Indian Institute of Technology, Madras
Department of Applied Mechanics & Biomedical Engineering
Computational Tools: Algorithms, Data Structures and Programams -
ID6105

Assignment set - V

To be submitted by: Thursday 30^{th} October, 2025.

1 Problem 1

Number representation: In class we looked at representing real numbers using different data types on a computer. Let us work out a few examples to understand them,

(a) Convert the following base-2 numbers to base-10: (i) 1011001, (ii) 110.00101 and (iii) 0.01011.

(b) Convert the following base-8 numbers to base -10: (i) $71,563$ and (ii) 3.14.

(c) The infinite series $f(n) = \sum_{i=1}^{n} i^{-4}$, converges on a value of $f(n) = \pi^4/90$ as n approaches infinity. Write a program in single precision to calculate $f(n)$ for $n = 10,000$ by computing the sum from $i = 1$ to $10,000$. Then repeat the calculation but in reverse order-that is, from $i = 10,000$ to 1 using increments of -1. In each case, compute the true percent relative error. Explain the results.

(d) Calculate the random access memory (RAM) in gigabytes necessary to store a multidimensional array that is $200 \times 400 \times 520$. This array is double precision, and each value requires a 64-bit word. Recall that a 64-bit word = 8 bytes and 1 kilobyte = 2^{10} bytes. Assume that the index starts at 1.

(e) What is the maximum size of a multidimensional array $M \times N \times P$ that can be stored on your laptop. Does it tally with your RAM configuration on your laptop? Explain your result.

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2 Problem 2

Determine the roots of the simultaneous nonlinear equations using (a) fixed-point iteration and (b) the Newton-Raphson method,

$$y = -x^2 + x + 0.75, \tag{1}$$

$$y + 5xy = x^2. \tag{2}$$

Employ initial guesses of $x = y = 1.2$ and discuss the results.

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3 Problem 3

The “divide and average” method, an old-time method for approximating the square root of any positive number a, can be formulated as

$$x = \frac{x + a/x}{2}. \tag{3}$$

Proove that this is equivalent to the Newton-Raphson algorithm.

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4 Problem 4

You are designing a tank to hold water as shwn in Fig. 1. The volume of liquid it can hold can be computed as,

$$V = \pi h^2 \frac{(3R - h)}{3}, \tag{4}$$

where V is the volume of the tank (in m^3), h = depth of water in tank (in m), and R = the tank radius (in m). If $R = 3m$, what depth must the tank be filled to so that it holds $30m^3$? Use three iterations of the Newton-Raphson method to determine your answer. Determine the approximate relative error after each iteration.

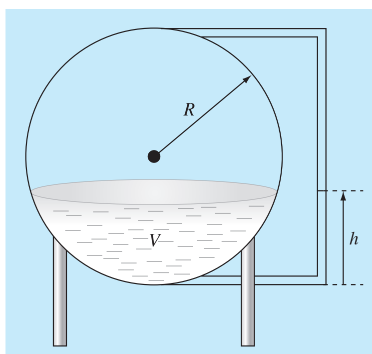

Figure 1: Water tank

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5 Problem 5

The polynomial $f(x) = 0.0074x^4 - 0.284x^3 + 3.355x^2 - 12.183x + 5$ has a real root between 15 and 20. Apply the Newton-Raphson method to this function using an initial guess of $x_0 = 16.15$. Explain your results.

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6 Problem 6

Use the secant method on the circle function $(x + 1)^2 + (y - 2)^2 = 16$ to find a positive real root. Set your initial guess to $x_i = 3$ and $x_{i-1} = 0.5$. Approach the solution from the first and fourth quadrants. When solving for f (x) in the fourth quadrant, be sure to take the negative value of the square root. Why does your solution diverge?

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7 Problem 7

In control systems analysis, transfer functions are developed that mathematically relate the dynamics of a system's input to its output. A transfer function for a robotic positioning system is given by,

$$G(s) = \frac{C(s)}{N(s)} = \frac{s^3 + 12.5s^2 + 50.5s + 66}{s^4 + 19s^3 + 122s^2 + 296s + 192} \tag{5}$$

where $G(s)$ = system gain, $C(s)$ = system output, $N(s)$ = system input, and s = Laplace transform complex frequency. Use a numerical technique to find the roots of the numerator and denominator and factor these into the form,

$$G(s) = \frac{(s + a_1)(s + a_2)(s + a_3)}{(s + b_1)(s + b_2)(s + b_3)(s + b_4)} \tag{6}$$

where a_i and b_i are roots of the numerator and denominator respectively.

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8 Problem 8

The general form for a three-dimensional stress field is given by,

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \tag{7}$$

where the diagonal terms represent tensile or compressive stresses and the off-diagonal terms represent shear stresses. A stress field (in MPa) is given by

$$\begin{bmatrix} 10 & 14 & 25 \\ 14 & 7 & 15 \\ 25 & 15 & 16 \end{bmatrix} \tag{8}$$

To solve for the principal stresses, it is necessary to construct the following matrix (again in MPa):

$$\begin{bmatrix} 10-\sigma & 14 & 25 \\ 14 & 7-\sigma & 15 \\ 25 & 15 & 16-\sigma \end{bmatrix} \tag{9}$$

σ_1, σ_2 and σ_3 can be solved from the equation,

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \tag{10}$$

where

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \tag{11}$$

$$I_2 = \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \sigma_{xy}^2 - \sigma_{yz}^2 - \sigma_{zx}^2 \tag{12}$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{xy}\sigma_{yz}\sigma_{zx} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2, \tag{13}$$

I_1, I_2, I_3 are the stress invariants. Find σ_1, σ_2, and σ_3 using a root-finding technique.

