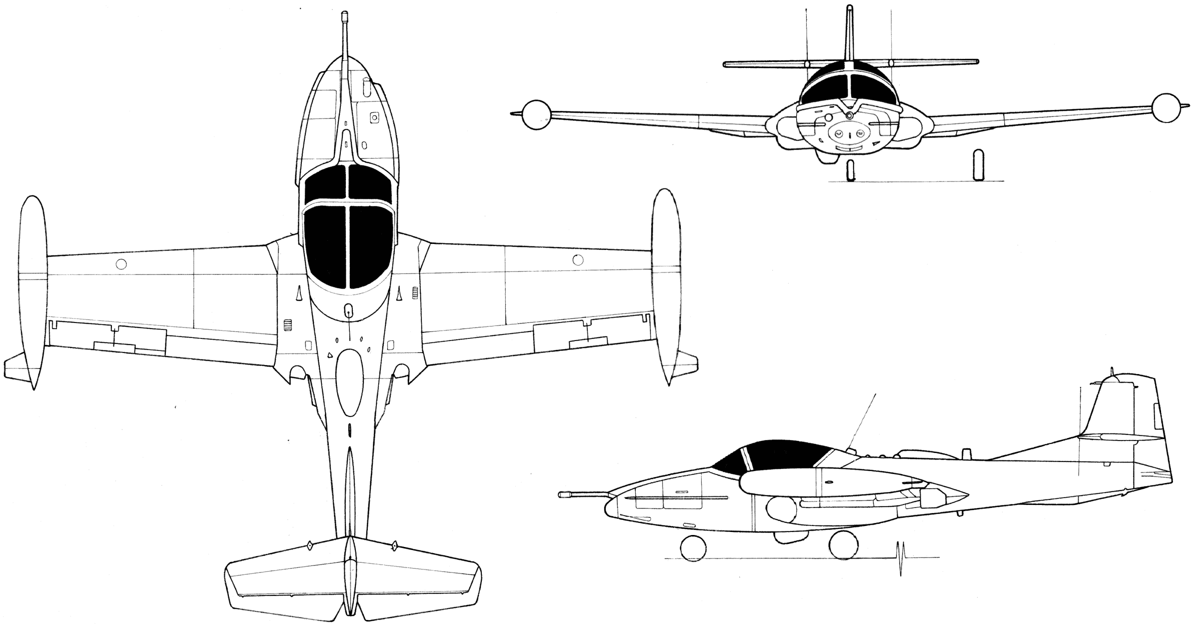

Cessna T-37, Image Source - Drawing Database

Flight Dynamics Project 2023 - 2024

Stability Analysis of the Cessna T-37

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Contents

1 Introduction

The following report outlines the methodology and the results of the stability analysis conducted on the Cessna T-37 as a part of the Flight Dynamics coursework for the academic year 2023-2024 at the Manipal Institute of Technology, Karnataka, India.

The stability analysis was conducted using MATLAB. LaTeX was used to formulate the report. Once the results were obtained, they were analysed, and suitable conclusions were drawn from the respective values.

The aircraft's initial geometric parameters were taken from Appendix C of reference [1]. These values are outlined in the Appendix of this report. These values were built upon to conduct these analyses.

2 Flight Conditions

The fight conditions chosen to analyse the static stability of the aircraft are as follows:

Parameter [unit]	**Value**	**Parameter [unit]**	**Value**
Altitude [ft]	30,000	Mach Number	0.459
V_{P_1} [ft/s]	456	q_1 [lbs/ft^2]	92.7
α_1 [deg]	2°	S [ft^2]	182
$\bar{c}$ [ft]	5.1	b [ft]	33.8
X_{CG} [ft]	0.27	W [lbs]	6,360

3 Modelling the Wing Lift Slope Coefficient

The empirical relations in reference [1] can be applied for subsonic operation, moderate sweep angles and reasonably moderate aspect ratios. This particular aircraft satisfies these conditions, and the following relations were used,

$$C_{L_\alpha} = \frac{2\pi AR}{2 + \sqrt{\left(\left[\frac{AR^2(1-M^2)}{k^2}\left(1 + \frac{tan^2(\Delta_{0.5})}{1-M^2}\right)\right] + 4\right)}}$$

where k for AR $\geq$ 4 is given by,

$$k = 1 + \frac{[(8.2 - 2.3\Lambda_{LE}) - AR(0.22 - 0.153\Lambda_{LE})]}{100}$$

On calculation, this turns out to be,

$$C_{L_\alpha} = 5.5746$$

The empirical relations outlined above are called the Polhamus formulae and have been modelled from correlation studies conducted over extensive wind tunnel data.

4 Modelling the Effect of Downwash

An important longitudinal aerodynamic effect is the downwash effect. In general, this effect can be considered an aerodynamic "interference" generated by the wing on the horizontal tail due to the system of vortices created by the wing.

The following relationship is used to model the downwash effect,

$$\left(\frac{d\epsilon}{d\alpha}\right) = f(M, m, r, \Lambda_{LE}, \lambda, AR)$$

The closed form expression is given by,

$$\left.\left(\frac{d\epsilon}{d\alpha}\right)\right|_{M} = \left.\left(\frac{d\epsilon}{d\alpha}\right)\right|_{M=0} \sqrt{1-M^2}$$

where,

$$\left.\left(\frac{d\epsilon}{d\alpha}\right)\right|_{M=0} = 4.44 \left(K_{AR} K_{\lambda} K_{mr} \sqrt{cos(\Lambda_{0.25})}\right)$$

with,

$$K_{AR} = \frac{1}{AR} - \frac{1}{1+AR^{1.7}}, \quad K_{\lambda} = \frac{10-3\lambda}{7}, \quad K_{mr} = \frac{1-{}^{m}/_{2}}{r^{0.333}}$$

