Maslen's Method: An Overview

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1 Introduction

The Maslen's method is a theory based on the assumption of a thin shock layer. It is a comparatively simple method and has frequent application even today for the approximate analysis of hypersonic inviscid shock layers. Moreover, Maslen's method gives results for the flow field over blunt as well as slender bodies.

2 MATLAB Code

```
% HSA project
clear all
clc
format long;
%% free stream properties
Y=1.4; % free stream specific heat ratio (gamma)
r=287; % gas constant
M=10; %mach no. is condider to be infinity
P=10000; % free stream pressure in pa
T=300; % free stream temperature in  k
rho=P/(287*T); % free stream density in kg/m^3
cp=Y*r/(Y-1); %cp
V=M*(Y*r*T)^0.5; % velocity
%% assuming shock shape and parameters
R=1; %nose radius
d=(0.386*exp(4.67/M^2))*R; %d-shock standoff distance
Rc=(1.386*exp(1.8/(10-1)^0.1))*R; %Rc- shock radius of curvature at vertex
b=pi/20; % b= asymptotic shock wave angle
y=linspace(0,5,50);
x=-(R+d-Rc*(cot(b)^2)*((((1+(y.^2*tan(b)^2)/Rc^3)).^.85)-1));
for i=1:numel(y)-1
 dx(i)=x(i+1)-x(i);
 dy(i)=y(i+1)-y(i);
 sl(i)=dy(i)/dx(i);% sl=slop
end
beta=atan(sl);
n=numel(sl);
for i=1:n-1
 dX(i)=x(i+1)-x(i);
```

```
 dY(i)=sl(i+1)-sl(i);
 dsl(i)=dY(i)/dX(i);
end
beta1=atan(dsl);
for i=1:numel(dsl)
    if abs(dsl(1,i))>0
        s(i)=(1+(sl(i))^2)^1.5/abs(dsl(i));
    else
        break
    end
end
% shock defn ends
%Properties calculated just after the shock using approximated oblique %shock relations
P1=(1+((2*Y/(Y+1))((M^2(sin(beta)).^2)-1)))*P;
rho1=(1+((Y+1)M^2(sin(beta)).^2)./(((Y-1)M^2(sin(beta)).^2)+2))*rho; % density ratio
T1=((P1/P).*(rho./rho1))*T; %temperature ratio
u1=V*cos(beta);
P0=P*((1+(Y-1)*M^2/2)^(Y/(Y-1)));
T0=T*(1+(Y-1)*M^2/2);
Mn=M*sin(beta);
Mn1=[(Mn.^2+(2/(Y-1)))./([2*Y*Mn.^2/(Y-1)]-1)].^0.5;
theta=atan(2*cot(beta).[((M^1(sin(beta)).^2)-1)]./((M^2*(Y+cos(2*beta)))+2));
M1=Mn1./sin(beta-theta);
P01=P1.*((1+(Y-1)*M1.^2/2).^(Y/(Y-1)));
dels=-8.314.*log(P01./P0);
q1= 0.5*rho1*V^2;
W=rho.*V.*y;
%% iterartive meslon's method
for i=1:numel(s)
    for j=1:i
        if j==1
            P2(i,j)=P1(i);
            T2(i,j)=T1(i);
            rho2(i,j)=P1(i)/(287*T1(i));
            u2(i,j)=(2*cp*(T0-T1(i)))^0.5;
        else
            P2(i,j)=P2(i,j-1)+((u1(i)/s(i))*(W(i-1)-W(i)));
            T2(i,j)=(exp((r*log(P2(i,j)/P2(i,j-1))-dels(i))/cp))*T1(i);
            rho2(i,j)=P2(i,j)/(287*T2(i,j));
            u2(i,j)=(2*cp*(T0-T2(i,j)))^0.5;
        end
        if j<i
            dn(i,j)=(2*(W(i)-W(i-j)))/(rho1(i)*u1(i)+rho2(i,j)*u2(i,j));
            ang(i,j)=pi/2-beta(i);
            x2(i,j)=x(i)+dn(i,j)*cos(ang(i,j));
            y2(i,j)=y(i)-dn(i,j)*sin(ang(i,j));
        end
    end
end
for i=1:47
 xb(i)=x2(i+1,i);
 yb(i)=y2(i+1,i);
 Pb(i)=P2(i+1,i);
end
%% Plots
figure(1),plot(x,y,'.',xb,yb,'-');
title('Shock and body shape');
xlabel('x-cordinate');
ylabel('y-cocrdinate');
legend('shock shape','Body');
figure(2),plot(xb(2:numel(xb)),Pb(2:numel(xb)),'--');
title('Pressure distribution');
xlabel('x-cordinate');
ylabel('Pressure (pa)');
```

