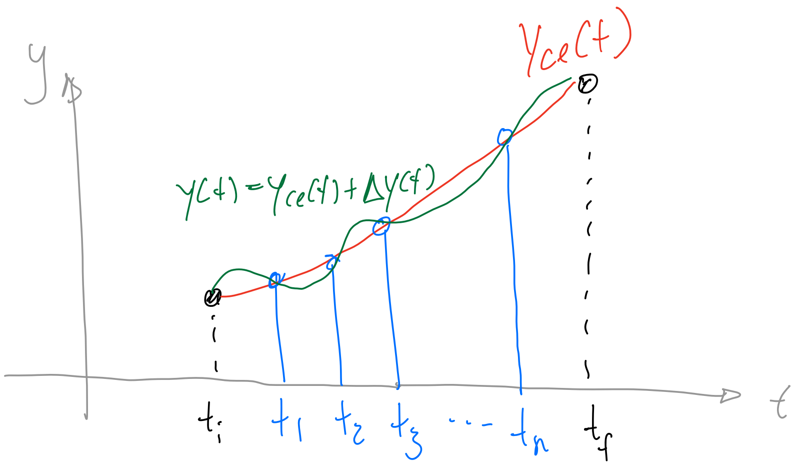
Local vs Global Minimization

<u>Idea ①</u>: consider 1D problem with classical solution $y_{cl}(t)$. we can parametrize a path by

$$y(t) = y_{cl}(t) + \Delta y(t)$$

where $\Delta y(t) = 0$ at $t_{initial}$ and t_{final} to satisfy boundary conditions.

Pick a set of N times, $\{t_1, \ldots, t_N\}$, where we constrain $\Delta y(t_k) = 0$ for $k = 1, \ldots, N$.



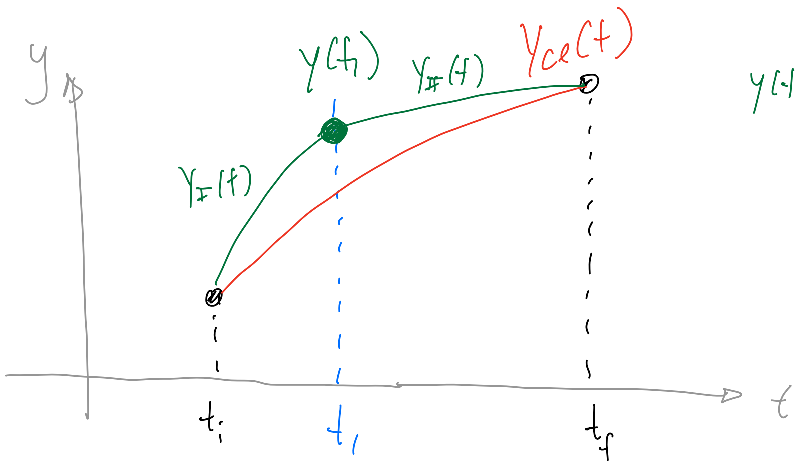
we can parametrize Δy as

$$\Delta y(t) = \alpha\big(\pm(t-t_1)\big)\big(\pm(t-t_2)\big)\ldots\big(\pm(t-t_n)\big)$$

where maybe the user can input the signs and the values of $t_1, \ldots, t_n$, and then vary α. to see changes in various quantities (like action, $\frac{dE}{dt}$, etc)

<u>Idea ②</u>: sequential path construction from L to R

user starts by specifying $y(t_1) = y_1$ as any value



$$y(t) = \begin{cases} y_I(t), & t_i < t < t_1 \\ y_{II}(t), & t_1 < t < t_f \end{cases}$$

we then define $y(t)$ as the path that minimizes the action piecewise for $\underbrace{t_i < t < t_1}_{\text{region I}}$ and $\underbrace{t_1 < t < t_f}_{\text{region II}}$.

For example, for uniform gravity, we solve for v_i satisfying BCs in each region:

$$y_1 - y_i = v_i^{(I)}(t_1 - t_i) - \frac{1}{2}g(t_1 - t_i)^2$$

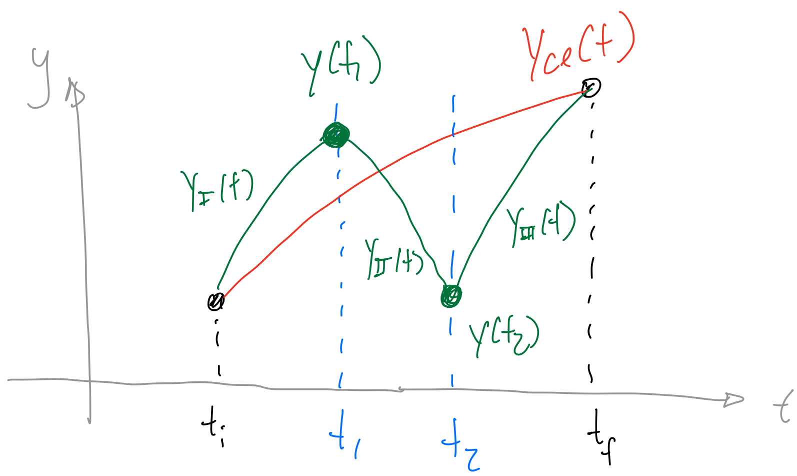
$$\hookrightarrow \quad v_i^{(I)} = \frac{y_1 - y_i + \frac{1}{2}g(t_1 - t_i)^2}{t_1 - t_i}$$

Thus, $$y_I(t) = y_i + \left[\frac{y_1 - y_i + \frac{1}{2}g(t_1 - t_i)^2}{t_1 - t_i}\right](t - t_i) - \frac{1}{2}g(t - t_i)^2$$

similarly, $$y_{II}(t) = y_1 + \left[\frac{y_f - y_1 + \frac{1}{2}g(t_f - t_1)^2}{t_f - t_1}\right](t - t_1) - \frac{1}{2}g(t - t_1)^2$$

For starters, maybe pick t_1 ourselves and then have a slider that allows user to vary y_1 and see change in path, action, etc.

There is then a button that allows us to fix value of y_1. We now do this over again for a new point $y(t_2) = y_2$



$y_I(t)$ is fixed but we now split the remainder into a new $y_{II}(t)$ and $y_{III}(t)$, which can be found analogously to before:

$$y_{II}(t) = y_1 + \left[\frac{y_2 - y_1 + \frac{1}{2}g(t_2-t_1)^2}{t_2 - t_1}\right](t-t_1) - \frac{1}{2}g(t-t_1)^2$$

$$y_{III}(t) = y_2 + \left[\frac{y_f - y_2 + \frac{1}{2}g(t_f-t_2)^2}{t_f - t_2}\right](t-t_2) - \frac{1}{2}g(t-t_2)^2$$

Do this for n intermediate points (for initial coding/debugging, keep it small like $n=3$, but eventually we will want something larger but not so large as to make it tedious (maybe 5–10)

At each step, we may want to add a "go back" feature that allows them to return to the previous step.

<u>Idea ③</u>: in some of our existing applets, allow user to consider & compare alternate versions of "action" like:

$$\int dt\, K$$

$$\int dt\, U$$

$$\int dt\,(K+U)$$

actual action $\quad S = \int dt\,(K-U)$

$\int dt\,(K^n \pm U^n)$ for some n, etc