7

Positions, Roles and Clusters

The network concepts that have been discussed so far in this book have mainly been concerned with the particular patterns of direct and indirect contacts which agents are able to maintain with one another. They have been concerned with such things as the abilities of agents to join with one another in cohesive social groupings, their abilities to influence the actions of those particular others to whom they are connected, and so on. But I have, at a number of points, alluded to the analysis of 'positions' rather than individual agents and their connections. Warner and Lunt (1942), for example, attempted to investigate the formation of distinct social positions, and Nadel (1957) argued that social roles were the central elements in social network analysis. The key concept in recent discussions of this problem is the idea of 'structural equivalence'. This involves a concern for the general *types* of social relations which are maintained by particular *categories* of agents. While two people may have direct connections to totally different individuals, the type of relations that they have with these others may, nevertheless, be similar. Two fathers, for example, will have different sets of children to whom they relate, but they might be expected to behave, in certain respects, in similar 'fatherly' ways towards them. The two men, that is to say, are 'structurally equivalent' to one another. They occupy the same social position – that of 'father' – and so are interchangeable so far as the sociological analysis of fathers is concerned. The idea behind structural equivalence, therefore, is that of identifying those uniformities of action which define social **positions**. Once the positions have been identified, the networks of relations which exist between the positions can be explored.

Social positions are occupied by agents which are 'substitutable' one for another, with respect to their relational ties (Sailer, 1978; Burt, 1982). They are, in certain important respects, interchangeable. Although social positions are manifest only in the particular relations which link specific agents, they cannot be reduced to these concrete connections. They involve more enduring relations which are reproduced over time. These enduring relations among social positions constitute a distinct area of structural analysis.

The Structural Equivalence of Points

It might appear, at first glance, that the analysis of structural equivalence is simply the analysis of social roles, but this is not the case. The example of the two fathers shows that the clearest cases of structural equivalence are, indeed, those which arise when people occupy institutionalized roles. The occupants of a clearly specified cultural role comprise a structurally equivalent category of agents: they do similar things in relation to similar others. But this is, of course, true only for fully institutionalized roles. If people do not act in conformity with standardized cultural expectations, but deviate or otherwise vary in the ways in which they perceive and enact their roles, very few uniformities of action may be found. In such circumstances, there will be no *position* of structurally equivalent agents corresponding to the culturally defined *role*. This is, no doubt, the case for many cultural roles, and the degree to which they are institutionalized into structured uniformities of action will be highly variable from one case to another.

Conversely, there may be structured uniformities of action which are neither culturally recognized nor identified in socially defined roles. Agents may occupy a distinct position in relation to other agents, acting in similar ways towards them, even though this fact is not recognized by the various participants. Indeed, this may be one of the ways in which new roles emerge: new forms of action arise and relations between more or less clearly defined categories of agents begin to crystallize long before people come to perceive what is going on and to give a name to it. In this sense, the identification of structurally equivalent categories of agents may be one basis for identifying emergent roles.

It is important, therefore, to see the concept of structural equivalence as applying to social positions *per se*, and not simply to roles or proto-roles. A social class, for example, could be identified in precisely these terms as a group of agents occupying an equivalent position with respect to the distribution of economic resources and, therefore, as having equivalent structurally determined interests and life chances in relation to the members of other classes.

The starting point for all formal discussions of structural equivalence has been the influential paper of Lorrain and White (1971). They built their approach around the concept of role, seeing the occupants of each role as being structurally equivalent to one another. Structurally equivalent agents, they argued, play the same part in the network or have similar linkages to the occupants of other positions and so are interchangeable one with another. The paper described some of the limitations of graph theory as a complete model for network structure and outlined an alternative strategy based on algebraic ideas. Lorrain and White argued that their approach had two major defining features which set it apart from other approaches to social network analysis. First, all points and their connections were handled simultaneously, rather than attention being limited to the particular lines, paths and cycles which connected them. Second, the approach did not remain with the adjacency matrix, but undertook a combined analysis of both the rows and the columns of the original incidence matrix. People and the organizations of which they were members, for example, could be analysed together rather than separately.

