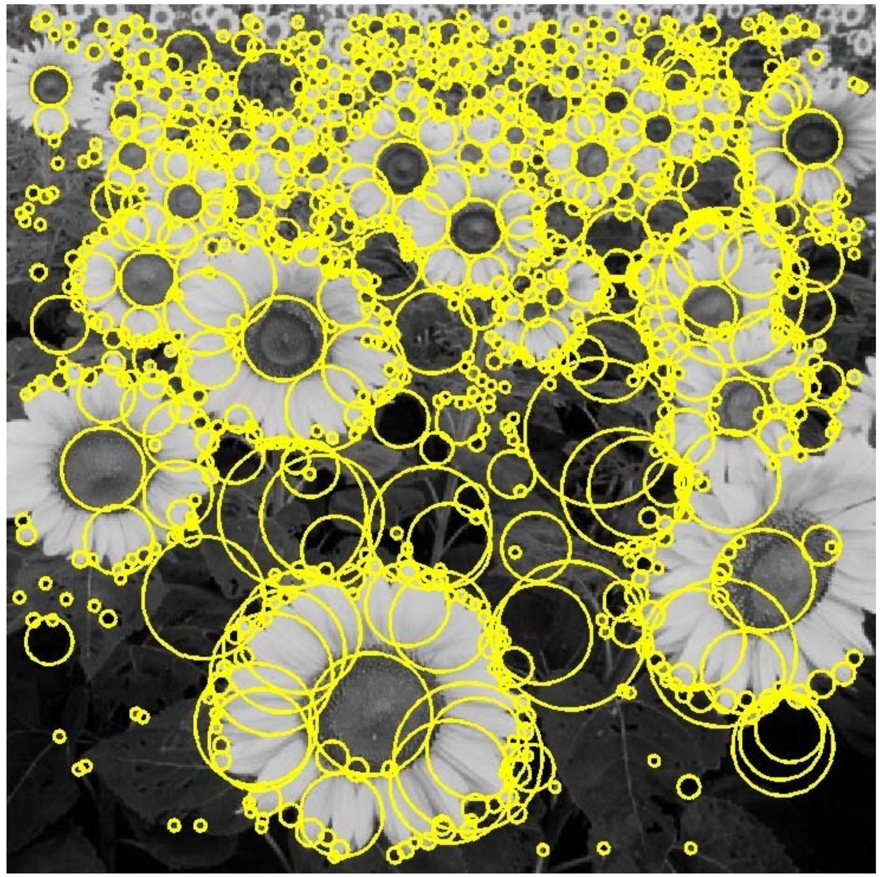
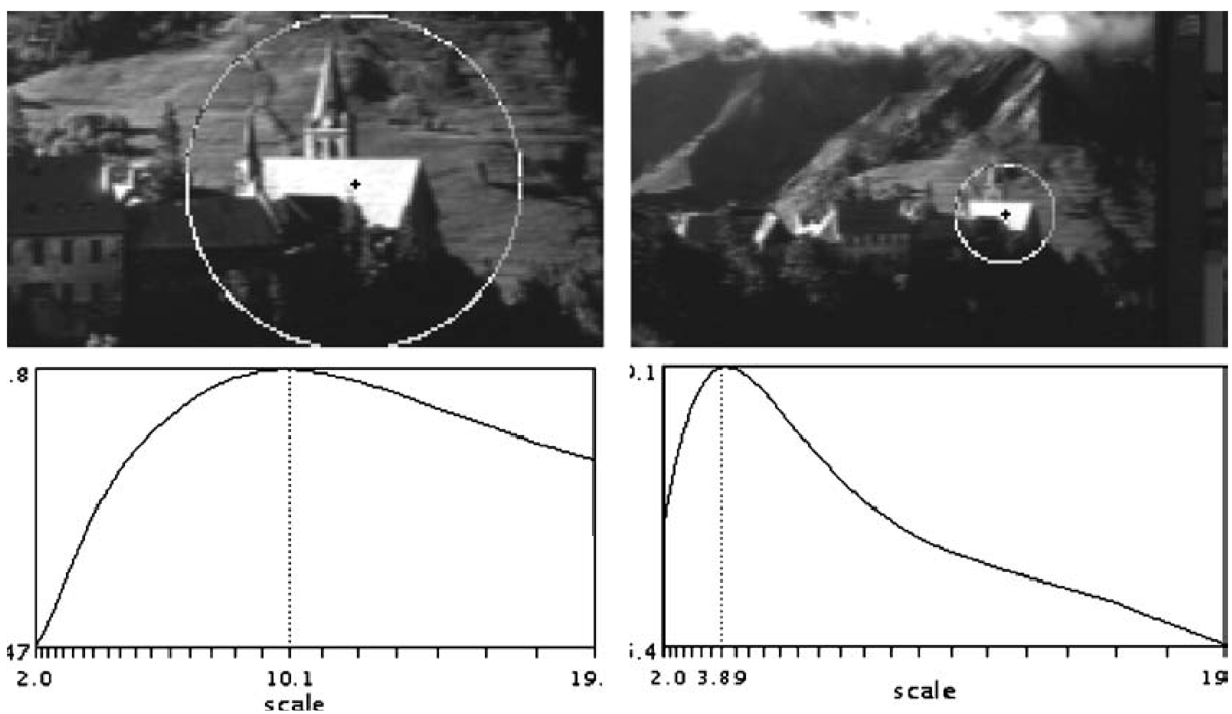
Blob detection



Achieving scale covariance

- Goal: independently detect corresponding regions in scaled versions of the same image
- Need *scale selection* mechanism for finding characteristic region size that is *covariant* with the image transformation

