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Bmat 2416

18/3/25 **Assignment 8**

Exercise 2.1: 2. Given system : $x_1 + x_2 = 2$
$\alpha x_1 + x_2 = 2 + \alpha$

Augmented matrix = $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ \alpha & 1 & 2+\alpha \end{array}\right]$

Performing Naive-Gaussian : R_1 unchanged
$R_2 \rightarrow R_2 - \alpha R_1$

$\therefore$ Matrix = $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1-\alpha & 2-\alpha \end{array}\right]$

Now, the computer gives problems when pivots are zero (or close to 0)
If $\alpha = 1$, pivot of the second column becomes 0, and thus during back substitution we will be dividing by 0, which is problematic

$\therefore$ For $\underline{\underline{\alpha \approx 1}}$, ~~computer fails to~~ Gaussian method fails due to division by zero and Numeric instability.

4. System : $0.1036\, x_1 + 0.2122\, x_2 = 0.7381$
$0.2081\, x_1 + 0.4247\, x_2 = 0.9327$

} Augmented matrix = $\left[\begin{array}{cc|c} 0.1036 & 0.2122 & 0.7381 \\ 0.2081 & 0.4247 & 0.9327 \end{array}\right]$

4 significant digits :

m = Multiplier = $\dfrac{0.2081}{0.1036}$ = 2.008687 .... $\simeq$ 2.008 [←]

$R_2 \rightarrow R_2 - mR_1 \Rightarrow$ $0.2081 - (0.1036 \times 2.008) \simeq 0.0001$
$0.4247 - (0.2122 \times 2.008) \simeq -0.0016$
$0.9327 - (0.7381 \times 2.008) \simeq -0.5493$

$\therefore$ Matrix = $\left[\begin{array}{cc|c} 0.1036 & 0.2122 & 0.7381 \\ 0.0001 & -0.0016 & -0.5493 \end{array}\right]$

Backsubstitution $\Rightarrow$ $x_2 = \dfrac{-0.5493}{-0.0016} \simeq 343.3$ [keeping only 4 significant digits]

$0.1036\, x_1 + 0.2122(343.3) = 0.7381$

$\Rightarrow x_1 = \dfrac{-72.12}{0.1036} \simeq -696.4$

8 significant digits :

$$m = \frac{0.2081}{0.1036} \approx 2.0086873$$

$R_2 \to R_2 - mR_1 \Rightarrow 0.2081 - (0.1036 \times 2.0086873) \approx 0.0000182$

$0.4277 - (0.2122 \times 2.0086873) \approx -0.0015063$

$0.9327 - (0.7381 \times 2.0086873) \approx -0.549509$

$$\text{New matrix} = \left[\begin{array}{cc|c} 0.1036 & 0.2122 & 0.7381 \\ 0.0000182 & -0.0015063 & -0.549509 \end{array}\right]$$

Back substitution : $x_2 = \frac{-0.549509}{-0.0015063} = 364.84031$

$0.1036x_1 + 0.2122(364.8403) = 0.7381$

$\Rightarrow x_1 = -740.45352$

$\therefore$

	x_1	x_2
4 significant digit	-694.4	343.3
8 significant digit	-740.45352	364.84031

} We can see the big difference in the answers obtained.

Exercise 2.2 :

19. Dominant term in operation count for Gaussian elimination is $\frac{1}{3}n^3$

given execution time for 1 operation = 1 μs = 10^{-6} sec

(This is for forward elimination phase)

$\therefore$ Time for n-sized system = $T(n) = \frac{1}{3}n^3 \times 10^{-6}$ sec

Cost = $C(n) = T(n) \times 500.00 = \frac{500}{3} n^3 \times 10^{-6}$ \$

[we ignore the back substitution phase as its dominant term n^2 is negligible when compared to $\frac{n^3}{3}$, makes calculation easier]

n	10	100	10^3	10^4
Time	$\frac{1}{3} \times 10^{-3}$ sec	$\frac{1}{3}$ sec	$\frac{1}{3} \times 10^3$ sec $\simeq$ 5.56 min	$\frac{1}{3} \times 10^6$ sec $\approx$ 3.86 days
Cost (approx)	0.005 cents	5 cents	46.30 \$	46296.30 \$

PTO

d.

$$A = (a_{ij}) = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{bmatrix}, \quad l = [1, 2, 3, 4]. \Rightarrow \text{index row}$$

$S = [3, 3, 6, 6] \longrightarrow$ Max absolute value row

Scaled values : $\begin{bmatrix} 1/3 \\ 0/3 \\ 3/6 \\ 0/6 \end{bmatrix} = \begin{bmatrix} 0.33\ldots \\ 0 \\ 0.5 \\ 0 \end{bmatrix} \Rightarrow$ Row 3 is pivot $\Rightarrow l = [3, 1, 2, 4]$

$\therefore R_1 \longrightarrow R_1 - \frac{1}{3}R_3 \Rightarrow$ Matrix is now $\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{bmatrix}$

Scaled values : $\begin{bmatrix} 1/3 \\ 1/3 \\ 2/6 \end{bmatrix} = \begin{bmatrix} 0.33\ldots \\ 0.33\ldots \\ 0.33\ldots \end{bmatrix} \Rightarrow$ Row 1 is pivot $\Rightarrow l = [3, 1, 2, 4]$

$R_2 \rightarrow R_2 - R_1$ & $R_4 \rightarrow R_4 - 2R_1 \Rightarrow$ Matrix is now $\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 \\ 3 & -3 & 0 & 6 \\ 0 & 0 & -2 & -2 \end{bmatrix}$

Matrix is ready for back substitution