According to Lorrain and White, the overall pattern of connections in a network must be converted into a system of structurally equivalent positions by aggregating the individual points into larger sets of points. The underlying structure of the system is more apparent in the relations that exist between the sets than it is in the more numerous and more concrete relations that exist between the individual agents which make up these sets. Figure 7.1 shows Lorrain and White's view of the 'reduction' of a complex network to its 'block model' or 'image matrix'. The points of the original incidence matrix are re-arranged through a method of cluster analysis to form the structurally equivalent sets of the image matrix. In Figure 7.1, for example, the set *M*1 comprises those of the row points which are regarded as being structurally equivalent to one another, yet structurally divergent from those structurally equivalent points which make up set *M*2. The most fundamental features of a network, argued Lorrain and White, are apparent in the relations among the sets, and the nature of these relations is shown by the values in the cells, the blocks, of the image matrix. The aim of much of White's subsequent work was to suggest how such block models might be produced.[1]

The concept of structural equivalence holds, in its strongest sense, that the members of a set are identical with one another as regards their relations to other members of the network. It is, however, very unusual to find agents which are perfectly equivalent in this strong sense. Most analysts of structural equivalence have, therefore, argued that the criterion needs to be weakened if it is to be of use in the study of real social networks. Instead of searching for those agents which are identical in their social relations, the aim is to identify those which are sufficiently *similar* to be regarded as structurally equivalent. Whatever the chosen measure of similarity, the researcher must decide on a cut-off threshold above which agents are to be regarded as being sufficiently similar to be, in effect, 'substitutable' for one another. This 'fuzzy' measure of

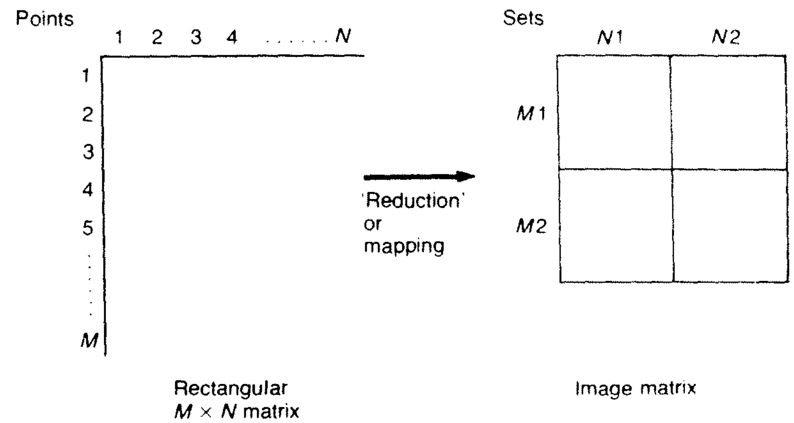


Figure 7.1 *A network and its block model*

structural equivalence is likely to be of greater use in real situations, though the cut-off level for identifying equivalence is, of course, a decision made by the researcher, and must be rigorously assessed for its substantive validity.

The major areas of disagreement among writers on structural equivalence concern the particular measure of 'similarity' that is to be used, the method of clustering by which points are to be grouped into sets, and the methods to be used for identifying the boundaries of the sets. In the next section, I will briefly review the main methods of cluster analysis which are available and, in the following section, I will outline in greater detail two particular approaches to structural equivalence. I will then return once more to the choice of clustering method and measure of 'similarity' through considering some alternative approaches.

Clusters: Agglomerative and Divisive

The words 'cluster' and 'clique' are often used interchangeably, as in the early discussions of sociometric 'cliques' in Old City and in Yankee City. Even some recent methodological commentators have not distinguished between the two ideas (see Lankford, 1974). I showed in the previous chapter, however, that the concept of the clique can be given a strict sociometric definition from which a whole family of related concepts can be derived. The concept of the cluster needs also to be clearly defined as a separate and very distinct idea. The intuitive idea of a cluster corresponds to the idea of an area of relatively high density in a graph. This idea of the cluster is applicable to relational and attribute data alike, and can be illustrated through scatter diagrams such as those in Figure 7.2. In a scatter diagram, the individual cases are plotted against the two variables which comprise the axes of the diagram. The scatter of the cases across the page gives an indication of how similar or different they are from one another in terms of these variables.

In each of the diagrams of Figure 7.2 there are two distinct clusters. In diagram (i), the clusters consist of points which are more 'similar' to one another than they are to other points. They form areas of high density in the overall scatter plot. While these clusters are apparent by simple visual inspection, computerized procedures are required for larger data sets. But the researcher must then choose a particular method of cluster analysis. Most available methods would recognize the clusters in diagram (i), but not all would recognize such clusters as those of diagram (ii). The points in this diagram spread across elongated areas of the distribution, and points at opposite 'ends' of each cluster are quite 'distant' from one another. Clusters are defined in terms of their *contiguity* in the diagram and their *separation* from other clusters, but not all clusters will consist of points which are equally 'close' to one another in the scatter plot. Most techniques of cluster analysis assume compact 'spherical' clusters and would have great difficulty in finding the kinds of clusters depicted in diagram (ii).

