.

The main idea of this paper is to transform the design space adaptive in the optimization. Whenever generating a new airfoil which is better than the current optimal airfoil, the shape and size of the design space are transformed according to the new design variables and a new factor. It can make the design space move towards the direction of the optimal solution. On the basis above, two different strategies for adaptive design space are proposed: 1) the shape and size of design space changes at the beginning of the optimization, named method 1; 2) the shape and size of the design space doesn't change at first, it changes when the optimization is converge to a certain extent then attenuate the locality of size of the design space, named method 2. In present study, RAE2822 airfoil is utilized as the baseline. Single- or multi-points optimizations for drag minimization of transonic airfoil are studied by the above strategies. The results show that compared with fixed design space, both of the adaptive design space methods improve the efficiency of optimization, method 2 leads to better optimization result than method 1.

CST parameterization method and application

CST parameterization methods specific implementation process are given below. CST equation expressions are as follows:
On the upper surface:

$$y_u = C(x) \bullet S_u(x) + x \bullet y_{TEu} \tag{1}$$

Under the lower surface:

$$y_l = C(x) \bullet S_l(x) + x \bullet y_{TEl} \tag{2}$$

The subscripts u, l denote the upper and lower surfaces of the airfoil. y_{TEu} and y_{TEl} represent the upper and lower surfaces of the rear edge of the coordinates.

Class function C(X) is defined as follows:

$$C(x) = x^{N1} \bullet (1-x)^{N2} \tag{3}$$

Shape function $S_u(x)$, $S_l(x)$ is defined as follows

$$Su(x) = \sum_{i=0}^{N} A_{u_i} \bullet S_i(x) \tag{4}$$

$$Sl(x) = \sum_{i=0}^{N} A_{l_i} \bullet S_i(x) \tag{5}$$

$$S_i(x) = \frac{N!}{i!(N-i)!} x^i (1-x)^{N-i} \tag{6}$$

For general airfoil. N1and N2 value 0.5 and 1.0. A_{u_i} and A_{l_i} are coefficients to be determined. $S_i(x)$ is Bernstein polynomials. Through the above analysis it can be seen, as long as the coefficient of (4) and (5) are solved the entire airfoil is determined. According to reference which pointed out that the first term of undetermined coefficients A_0 associated with the airfoil leading edge radius, in order to ensure that the upper and lower surfaces of the same radius at the leading edge, so that, $A_{u_0} = -A_{l_0}$,in the above formula, the first CST equation is separated listed, $A_0 = A_{u_0}$. Using the least squares method calculates coefficient, which determines the entire airfoil. Typically, the number of known airfoil coordinate points are greater than the number of polynomial order, and therefore need to solve the following equation of the minimum.

$$\begin{aligned} F = &\sum_{j=1}^{nup}\left(\left(x^{N1}\bullet(1-x)^{N2}\left(A_0 S_0(x)+\sum_{i=1}^{N} A_{u_i}\bullet S_i(x)\right)+x\bullet y_{TEu}\right)-y_{u,j}\right)^2 \\ &+\sum_{j=1}^{nlow}\left(\left(x^{N1}\bullet(1-x)^{N2}\left(-A_0 S_0(x)+\sum_{i=1}^{N} A_{l_i}\bullet S_i(x)\right)+x\bullet y_{TEl}\right)-y_{l,j}\right)^2 \end{aligned} \quad (7)$$

nup represents the number of the upper surface of airfoil coordinate, nlow represents the number of the lower surface of airfoil coordinate.

Adaptive design space

As is well known that, design space has significant influence on the optimization design efficiency and result. The main idea of this paper is to transform the design space adaptive in the optimization. Whenever generating a new airfoil which is better than the current optimal airfoil, the shape and size of the design space are transformed according to the new design variables and a new factor. It can make the design space move towards the direction of the optimal solution.