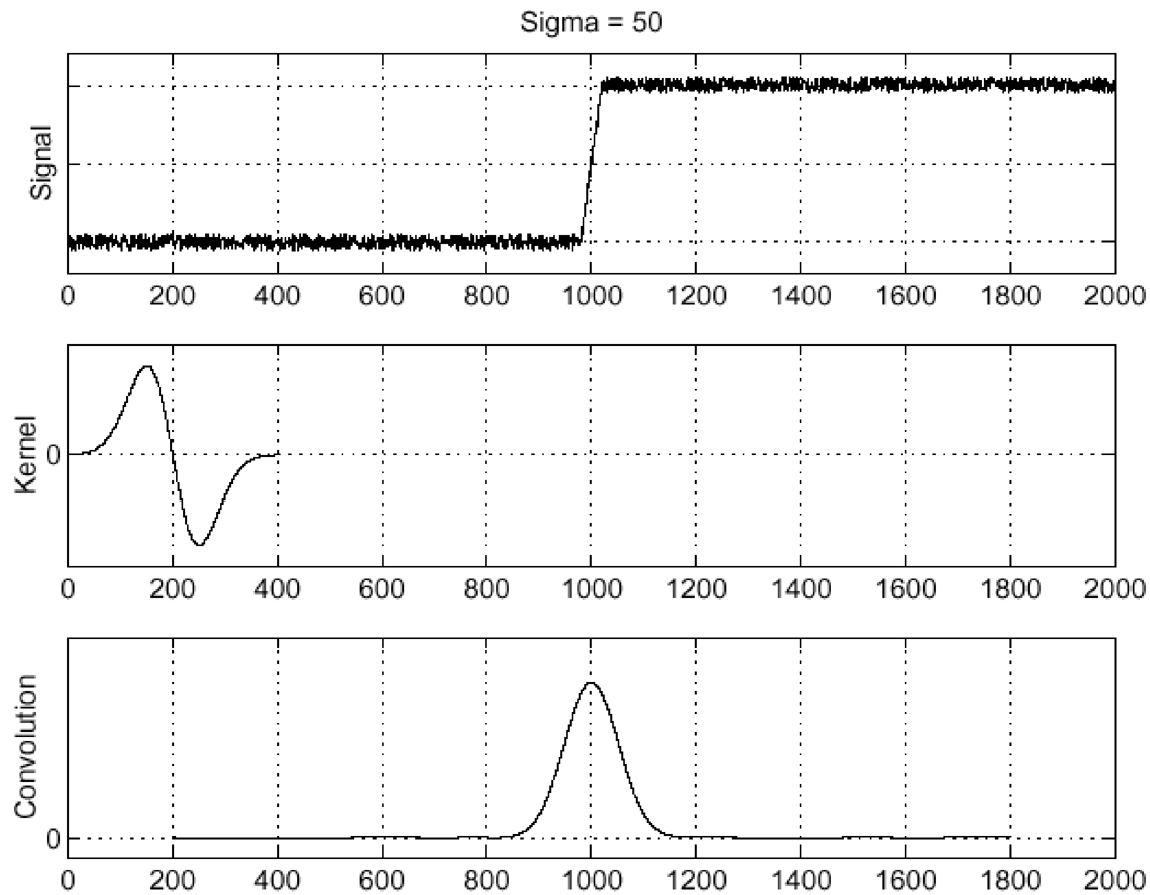


Recall: Edge detection

f

$\frac{d}{dx} g$

$f * \frac{d}{dx} g$



Edge

Derivative of Gaussian

Edge = maximum of derivative

Source: S. Seitz

Edge detection, Take 2

$$f$$

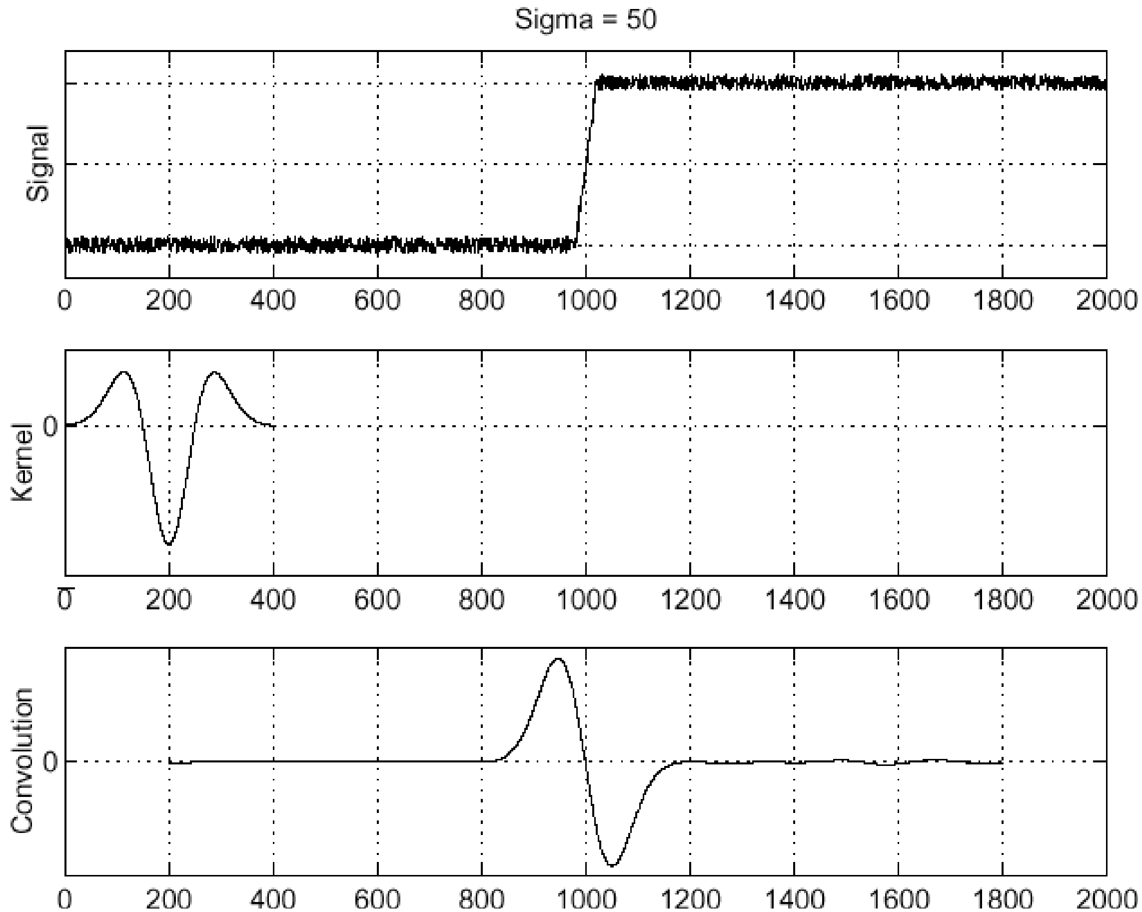
Edge

$$\frac{d^2}{dx^2} g$$

Second derivative of Gaussian (Laplacian)

$$f * \frac{d^2}{dx^2} g$$

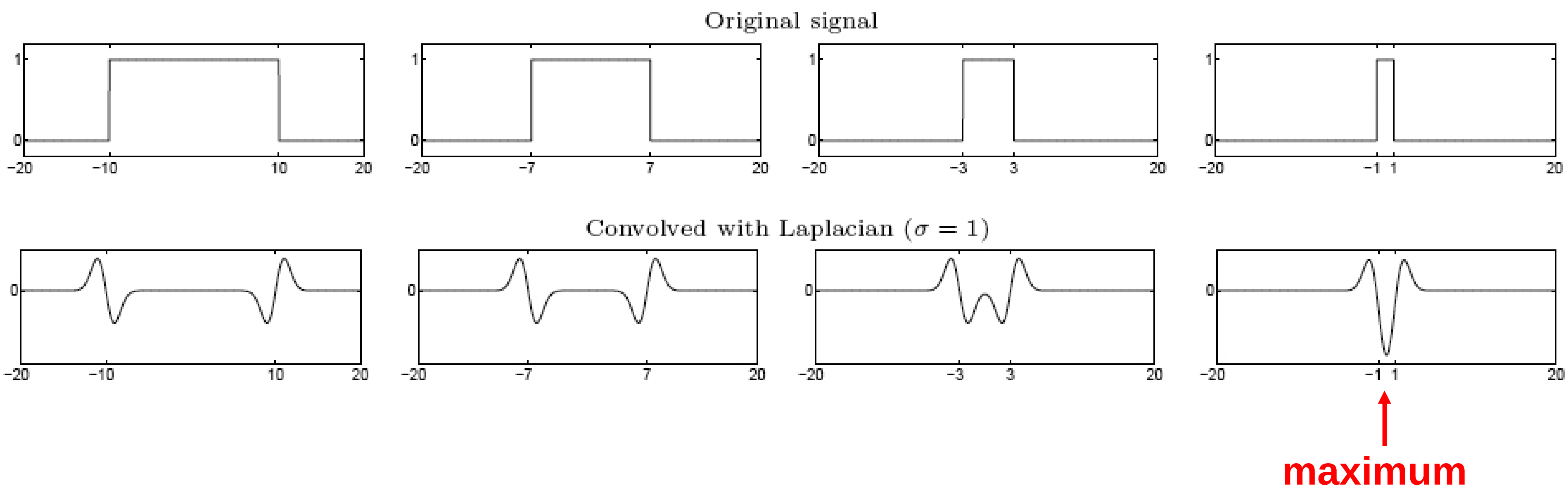
Edge = zero crossing of second derivative



Source: S. Seitz

From edges to blobs

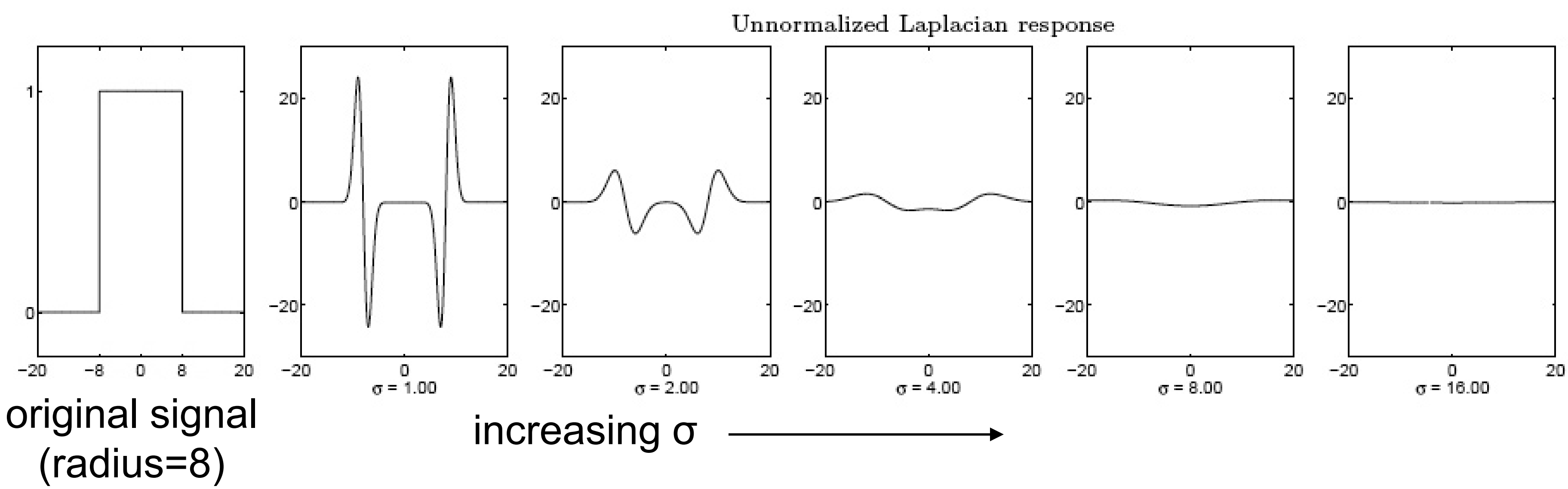
- Edge = ripple
- Blob = superposition of two ripples



Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

Scale selection

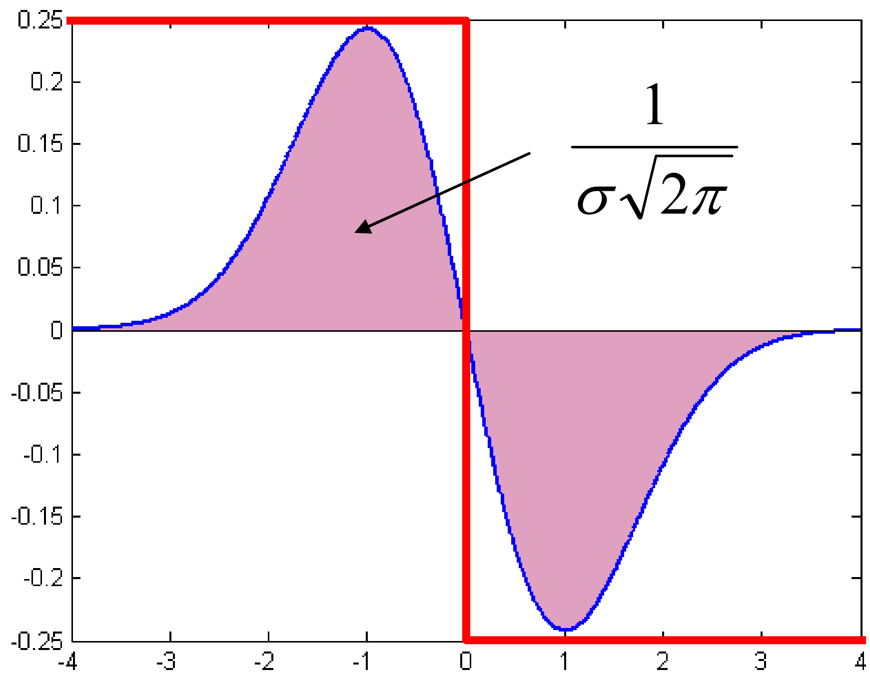
- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



Why does this happen?

Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



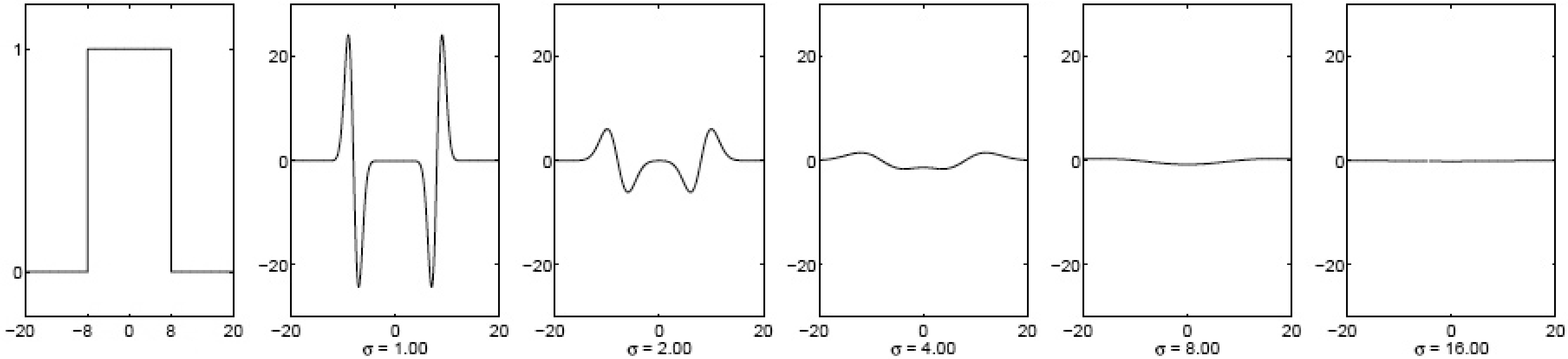
Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

