16/8/25 Assignment 7

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Exercise 1.1 : 3. Actual value = 4.9997
Rounded value = 5.000
Relative error = $\frac{|5.000 - 4.9997|}{|5.000|} = 6 \times 10^{-5}$

Exercise 1.2: 30. $f(x) \approx f(a) + f'(a)(x-a)$ [linear approximation]

$\Rightarrow \cos x \approx \cos a + (x-a)(-\sin a)$

$a = 5\pi/6$

$\Rightarrow \cos x \approx \left(-\frac{\sqrt{3}}{2}\right) + \left(x - \frac{5\pi}{6}\right)\left(\frac{-1}{2}\right)$

$\Rightarrow \cos x \approx -\frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{5\pi}{6}\right)$

Exercise 1.3: 15. In case of underflow, x is replaced by 0

$\therefore$ Actual value = x, Approximate value = 0

$\therefore$ Relative error = $\frac{|x-0|}{|x|} = 1$

Exercise 1.4 : 13. $z \leftarrow \sqrt{x^4+4} - 2$

When we have x near zero, $\sqrt{x^4+4}$ is very close to 2

$\Rightarrow$ While subtracting, there will be a loss in precision

To overcome this, we do the following:

$$z \leftarrow \frac{\left(\sqrt{x^4+4} - 2\right)\left(\sqrt{x^4+4} + 2\right)}{\sqrt{x^4+4} + 2} = \frac{x^4 + 4 - 4}{\sqrt{x^4+4} + 2}$$

$$\Rightarrow z \leftarrow \frac{x^4}{\sqrt{x^4+4} + 2}$$: No more subtraction of close by values