Design space bounds expressions are as follows:

On the upper surface:

$$\begin{aligned} BL(i_1) &= (1-\alpha)*VAR(i_1) \\ BU(i_1) &= (1+\alpha)*VAR(i_1) \end{aligned} \quad (8)$$

Under the lower surface:

$$\begin{aligned} BL(i_2) &= VAR(i_2)+\alpha*|VAR(i_2)| \\ BU(i_2) &= VAR(i_2)-\alpha*|VAR(i_2)| \end{aligned} \quad (9)$$

In the above formula B, C represent lower and upper bounds of the design space, VAR represents the design variables, $i_1 + i_2$ represents the number of design variables,based on experience α is generally valued 0.1 to 0.8. In this paper, Using Latin Hypercube Sampling method to select the entire design space sample points, and then calculate the response of the sample points, in response to the calculated values, select the response value corresponding to the minimum sample point. On this basis to give a new design variable value:

$$VAR(i) = SAMPLE(i,\min)*(BU(i)+BL(i))+BL(i) \quad (10)$$

$SAMPLE(i,\min)$ is response value which is corresponding to the minimum sample point. Equation (10) is substituted into (8) (9), which got a new design space, and by changing the α value, the new design space appropriately reduced. This is because on the one hand design space optimization are inefficiency caused by excessive, abnormal geometry appears during optimization process, thereby affecting the quality of optimization. On the other hand, new optimization design space is determined by the airfoil which corresponding the minimum response value, so the new design space is more reasonable than the initial design space, easier and more efficient to find the optimal value.In this paper, Surrogate-based optimization is adopted in this paper ,the Kriging models are refined by infilling samples to converge to the optimum, The aerodynamic force are solved by Reynolds-Averaged Navier-Stokes equations combined with S-A(Spalart-Allmaras) turbulence model.

Fitting Accuracy of CST Methods

Reference noted that for most NACA airfoil series, exponential of class functions N1 and N2 value 0.5 and 1.0. But for the other series of airfoil value 0.5 and 1.0 may cause large errors. Approach of this paper is to solve the equation (7) minimum problem, N1 and N2 as unknowns, the coefficients and N1, N2 are simultaneously optimized for the minimum value of the function F.

For DU93-W-210 wind turbine airfoils and NACA0012 airfoil are using the basic method and an improved CST. Figures 1 and 2 shows the fitting error under different orders. Obviously, in all orders, improved CST method have been improved fitting accuracy.

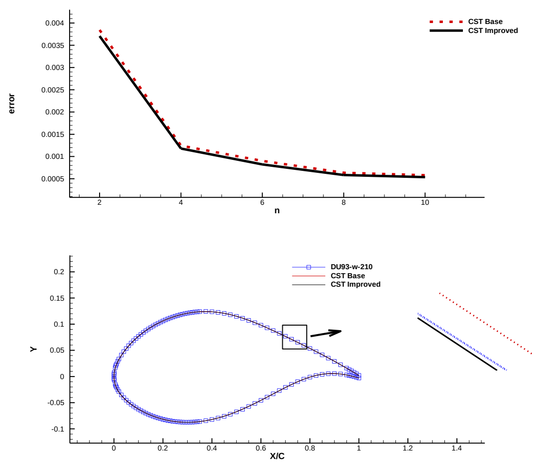

Fig. 1 DU93-W-210 CST method and improved method fitting error comparison under different orders

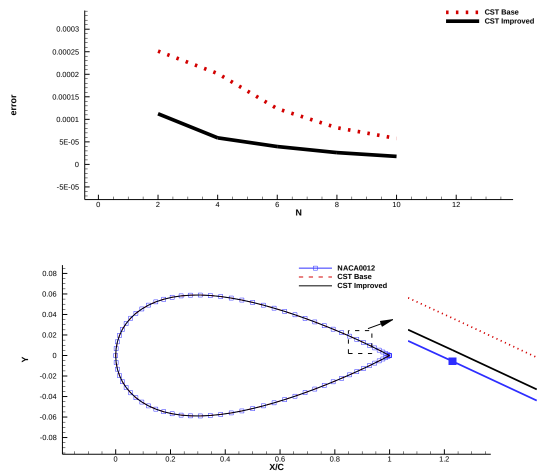

Fig. 2 NACA0012 CST method and improved method fitting error comparison under different orders

To study the impact of the order on the fitting airfoil aerodynamic characteristics, RAE2822 airfoil is studied, Calculation conditions: $Ma = 0.729$, $\mathrm{Re} = 6.5\times10^6$, $\alpha = 2.79^\circ$, It should be noted that all calculations in this paper are running on the Intel Core i7 3.4G computer.

Figure 3 shows C-grid used for the flow solver.