The sweep angle at any point on the wing can be found using,

$$tan(\Lambda_x) = tan(\Lambda_{LE}) - \frac{4x(1-\lambda)}{AR(1+\lambda)}$$

The downwash effect is calculated to be,

$$\left.\left(\frac{d\epsilon}{d\alpha}\right)\right|_{M} = 0.3758$$

5 Aerodynamic Center of the Wing

The Aerodynamic Center of the wing can be found using the relation,

$$\bar{x}_{AC_W} = K_1 \left(\frac{x'_{AC}}{c_R} - K_2\right)$$

where K_1 and K_2 are geometric parameters of the wing and can be determined from the empirical plots. This turned out to be,

$$\bar{x}_{AC_W} = 0.1150$$

6 Effect of the Fuselage on the Aerodynamic Center

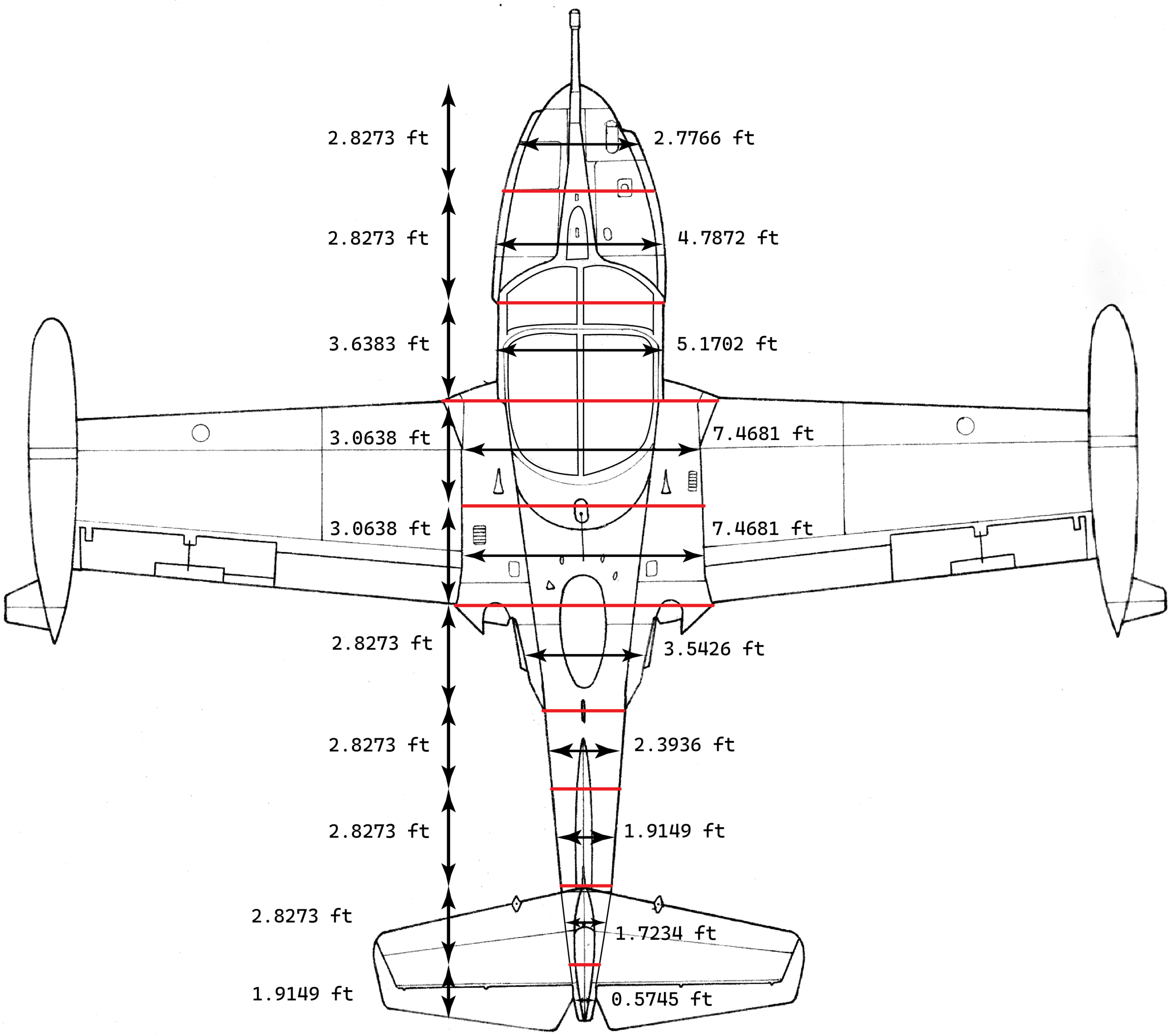
The shift in the aerodynamic center due to the addition of the fuselage is given by,

$$\Delta\bar{x}_{AC_B} = \frac{\left(\frac{\bar{q}}{36.5}\frac{C_{L_{\alpha_W}}}{0.08}\right)}{\bar{q}S\bar{c}C_{L_{\alpha_W}}} \sum_{i=1}^{N} w_{B_i}^2 \left(\frac{d\epsilon}{d\alpha}\right)_i \Delta x_i$$

where,

$$\left(\frac{d\epsilon}{d\alpha}\right)_i = \left(\frac{x_i}{x_H}\right)\left(1 - \frac{d\epsilon}{d\alpha}\bigg|_{m=0}\right)$$

w_{B_i} and x_i are geometric parameters for the discretised aircraft sections in accordance with Munk's theory. These parameters are shown below,



The final shift of the aerodynamic center due to the body is,

$$\Delta\bar{x}_{AC_B} = -0.0011$$

7 Modelling the Horizontal Tail Lift Slope Coefficient

The empirical relations in reference [1] can be applied for subsonic operation, moderate sweep angles and reasonably moderate aspect ratios. This particular aircraft satisfies these conditions, and the following relations were used,

$$C_{L_{\alpha_H}} = \frac{2\pi AR_H}{2 + \sqrt{\left(\left[\frac{AR_H^2(1-M^2)}{k^2}\left(1 + \frac{tan^2(\Delta_{0.5_H})}{1-M^2}\right)\right] + 4\right)}}$$

where k for $AR_H \geq 4$ is given by,

$$k = 1 + \frac{[(8.2 - 2.3\Lambda_{LE_H}) - AR_H(0.22 - 0.153\Lambda_{LE_H})]}{100}$$

On calculation, this turns out to be,

$$C_{L_{\alpha_H}} = 4.3335$$

The empirical relations outlined above are called the Polhamus formulae and have been modelled from correlation studies conducted over extensive wind tunnel data. The results conforms to the trend of the $C_{L_{\alpha_H}}$ being lower of the C_{L_α} value.

8 Aircraft Aerodynamic Center

The aircraft's aerodynamic center can be estimated using the relation,

$$\bar{x}_{AC} = \frac{\bar{x}_{AC_{WB}} + \frac{C_{L_{\alpha_H}}}{C_{L_{\alpha_H}}}\eta_H \frac{S_H}{S}\left(1 - \frac{d\epsilon}{d\alpha}\right)\bar{x}_{AC_H}}{1 + \frac{C_{L_{\alpha_H}}}{C_{L_{\alpha_H}}}\eta_H \frac{S_H}{S}\left(1 - \frac{d\epsilon}{d\alpha}\right)} = 0.4486$$

9 Static Margin

The static margin is given by,

$$SM = \bar{x}_{CG} - \bar{x}_{AC} = -0.0327$$

Since it is a negative value, the CG is ahead of the AC, and hence the aircraft is stable.

