31/3/25 Assignment 9

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Chapter 3

Exercise 3.1 : 8. $n > \dfrac{\log(b-a) - \log(2\varepsilon)}{\log 2}$

$b = 1$
$a = 0.1$
$\varepsilon = \frac{1}{2} \times 10^{-8}$

$\therefore n \geq \dfrac{\log(0.9) - \log(10^{-8})}{\log 2} = 26.423..$

$\therefore$ Minimum number of steps = 27

Exercise 3.2 : 35. Modified Newton's method : $x_{n+1} = x_n - m\dfrac{f(x_n)}{f'(x_n)}$

Let r be a root of $f(x)$ with multiplicity m

$\Rightarrow f(x) = (x-r)^m g(x)$

$\Rightarrow f'(x) = m(x-r)^{m-1} g(x) + (x-r)^m g'(x)$

$= (x-r)^{m-1}\left[(x-r)g'(x) + mg(x)\right]$

$\varepsilon_{n+1} = (x_{n+1} - r) = \overbrace{(x_n - r)}^{\varepsilon_n} - m\dfrac{f(x_n)}{f'(x_n)}$

$\Rightarrow \varepsilon_{n+1} = \varepsilon_n - m\dfrac{(x_n-r)^m g(x_n)}{(x_n-r)^{m-1}\left[(x_n-r)g'(x_n) + mg(x_n)\right]}$

$= \varepsilon_n\left[1 - \dfrac{m\,g(x_n)}{mg(x_n) + (x_n-r)g'(x_n)}\right]$

Now for x_n in neighbourhood of r, $g(x_n) \approx g(r) + (x_n - r)g'(r)$

$\therefore mg(x_n) + (x_n-r)g'(x_n) \approx mg(r) + (x_n-r)\left[g'(x_n) + mg'(r)\right]$

$\simeq mg(r)$ [as x_n is close to r, $x_n - r$ is close to 0]

$\therefore 1 - \dfrac{mg(x_n)}{mg(x_n) + (x_n-r)g'(x_n)} = \dfrac{(x_n-r)g'(x_n)}{mg(x_n) + (x_n-r)g'(x_n)} \approx \dfrac{(x_n-r)g'(x_n)}{mg(r)} = \dfrac{\varepsilon_n g'(x_n)}{mg(r)}$

$\therefore \varepsilon_{n+1} \approx \varepsilon_n^2 \cdot C \Rightarrow$ quadratic convergence.

Chapter 4

Exercise 4.1: 6.

x	$f(x)$			
$x_0 = 0$	$f(x_0) = 2$			
		$f[x_0, x_1] = 1$		
$x_1 = 2$	$f(x_1) = 4$		$f[x_0, x_1, x_2] = -3$	
		$f[x_1, x_2] = -8$		$f[x_0, x_1, x_2, x_3] = 4$
$x_2 = 3$	$f(x_2) = -4$		$f[x_1, x_2, x_3] = 17$	
		$f[x_3, x_2] = 43$		
$x_3 = 5$	$f(x_3) = 82$			

$$\therefore\ p(x) = 2 + 1(x-0) + (-3)(x-0)(x-2) + 4(x-0)(x-2)(x-3).$$

Exercise 4.2: 9. Given $f(x) = e^{-x}$, $n = 20$, $[a,b] = [0,2]$

$$f'(x) = -e^{-x}$$

$$f''(x) = e^{-x} \quad \text{\& so on}$$

$$\therefore\ |f^{(n+1)}(x)| = e^{-x} \le 1 \implies M = \max f^{(n+1)}(x) = 1$$

Now by theorem 2, an upper bound on the error is :

$$\varepsilon \le \frac{1}{4(n+1)} M h^{n+1}, \quad h = \frac{b-a}{n} = \frac{2-0}{20} = 10^{-1}$$

$$= \frac{1}{4(21)} \cdot 1 \cdot (10^{-1})^{21}$$

$$\implies \varepsilon \le 1.19 \times 10^{-23}.$$

Exercise 4.3: 1. $f(x+3h) = f(x) + (3h) f'(x) + \frac{(3h)^2}{2!} f''(x) + \cdots$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!} f''(x) + \cdots$$

$$\implies f'(x) = \frac{1}{4h}\left[f(x+3h) - f(x-h)\right] - hf''(x) + \cdots$$

$$\therefore\ \text{Error term} = -hf''(\xi) \text{ for } \xi \in (x-h, x+3h)$$