Clearly, the boundaries of clusters cannot be drawn sharply. Diagram (iii) in Figure 7.2, for example, shows what might appear to be two large clusters, each of which contains a smaller cluster. But an alternative view is to see only the smaller clumps of points as being clusters. The composition of the clusters identified in a cluster analysis will depend on the density level that is chosen by the researcher, and on the assumptions made by the particular clustering method.

This arbitrariness in determining the boundaries of clusters indicates that clustering methods may be seen as using a variant of the nesting procedure. There is a hierarchical structure to clusters, which can be represented in 'dendrograms', or tree diagrams which show the clusters that exist at each level of similarity. This idea is illustrated in Figure 7.3. This diagram shows that points C and D are linked into a cluster at the first step in the analysis, points G and H are linked at the second step, points E and F at the third step, and points A and B at the fourth step. If the analysis stopped at this point, four clusters would have been identified. If the analysis is moved to a fifth step, however, points E, F, G and H are all identified as being members of a single cluster. Similarly points A, B, C and D are clustered together at step six. Finally, at step seven,

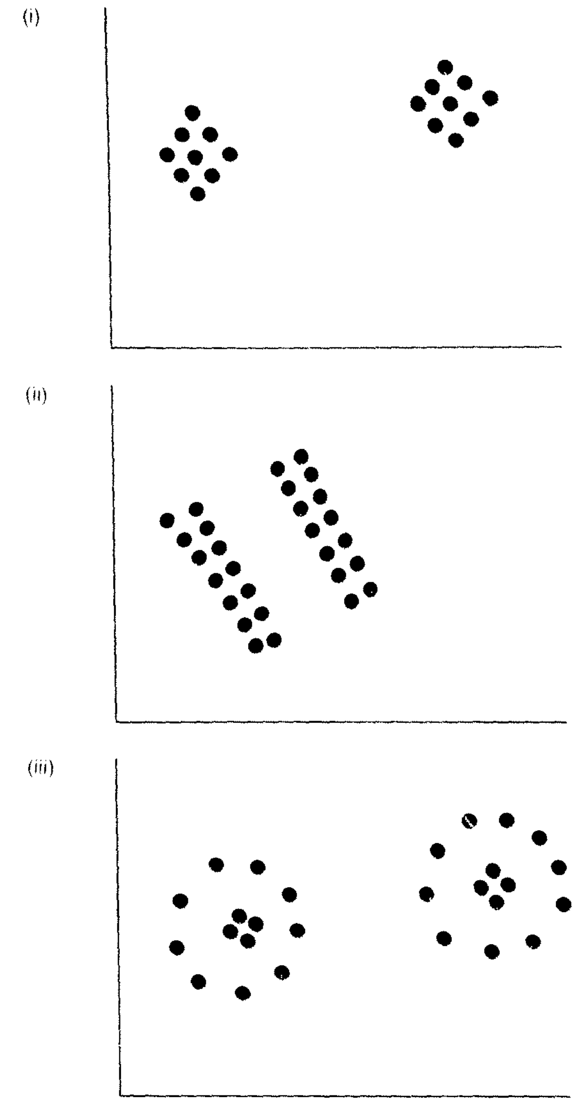


Figure 7.2 *Clusters*

all points are aggregated into the same cluster. The number and composition of clusters found in any investigation will depend on the step at which the analysis is stopped.

Although these ideas have been illustrated through the more familiar type of attribute data, they are equally applicable to relational data. Here, for example, points might be clustered according to their path distance or density within a sociometric graph. The members of a cluster might be those which are similar to one another in terms of some graph theoretical criterion of closeness or distance from other points.