10 Contribution to the Dihedral Effect

The individual contributions to the dihedral effect are evaluated separately and combined into one integrated term. The contributions to the dihedral effect are as follows,

* Wing contribution due to the geometric dihedral angle
* Wing contribution due to the wing-fuselage positions
* Wing contribution due to the sweep angle
* Wing contribution due to the aspect ratio
* Wing contribution due to the twist angle
* Body (fuselage) contribution

$C_{L_{\beta_{WB}}}$ is given by,

$$C_{L_{\beta_{WB}}} = 57.3 \cdot C_{L_1}\left[\left(\frac{C_{L_\beta}}{C_{L_1}}\right)_{\Lambda_{c/2}} K_{M_\Lambda} K_f + \left(\frac{C_{L_\beta}}{C_{L_1}}\right)_{AR}\right] +$$

$$57.3\left\{\Gamma_W\left[\frac{C_{L_\beta}}{\Gamma_W}K_{M_\Gamma} + \frac{\Delta C_{L_\beta}}{\Gamma_W}\right] + \left(\Delta C_{L_\beta}\right)_{Z_W} + \epsilon_W tan\Lambda_{c/4}\left(\frac{\Delta C_{L_\beta}}{\epsilon_W tan\Lambda_{c/4}}\right)\right\}$$

$$= -0.0615$$

11 Modelling Stability Derivatives

11.1 C_{L_α}

Assuming a η_H of 0.9, the derivative is obtained using the relation,

$$C_{L_\alpha} = C_{L_{\alpha_W}} + C_{L_{\alpha_H}} \eta_H \frac{S_H}{S} \left(1 - \frac{d\epsilon}{d\alpha}\right) = 5.5746$$

11.2 C_{m_α}

This derivative is calculated using,

$$C_{m_\alpha} = C_{L_{\alpha_W}} (\bar{x}_{CG} - \bar{x}_{AC_{wb}}) + C_{L_{\alpha_H}} \eta_H \frac{S_H}{S} \left(1 - \frac{d\epsilon}{d\alpha}\right) (\bar{x}_{AC_H} - \bar{x}_{AC_{WB}}) = -1.1815$$

11.3 C_{L_q}

The modelling of this derivative is given by,

$$C_{L_q} = C_{L_{q_W}} + C_{L_{q_H}}$$

where $C_{L_{q_H}}$ is,

$$C_{L_{q_H}} = 2C_{L_{\alpha_H}} \eta_H \frac{S_H}{S} (\bar{x}_{AC_H} - \bar{x}_{CG})$$

and $C_{L_{q_W}}$ is given by,

$$C_{L_{q_W}} = \left[\frac{AR + 2cos\Lambda_{c/4}}{AR \cdot B + 2cos\Lambda_{c/4}}\right] \cdot C_{L_{q_W}}\Big|_{M=0}$$

where B is,

$$B = \sqrt{1 - M^2 (cos\Lambda_{c/4})^2}$$

Finally the value of C_{L_q} is calculated to be,

$$C_{L_q} = 8.7064$$

11.4 C_{m_q}

This derivative is evaluated using the relation,

$$C_{m_q} = C_{m_{q_H}} + C_{m_{q_W}}$$

where,

$$C_{m_{q_H}} = -2C_{L_{\alpha_H}} \eta_H \frac{S_H}{S} (\bar{x}_{AC_H} - \bar{x}_{CG})^2$$

and,

$$C_{m_{q_W}} = \left[\frac{\frac{AR^3 tan^2\Lambda_{c/4}}{AR \cdot B + 6cos\Lambda_{c/4}} + \frac{3}{B}}{\frac{AR^3 tan^2\Lambda_{c/4}}{AR + 6cos\Lambda_{c/4}} + 3}\right] \cdot C_{m_{q_W}}\Big|_{M=0}$$

where,

$$C_{m_{q_W}}\Big|_{M=0} = -K_q C cos\Lambda_{c/4}\, C_{L_{\alpha_W}}\Big|_{M=0}$$

$$B = \sqrt{1 - M^2(cos\Lambda_{c/4})^2}$$

$$C = \left\{ \frac{AR(0.5|(\bar{x}_{AC_W} - \bar{x}_{CG})| + 2|(\bar{x}_{AC_H} - \bar{x}_{CG})|^2)}{AR + 2cos\Lambda_{c/4}} + \frac{1}{24}\left(\frac{AR^3 tan\Lambda_{c/4}}{AR + 6cos\Lambda_{c/4}}\right) + \frac{1}{8} \right\}$$

Therefore,

$$C_{m_q} = -20.4800$$

11.5 C_{Y_β}

The modelling of this derivative is given by,

$$C_{Y_\beta} = C_{Y_{\beta_W}} + C_{Y_{\beta_B}} + C_{Y_{\beta_H}} + C_{Y_{\beta_V}}$$

$C_{Y_{\beta_H}} = 0$, since the dihedral angle of the horizontal tail is 0. Further,

$$C_{Y_{\beta_W}} = -0.0001|\Gamma_W| \cdot 57.3$$

$$C_{Y_{\beta_B}} = -2 \cdot K_{int} \cdot \frac{S_{P \to V}}{S}$$

$$C_{Y_{\beta_V}} = -K_{Y_V} \cdot |C_{L_{\alpha_V}}|\eta_V \cdot \left(1 + \frac{d\sigma}{d\beta}\right)\frac{S_V}{S}$$

Therefore,

$$C_{Y_\beta} = -0.5151$$

11.6 C_{n_β}

This derivative is calculated using the relation,

$$C_{n_\beta} = C_{n_{\beta_W}} + C_{n_{\beta_B}} + C_{n_{\beta_H}} + C_{n_{\beta_V}}$$

At small angles of attack, $C_{n_{\beta_W}} = 0$. Also, due to the zero dihedral angle of the horizontal tail,$C_{n_{\beta_H}} = 0$. Further,

$$C_{n_{\beta_B}} = -57.3 \cdot K_N K_{R_t} \frac{S_{B_s}}{S}\frac{l_B}{b}$$

$$C_{n_{\beta_V}} = -C_{Y_{\beta_V}} \cdot \frac{X_V cos\alpha_1 + Z_V sin\alpha_1}{b}$$

