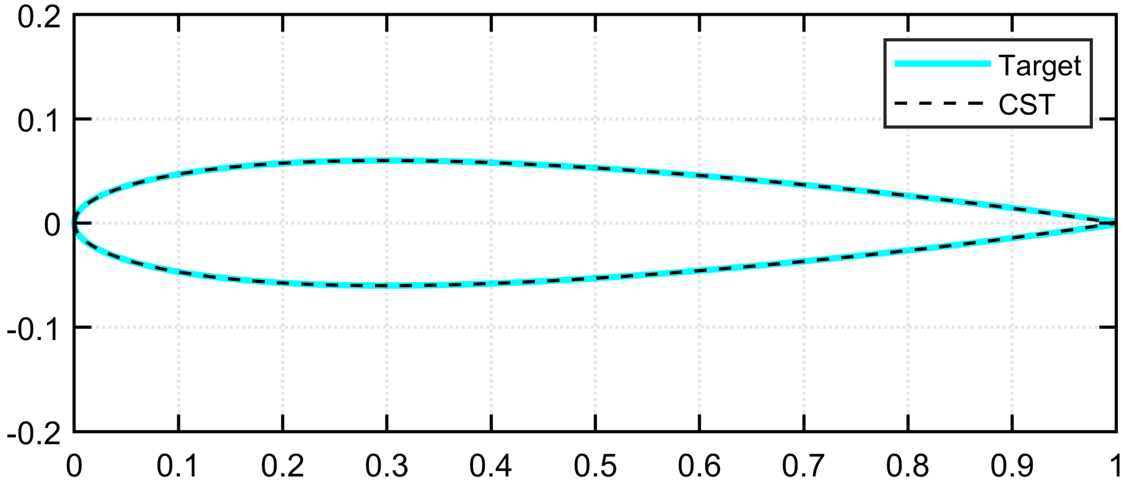

NACA 0012

Class Shape Transformation

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Contents

1 Class Shape Transformation (CST)

1.1 Introduction

The Class Shape Transformation (CST) parametrisation technique was developed by Brenda Kulfan, an aerodynamic engineer at Boeing, in 2008. It incorporates a geometric class/shape function transformation technique. Any geometric representation model should have the following desirable characteristics:

* It should be well-behaved and produces smooth and realistic shapes.
* It should be mathematically efficient, numerically stable, fast, accurate, and consistent.
* Requires relatively few variables to represent a large domain to contain optimum aerodynamic shapes for varying design conditions and constraints.
* Allows specification of design parameters such as leading-edge radius, boat-tail angle and aerofoil closure.
* Provides easy control for designing and editing the shape of a curve.
* A geometry algorithm with an intuitive and geometric interpretation.

A well-behaved mathematical model indicates a model that doesn't violate any assumptions made for that particular analysis. The leading edge radius is the minimum radius of the leading edge of the aerofoil. The aerofoil closure refers to the aerofoil's leading edge and trailing edge closure. The boat tail angle is the trailing edge angle usually used in parametrisation techniques.

The CST model is an optimisation technique where a shape can be "optimised" for analysis using a mathematical model. This optimisation is carried out to obtain an optimum and smooth aerofoil curve. It consists of 5 essential steps:

* The development of a general-purpose mathematical equation that can describe the geometry of any round-nose or sharp aft-ended aerofoil.
* Examine the nature of the expression to determine the elements that are the source of the numerical singularity.
* Rearrange or transform the parts of the expression to eliminate this numerical singularity.
* This resulted in the identification and the definition of a "shape function" such that the definition of an aerofoil using this function becomes a simple analytical function with physical design features that are easily controlled.
* Finally, a "class function" was introduced to generalise this methodology for various applications, including two-dimensional aerofoils.

A singularity is a point on the mathematical function where its value isn't defined, or a point of discontinuity. The mathematical approach of this method is discussed below.

1.2 Mathematical Model

1.2.1 Fundamental Approach

In round-nose aerofoils defined in a fixed Cartesian coordinate system, the slopes and second derivatives of the surface geometry are infinite at the nose, and large changes in curvature occur over the entire aerofoil.