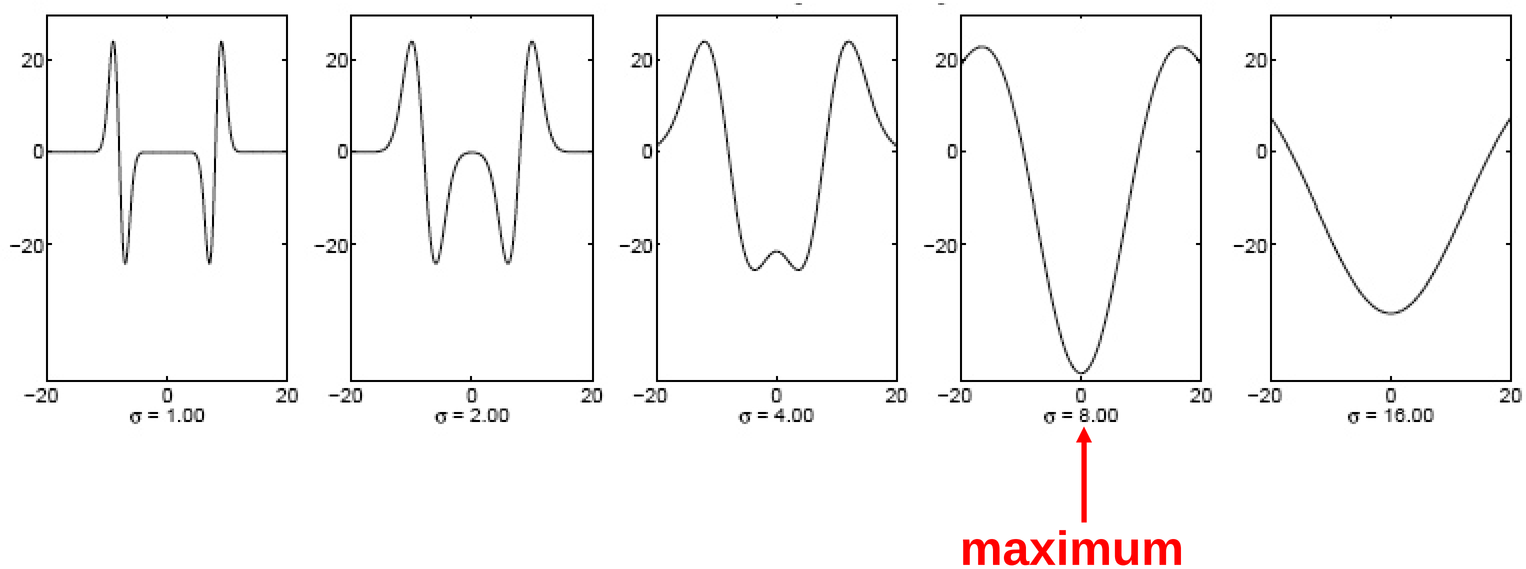
Effect of scale normalization

Original signal

Unnormalized Laplacian response

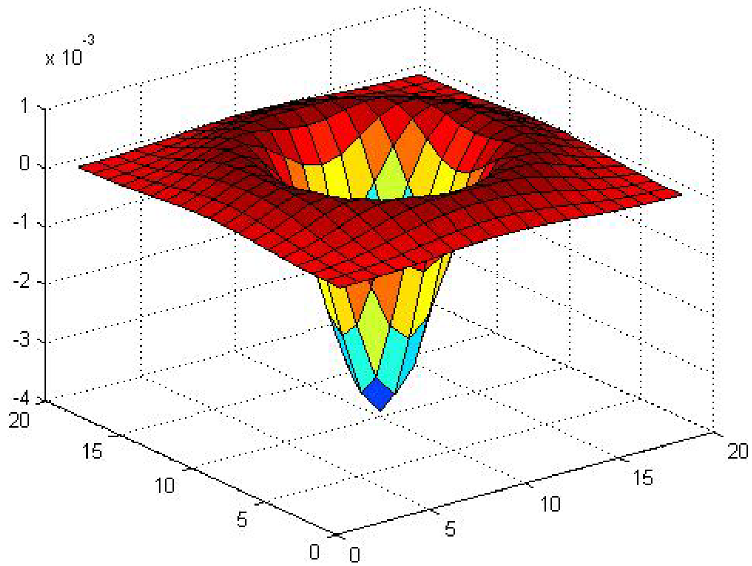


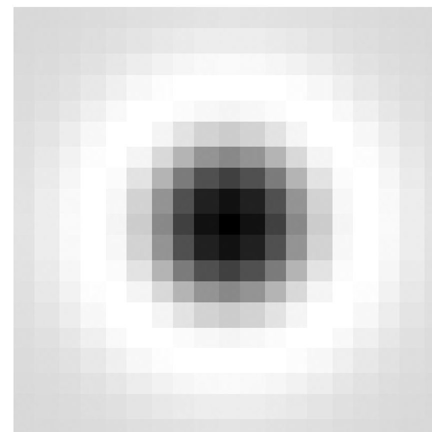
Scale-normalized Laplacian response



Blob detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

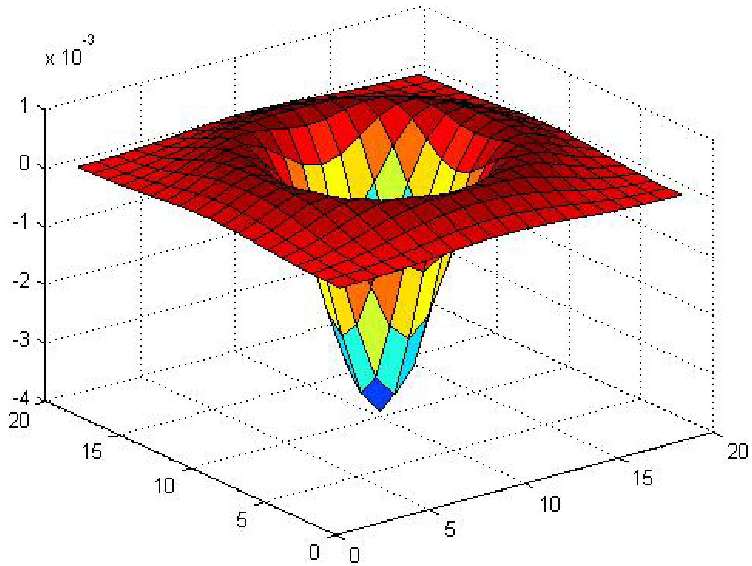


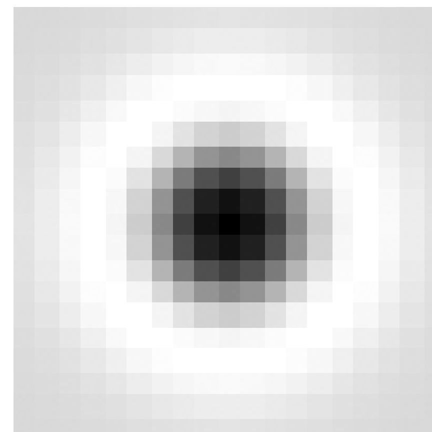


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Blob detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

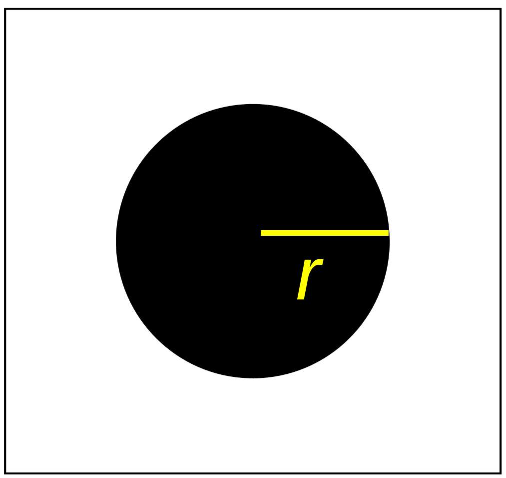




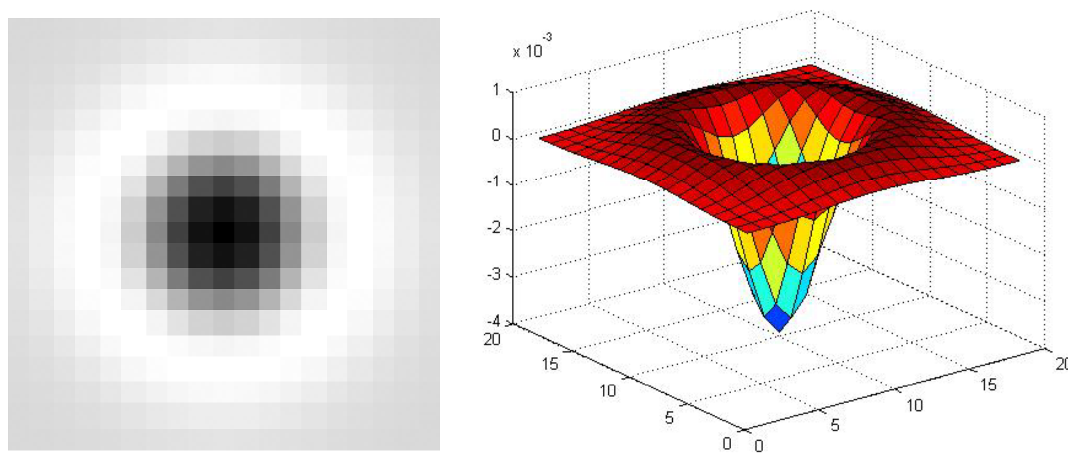
Scale-normalized: $$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

Scale selection

- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?



image



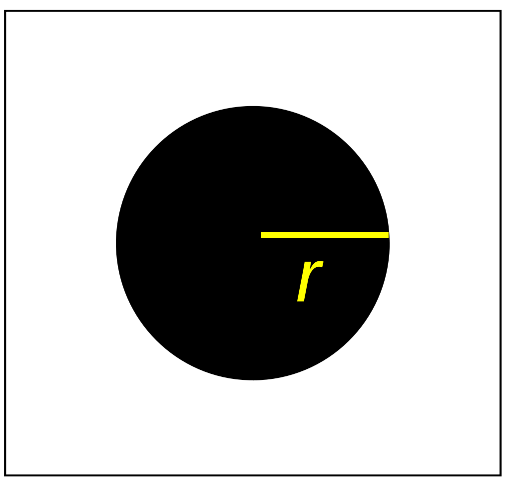
Laplacian

Scale selection

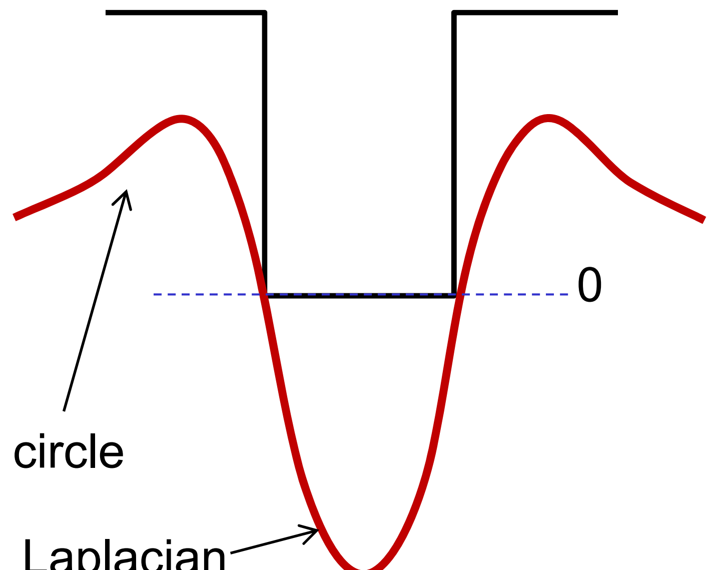
- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?
- To get maximum response, the zeros of the Laplacian have to be aligned with the circle
- The Laplacian is given by (up to scale):

$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2}$$

- Therefore, the maximum response occurs at $\sigma = r/\sqrt{2}$.