Therefore,

$$C_{n_\beta} = 0.5658$$

11.7 C_{L_p}

The derivative is modelled using the following relation,

$$C_{l_p} = C_{l_{p_{WB}}} + C_{l_{p_H}} + C_{l_{p_V}}$$

where,

$$C_{l_{p_{WB}}} = C_{l_{p_W}} = RDP \cdot \frac{k}{\beta}$$

$$C_{l_{p_H}} = \frac{1}{2}\left(C_{L_{p_W}}\right)\Big|_H \frac{S_H}{S}\left(\frac{b_H}{b}\right)^2$$

and,

$$\left(C_{l_{p_W}}\right)\Big|_H = RDP_H \cdot \frac{k_H}{\beta_H}$$

where,

$$k_H = \frac{\left(C_{L_{\alpha_H}}\right)_W\Big|_M \cdot \beta_H}{2\pi}$$

Further,

$$C_{l_{p_V}} = 2C_{Y_{\beta_V}}\left(\frac{z_V}{b}\right)^2$$

Therefore,

$$C_{n_\beta} = -0.5432$$

11.8 C_{L_β}

This derivative is evaluated using,

$$C_{L_\beta} = C_{L_{\beta_{WB}}} + C_{L_{\beta_H}} + C_{L_{\beta_V}}$$

$C_{L_{\beta_H}} = 0$, since the dihedral angle of the horizontal tail is 0. Further,

$$C_{L_{\beta_V}} = -K_{Y_V} \cdot |C_{L_{\alpha_V}}|\eta_V \cdot \left(1 + \frac{d\sigma}{d\beta}\right)\frac{S_H}{S} \cdot \frac{Z_V cos\alpha_1 - X_V sin\alpha_1}{b}$$

Therefore,

$$C_{L_\beta} = -0.0996$$

11.9 C_{n_r}

This derivative is modelled using the relation,

$$C_{n_r} = C_{n_{r_w}} + C_{n_{r_V}}$$

where,

$$C_{n_{r_W}} = \left(\frac{C_{n_r}}{C_{L_1}}\right) \cdot C_{L_1}^2$$

and,

$$C_{n_{r_V}} = 2C_{Y_{\beta_V}} \cdot \frac{(X_V cos\alpha_1 + Z_V sin\alpha_1)^2}{b^2}$$

Therefore,

$$C_{n_r} = -0.1118$$

11.10 C_{m_u}

This parameter is negligible at the subsonic conditions associated with the operating Mach number of this aircraft. In general, this parameter plays a significant role only during transonic operation. Therefore,

$$C_{m_u} = 0$$

11.11 $C_{T_{X_u}}$

It is the coefficient modelling the thrust variation along X_S associated with small variations in the linear speed in the forward direction. The quantification of this effect depends on the specific propulsion system used onboard the aircraft. Further,

$$C_{T_{X_u}} = -0.07$$

11.12 C_{D_u}

For this particular aircraft, it is given that,

$$C_{D_u} = 0$$

12 Static Stability Criteria

Stability Criteria	Value	Conclusion
SC #1: $\left(C_{T_{X_u}} - C_{D_u}\right) < 0$	$C_{T_{X_u}}$ = -0.07, $C_{D_u} = 0$	**STABLE**
SC #2: $C_{Y_\beta} < 0$	C_{Y_β} = -0.5151	**STABLE**
SC #3: $C_{L_\alpha} > 0$	C_{L_α} = 5.5746	**STABLE**
SC #4: $C_{m_\alpha} < 0$	C_{m_α} = -1.1815	**STABLE**
SC #5: $C_{n_\beta} > 0$	C_{n_β} = 0.5658	**STABLE**
SC #6: $C_{l_p} < 0$	C_{l_p} = -0.5432	**STABLE**
SC #7: $C_{m_q} < 0$	C_{m_q} = -20.4800	**STABLE**
SC #8: $C_{n_r} < 0$	C_{n_r} = -0.1118	**STABLE**
SC #9: $C_{L_\beta} < 0$	C_{L_β} = -0.0996	**STABLE**
SC #10: $C_{m_u} > 0$	$C_{m_u} = 0$	**MARGINALLY STABLE**

13 Conclusion

The aircraft in this particular flight condition meets all the static stability criteria. Hence, it can be concluded that the aircraft is statically stable for this specific flight condition.

14 References

[1] Marcello R. Napolitano, "Aircraft Dynamics From Modelling to Simulation", *Wiley - 2012*

15 Appendix

15.1 Geometric Parameters of the Aircraft (Cessna T-37)

Geometric Parameters [unit]	**Value**	**Geometric Parameters [unit]**	**Value**
A [ft]	12.4	X_{HV} [ft]	1.5
b [ft]	33.8	X_{WHr} [ft]	15.9
b_H [ft]	14.0	X_1 [ft]	26.6
b_V [ft]	14.0	y_{A_1} [ft]	9.9
$\bar{c}$ [ft]	5.47	y_{A_0} [ft]	16.6
$\bar{c}_{Aileron}$ [ft]	1.2	y_{R_I} [ft]	0
$\bar{c}_R$ [ft]	1.4	y_{R_F} [ft]	4.4
$\bar{c}_{wing\,(At\,aileron)}$ [ft]	4.9	y_V [ft]	1.7
c_r [ft]	6.2	Z_H [ft]	-3.1
c_{r_H} [ft]	4.6	Z_{R_S} [ft]	3.6
c_{r_V} [ft]	6	z_1 [ft]	4.3
c_T [ft]	4.5	z_2 [ft]	2.1
c_{T_H} [ft]	2.2	Z_{H_S} [ft]	-3.1
c_{T_V} [ft]	2.5	z_{max} [ft]	4.4
d [ft]	4	Z_W [ft]	0
l_b [ft]	29.2	Z_{WHr} [ft]	3
l_{cg} [ft]	11.4	Γ_H [deg]	0
r_1 [ft]	2.2	Γ_W [deg]	3
S [ft^2]	182	ϵ_H [deg]	0
S_{B_S} [ft^2]	80.2	ϵ_W [deg]	0
$S_{f_{avg}}$ [ft^2]	8.7	Δ_{LE} [deg]	1.5
$S_{P\to V}$ [ft^2]	1.9	Δ_{LE_H} [deg]	12.5
w_{max} [ft]	9	Δ_{LE_V} [deg]	33
X_{AC_R} [ft]	5.1		