Therefore, the mathematical characteristics of the aerofoil surfaces are non-analytic functions with singularities in the derivatives at the nose. The general form of the mathematical expression that represents a typical aerofoil geometry is:

$$\zeta(\psi) = \sqrt{\psi}(1-\psi)\sum_{i=0}^{N} A_i \psi^i + \psi \zeta_T \qquad (1)$$

where $\psi = x/c$, $\zeta = z/c$ and $\zeta_T = \Delta Z_{TE}/c$. The term $\sqrt{\psi}$ corresponds to the round nose of the aerofoil. Similarly, the term $(1-\psi)$ corresponds to the sharp trailing edge of the aerofoil. The term $\psi\zeta_T$ controls the thickness of the trailing edge, and the summation term is a general function that describes the shape of the aerofoil between the round nose and the sharp trailing edge. It has been represented as a power series for convenience but can be represented by any well-behaved analytic mathematical function.

1.2.2 Aerofoil Shape & Class Function

The shape function $S(\psi)$ is given by the relation,

$$S(\psi) = \sum_{i=0}^{N} \left[A_i \psi^i\right] \tag{2}$$

The leading-edge radius, the trailing-edge thickness, and the boat-tail angle are related directly to the unique bounding values of the $S(\psi)$ function. The value of the shape function at $x/c = 0$ is related to the aerofoil leading-edge nose radius R_{LE} and the chord length C of the aerofoil by the relation,

$$S(0) = \sqrt{2\left[R_{LE}/C\right]} \tag{3}$$

The value of the shape function at $x/c = 1$ is related directly to the aerofoil boat-tail angle β and trailing-edge thickness by the relation,

$$S(1) = tan(\beta) + \frac{\Delta Z_{TE}}{c} \tag{4}$$

Hence the parameters of the aerofoil (R_{LE}, β and Δz_{TE}) can be controlled by specifying the end-points of the shape function.The term $\psi[1-\psi]$ defines the class function $C(\psi)$ given by,

$$C_{N2}^{N1}(\psi) \triangleq (\psi)^{N1}[1-\psi]^{N2} \tag{5}$$

Generally, for a round-nose aerofoil, the values of N1 and N2 are 0.5 and 1, respectively. The class function is used to define general classes of geometries, whereas the shape function is used to define specific shapes within a class of geometries. Defining an aerofoil shape function and specifying its geometry class is equivalent to defining the actual aerofoil coordinates, which can then be obtained from the combination of the shape and class functions from the relation,

$$\zeta(\psi) = C_{N2}^{N1}(\psi)S(\psi) + \psi\zeta_T \tag{6}$$

The aerofoil is then defined using the Bernstein polynomials, which is defined for the order n as follows,

$$S_{r,n}(x) = K_{r,n} x^r (1-x)^{n-r} \tag{7}$$

The binomial coefficients are defined as,

$$K_{r,n} = \binom{n}{r} = \frac{n!}{r!(n-r)!} \tag{8}$$

The first term of the polynomial defines the leading-edge radius, and the last term is the boat-tail angle. The other terms in between are "shaping terms" that don't affect the leading edge radius or the trailing-edge boat-tail angle. In our case, we take a Bernstein polynomial of the order 8, and hence eight weighted coefficients are obtained at the end of the CST process. The upper surface of the aerofoil is defined by the equations,

$$(\zeta)_{upper} = C_{N2}^{N1}(\psi)Sl(\psi) + \psi\Delta\xi_{upper} \tag{9}$$

$$Su(\psi) = \sum_{i=1}^{N} Au_i S_i(\psi) \tag{10}$$

The lower surface of the aerofoil is defined by the equations,

$$(\zeta)_{lower} = C_{N2}^{N1}(\psi)Sl(\psi) + \psi\Delta\xi_{lower} \tag{11}$$

$$Sl(\psi) = \sum_{i=1}^{N} Al_i S_i(\psi) \tag{12}$$

where,

$$\Delta\xi_U = \frac{zu_{TE}}{C} \text{ and } \Delta\xi_L = \frac{zl_{TE}}{C} \tag{13}$$

The drag predictions and pressure distribution for a Bernstein's polynomial of the 8th order agree exactly with experimental data, and hence a Bernstein polynomial of the 8th order was chosen for this project.