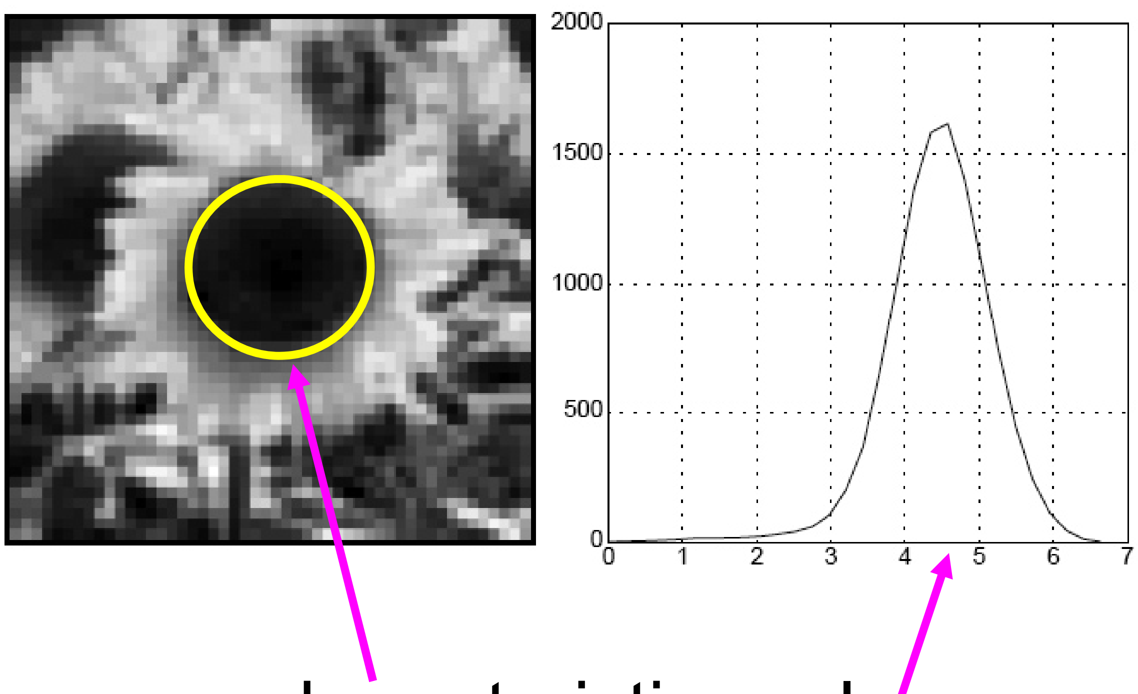


image



Characteristic scale

- We define the characteristic scale of a blob as the scale that produces peak of Laplacian response in the blob center



characteristic scale

T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#) *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

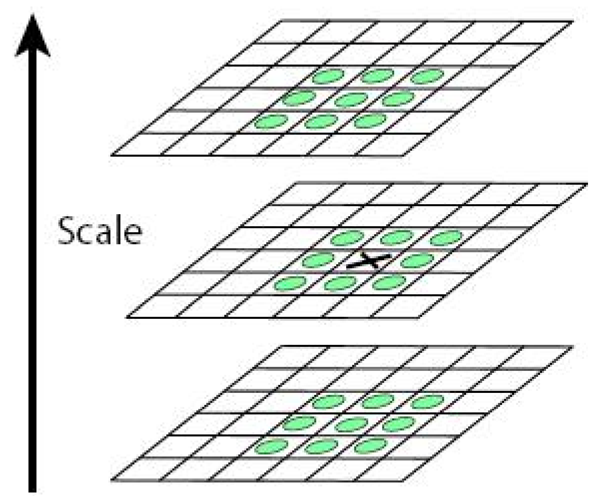
Scale-space blob detector: Example

Scale-space blob detector: Example

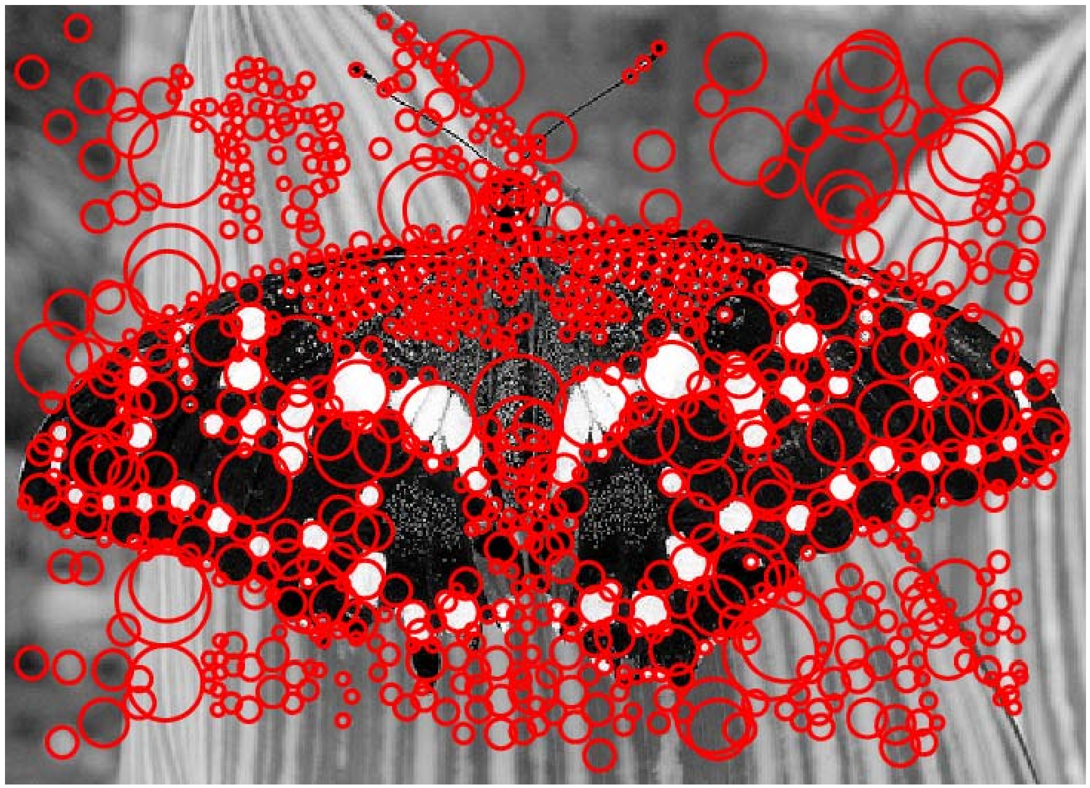
sigma = 11.9912

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Scale-space blob detector: Example



Efficient implementation

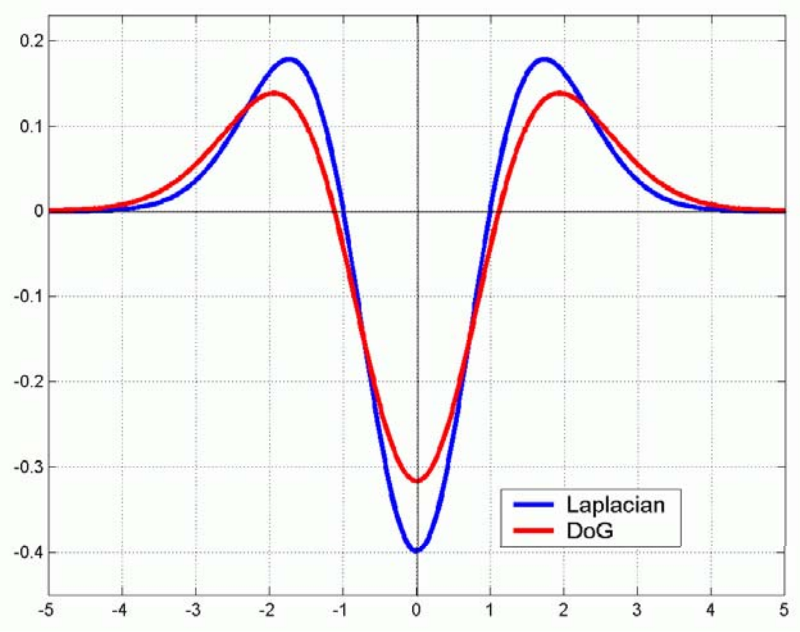
Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