15.2 MATLAB Code

```matlab
%% Aircraft DATA Cessna T37 A
AR = 6.27;                % Aspect Ratio
A = 12.4;                 % ft - Distance between Nose and AC of C_tip
b = 14;                   % ft - wingspan
b_H = 14;                 % ft - Horizontal Tail span
b_V = 4.8;                % ft - Vertical Tail span
mac_w = 5.47;             % ft - MAC wing
c_ail = 1.2;              % ft - Aileron chord
c_R = 1.4;                % ft - Rudder Chord
C_r = 6.2;                % ft - Root Chord Wing
C_r_H = 4.6;              % ft - Root Chord Horizontal Tail
C_r_V = 4.6;              % ft - Root Chord Vertical Tail
C_t = 4.5;                % ft - Tip Chord Wing
C_t_H = 2.2;              % ft - Tip Chord Horizontal Tail
C_t_V = 2.5;              % ft - Tip Chord Vertical Tail
d = 4;                    % ft - Maximum Fuselage height at Wing Body intersection
l_b = 29.2;               % ft - Horizontal length of the aircraft
l_cg = 11.4;              % ft - Moment arm of CG
r1 = 2.2;                 % ft - Fuselage Height (including fin) at Root Chord AC of Vertical tail
S_w = 182;                % sq. ft - Planform Area of the Wing
S_B_s = 80.2;             % sq. ft - Fuselage Side Surface Area
S_f_avg = 8.7;            % sq. ft - Average Fuselage Cross Sectional Area
S_p_v = 1.9;              % sq. ft - Fuselage Cross-sectional Area where flow turns from Potential to Viscous
w_max = 9;                % ft - Max Fuselage width (due to engine intakes)
X_AC_R = 5.1;             % ft - Distance between LE of C_r_V and AC of Rudder
X_HV = 1.5;               % ft - Distance between LE of C_r_V and LE of C_r_H
X_WH_r = 15.9;            % ft - Distance between LE of C_r and LE of C_r_H
X_WV_r = X_WH_r-X_HV;     % ft - Distance between LE of C_r and LE of C_r_V
X_1 = 26.6;               % ft - Location on the Fuselage where flow turns from Potential to Viscous
y_A_1 = 9.9;              % ft - Inboard Location of aileron on the wing
y_A_0 = 16.6;             % ft - Outboard Location of aileron on the wing
y_R_I = 0;                % ft - Inboard Location of Rudder
y_R_F = 4.4;              % ft - Outboard Location of Rudder
y_V = 1.7;                % ft - Vertical distance between FRL and C_r_V
Z_H = -3.1;               % ft - Vertical distance between FRL and C_r_H
Z_R_s = 3.6;              % ft - Moment Arm for Rudder
z_1 = 4.3;                % ft - Fuselage Diameter before canopy
z_2 = 2.1;                % ft - Fuselage Diameter just before Tail
Z_H_s = -3.1;             % ft - Vertical Distance Horizontal Stabiliser
z_max = 4.4;              % ft - Max Fuselage height including canopy
Z_W = 0;                  % ft - Vertical Distance between Wing Root and FRL
Z_WH_r = 3;               % ft - Vertical Distance between Wing Root axis and Horizontal Tail root axis
gamma_H = 0;              % deg - Horizontal Tail Dihedral Angle
gamma_W = 3;              % deg - Wing Dihedral Angle
epsilon_H = 2;            % deg - Twist Angle Horizontal Stabliser
epsilon_W = 2;            % deg - Wing twist angle (aerodynamic twist) - angle between zero lift lines
Lambda_LE = 1.5;          % deg - Sweep Angle Wing
Lambda_LE_H = 12.5;       % deg - Sweep Angle Horizontal Stabiliser
Lambda_LE_V = 33;         % deg - Sweep Angle Vertical Stabiliser
Z_WH = 3.1;               % ft - Vertical Distance between FRL and ftAC of Horizontal Tail

%% Flight Conditions DATA
h = 30000;                            % ft - Altitude
[T,a,P,rho] = atmosisa(h*.3048);      % Flow Properties at 30,000 ft
M = 0.459;                            % Mach Number
Vp1 = 456;                            % ft/s - Perturbation Velocity
q = 92.7;                             % lbs/sq. ft - Dynamic Pressure
X_cg = 0.27;                          % cg location as a fraction of fuselage length
W = 6360;                             % lbs - Weight
alpha = 2;                            % deg - AoA
mu = 3.823*10^-4;                     % Dynamic Viscosity at 30k ft