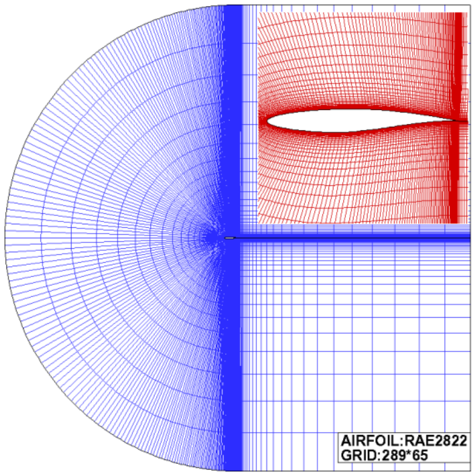

Fig. 3 RAE2822 global and local grid chart

Figure 4 shows a comparison of the lift coefficient and drag coefficient under different order of fitting airfoil. As can be seen, with the increase order of N, the aerodynamic characteristics of the fitting airfoil obtained converge to the initial airfoil aerodynamic characteristics. Figure 5 shows the comparison of fitting airfoil and initial airfoil pressure distribution under order of N= 2,4,6. Results show that when N=4, pressure distribution obtained by fitting airfoil is very close to the initial RAE2822 airfoil pressure distribution. Airfoil design optimization are used 4th order 11 design variables CST parameterization.

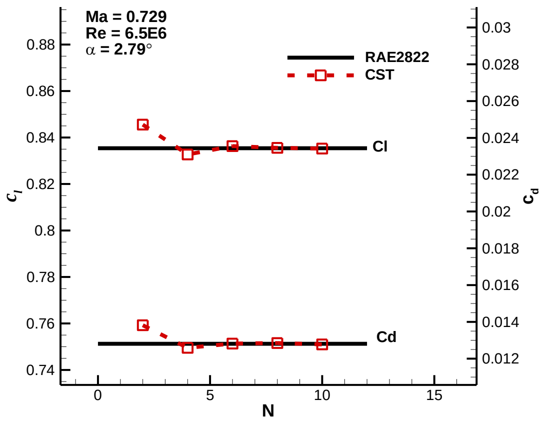

Fig. 4 Lift and drag coefficients of different order compared with baseline

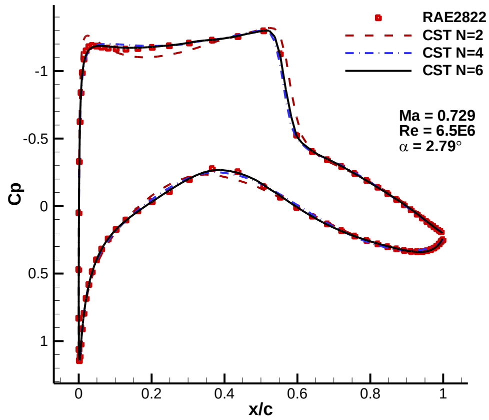

Fig. 5 Different order of fitting RAE2822 airfoil pressure distribution compared with baseline

Influences of adaptive design space for optimization design efficiency and result

Two different strategies for adaptive design space are proposed:

1) The shape and size of design space changes at the beginning of the optimization, named method 1;

2) The shape and size of the design space doesn't change at first, it changes when the optimization is convergent to a certain extent then attenuate the locality of size of the design space, named method 2.

A. RAE2822 airfoil single-point optimizations

RAE2822 airfoil is utilized as the baseline. Single-points optimizations for drag minimization of transonic airfoils is studied by the above strategies .design conditions, design goal and constraints are described:

Design conditions: $Ma = 0.729$, $\mathrm{Re} = 6.5 \times 10^6$, $\alpha = 2.79^\circ$

Design goal and constraints:

$$\begin{aligned} \min\ & C_d \\ s.t.\ & \frac{T}{T_0} - 0.995 \geq 0 \\ & \frac{c_l}{c_{l0}} - 0.995 \geq 0 \\ & \left|\frac{c_m}{c_{m0}}\right| - 0.9 \leq 0 \end{aligned}$$

T, c_l, c_m is the maximum thickness of airfoil , lift coefficient and torque coefficient after optimization, respectively; T_0, c_{l0}, c_{m0} is maximum thickness of airfoil , lift coefficient and torque coefficient before optimization, respectively. Selecting 20 initial sample points. Here, initial α takes 0.25, after the change of the design space , α takes 0.2.