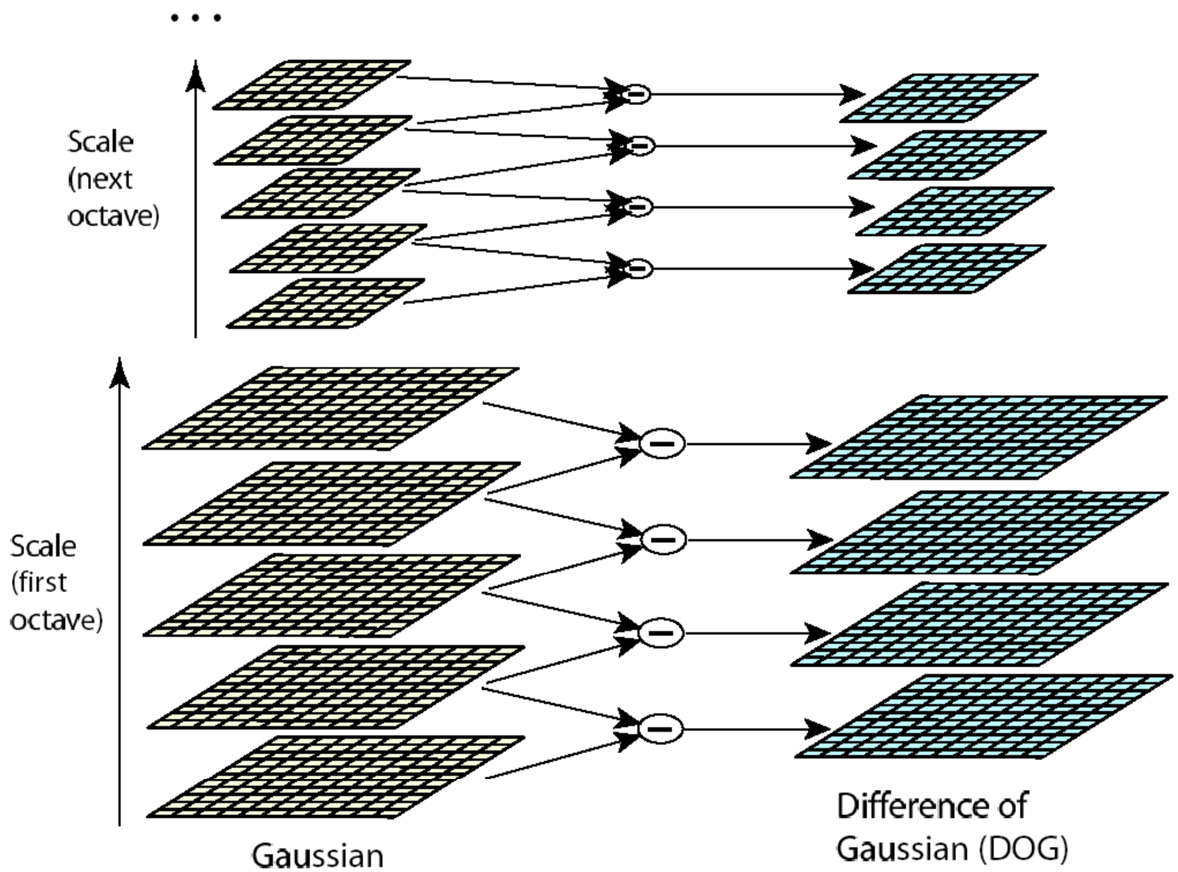
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



Efficient implementation



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Invariance and covariance properties

- Laplacian (blob) response is *invariant* w.r.t. rotation and scaling
- Blob location and scale is *covariant* w.r.t. rotation and scaling
- What about intensity change?

Achieving affine covariance

- Affine transformation approximates viewpoint changes for roughly planar objects and roughly orthographic cameras

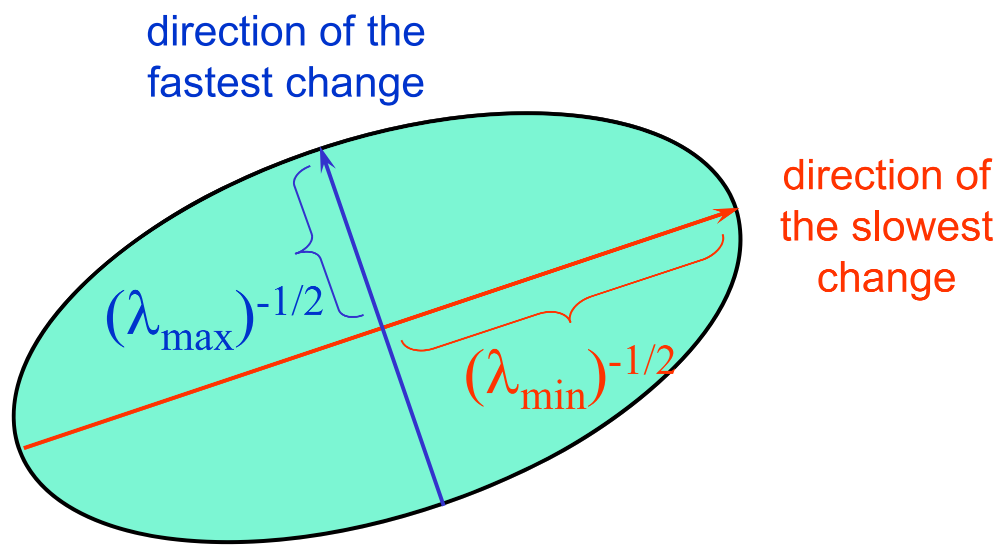
Achieving affine covariance

Consider the second moment matrix of the window containing the blob:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

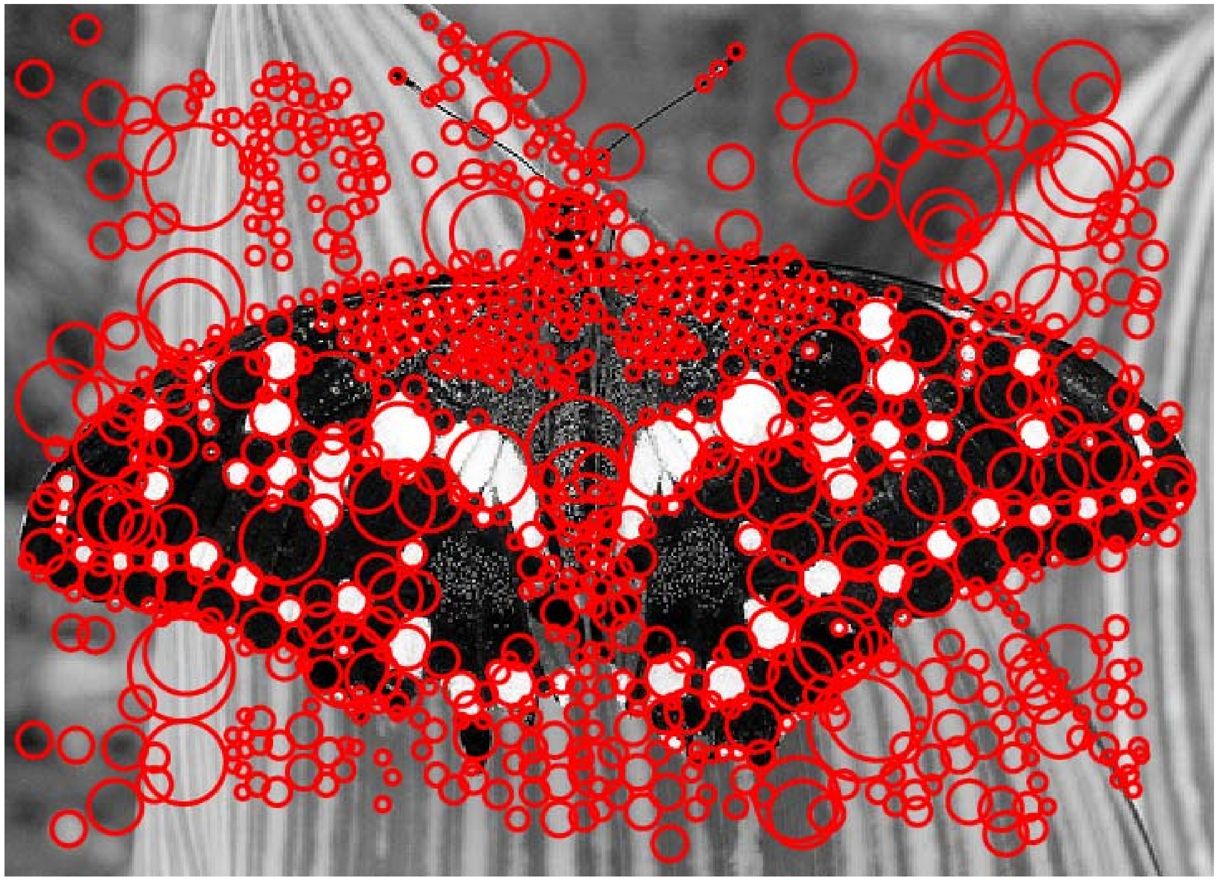
Recall:

$$[u \;\; v] \;\; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



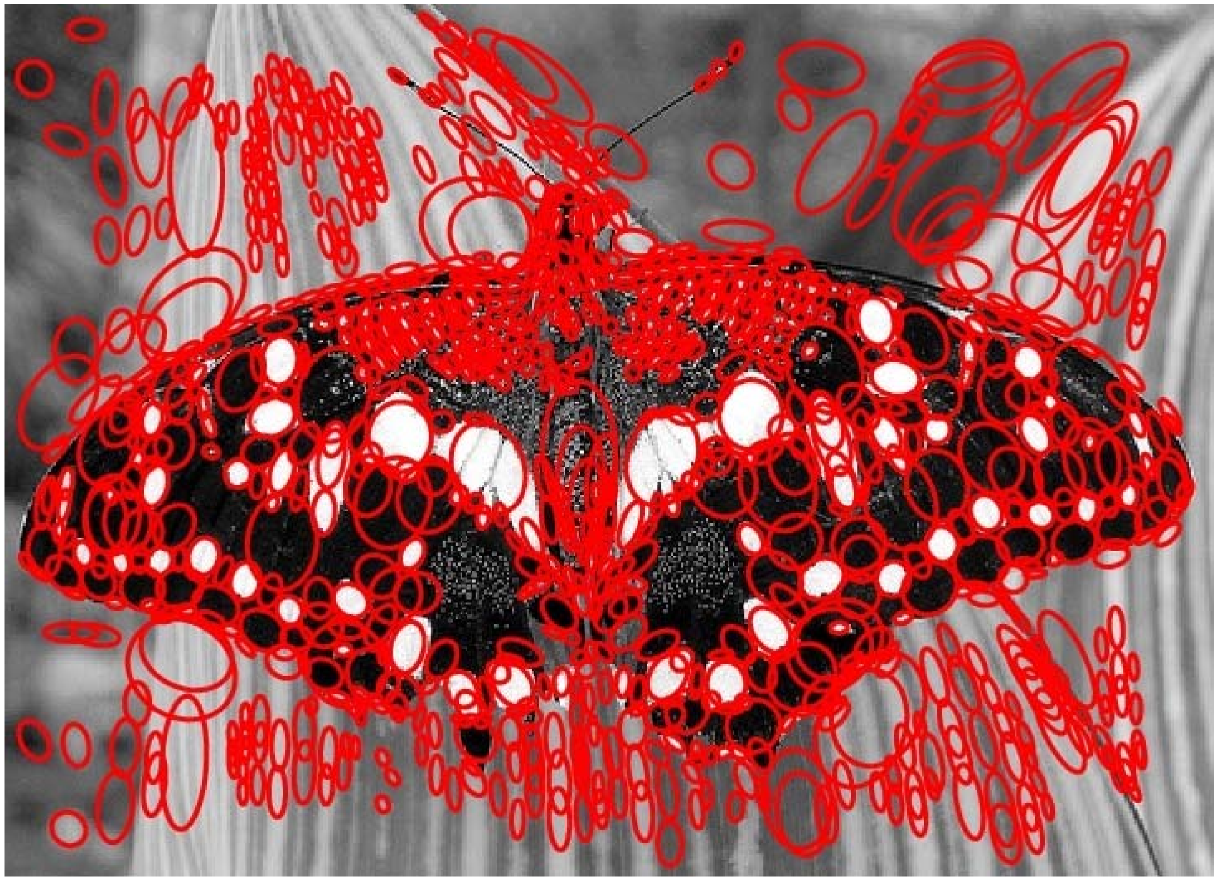
This ellipse visualizes the “characteristic shape” of the window

Affine adaptation example



Scale-invariant regions (blobs)

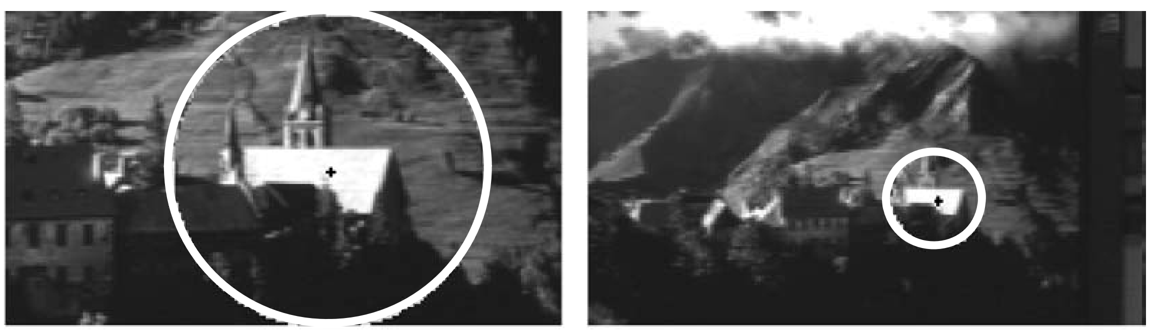
Affine adaptation example

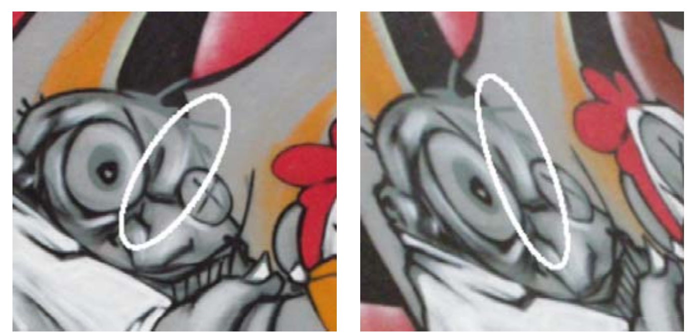


Affine-adapted blobs

From covariant detection to invariant description

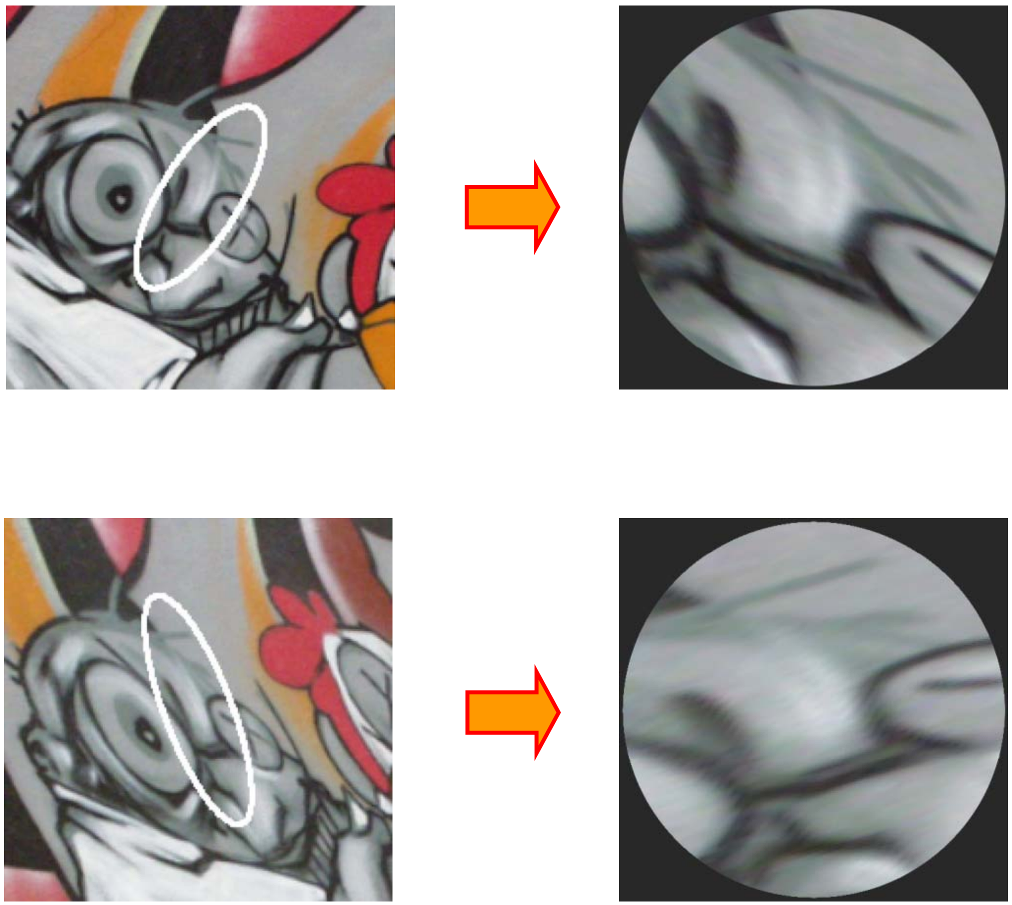
- Geometrically transformed versions of the same neighborhood will give rise to regions that are related by the same transformation
- What to do if we want to compare the appearance of these image regions?
  - *Normalization*: transform these regions into same-size circles





