# FIR Filter Design
# Using Kaiser Window

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# 1 Bandpass Filter Design

## 1.1 Un-normalized Discrete Time Filter

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**Specifications:**

- Passband: 100 kHz to 175 kHz,
- Stopband: 0 to 95 kHz and 180 kHz to 300 kHz (since $f_s = 600$ kHz),
- Passband and Stopband tolerance: 0.15,
- Passband and Stopband nature: we do not have a choice of nature.

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## 1.2 Normalized Digital Filter

| Discrete-time frequency $f$ (kHz) | Normalized digital frequency $\omega$ (radians) |
|---|---|
| 0 | 0 |
| 95 | $\omega_{s1} = 0.9948377$ |
| 100 | $\omega_{p1} = 1.0471976$ |
| 175 | $\omega_{p2} = 1.8325957$ |
| 180 | $\omega_{s2} = 1.8849556$ |
| 300 | $\pi$ |

Table 1: Normalizing frequency.

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**Specifications:**

- Passband: 1.0471976 to 1.8325957,
- Stopband: 0 to 0.9948377 and 1.8849556 to $\pi$,
- Passband and Stopband tolerance: 0.15,
- Passband and Stopband nature: we do not have a choice of nature.

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### 1.3 $\mathbf{h_{ideal}[n]}$

The ideal impulse response of the bandpass filter with the specifications can be made by using two ideal low-pass filters having cut-off frequencies in the middle of the transition band:

$$\omega_{c1} = \frac{\omega_{s1} + \omega_{p1}}{2} = 1.0210176,$$
$$\omega_{c2} = \frac{\omega_{p2} + \omega_{s2}}{2} = 1.8587757.$$
$$\mathrm{h_{ideal}[n]} = sinc(\omega_{c2}.n)\frac{\omega_{c2}}{\pi} - sinc(\omega_{c1}.n)\frac{\omega_{c1}}{\pi}$$

### 1.4 Kaiser window

$$A = -20.\log_{10}(\delta) = -20.\log_{10}(0.15) = 16.478175.$$

$$\begin{aligned}\alpha &= 0.1102(A - 8.7)\, for A > 50 \\ &= 0.5842(A-21)^{0.4} + 0.07886(A-21)\, for\, 50 \geq A \geq 21 \\ &= 0\, for\, 20 > A\end{aligned}$$

hence,

$$\alpha = 0.$$
$$\beta = \frac{\alpha}{N} = 0.$$

For $\beta = 0$,

$$I_o(\beta) = 1.$$

Thus, the kaiser window function is simply the rectangular window function from -N to +N.

$$v[k] = \frac{I_o\left(\beta N\sqrt{1-(\frac{k}{N})^2}\right)}{I_o(\beta N)} = 1;\;\; k\,\epsilon\,[-N, +N]$$

Choosing N:

$$2N + 1 \geq 1 + \frac{A - 8}{2.285\Delta\omega_T}$$
$$\Delta\omega_T = \omega_{s1} - \omega_{p1} = \omega_{p2} - \omega_{s2} = 0.0523599$$
$$2N + 1 \geq 71.862675$$
$$N \geq 35.4313375$$