3 Preliminary Results

Some preliminary results are given below,

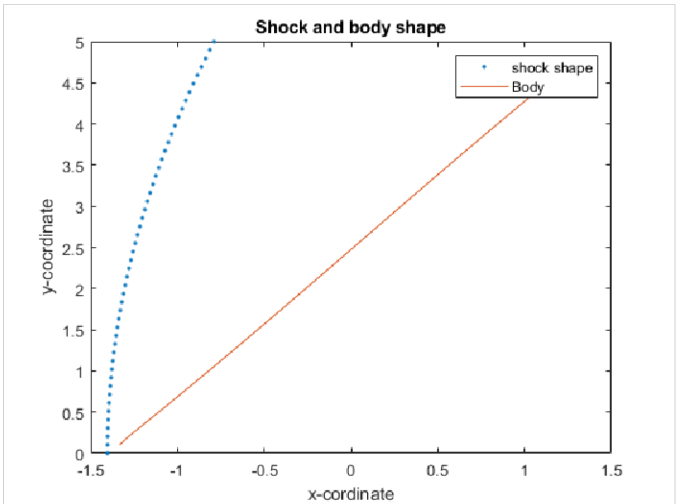

Figure 1: Shock and Body Shape

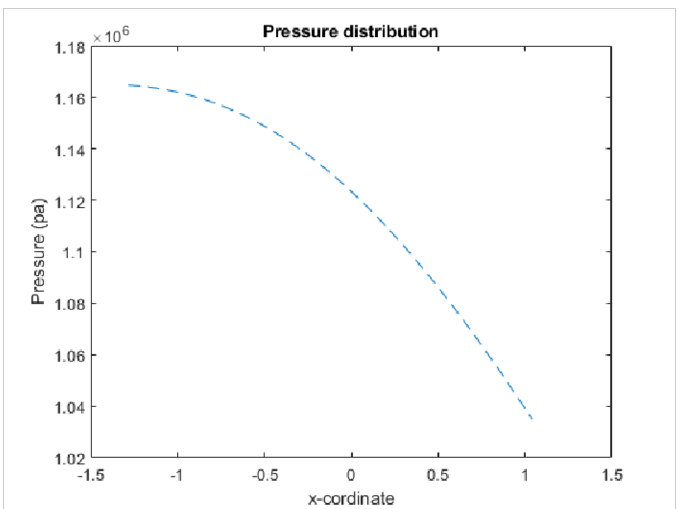

Figure 2: Pressure Distribution

4 Methodology

Consider the curvilinear coordinate system, where x and y respectively, are parallel and perpendicular to the shock, and u and v are corresponding components of velocity. For simplicity we will assume a two dimensional flow; however, Maslen's method also applies to axisymmetric flow. Now we assume that the shock layer is thin and hence the streamlines are essentially parallel to the shock wave. In a streamline-based coordinate system, The momentum equation for the present coordinate system is given by,

$$\rho \frac{u^2}{R} = \frac{\partial p}{\partial y} \tag{1}$$

Where R is the local streamline radius of curvature. For thin shock-layer assumptions,

$$\rho \frac{u^2}{R_S} = \frac{\partial p}{\partial y} \tag{2}$$

Where R_S is the local shock radius of curvature. Further simplifying,

$$\rho \frac{u^2}{R_S} = \left(\frac{\partial p}{\partial \psi}\right) \rho u \tag{3}$$