Figure 6 shows method 1 and method 2 final design space and the initial design space comparison chart, it can be seen from figure 6 two methods at the leading edge of the airfoil design space both become narrowing, which in line with airfoil drag reduction law, that is the leading edge of the airfoil thickness decreases, shock weakened.
Figure 7, Figure 8 shows the comparison of geometry and pressure distribution by three methods. Airfoil pressure distribution obtained by all methods can weaken the shock wave which reached the drag reduction.
Figure 9 shows a comparison of the number of iterations as the objective function changes, it can be seen two ways to change the design space fell faster than fixed design space to the objective function value, indicating an efficient variant of the adaptive design space.
Table 1 shows the comparison of drag reduction and optimization efficiency of the three methods, Optimization design time of fixed design space as the reference time, the reference time is 6 hours 23 minutes. As can be seen from Table 1, adaptive design space method has improved efficiency relative to the fixed design space, it is because the design adaptive design space method filters out unnecessary space, so that it can optimize in a more rational, more quickly way to find the objective function.
Comparison of adaptive design space method, method 1 has better optimization efficiency than method 2 but worse optimize performance than method 2.

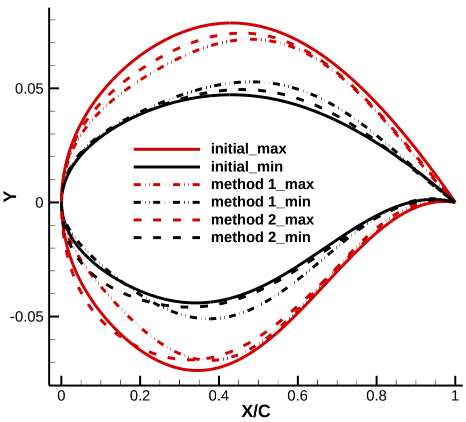

Fig. 6 Comparison of three methods of design space

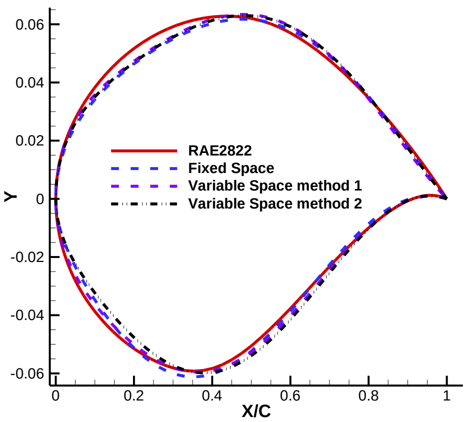

Fig. 7 Comparison of optimization airfoil and initial airfoil geometry of three methods

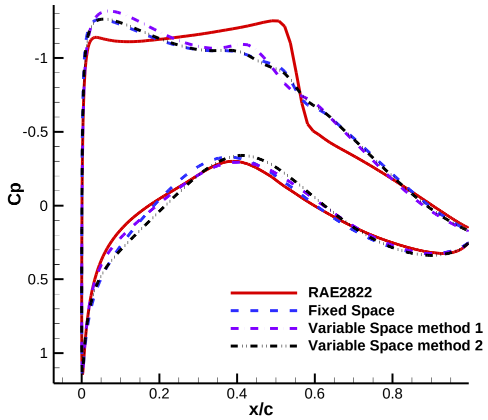

Fig. 8 Comparison of optimization airfoil and initial airfoil pressure distribution of three methods

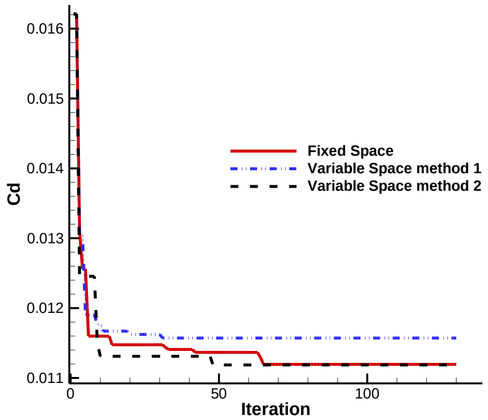

Fig. 9 Comparison of the number of iterations as the objective function changes

Tab. 1 *Comparison of fixed design space and adaptive design space for optimized results and efficiency of optimization*

	Baseline	Fixed Space	Adaptive Space method 1	Adaptive Space method 2
c_d (10^{-2})	1.6811	1.1214	1.1572	1.1186
Δc_d (%)		33.29	31.16	33.46
Time		T	0.64T	0.83T

B. RAE2822 airfoil multi -point optimizations

RAE2822 airfoil is utilized as the baseline. Multi -points optimizations for drag minimization of transonic airfoils is studied to further verify optimize performance and efficiency of two methods. Design conditions, design goals and constraints are as follows:

Design conditions: design point (1) $Ma = 0.729$, $\mathrm{Re} = 6.5\times10^6$, $\alpha = 2.44^\circ$

design point (2) $Ma = 0.729$, $\mathrm{Re} = 6.5\times10^6$, $\alpha = 2.0^\circ$

Design goals and constraints:

$$\begin{aligned} \min \quad & design\ (1)\ C_d \\ & design\ (2)\ C_d \\ s.t. \quad & \frac{T}{T_0} - 0.995 \geq 0 \\ & design(1)\frac{C_{l1}}{C_{l10}} - 0.995 \geq 0 \\ & \mathrm{design}(2)\frac{C_{l2}}{C_{l20}} - 0.995 \geq 0 \end{aligned}$$

T , c_{l1} , c_{l2} is the maximum thickness of airfoil , lift coefficient at design point(1) and lift coefficient at design point(2) after optimization, respectively; T_0 , c_{l0} , c_{l20} is maximum thickness of airfoil , lift coefficient at design point(1) and lift coefficient at design point(2) before optimization, respectively.

Selecting 20 initial sample points. Figure 10 shows method 1 and method 2 final design space and the initial design space comparison chart, Similar with the single-objective optimization case, both methods reduce the design space which is always changed toward to optimal airfoil. Figure 11, Figure 12 shows the comparison of geometry and pressure distribution by three methods. Figure 13 shows a comparison of the number of iterations as the objective function changes, it can be seen two ways to change the design space fell faster than fixed design space to the objective function value. Table 2 shows the comparison of drag reduction and optimization efficiency of the three methods, Optimization design time of fixed design space as the reference time, the reference time is 21 hours 16 minutes. As can be seen from Figure 13 and Table 2, considering the optimization effect and optimization efficiency of two design points, Method 2 is better than the other two methods.

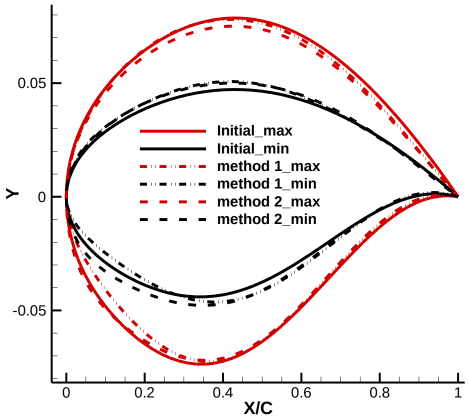

Fig. 10 Comparison of three methods of design space

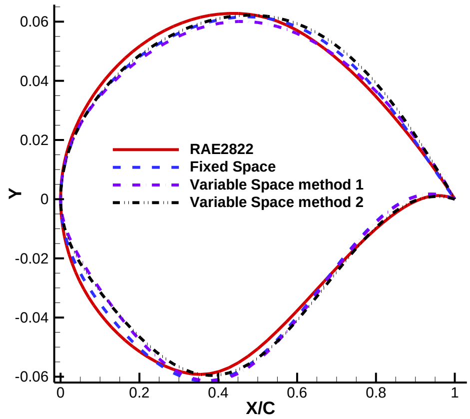

Fig. 11 Comparison of optimization airfoil and initial airfoil geometry of three methods

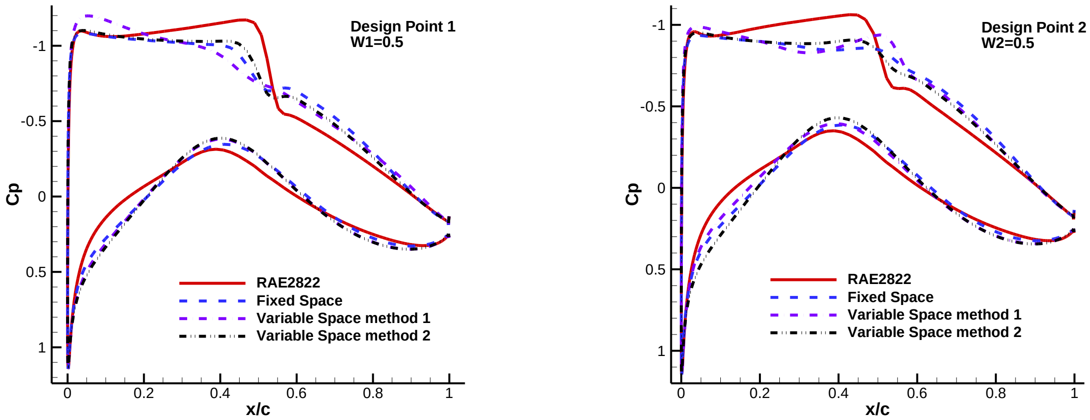

Fig. 12 Comparison of optimization airfoil and initial airfoil pressure distribution of three methods