Since N should be a positive integer,

$$N \geq 36.$$

### 1.5 Filter Transfer Function

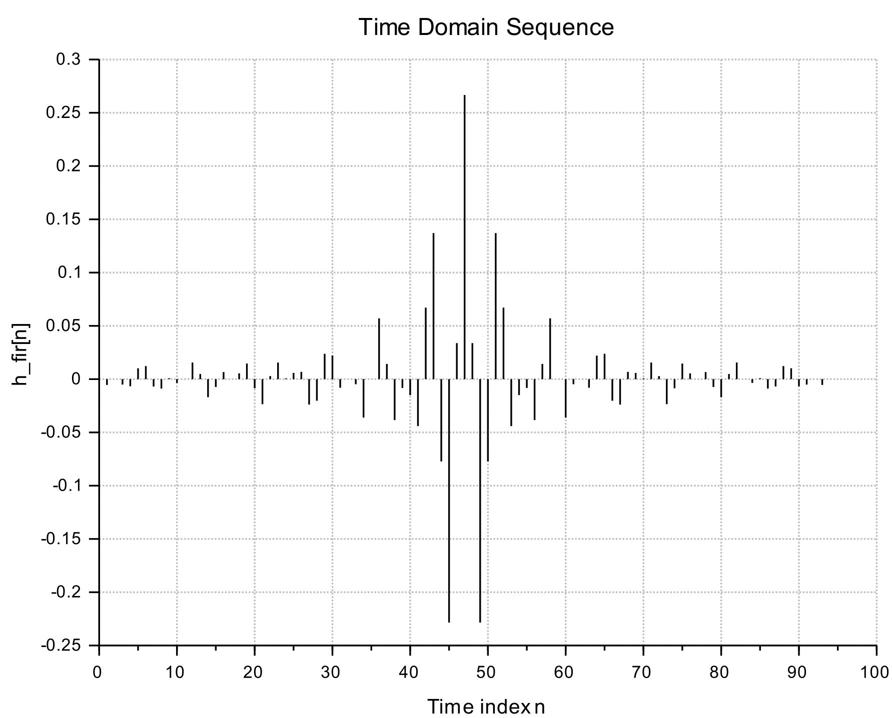
For the exact value of N, we must evaluate whether the magnitude tolerances at the transition band edges are satisfied. For this, we would have to iterate over N and find the magnitude of the FIR filter transfer function at the passband and stopband edges. Shifting the response for causality,

$$\mathrm{h_{fir}[n + N] = h_{ideal}[n].v[n]},$$

$$\mathrm{H_{fir}[z]} = \sum_{n=0}^{2N} \mathrm{h_{fir}[n]z}^{-n}.$$

Note that the window length is 2N + 1 for a given N. The smallest value of N that satisfies all the requirements of the Bandpass filter is:

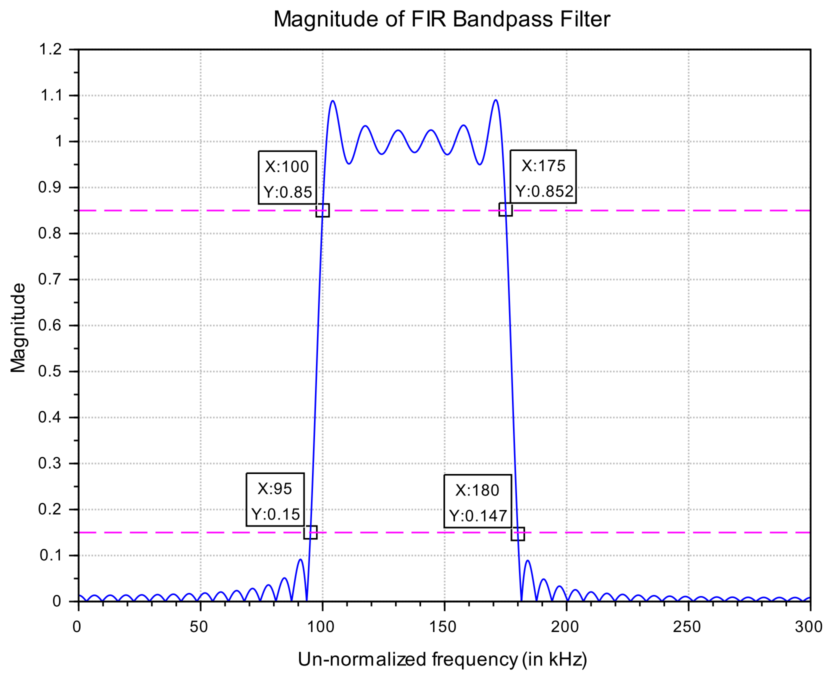
$$N = 46.$$

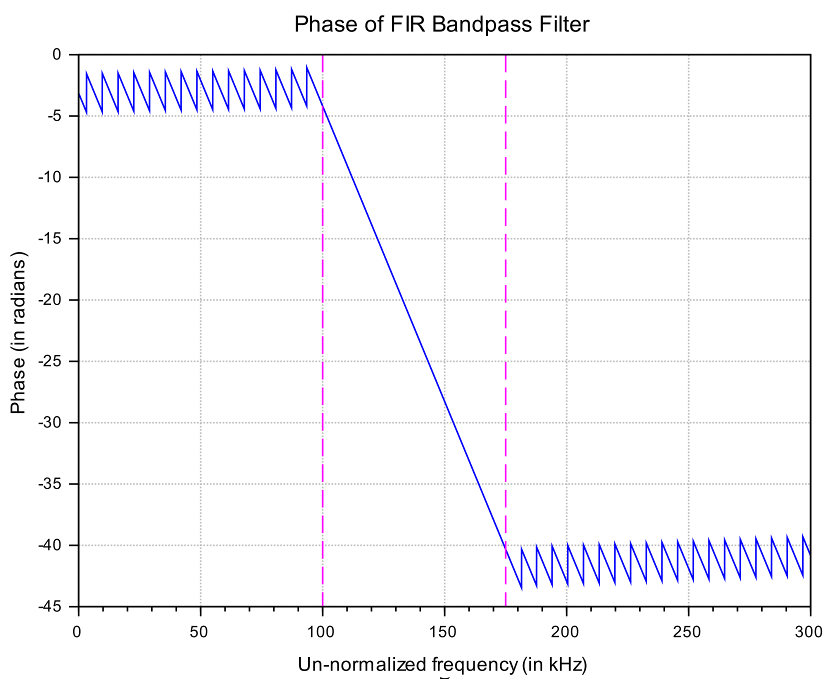


```
       column 1 to 7
-0.0054373   4.250D-17  -0.0050965  -0.0066978   0.0101935   0.0122512  -0.0068916
       column 8 to 14
-0.0088644   0.0009015  -0.0035467   3.990D-17   0.0156175   0.0048196  -0.0169507
       column 15 to 21
-0.0073922   0.0066267  -3.296D-17   0.0054355   0.0146328  -0.0085815  -0.0235215
       column 22 to 28
 0.0028785   0.0155915   0.0007512   0.0058114   0.006819  -0.0238732  -0.0202856
       column 29 to 35
 0.0237848   0.0220786  -0.0080918   4.510D-17  -0.004787  -0.036081   7.980D-17
       column 36 to 42
 0.0570651   0.0142694  -0.0384124  -0.0082725  -0.0150013  -0.0440994   0.0671256
       column 43 to 49
 0.1370772  -0.0772333  -0.2284901   0.033798   0.2666667   0.033798  -0.2284901
       column 50 to 56
-0.0772333   0.1370772   0.0671256  -0.0440994  -0.0150013  -0.0082725  -0.0384124
       column 57 to 63
 0.0142694   0.0570651   7.980D-17  -0.036081  -0.004787   4.510D-17  -0.0080918
       column 64 to 70
 0.0220786   0.0237848  -0.0202856  -0.0238732   0.006819   0.0058114   0.0007512
       column 71 to 77
 0.0155915   0.0028785  -0.0235215  -0.0085815   0.0146328   0.0054355  -3.296D-17
       column 78 to 84
 0.0066267  -0.0073922  -0.0169507   0.0048196   0.0156175   3.990D-17  -0.0035467
       column 85 to 91
 0.0009015  -0.0088644  -0.0068916   0.0122512   0.0101935  -0.0066978  -0.0050965
       column 92 to 93
 4.250D-17  -0.0054373
```

Time index n and column n in the above figures correspond to the coefficient of $z^{-n+1}$

## 1.6 Frequency Response



# 2 Bandstop Filter Design

## 2.1 Un-normalized Discrete Time Filter

**Specifications:**

- Passband: 0 to 85 kHz and 135 kHz to 212.5 kHz (since $f_s = 425$ kHz),
- Stopband: 90 kHz to 130 kHz,
- Passband and Stopband tolerance: 0.15,
- Passband and Stopband nature: we do not have a choice of nature.

## 2.2 Normalized Digital Filter

| Discrete-time frequency $f$ (kHz) | Normalized digital frequency $\omega$ (radians) |
|---|---|
| 0 | 0 |
| 85 | $\omega_{p1} = 1.2566371$ |
| 90 | $\omega_{s1} = 1.3305569$ |
| 130 | $\omega_{s2} = 1.9219155$ |
| 135 | $\omega_{p2} = 1.9958353$ |
| 212.5 | $\pi$ |

Table 2: Normalizing frequency.

**Specifications:**

- Passband: 0 to 1.2566371 and 1.9958353 to $\pi$,
- Stopband: 1.3305569 to 1.9219155,
- Passband and Stopband tolerance: 0.15,
- Passband and Stopband nature: we do not have a choice of nature.