2 Aerofoil Data

2.1 fx73cl3152

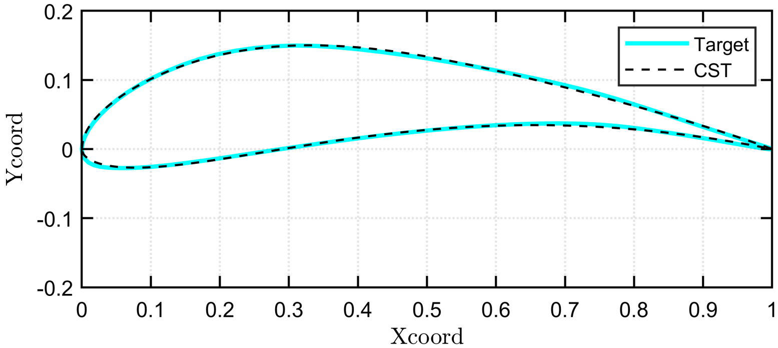

Figure 1: CST on the fx73cl3152 Aerofoil

Name of the Aerofoil	W	Values
fx73cl3152	W1	0.3120234682
	W2	0.4954393387
	W3	0.2868077264
	W4	0.3696218008
	W5	-0.1499320592
	W6	0.0534256903
	W7	0.1394470570
	W8	0.1986355486

Table 1: Weighted Coefficients

2.2 fx73k170

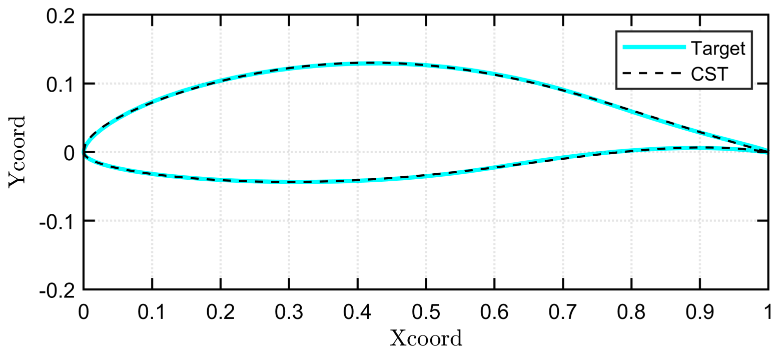

Figure 2: CST on the fx73k170 Aerofoil

Name of the Aerofoil	W	Values
fx73k170	W1	0.2145805658
	W2	0.3480156817
	W3	0.4491558798
	W4	0.2576306351
	W5	-0.1093967728
	W6	-0.1178363634
	W7	-0.1488996555
	W8	0.1509693371

Table 2: Weighted Coefficients

2.3 fx74cl5140

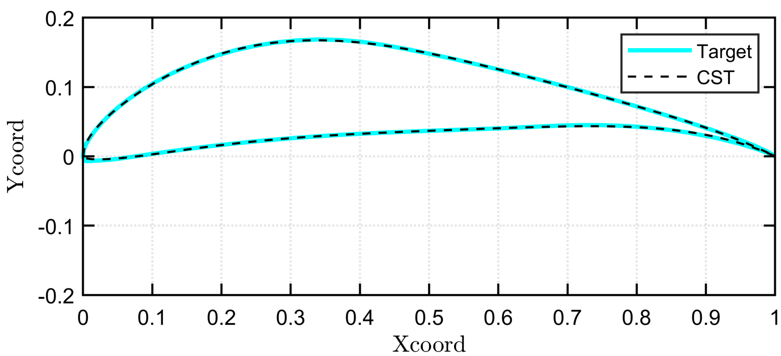

Figure 3: CST on the fx74cl5140 Aerofoil

Name of the Aerofoil	W	Values
fx74cl5140	W1	0.2793438795
	W2	0.6404946180
	W3	0.2207820067
	W4	0.4976161492
	W5	-0.0441807820
	W6	0.1760231627
	W7	-0.0342550828
	W8	0.4520180627

