# A simulation to verify the normality of metric spaces

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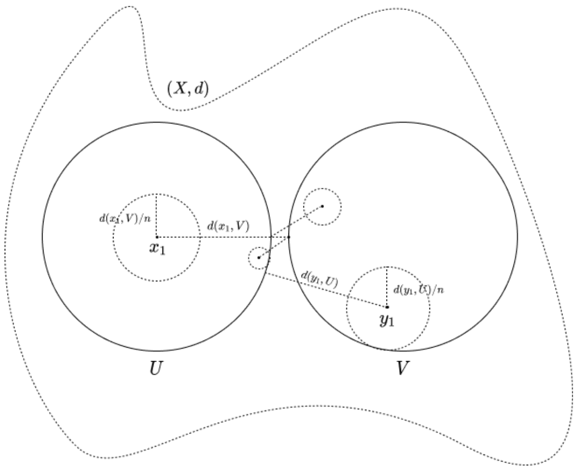
**Definition 0.1.** (Normal Space): A topological space $(X, \tau)$ is said to be a normal space if for any two closed sets $U, V$ such that $U \cap V = \emptyset$ there exist open sets $A, B$ such that $A \cap B = \emptyset$ and $U \subseteq A$ and $V \subseteq B$.

**Proposition 0.1.** All metric spaces are normal.

*Proof.* Let $(X, d)$ be a metric space. Consider two closed sets $U, V$ such that $U \cap V = \emptyset$. For an appropriate choice of $n > 1 \in \mathbb{N}$ and each $x \in U$ let $U_x = B(x, d(x, V)/n)$. Similary define $V_y$ for each $y \in V$. Now $\bigcup_{x \in U} U_x$ is an open set containing $U$ and $\bigcup_{y \in V} V_x$ is an open set containing $V$. Lastly, $\bigcup_{x \in U} U_x \bigcap \bigcup_{y \in V} V_x = \emptyset$ grants that $(X, d)$ is a normal space.

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