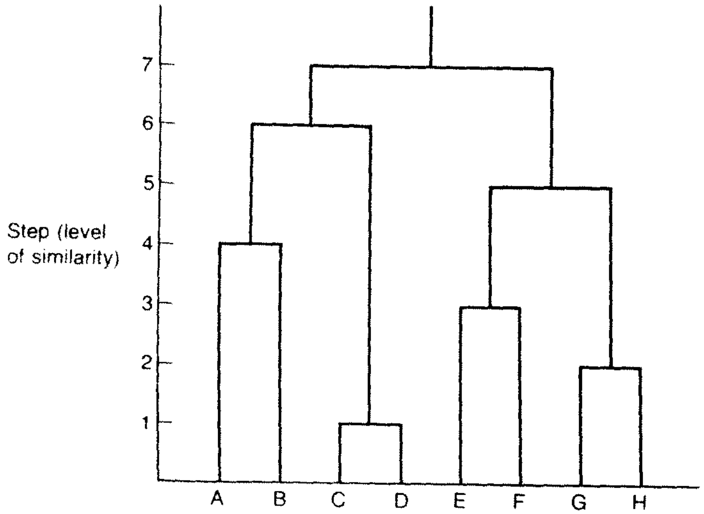


Figure 7.3 *A dendrogram*

There are two principal families of cluster analysis methods: 'the agglomerative' and the 'divisive' (Everitt, 1974; Bailey, 1976). Each is hierarchical, in the sense that it 'nests' small clusters within larger clusters, but the principles which are used in constructing a hierarchy of clusters vary between the two cases. The discussion above was couched in terms of an agglomerative model in which individual points are gradually aggregated into larger and larger sets. Points are compared for their 'similarity' or 'distance' from one another, and are grouped together with those to which they are closest or most similar. Agglomerative methods can be of the 'single linkage' or the 'complete linkage' type (Johnson, 1967). In a **single linkage** method, points are fused into a cluster with their nearest neighbours. In a study of interlocking directorships, for example, enterprises might be merged into clusters on the basis of the number of directors which they have in common. Initially, the two closest points are fused into a cluster and later steps fuse successively more distant points and clusters. Two clusters would be fused, for

example, if their most similar members were closer to one another than were any other pair of points in the set. A **complete linkage** method follows the same general approach, but measures the similarity between two clusters not by their closest but by their remotest members.[2] While the single linkage method tends to 'chain' points together into existing clusters, the complete linkage method is more likely to initiate new clusters at early stages in the analysis. The single linkage method, therefore, is less likely to identify the compact and homogeneous clusters of the kind found through complete linkage. In emphasizing the connections between clusters, the single linkage method can mask the existence of important divisions in the network (Alba, 1982; 55–6).

With both methods of agglomerative cluster analysis, it is for the analyst to decide on the level of similarity at which clusters are to be identified. In a connected graph, all points will, eventually, fuse into a single cluster, so the number and size of clusters identified will depend upon the cut-off threshold which is chosen. It follows that the choice of a cut-off threshold, as in so many areas of social network analysis, is a matter for the informed judgement of the researcher, though some measures of goodness of fit have been suggested as aids in this task.

In a divisive, or partitioning approach, the opposite strategy is followed. Starting from the graph as a whole, regarded as a single cluster, sub-sets are split off at reducing levels of similarity. There are two approaches to divisive clustering, the 'single attribute' and the 'all attribute' methods. **Single attribute** methods begin by differentiating those points which possess a particular indicator or value from those which do not, and the initial cluster is split into two on the basis of the possession or non-possession of this indicator. The same procedure is followed within each cluster at subsequent steps, in order to sub-divide each of them further.[3] The single attribute procedure, therefore, consists of a series of binary splits aimed at producing mutually exclusive sets of points. In an **all attribute** method, on the other hand, the first and subsequent splits are based on the average similarity of a set of points to all other points in the graph.

The methods of cluster analysis may seem a little vague in the abstract, but I hope that their general principles are clear. It should be apparent that the clusters which are identified in a particular graph will depend upon the choice of the method and the choice of the measure of similarity on which it works. The implications of this can be pursued by considering a particular approach which builds on the work of Lorrain and White (1971).

Block Models: CONCOR and BURT

The first workable algorithm for investigating structural equivalence along the lines which had been suggested by Lorrain and White was formulated by Breiger and Schwartz, two of White's students, who independently rediscovered the matrix clustering method proposed by McQuitty (1968).[4] Their algorithm, called CONCOR (standing for 'CONvergence of iterated CORrelations'), involves a rather complex and cumbersome procedure, although its general principles are fairly straightforward. The CONCOR algorithm operates on sociometric incidence matrices of cases and affiliations, and can be applied to the rows, to the columns, or simultaneously to both the rows and the columns of the matrix. Its general logic can, however, best be understood by following through the steps involved in an analysis of the rows alone. Such an analysis investigates the structural equivalences among the cases.