Table 3: Weighted Coefficients

2.4 fx74cl6140

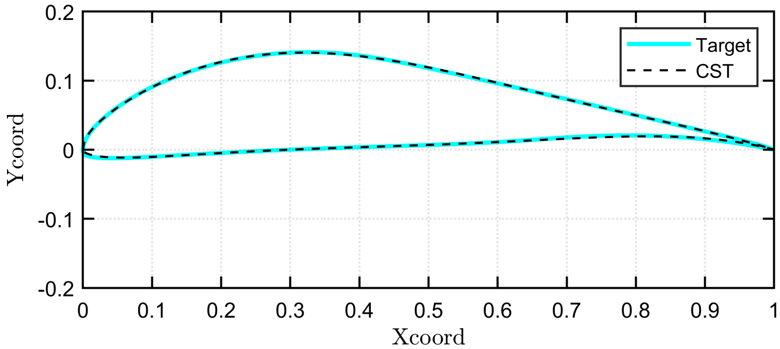

Figure 4: CST on the fx74cl6140 Aerofoil

Name of the Aerofoil	W	Values
fx74cl6140	W1	0.2474621678
	W2	0.5467683613
	W3	0.1653124231
	W4	0.3125454222
	W5	-0.0747507489
	W6	0.0823636401
	W7	-0.0935304836
	W8	0.2658792059

Table 4: Weighted Coefficients

2.5 fx74modsm

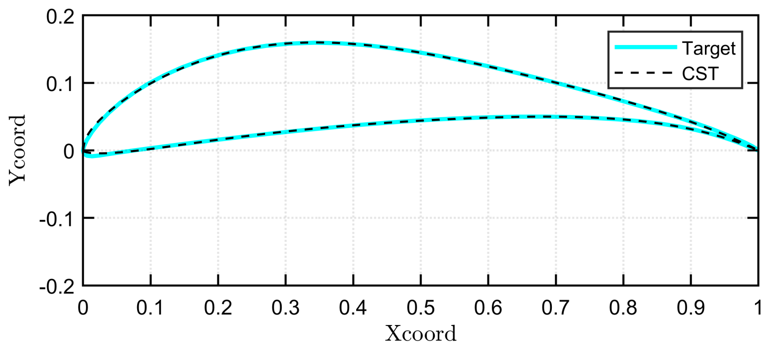

Figure 5: CST on the fx74modsm Aerofoil

Name of the Aerofoil	W	Values
fx74modsm	W1	0.2719184200
	W2	0.5892798381
	W3	0.2415881098
	W4	0.5084197654
	W5	-0.0420332980
	W6	0.1485538418
	W7	0.0532259196
	W8	0.4327490194

Table 5: Weighted Coefficients

2.6 fx75vg166

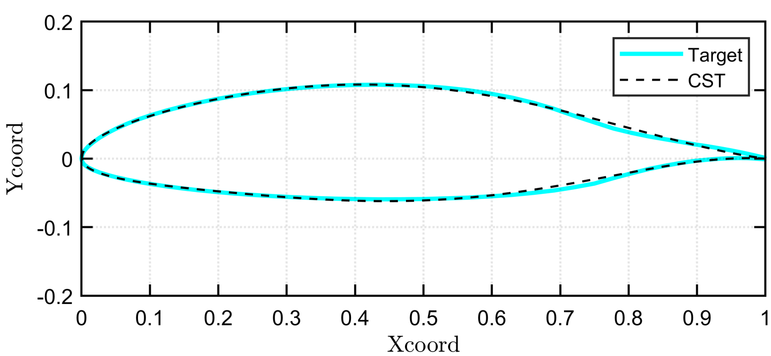

Figure 6: CST on the fx75vg166 Aerofoil

Name of the Aerofoil	W	Values
fx75vg166	W1	0.1928762932
	W2	0.2739563905
	W3	0.4041799497
	W4	0.1328298888
	W5	-0.1372378072
	W6	-0.0762956812
	W7	-0.3577898915
	W8	0.0596583198