## 2.3 $\mathbf{h_{ideal}[n]}$

The ideal impulse response of the bandstop filter with the specifications can be made by using an all-pass filter and two ideal low-pass filters having cut-off frequencies in the middle of the transition band:

$$\omega_{c1} = \frac{\omega_{p1} + \omega_{s1}}{2} = 1.2935970,$$
$$\omega_{c2} = \frac{\omega_{s2} + \omega_{p2}}{2} = 1.9588754.$$

$$\mathrm{h_{ideal}[n]} = sinc(\pi.n) - sinc(\omega_{c2}.n)\frac{\omega_{c2}}{\pi} + sinc(\omega_{c1}.n)\frac{\omega_{c1}}{\pi}$$

## 2.4 Kaiser window

For the Bandstop filter, we again obtain the shape parameter $\beta = 0$, hence a rectangular window with the bound on N being:

$$N \geq 26.$$

## 2.5 Filter Transfer Function

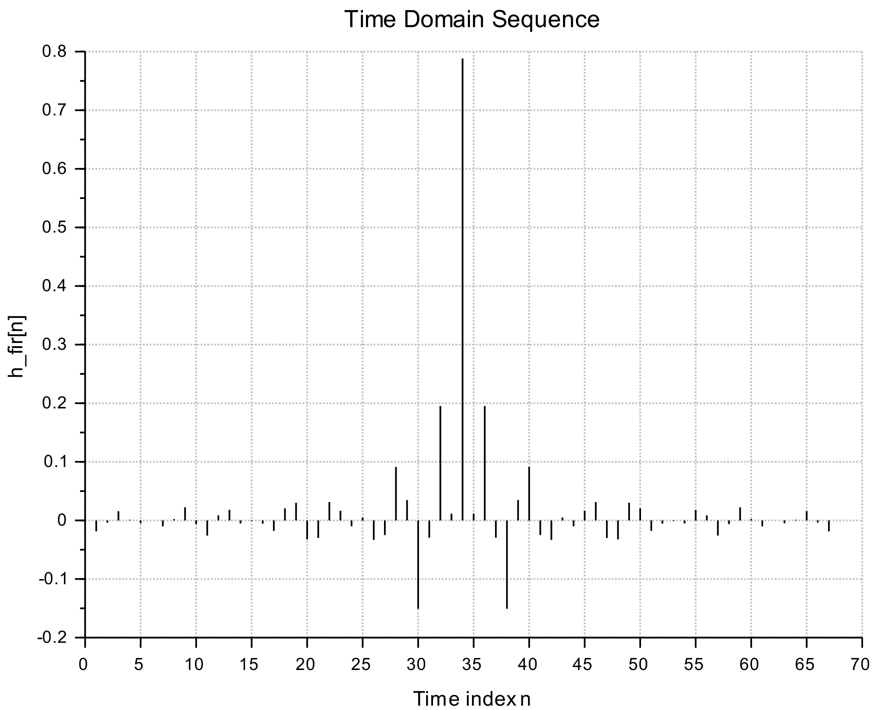
For the exact value of N, we must evaluate whether the magnitude tolerances at the transition band edges are satisfied. For this, we would have to iterate over N and find the magnitude of the FIR filter transfer function at the passband and stopband edges. Shifting the response for causality,

$$\mathrm{h_{fir}[n + N] = h_{ideal}[n].v[n]},$$

$$\mathrm{H_{fir}[z]} = \sum_{n=0}^{2N} \mathrm{h_{fir}[n]z}^{-n}.$$

Note that the window length is 2N + 1 for a given N. The smallest value of N that satisfies all the requirements of the Bandpass filter is:

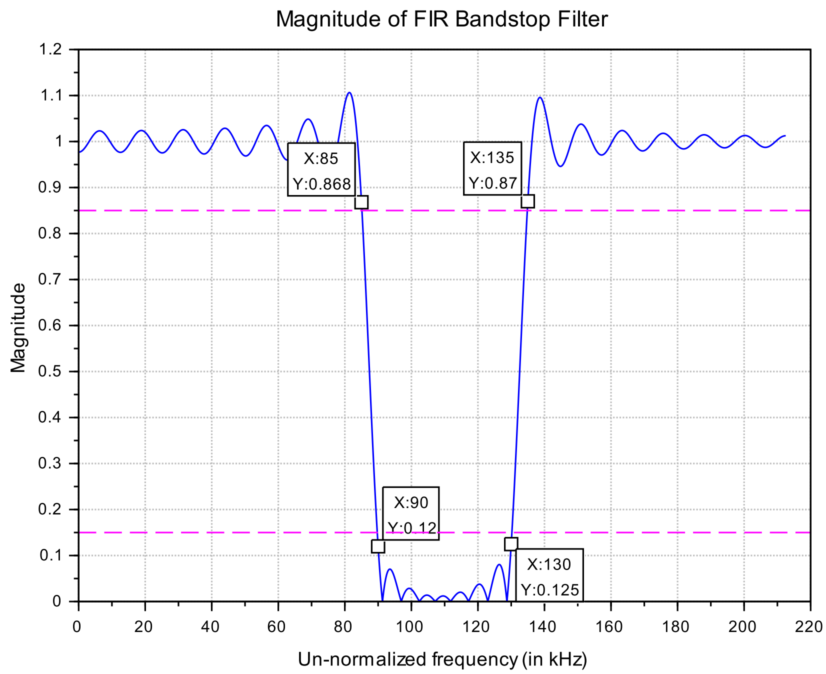
$$N = 33.$$

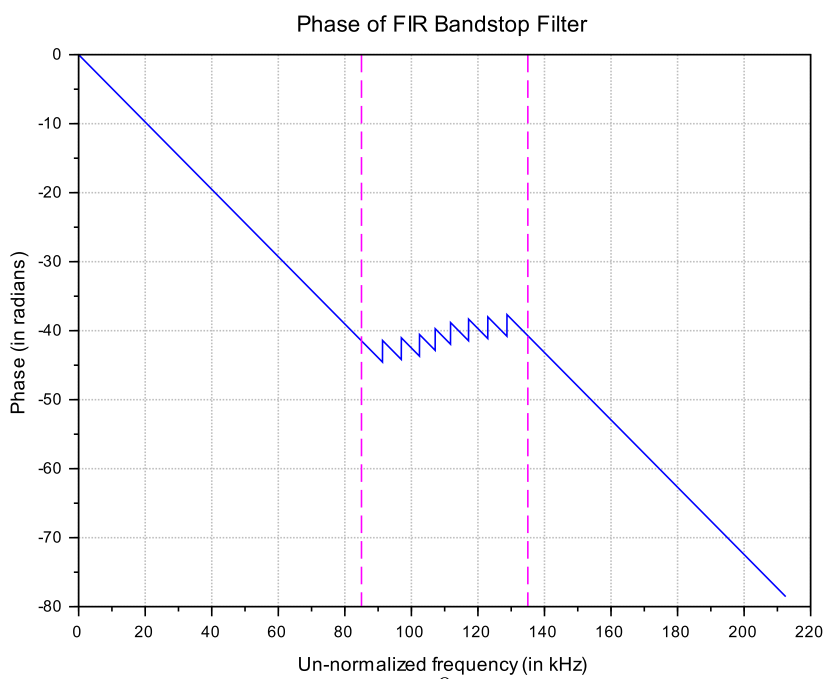



```
       column 1 to 7
-0.0186463  -0.0037713   0.015746   0.0010308  -0.0048251  -0.0000465  -0.0100904
       column 8 to 14
 0.0022127   0.022407  -0.0062587  -0.0259328   0.0085575   0.0179853  -0.0051254
       column 15 to 21
-0.001076  -0.0055863  -0.0178077   0.0206042   0.0301672  -0.0324  -0.0299138
       column 22 to 28
 0.0313592   0.0163962  -0.0099457   0.0049858  -0.0332224  -0.0250001   0.0913638
       column 29 to 35
 0.0346952  -0.1507987  -0.0295227   0.1952792   0.011519   0.7882353   0.011519
       column 36 to 42
 0.1952792  -0.0295227  -0.1507987   0.0346952   0.0913638  -0.0250001  -0.0332224
       column 43 to 49
 0.0049858  -0.0099457   0.0163962   0.0313592  -0.0299138  -0.0324   0.0301672
       column 50 to 56
 0.0206042  -0.0178077  -0.0055863  -0.001076  -0.0051254   0.0179853   0.0085575
       column 57 to 63
-0.0259328  -0.0062587   0.022407   0.0022127  -0.0100904  -0.0000465  -0.0048251
       column 64 to 67
 0.0010308   0.015746  -0.0037713  -0.0186463
```

Time index n and column n in the above figures correspond to the coefficient of $z^{-n+1}$

## 2.6 Frequency Response



# 3 Comparison with IIR realizations

The plots comparing the responses of the IIR and FIR filters may be viewed from the next page onwards.