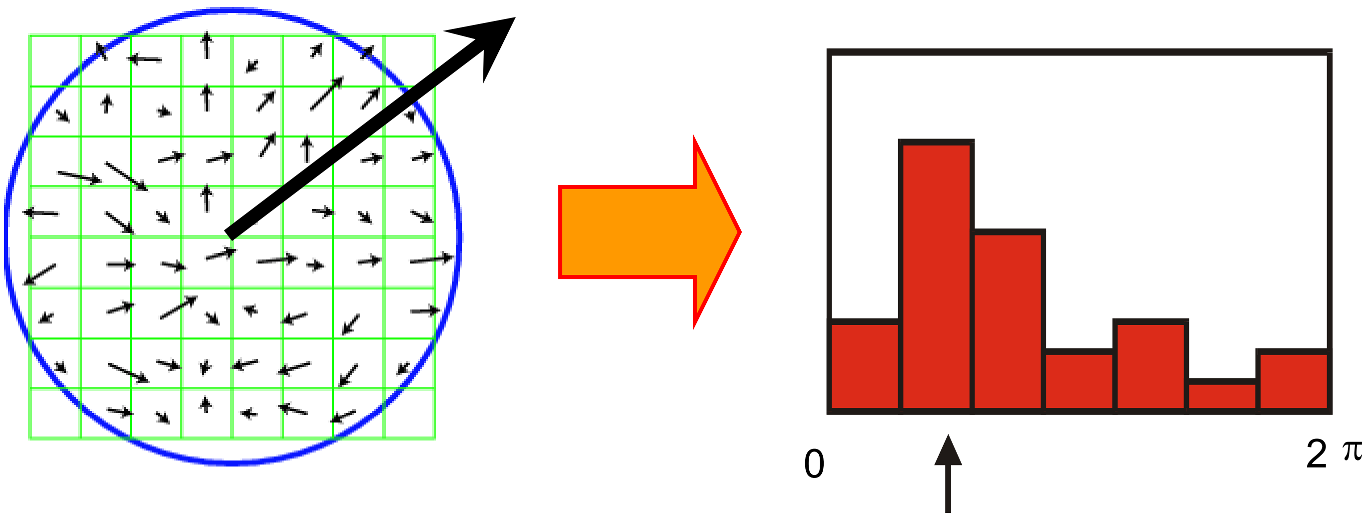
Affine normalization

- Problem: There is no unique transformation from an ellipse to a unit circle
  - We can rotate or flip a unit circle, and it still stays a unit circle

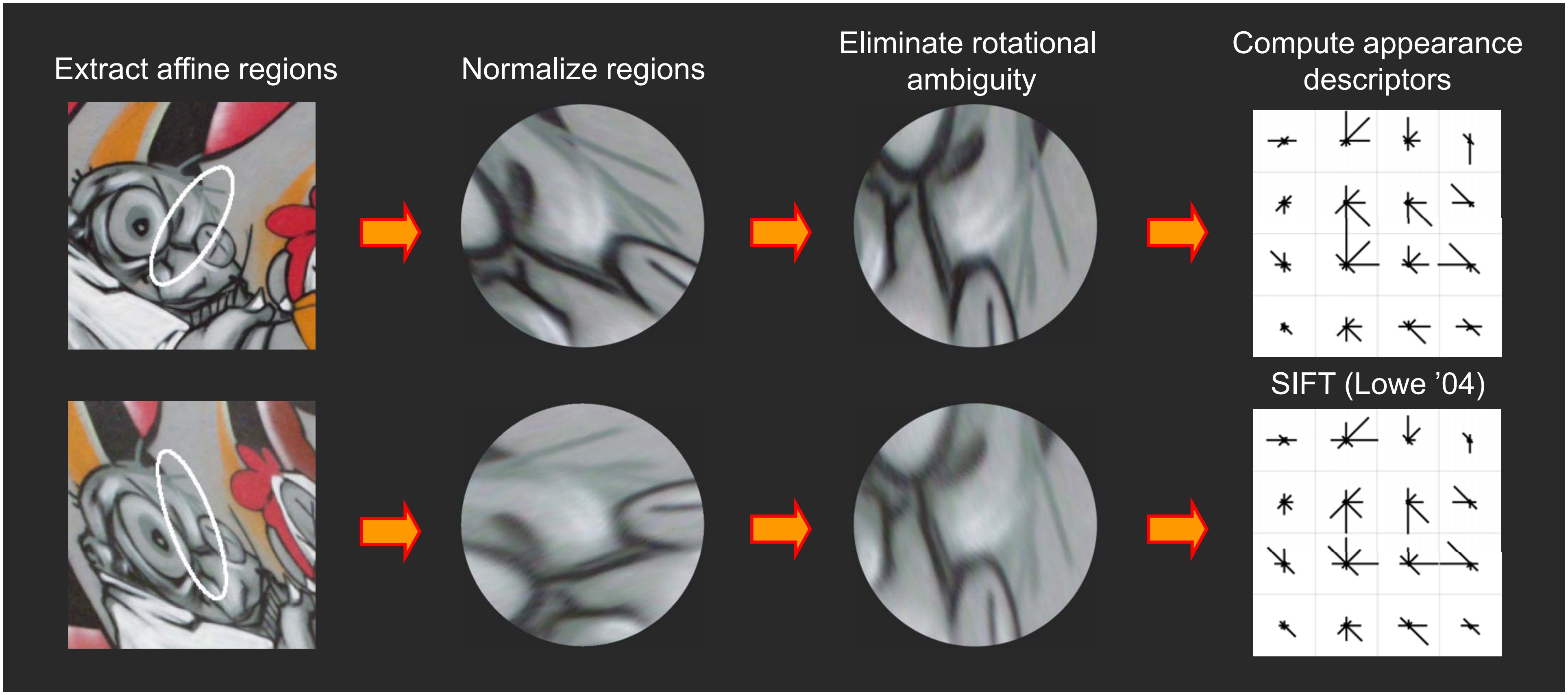


Eliminating rotation ambiguity

- To assign a unique orientation to circular image windows:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram



From covariant regions to invariant features



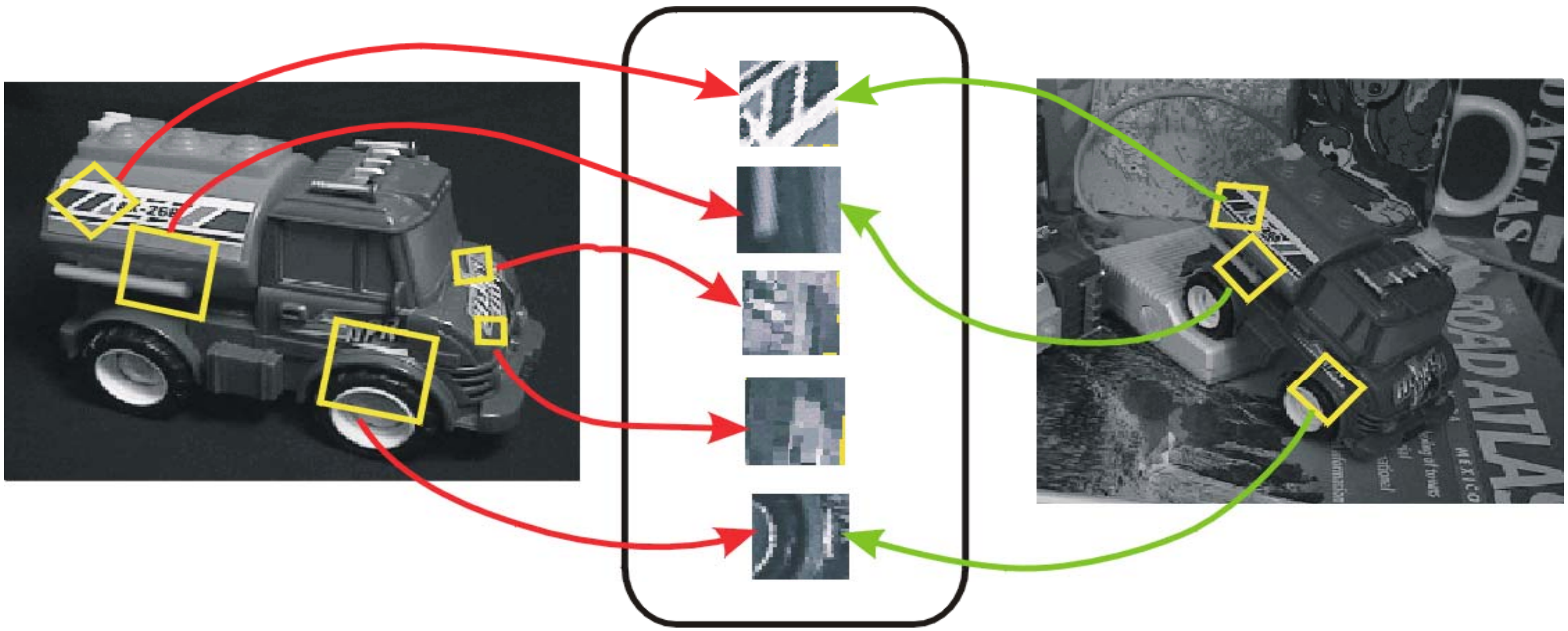
Invariance vs. covariance

Invariance:

- features(transform(image)) = features(image)

Covariance:

- features(transform(image)) = transform(features(image))



Covariant detection => invariant description