Table 6: Weighted Coefficients

2.7 fx76mp120

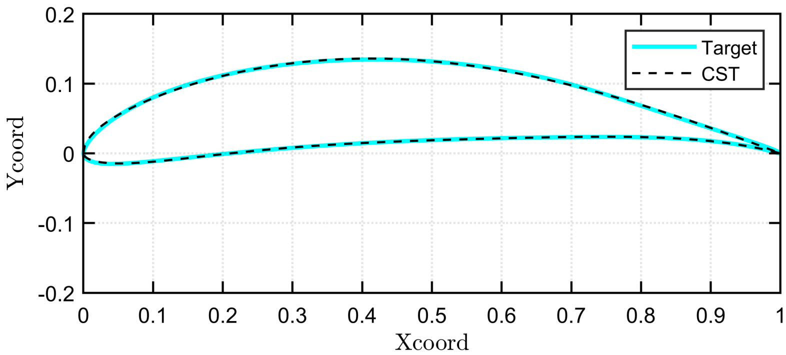

Figure 7: CST on the fx76mp120 Aerofoil

Name of the Aerofoil	W	Values
fx76mp120	W1	0.2391337987
	W2	0.3787341677
	W3	0.4122211027
	W4	0.3756148837
	W5	-0.0992550568
	W6	0.1286009403
	W7	-0.0416638785
	W8	0.2630216670

Table 7: Weighted Coefficients

2.8 fx74080

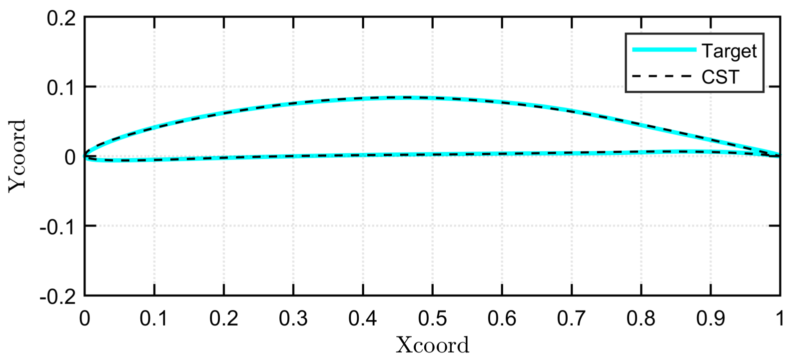

Figure 8: CST on the fx74080 Aerofoil

Name of the Aerofoil	W	Values
fx74080	W1	0.1088665386
	W2	0.2251047758
	W3	0.2902885064
	W4	0.2366312738
	W5	-0.0408313348
	W6	0.0479400052
	W7	-0.0505345533
	W8	0.1006646506

Table 8: Weighted Coefficients

2.9 fx74130wp1

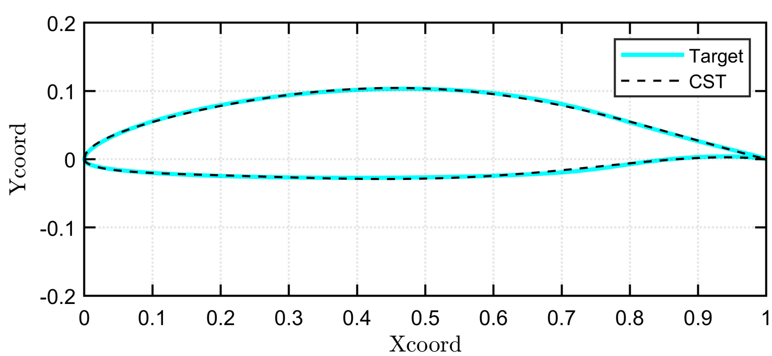

Figure 9: CST on the fx74130wp1 Aerofoil

Name of the Aerofoil	W	Values
fx74130wp1	W1	0.1680515252
	W2	0.2385629706
	W3	0.4031322334
	W4	0.2507703459
	W5	-0.0887443552
	W6	-0.0014255633
	W7	-0.2160149988
	W8	0.0982174203