## 3.1 Magnitude Response

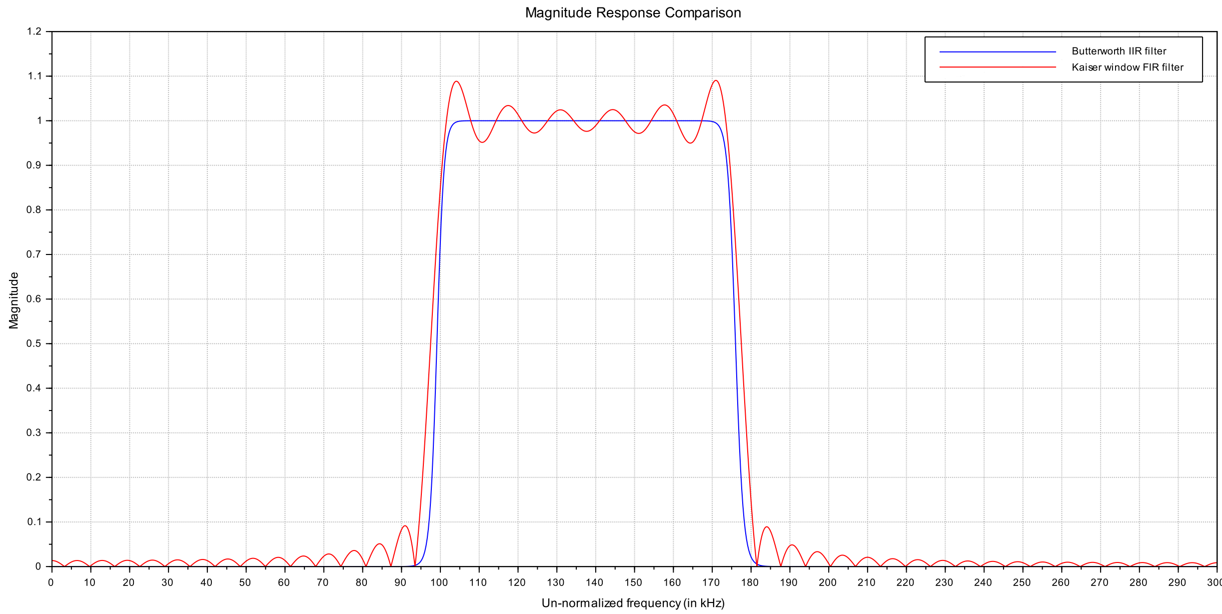
The FIR filters have a sharper transition band than the IIR filters. However, the nature of the response in the passband and stopband is preferred in the following order: Constant > Monotonic > Equiripple. The FIR filter is unable to achieve either of these and gives a rippled response where the ripples converge toward the cut-off frequencies as we increase N (for the rectangular window) but their heights remain unchanged.