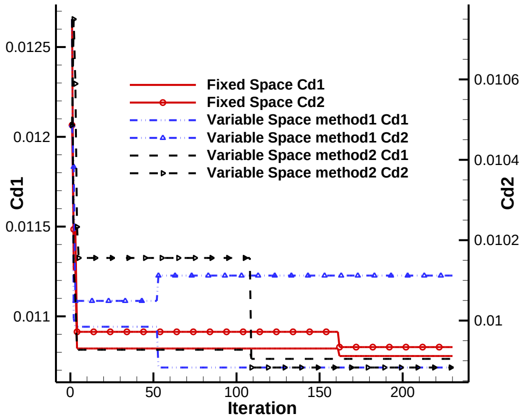

Fig. 13 Comparison of the number of iterations as the objective function changes

Tab. 2 Comparison of fixed design space and adaptive design space for optimized results and efficiency of optimization

	Baseline	Fixed Space	Adaptive Space method 1	Adaptive Space method 2
c_{d1} (10^{-2})	1.2655	1.0772	1.0715	1.0763
Δc_{d1} (%)		14.88	15.33	14.95
c_{d2} (10^{-2})	1.0486	0.9934	1.0112	0.9883
Δc_{d2} (%)		5.2625	3.5705	5.7496
time		T	0.36T	0.78T.

Conclusions

The obtained conclusions are as following:

(1) By simultaneously optimizing the exponential of class functions N1 and N2 value, improved CST method has improved fitting airfoil accuracy.

(2)Single- or multi-point optimizations for drag minimization of transonic airfoils are studied, the results show compared with fixed design space, both of the adaptive design space methods improve the efficiency of optimization. But method 1 has worse optimize performance than baseline and method 2.

(2) Method 2 leads to better optimization result than method 1.

(3) Adaptive design space method 2 and fixed design space have nearly same optimized results.

References

1. Brenda M. Kulfan. “A Universal Parametric Geometry Representation Method – “CST”,” AIAA Paper: 2007-62, 2007
2. Han, Z. –H, Görtz S. “A Hierarchical Kriging Model for Variable-Fidelity Surrogate Modeling,” AIAA Journal, Vol.50, No.9, pp.1885-1896, 2012
3. Hicks, R. M, and Henne, P. A., “Wing Design by Numerical Optimisation, Journal of Aircraft,” Vol. 15, No. 7, 1978, pp. 407-413.
4. David Bogue and Nick Crist,"CST Transonic Optimization using Tranair++",AIAA-2008-321-509,2008
5. Marco Ceze"A Study of the CST Parameterization Characteristics",AIAA-2009-3767-676,2009
6. Kevin A. Lane and David D. Marshall,"Inverse Airfoil Design Utilizing CST Parameterization".AIAA-2010-1228-575,2010
7. Zhang, K. -S., Han Z. –H, “Support Vector Regression-based multidisciplinary Design Optimization in Aircraft Conceptual Design” 51st AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, AIAA paper 2013-1160 , Jan. 7-10, Grapevine, Texas, 2013.
8. Han, Z. -H., Görtz, S., and Zimmermann, R., “Improving Variable-Fidelity Surrogate Modeling via Gradient-Enhanced Kriging and a Generalized Hybrid Bridge Function,” Aerospace Science and Technology, Vol. 25, No. 1, pp. 177-189 ,2012.01.006, 2013.
9. Timothy Takahashi. "The Search for the OptimalWing Configuration for Small Subsonic Air Vehicles", AIAA-2008-5915-796,2008
10. R.Hick, Moffett Field and P.A.Henne "Wing Design by Numerical Optimization", AIAA-1977-1247-526.1977
11. Brenda M.Kulfan."Recent Extensions and Applications of the CST Universal Parametric Geometry Representation Method",AIAA-2007-7709-403,2007
12. Sharon L.Padula,"Options for Robust Airfoil Optimization under Uncertainty",AIAA-2002-5602-721,2002