This leads to the expression,

$$\frac{\partial p}{\partial \psi} = \frac{u}{R_S} \tag{4}$$

According to the earlier assumptions made of thin shock layer, we can consider u $\approx u_S$ where u_S is the the velocity just behind the shock. By this assumption we are re-asserting the assumption that all the streamlines are parallel to the shock, therefore,

$$\frac{\partial p}{\partial \psi} = \frac{u_S}{R_S} \tag{5}$$

We can integrate the above equation between a point in the shock layer where the value of the stream function as ψ and just behind the shock layer where $\psi = \psi_S$. Thus,

$$p(x, y) = p_S(x) + \frac{u_S(x)}{R_S(x)} \left[\psi - \psi_S(x)\right] \tag{6}$$

Using the above equation we can build an inverse method where a shock wave shape will be assumed for a body to solve the above equation and then obtain the shape and pressure distribution over the body. When the obtained body shape matches with the real shape then we can get the shock shape and pressure distribution. The procedure described by Maslen can be summarized as,

- Assume a shock-wave shape. In a sense, Maslen's method is an inverse method, where a shock wave is assumed and the body that supports thin shock is calculated.
- Hence, all flow quantities are known at a point just behind the shock, from the oblique shock relations. The value for ψ_1 is known from,

$$\psi = \rho_\infty V_\infty h \tag{7}$$

- Choose a value of ψ_2, where $0 < \psi_2 < \psi_1$. This identifies a point 2 inside the flowfield along the y axis, where $\psi = \psi_2$. (The precise value of the physical coordinate y_2 will be found in a subsequent step).
- Calculate the pressure at point 2 from the equation,

$$p_2 = p_1 + \frac{u_1}{R_S} \left(\psi_2 - \psi_1\right) \tag{8}$$

- The entropy at point 2, s_2 is known because the streamline at point 2, corresponding to $\psi = \psi_2$, has come through that point on the shock wave, point 2', where $\psi_{2'} = \psi_2$, and where

$$\psi_{2'} = \psi_2 = \rho_\infty V_\infty h_2 \tag{9}$$

or,

$$h_2 = \frac{\psi_2}{\rho_\infty V_\infty} \tag{10}$$

Therefore, h_2 is obtained from the above equation, which locates point 2' on the shock. In turn, $s_{2'}$ is known from the oblique shock relations, and because the flow is isentropic along any given streamline $s_2 = s_{2'}$. Calculating the enthalpy h_2 and density ρ_2 from the thermodynamics equations of state,

$$h_2 = h(s_2, p_2) \qquad\qquad \rho_2 = \rho(s_2, p_2) \tag{11}$$

- Calculating the velocity at point 2 from the adiabatic equation (total enthalpy is constant),

$$h_o = h_\infty + \frac{v_\infty^2}{2} \tag{12}$$

  Where h_o is the total enthalpy, which is constant throughout the adiabatic flow-field. In turn,

$$h_o = h_2 + \frac{u_2^2}{2} \tag{13}$$

  Thus,

$$u_2 = \sqrt{2(h_o - h_2)} \tag{14}$$

- All of the flow quantities are now known at point 2. Now repeat the preseding steps for all points along the y axis between the shock (point 1) and the body (point 3). The body surface is defined by $\psi = 0$.

- The physical coordinates y, which corresponds to a particular value of ψ, can now be found by integrating the definitions of the stream function (which is essentially the continuity equation). Since,

$$\frac{d\psi}{dy} = \rho u \tag{15}$$

  Then,

$$y = \int_{\psi}^{\psi_S} \frac{d\psi}{\rho u} \tag{16}$$

  Where ρ and u are known as a function of ψ from the preceeding steps. This also locates the body coordinate, where

$$y_b = \int_{0}^{\psi_S} \frac{d\psi}{\rho u} \tag{17}$$

- This procedure is repeated for any desired number of points along the specified shock wave, hence generating the flowfield and body shape which supports that particular shock.