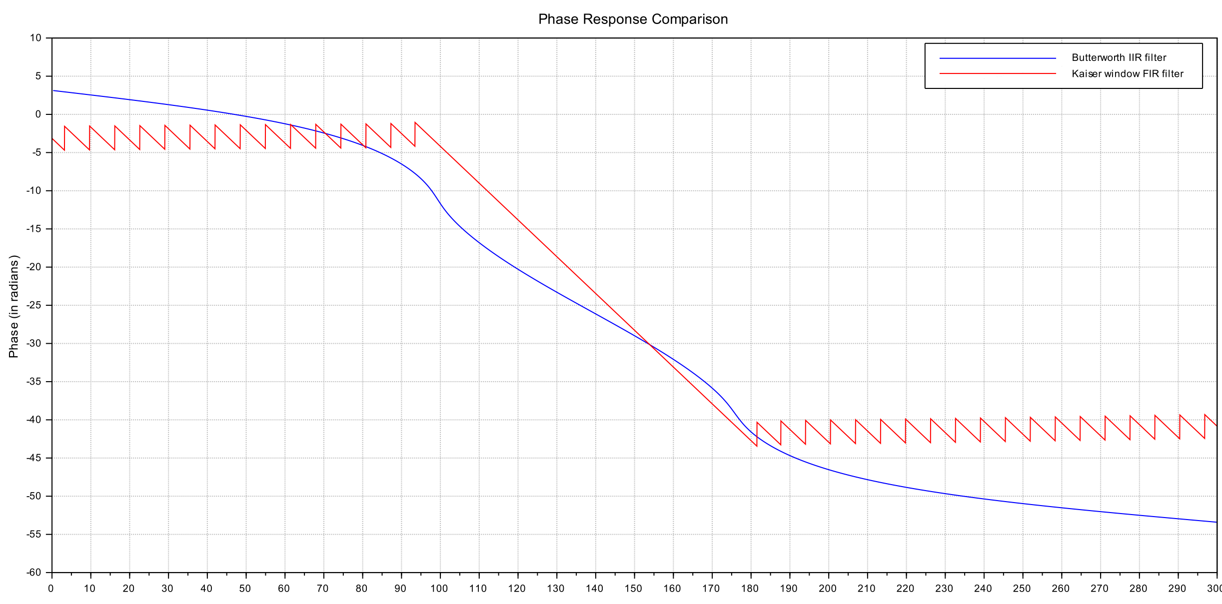
## 3.2 Phase Response

The adjustable phase response is the main advantage of the FIR filters and we observe that in the passband of both filters, the phase response is linear whereas that of the IIR filters is non-linear. Outside the pass band, however, the phase response shows a sawtooth-like pattern.

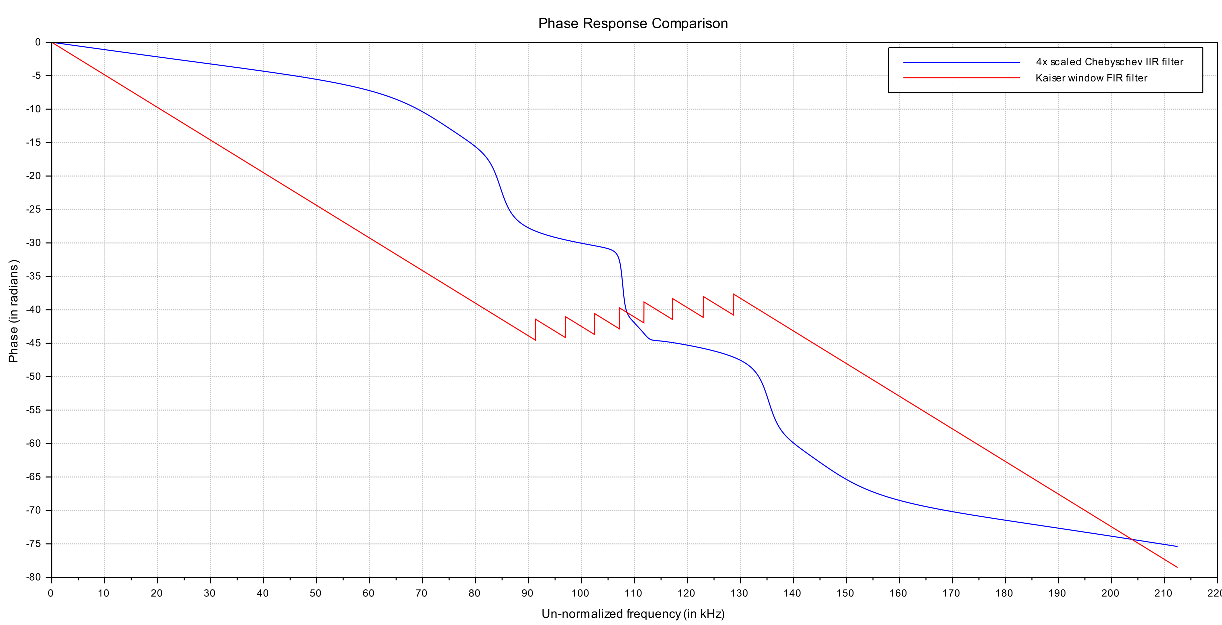
# 4 Review

I have reviewed Abhijeet's (200100107) report and certify it to be correct.


Magnitude Response Comparison
Butterworth IIR filter
Kaiser window FIR filter
Magnitude
Un-normalized frequency (in kHz)


Phase Response Comparison
Butterworth IIR filter
Kaiser window FIR filter
Phase (in radians)
Un-normalized frequency (in kHz)


Magnitude Response Comparison
Chebyschev IIR filter
Kaiser window FIR filter
Magnitude
Un-normalized frequency (in kHz)


Phase Response Comparison
4x scaled Chebyschev IIR filter
Kaiser window FIR filter
Phase (in radians)
Un-normalized frequency (in kHz)