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9 Problem 9

In the thermos shown in Fig. 2, the innermost compartment is separated from the middle container by a vacuum. There is a final shell around the thermos. This final shell is separated from the middle layer by a thin layer of air. The outside of the final shell comes in contact with room air. Heat transfer from the inner compartment to the next layer q_1 is by radiation only (since the space is evacuated). Heat transfer between the middle layer and outside shell q_2 is by convection in a small space. Heat transfer from the outside shell to the air q_3 is by natural convection. The heat flux from each region of the thermos must be equal—that is, $q_1 = q_2 = q_3$. Find the temperatures T_1 and T_2 at steady state. T_0 is $450C$ and $T_3 = 25C$.

$$q_1 = 10^{-9}\left[(T_0 + 273)^4 - (T_1 + 273)^4\right] \tag{14}$$

$$q_2 = 4(T_1 - T_2) \tag{15}$$

$$q_3 = 1.3(T_2 - T_3)^{4/3} \tag{16}$$

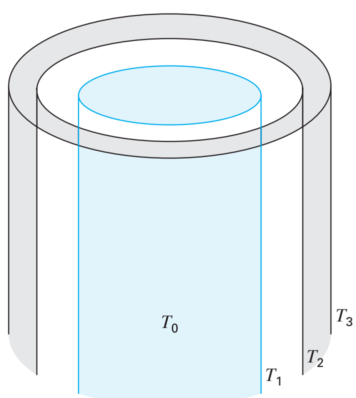

Figure 2: Thermos

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10 Problem 10

Of the following three sets of linear equations, identify the set(s) that you could not solve using an iterative method such as Gauss-Seidel. Show using any number of iterations that is necessary that

Set One	Set Two	Set Three
$9x + 3y + z = 13$	$x + y + 6z = 8$	$-3x + 4y + 5z = 6$
$-6x + 8z = 2$	$x + 5y - z = 5$	$-2x + 2y - 4z = -3$
$2x + 5y - z = 6$	$4x + 2y - 2z = 4$	$2y - z = 1$

Table 1: Systems of Linear Equations: Set One, Set Two, and Set Three

your solution does not converge. Clearly state your convergence criteria (how you know it is not converging).

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11 Problem 11

Consider the three mass-four spring system in Fig. 3. Determining the equations of motion from $\sum F_x = ma$, for each mass using its free-body diagram results in the following differential equations:

$$\begin{aligned}\ddot{x}_1 + \frac{k_1 + k_2}{m_1}x_1 - \frac{k_2}{m_1}x_2 &= 0,\\ \ddot{x}_2 - \frac{k_2}{m_2}x_1 + \frac{k_2 + k_3}{m_2}x_2 - \frac{k_3}{m_2}x_3 &= 0,\\ \ddot{x}_3 - \frac{k_3}{m_3}x_2 + \frac{k_3 + k_4}{m_3}x_3 &= 0.\end{aligned} \tag{17}$$

where $k_1 = k_4 = 10N/m$, $k_2 = k_3 = 30N/m$, and $m_1 = m_2 = m_3 = 2kg$. Write the three equations in matrix form:

0= [Acceleration vector] + [k/m matrix][displacement vector x]

At a specific time where $x_1 = 0.05m, x_2 = 0.04m,$ and $x_3 = 0.03m$, this forms a tridiagonal matrix. Solve for the acceleration of each mass.

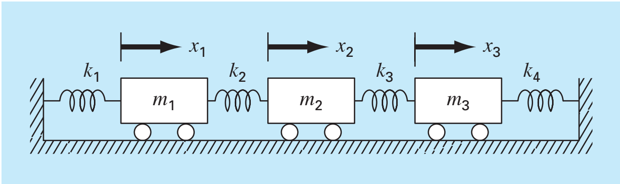

Figure 3: Spring mass system

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12 Problem 12

The steady-state distribution of temperature on a heated plate can be modeled by the Laplace equation,

$$0 = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{18}$$

If the plate is represented by a series of nodes (Fig. 4), centered finite-divided differences can be substituted for the second derivatives, which results in a system of linear algebraic equations. Use the Gauss-Seidel method to solve for the temperatures of the nodes in Fig. 4.

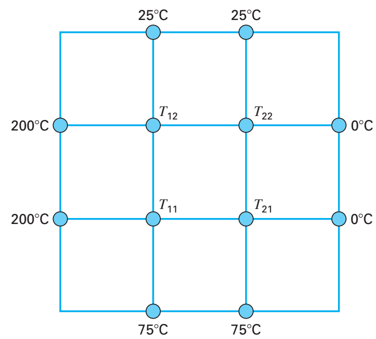

Figure 4: Temperature grid.