```

```
%% Wing Parameters
lambda = C_t/C_r;                                                    % Taper Ratio Wing
Lambda_LE = Lambda_LE*pi/180;                                        % rad
Lambda_LE_half = atan((tan(Lambda_LE))-((2*(1-lambda))/(AR*(1+lambda))));   % rad - Geometric Relation
Lambda_LE_quarter = atan((tan(Lambda_LE))-((1-lambda)/(AR*(1+lambda))));    % rad - Geometric Relation
X_mac_w = ((b*(1+(2*lambda)))/(6*(1+lambda)))*tan(Lambda_LE);
% ft - Distance between LE of C_r and LE of mac

%% Horizontal Tail Parameters
Lambda_LE_H = Lambda_LE_H*pi/180;                                              % rad
lambda_H = C_t_H/C_r_H;
% Taper Ratio Horizontal Tail
mac_H = (2/3)*C_r_H*((1+lambda_H+(lambda_H)^2)/(1+lambda_H));
% ft - MAC of Horizontal Tail
X_mac_H = ((b_H*(1+(2*lambda_H)))/(6*(1+lambda_H)))*tan(Lambda_LE_H);
% ft - Distance between LE of C_r_H and LE of mac_H
S_H = (b_H/2)*C_r_H*(1+lambda_H);
% sq. ft - Planform Area Horizontal Tail
AR_H = (b_H^2)/S_H;
% Aspect Ratio Horizontal Tail
Lambda_LE_half_H = atan((tan(Lambda_LE_H))-((2*(1-lambda_H))/(AR_H*(1+lambda_H))));
% rad - Geometric Relation
Lambda_LE_quarter_H = atan((tan(Lambda_LE_H))-(((1-lambda_H))/(AR_H*(1+lambda_H))));
% rad - Geometric Relation

%% Wing Horizontal Tail Geometric Properties
X_WH = X_WH_r - (C_r/4) + (C_r_H/4);
% ft - distance between Wing root AC and Horizontal Tail root AC
m = Z_WH*2/b;                                   % Geometric Parameter
r = X_WH*2/b;                                   % Geometric Parameter
X_AC_H = X_WH_r + X_mac_H + (mac_H/4)-X_mac_w;
% ft - distance between Wing mac AC and Horizontal Tail mac AC

%% Wing Lift Slope Coefficient
k = 1+(((8.2-(2.3*Lambda_LE))-AR*(0.22-(0.153*Lambda_LE)))/100);
% From Polhamus Formula
C_L_alpha_w = (2*pi*AR)/(2+sqrt(((AR^2*(1-M^2)/k^2)*(1+(((tan(Lambda_LE_half))^2)/(1-M^2))))+4));
% From Polhamus Formula Mach = M
C_L_alpha_w_o= (2*pi*AR)/(2+sqrt(((AR^2*(1)/k^2)*(1+(((tan(Lambda_LE_half))^2)/(1))))+4));
% From Polhamus Formula Mach = 0

%% Downwash on Horizontal Tail
K_AR = (1/AR)-(1/(1+(AR^1.7)));
% Parameter to calculate d epsilon/d alpha at Mach = 0
K_lambda = (10-(3*lambda))/7;
% Parameter to calculate d epsilon/d alpha at Mach = 0
K_mr = (1 - (m/2))/(r^0.333);
% Parameter to calculate d epsilon/d alpha at Mach = 0
dEps_dalpha_o = 4.44*((K_AR*K_lambda*K_mr*sqrt(cos(Lambda_LE_quarter))^1.19));
% d epsilon/d alpha at Mach = 0
dEps_dalpha = dEps_dalpha_o*(C_L_alpha_w/C_L_alpha_w_o);
% d epsilon/d alpha at Flight Conditions
alpha_H = alpha*(1-dEps_dalpha);
% rad - Effective AoA at Horizontal Tail
K_mr_o = 1/(r^0.333);
dEps_dalpha_m_o = 4.44*((K_AR*K_lambda*K_mr_o*sqrt(cos(Lambda_LE_quarter))^1.19));
% d epsilon/d alpha at Mach = 0, m = 0

%% Wing Aerodynamic Center
p1 = (tan(Lambda_LE))/(sqrt(1-(M^2)));  % Parameter to determine K1 and K2 from the plots
p2 = AR*tan(Lambda_LE);                 % Parameter to determine K1 and K2 from the plots
k1 = 1.15;                              % Parameter to determine wing AC (from plots)
k2 = 0.1;                               % Parameter to determine wing AC (from plots)
xd_AC_Cr = 0.2;                         % Parameter to determine wing AC (from plots)
x_AC_w = k1*(xd_AC_Cr-k2);              % ft - Location of Wing AC
```

```
%% Effect of Body on AC Munk's Theory
X_H = X_mac_w + X_AC_H - C_r;
% ft - Distance between Wing TE and AC of Horizontal Tail MAC
w_i = [0.29 0.5 .54 .78 0.78 0.37 0.25 0.20 0.18 0.06];             % Fuselage Widths in inches
s_f = 9/0.94;                                                       % Scale Factor
w_i = w_i.*s_f;                                                     % in ft
x_i = [0.83 0.53 0.19 0.48 0.16 0.15 0.45 0.75 1.05 1.15];
% position of fuselage widths (middle of section)
x_i = x_i*s_f;                                                      % in ft
Del_x_i = [0.3 0.3 0.38 0.32 0.32 0.3 0.3 0.3 0.3 0.2];             % relative position of fuselage section
Del_x_i = Del_x_i*s_f;                                              % in ft
x_i_c_f = x_i./C_r;
x_i_x_H = x_i./X_H;
dEps_dalpha_M_m_o = dEps_dalpha_m_o*(C_L_alpha_w/C_L_alpha_w_o);
dEps_dalpha_i = [1.1 1.2 1.6 1.3 (x_i_x_H(5)*(1-dEps_dalpha_M_m_o)) ...
    (x_i_x_H(6)*(1-dEps_dalpha_M_m_o)) (x_i_x_H(7)*(1-dEps_dalpha_M_m_o)) ...
    (x_i_x_H(8)*(1-dEps_dalpha_M_m_o)) (x_i_x_H(9)*(1-dEps_dalpha_M_m_o)) ...
    (x_i_x_H(10)*(1-dEps_dalpha_m_o))];  % d eps/ d alpha m = 0 for every iteration

Del_x_AC_b = 0;                                                     % init
for i=1:10          % Number of Fuselage Sections
Del_x_AC_b = Del_x_AC_b - (1/(q*C_L_alpha_w*S_w*mac_w)*(w_i(i)^2)*dEps_dalpha_i(i)*Del_x_i(i));
% Change in AC due to body
end

%% Horizontal Tail Lift Slope Coefficient
k_H = 1+(((8.2-(2.3*Lambda_LE_H))-AR_H*(0.22-(0.153*Lambda_LE_H)))/100);
% From Polhamus Formula
C_L_alpha_H = (2*pi*AR_H)/(2+sqrt(((AR_H^2*(1-M^2)/k^2)*(1+(((tan(Lambda_LE_half_H))^2)/(1-M^2))))+4));
% From Polhamus Formula

%% Modelling of X_AC_wb
X_AC_wb = x_AC_w + Del_x_AC_b;

eta_H = 0.9;        % Assumption