Table 9: Weighted Coefficients

2.10 fx74130wp2

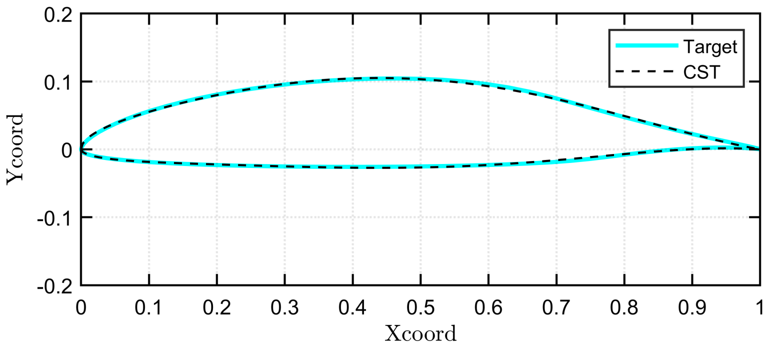

Figure 10: CST on the fx74130wp2 Aerofoil

Name of the Aerofoil	W	Values
fx74130wp2	W1	0.1668275455
	W2	0.2525215324
	W3	0.4115831082
	W4	0.1780203856
	W5	-0.0782736233
	W6	-0.0123109188
	W7	-0.1848804841
	W8	0.0662187593

Table 10: Weighted Coefficients

2.11 fx74130wp2mod

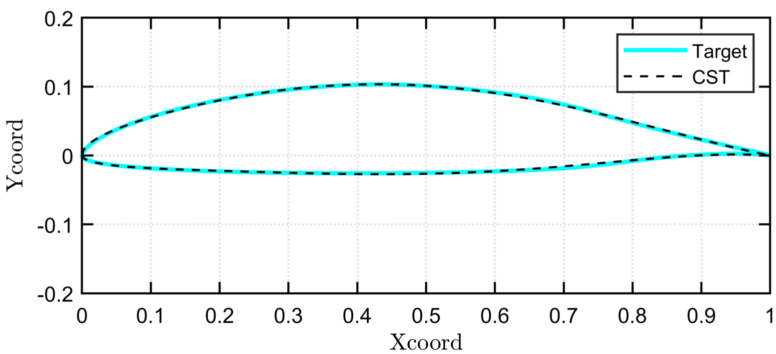

Figure 11: CST on the fx74130wp2mod Aerofoil

Name of the Aerofoil	W	Values
fx74130wp2mod	W1	0.1644785622
	W2	0.2659433403
	W3	0.3760310765
	W4	0.1990371114
	W5	-0.0778953604
	W6	-0.0139819490
	W7	-0.1815641352
	W8	0.0648338067

Table 11: Weighted Coefficients

2.12 fx75141

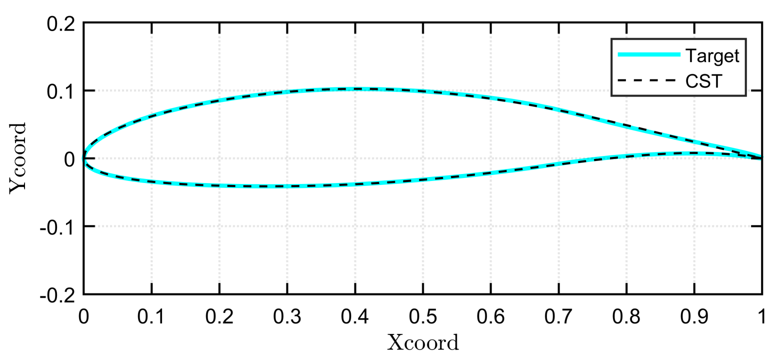

Figure 12: CST on the fx75141 Aerofoil

Name of the Aerofoil	W	Values
fx75141	W1	0.1955340983
	W2	0.2709250312
	W3	0.3319371559
	W4	0.2304357677
	W5	-0.1360338834
	W6	-0.0698735202
	W7	-0.1816524106
	W8	0.1782090620

