AM5801: Computational Lab

Assignment 1

Date: August 14, 2025

Deadline: August 20, 2025 Max mark: 50

Please refrain from using any in-built polynomial functions

1. Legendre polynomials $P_n(x)$ are solutions of degree n to the differential equation,

$$(1-x^2)P_n''(x) - 2xP_n'(x) + n(n+1)P_n(x) = 0.$$

They are defined on the interval [-1,1] and are orthogonal with respect to the standard inner product on this interval. Some methods to construct Legendre Polynomials are:

(i) Generating function: $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)\, t^n$

(ii) Recurrence formula: $(n+1)P_{n+1}(x) = (2n+1)\, x\, P_n(x) - n\, P_{n-1}(x)$

The first five Legendre Polynomials are:

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$$
$$P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$$
$$P_4(x) = \frac{1}{8}\left(35x^4 - 30x^2 + 3\right)$$

(a) Write a Python program to evaluate Legendre polynomials for degrees 0 to 10 from either generating function or recurrence formula and plot $P_n(x)$ versus x. Also plot the time complexity for different sizes. **(10 + 10)**

(b) Write a Python program that first uses Horner's method to evaluate the polynomial $f(x) = x^4+2x^3+2x^2-x$ at x=4. Then express *f(x)* in terms of Legendre polynomials $P_n(x)$. Plot the two forms of *f(x)*. Discuss the results. **(5 + 10 + 5)**

(c) Write a Python program (within 100 lines) that demonstrates an application of Legendre polynomials in one of their key application areas (e.g., Physics, Numerical Analysis, Engineering, or Machine Learning). The code should show how Legendre polynomials are used practically within that field. **(10)**