tau_e = 0.4;        % Assumption

%% Longitudinal Stability And Control Derivatives
C_L_alpha = C_L_alpha_w + (C_L_alpha_H*eta_H*(S_H/S_w)*dEps_dalpha);
C_L_delev = eta_H*(S_H/S_w)*C_L_alpha_H*tau_e;
C_L_i_H = eta_H*(S_H/S_w)*C_L_alpha_H;
C_m_alpha = (C_L_alpha_w*(X_cg - X_AC_wb)) - (C_L_alpha_H*eta_H*(S_H/S_w)* ...
(1-dEps_dalpha)*((X_AC_H - X_cg)/(mac_w)));
C_m_delev = -1*C_L_alpha_H*eta_H*(S_H/S_w)*tau_e*((X_AC_H - X_cg)/(mac_w));
C_m_i_H = -1*C_L_alpha_H*eta_H*(S_H/S_w)*((X_AC_H - X_cg)/(mac_w));

%% C_L_alpha_dot and C_m_alpha_dot
C_L_alpha_dot = 2*C_L_alpha_H*eta_H*(S_H/S_w)*dEps_dalpha*((X_AC_H - X_cg)/(mac_w));
C_m_alpha_dot = -2*C_L_alpha_H*eta_H*(((X_AC_H - X_cg)/(mac_w))^2)*dEps_dalpha;

%% Aircraft Aerodynamic Center
x_AC = (X_AC_wb + (C_L_alpha_H/C_L_alpha_w)*eta_H*(S_H/S_w)*(1-dEps_dalpha)* ...
X_AC_H/(mac_w))/(1 + (C_L_alpha_H/C_L_alpha_w)*eta_H*(S_H/S_w)*(1-dEps_dalpha));

%% Static Margin
SM = (X_cg - x_AC)/(mac_w);

%% Modelling C_L_q
B = sqrt(1 - ((M^2)*(cos(Lambda_LE_quarter)^2)));
C_L_q_w_o = (0.5 + 2*((x_AC_w - X_cg)/(mac_w)))*C_L_alpha_w_o;
C_L_q_w = ((AR + (2*cos(Lambda_LE_quarter)))/((AR*B) + (2*cos(Lambda_LE_quarter))))*C_L_q_w_o;
C_L_q_H = 2*C_L_alpha_H*eta_H*(S_H/S_w)*((X_AC_H - X_cg)/(mac_w));
C_L_q = C_L_q_H + C_L_q_w;
```

```
%% Modelling C_m_q
K_q = 0.7;              % Correlation Coefficient from plot
C = (((AR*0.5*abs((x_AC_w - X_cg)/mac_w)) + (2*(abs((x_AC_w - X_cg)/mac_w)^2)))/ ...
(AR + (2*cos(Lambda_LE_quarter)))) + (((1/24)*(AR^3)*tan(Lambda_LE_quarter))/ ...
(AR + (6*cos(Lambda_LE_quarter)))) + (1/8);
C_m_q_w_o = -1*K_q*C_L_alpha_w_o*cos(Lambda_LE_quarter)*C;
C_m_q_w = (((((AR^3)*tan(Lambda_LE_quarter))/((AR*B) + (6*cos(Lambda_LE_quarter)))) + ...
 (3/B))/((((AR^3)*tan(Lambda_LE_quarter))/((AR) + (6*cos(Lambda_LE_quarter)))) + (3)))*C_m_q_w_o;
C_m_q_H = -2*C_L_alpha_H*eta_H*(S_H/S_w)*(((X_AC_H - X_cg)/mac_w)^2);
C_m_q = C_m_q_w + C_m_q_H;

%% Vertical Tail Geometric Properties
b_2v = b_V*2;
% sq. ft - twice the span of vertical tail
lambda_V = C_t_V/C_r_V;
%  Taper Ratio of the vertical tail
S_2v = 0.5*b_2v*C_r_V*(1+lambda_V);
% sq. ft - Area of the Vertical Tail
AR_V = (b_2v^2)/S_2v;
% Aspect Ratio Vertical Tail
mac_V = (2/3)*C_r_V*((1+lambda_V+lambda_V^2)/(1+lambda_V));
% ft - MAC of the Vertical Tail
Lambda_LE_V = Lambda_LE_V*pi/180;                                     % rad
X_mac_V = ((b*(1+(2*lambda_V)))/(6*(1+lambda_V)))*tan(Lambda_LE_V);
% ft - Distance between LE of C_r_V and LE of mac_V
Y_mac_V = ((b*(1+(2*lambda_V)))/(6*(1+lambda_V)));
% ft - Distance between C_r_V and mac_V along Y
Lambda_LE_half_V = atan((tan(Lambda_LE_V))-((2*(1-lambda_V))/(AR_V*(1+lambda_V))));
% rad - Geometric Relation

%% Wing Horizontal Tail Geometric Properties
X_cg_r = X_mac_w + (X_cg*mac_w);                  % ft - Distance between LE of wing root and cg
X_V_s = X_WV_r + (mac_w/4) + X_mac_V - X_cg_r;    % ft - Distance between cg and AC of mac_V
Z_V_s = y_V + Y_mac_V;                            % ft - Distance between FRL and mac_V
X_R_s = X_WV_r +    X_AC_R - X_cg_r;              % ft - Distance between cg and Rudder AC

%% Vertical Tail Lift Slope Coefficient
p_1 = b_V/(2*r1);                                 % parameter for AR_V_eff
p_2 = Z_H/b_V;                                    % parameter for AR_V_eff
p_3 = S_H/S_2v;                                   % parameter for AR_V_eff
X_AC_H_V = X_mac_H + 0.25*mac_H;                  % parameter for AR_V_eff
p_4 = X_AC_H_V/mac_V;                             % parameter for AR_V_eff
c1 = 1.6;                                         % from plot
c2 = 0.82;                                        % from plot
K_HV = 1.05;                                      % from plot
AR_V_eff = c1*AR_V*(1+(K_HV*(c2 - 1)));           % Effect AR at Vertical Tail
k_V = 1+(((8.2-(2.3*Lambda_LE_V))-AR_V_eff* ...
(0.22-(0.153*Lambda_LE_V)))/100);                  % From Polhamus Formula
C_L_alpha_V_eff = (2*pi*AR_V_eff)/(2+sqrt(((AR_V_eff^2*(1-M^2)/k_V^2)* ...
(1+(((tan(Lambda_LE_half_V))^2)/(1-M^2))))+4));    % 1/rad - From Polhamus Formula
C_L_alpha_V_eff_o= (2*pi*AR_V_eff)/(2+sqrt(((AR_V_eff^2*(1)/k_V^2)* ...
(1+(((tan(Lambda_LE_half_V))^2)/(1))))+4));           % 1/rad - From Polhamus Formula Mach = 0

%% Wing Body Contribution

%% Wing Contribution to the Dihedral Effect to the Geometric Dihedral Angle
C_L_beta_gamma_w = -1.6*(10^-4);                                     % From Plot
par1 = AR/cos(Lambda_LE_half);                                       % Parameter for K_m_gamma
par2 = M*cos(Lambda_LE_half);                                        % Parameter for K_m_gamma
K_m_gamma = 1.08;                                                    % From Plot
C_L_beta_wb_gda = 57.3*gamma_W*(C_L_beta_gamma_w*K_m_gamma);
% Dihedral Effect due to geometric dihedral angle

%% Wing contribution to the dihedral effect due to wing-fuselage position
d_B = sqrt(S_f_avg/0.7854);                             % ft - Fuselage Average Diameter
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```
C_L_beta_wb_low_wing = (1.2*sqrt(AR)*Z_W*2*d_B)/(b^2);