Table 12: Weighted Coefficients

2.13 fx75193

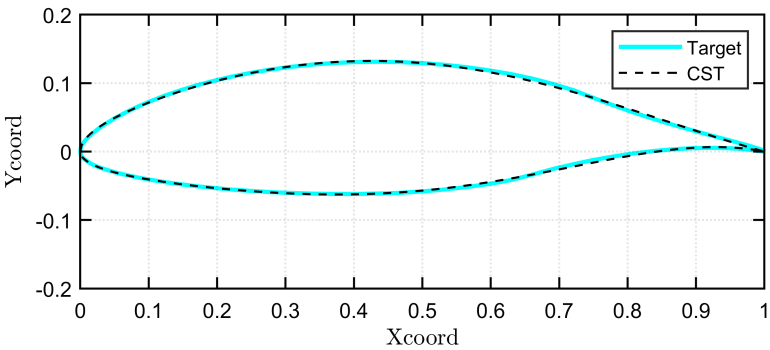

Figure 13: CST on the fx75193 Aerofoil

Name of the Aerofoil	W	Values
fx75193	W1	0.3513955068
	W2	0.4629492267
	W3	0.2703737031
	W4	-0.1452019864
	W5	-0.1179560579
	W6	-0.1179560579
	W7	-0.3290954029
	W8	0.1970838211

Table 13: Weighted Coefficients

2.14 fx76100

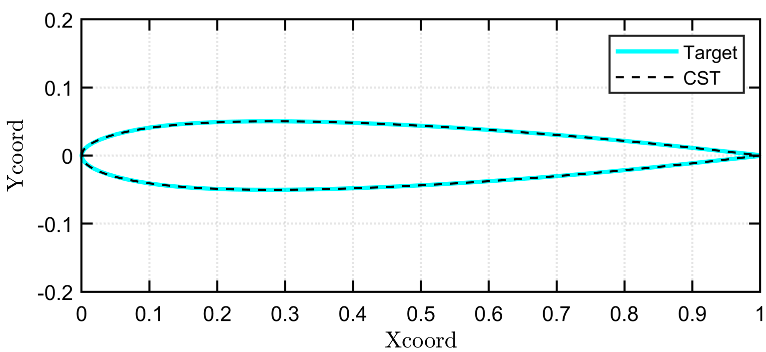

Figure 14: CST on the fx76100 Aerofoil

Name of the Aerofoil	W	Values
fx76100	W1	0.1501244936
	W2	0.1234408246
	W3	0.1159812423
	W4	0.1221237248
	W5	-0.1514514896
	W6	-0.1219561352
	W7	-0.1168798227
	W8	-0.1219901301

Table 14: Weighted Coefficients

2.15 fx76120

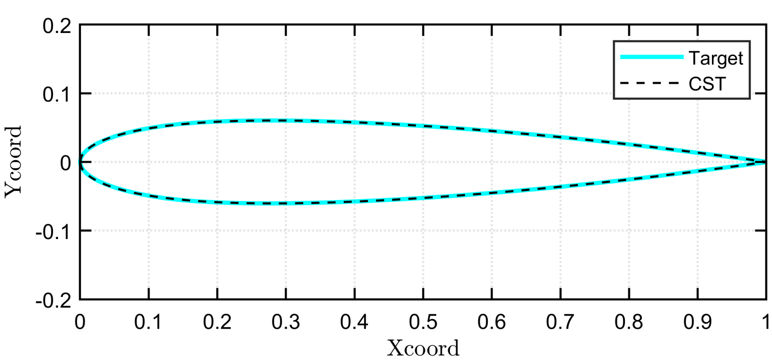

Figure 15: CST on the fx76120 Aerofoil

Name of the Aerofoil	W	Values
fx76120	W1	0.1807069824
	W2	0.1469782904
	W3	0.1406464747
	W4	0.1430230521
	W5	-0.1820253090
	W6	-0.1454053671
	W7	-0.1418527300
	W8	-0.1424241264

Table 15: Weighted Coefficients

3 Conclusion

The Class shape Transformation technique (CST) was carried out on 15 aerofoils using MATLAB. Further, the weighted coefficients, i.e., the coefficients of the eighth-order Bernstein polynomial, were calculated for each aerofoil. There is a minimal margin of error between the original and the obtained aerofoils. Extremely accurate results have been obtained from the mathematical model.

4 References

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