% Dihedral Effect due to wing-fuselage position

%% Wing contribution to the Dihedral effect due to the Wing Sweep Angle
C_L_1 = W/(q*S_w);                                          % C_L for the Flight Conditions
C_L1_C_L_beta = -0.5*(10^-3);                               % From plot
p1 = AR/(cos(Lambda_LE_half));                              % Parameter for K_M_Lambda
p2 = M*cos(Lambda_LE_half);                                 % Parameter for K_M_Lambda
K_M_Lambda = 1.05;                                          % From Plot
p = A/b;                                                    % Parameter for K_f
K_f = 0.9;                                                  % From Plot
C_L_beta_wb_Lambda_w = 57.3*C_L_1*(C_L1_C_L_beta*K_M_Lambda*K_f);
% Dihedral effect due to the Wing Sweep Angle

%% Wing contribution to the Dihedral effect due to the Wing AR
C_L_beta_C_L1 = -1*(10^-3);                    % From Plot using lambda and AR
C_L_beta_wb_AR_w = 57.3*C_L_1*C_L_beta_C_L1;    % Dihedral effect due to the Wing AR

%% Wing contribution to the dihedral effect due to wing twist angle
Del_C_L_beta_eps_tan_Del_c_4 = -2.25*(10^-5);        % From Plot using AR and lambda
C_L_beta_wb_twist = 57.3*epsilon_W*(pi/180)*tan(Lambda_LE_quarter)* ...
Del_C_L_beta_eps_tan_Del_c_4;     % Dihedral effect due to the Wing Twist

%% Fuselage contribution to the dihedral effect
Del_C_L_beta_gamma_w = -0.0005*AR*(d_B/b)^2;                             % Contribution
C_L_beta_wb_f_gda = 57.3*gamma_W*(C_L_beta_gamma_w*Del_C_L_beta_gamma_w);
% Dihedral Effect due to geometric dihedral angle


%% FINAL C_L_beta_wb
C_L_beta_wb = C_L_beta_wb_gda + C_L_beta_wb_low_wing + C_L_beta_wb_Lambda_w + ...
 C_L_beta_wb_AR_w + C_L_beta_wb_twist + C_L_beta_wb_f_gda;

%% Vertical Tail Contribution
eta_V = 0.9;                                                        % Vertical Tail Efficiency
p1 = b_V/(2*r1);                                                    % Parameter for K_Y_V
K_Y_V = 0.76;                                                       % K_Y_V
eta_V_d_sigma_d_beta = 0.724 + (3.06*((0.5*S_2v/S_w)/ ...
(1+cos(Lambda_LE_quarter)))) + (0.4*Z_W/d) + (0.009*AR);           % another term
C_L_beta_V = -1*K_Y_V*abs(C_L_alpha_V_eff)*eta_V_d_sigma_d_beta* ...
((0.5*S_2v/S_w)/(1+cos(Lambda_LE_quarter)))*(((Z_V_s*cos(alpha*pi/180)) ...
 - (X_V_s*sin(alpha*pi/180)))/b);         % Vertical Tail Contribution

%% FINAL C_L_beta
C_L_beta = C_L_beta_wb + C_L_beta_V;                % 1/rad - C_L_beta

%% Modelling C_Y_beta
C_Y_beta_w = -0.0001*abs(gamma_W)*57.3;     % Wing Contribution
p = Z_W/(d/2);                              % Parameter for K_int
K_int = 0;                                  % Mid Wing so 0
C_Y_beta_B = -2*K_int*(S_p_v/S_w);          % Body Contribution
C_Y_beta = 0;
% Horizontal Tail 0 due to negligible dihedral angle
C_Y_beta_V = -K_Y_V*abs(C_L_alpha_V_eff)*eta_V_d_sigma_d_beta*(S_2v/S_w);   % Vertical Tail Contribution
C_Y_beta = C_Y_beta_w + C_Y_beta_B + C_Y_beta_V;                            % C_Y_beta

%% Modelling C_n_beta
C_n_beta_w = 0;                  % Wing Contribution
p1 = l_cg/l_b;                   % Parameter for K_N
p2 = (l_b^2)/S_B_s;              % Parameter for K_N
p3 = sqrt(z_1/z_2);              % Parameter for K_N
p4 = z_max/w_max;                % Parameter for K_N
K_N = 0.0002;                    % From Plot
Re_fuselage = Vp1*l_b/mu;
% Reynolds Number of Fuselage
```

```
K_RE_i = 1.7;                         % From Plot
C_n_beta_B = -57.3*K_N*K_RE_i*(S_B_s/S_w)*(l_b/b);
% Fuselage/Body Contribution
C_n_beta_V = -C_Y_beta_V*(((X_V_s*cos(alpha*pi/180)) + (Z_V_s*sin(alpha*pi/180)))/b);
% Vertical Tail Contribution
C_n_beta = C_n_beta_B + C_n_beta_V;   % C_n_beta

%% Modelling C_L_p
Beta = sqrt(1 - M^2);                                       % Beta
k = C_L_alpha_w*Beta/(2*pi);                                % k
p1 = Beta*AR/k;                                             % Parameter for RDP
Lambda_beta = atan(tan(Lambda_LE_quarter)/Beta)*180/pi;     % deg - Parameter for RDP
RDP = -0.40;                                                % RDP
C_L_p_wb = RDP*k/Beta;                                      % Wing Body Contribution
k_H = (C_L_alpha_H*Beta)/(2*pi);                            % k_H
p2 = Beta*AR/k_H;                                           % Parameter for RDP_H
Lambda_beta_H = atan(tan(Lambda_LE_quarter_H)/Beta)*180/pi; % deg - Parameter for RDP_H
RDP_H = -0.49;                                              % RDP_H
C_L_p_H = (RDP_H*Beta/k_H)*0.5*(S_H/S_w)*(b_H/b)^2;         % Horizontal Tail Contribution
C_L_p_V = 2*C_Y_beta_V*((Z_V_s/b)^2);                       % Vertical Tail Contribution
C_L_p = C_L_p_V + C_L_p_H + C_L_p_wb;                       % C_L_p

%% Modelling C_n_r
C_n_r_C_L_1 = -0.2;                            % From Graph
C_n_r_w = C_n_r_C_L_1*C_L_1^2;
% Wing Contribution
C_n_r_V = 2*C_Y_beta_V*(((X_V_s*cos(alpha*pi/180)) + (Z_V_s*sin(alpha*pi/180)))/(b^2));
% Vertical Tail Contribution
C_n_r = C_n_r_V + C_n_r_w;                     % C_n_r
```