The first step in the analysis is to calculate the correlations between all pairs of cases in the matrix, measuring this by the similarity of the values which are contained in the row entries. For each pair of cases it is possible to measure their 'similarity' by the value of the Pearson correlation coefficient: two cases with exactly the same pattern of affiliations would show a correlation of +1, while a pair with completely different patterns of affiliation would have a correlation of −1. The result of this first step is a square case-by-case correlation matrix, a particular form of the adjacency matrix. The second step involves the use of a clustering procedure to group the cases into structurally equivalent sets, according to their measured similarity. If rows were either perfectly correlated or completely un-correlated, such a grouping would be easy. All values in the correlation matrix would be either +1 or −1, and a strong criterion of structural equivalence could be used to divide the matrix into two sets. The matrix would fall into two sets which were completely connected internally but had no connections with one another. Such a clustering would be possible for the data shown in Figure 2.6. As this kind of patterning is not normally the case with real data, a clustering method that works on a wider range of correlation values must be used as the basis for identifying 'fuzzy' sets of equivalent points.

CONCOR achieves a fuzzy clustering by converting the raw correlations into a tighter pattern. It does this by calculating, for each pair of cases, the correlation between their scores in the correlation matrix which has been constructed. That is, the correlations among the correlations scores are calculated and they are entered into a new correlation matrix. This process is repeated over and over again for each successive matrix – correlating the correlations of the correlations, and so on. Repeated correlations of this kind have been found to produce, eventually, a matrix in which all the cells will contain values of either +1 or −1. The iterated (repeated) correlations converge to a simple pattern and the rows can be partitioned into two clusters in much the same way as if a strong criterion of structural equivalence were being used, and each cluster constitutes a set of structurally equivalent cases.

Each of the two clusters can be divided into its constituent elements by using precisely the same method. To achieve this, the algorithm returns to the original matrix of raw values and divides this into two separate matrices, one for each of the clusters which have been identified. As in the first round of iterations, the raw group memberships within one of the clusters are converted into correlations, the correlations are correlated, and so on, until a pattern of +1 and −1 entries emerges within the cluster. At this point the cluster can be partitioned once more and the whole process repeated. Division and sub-division of clusters in this way can proceed for as long as the researcher wishes, though the larger the number of clusters, the more difficult it may be to interpret the final results.[5]

While the researcher must make an arbitrary decision about when to stop the process of division and sub-division within clusters, the emergence of a pattern of +1 and −1 values at each step does mean that there is a relatively unambiguous approximation to a strong criterion for identifying structural equivalence. The partitioning of the cases depends simply on the actual values which are produced in the final matrix.[6] Unfortunately, the reason why such a pattern should emerge is far from clear. This means that there is an unspecified, and partly obscure clustering principle at work in the CONCOR algorithm. It is the algorithm itself which, for reasons which are not entirely clear, produces the conversion of the raw data into structural equivalence categories. The clusters identified by CONCOR, therefore, are just as 'fuzzy' as those that might be produced through a procedure which does not result in such an elegant pattern.

This process of partitioning into clusters can be repeated for the columns of the original incidence matrix, so as to produce a separate grouping of the affiliations. If the cases were individuals and the affiliations were the organizations of which they were members, a partitioning of the organizations would cluster them according to similarities in their patterns of recruitment. For both the rows and the columns of the original incidence matrix, then, CONCOR can produce a hierarchical partitioning into structurally equivalent clusters – 'discrete mutually exclusive and exhaustive categories' (Knoke and Kuklinski, 1982: 73)

The clusters identified in these ways can be constructed into re-arranged image matrices of the type illustrated in Figure 7.1. It is possible to produce a square image matrix for the adjacency matrix of the cases or for the adjacency matrix of the affiliations. Each of the cells in the image matrix – they are termed the 'blocks' – contains a measure of the density of the connections between pairs of sets. If all density values were either 1 or 0, the pattern of relations would be clear. The 'zero-blocks' (the cells with density 0) would represent 'holes' in the network, the complete absence of connections; and the distribution of cells with density 1 would show the basic structure of the network. Such density patterns rarely occur in real data, and so a block modelling has to convert the actual range of density values into two categories of 'high' and 'low' values as approximations to the 1-blocks and zero-blocks. In the image matrix, the high values – those which are above a specified threshold value – are represented by 1, while low values are represented by 0. The most commonly used method for defining blocks with a high density is to take the average density of the whole matrix as a cut-off point: values at or above the mean are regarded as 'high', while those below it are 'low'. But this procedure, like so many in network analysis, involves a discretionary choice on the part of the researcher, and this choice must be grounded in theoretical or empirical considerations. It cannot be justified on any purely formal, mathematical principles alone.

Exactly the same procedure can be used to produce a block model for a combined analysis of the rows and columns. CONCOR will produce a clustering of the rows and a clustering of the columns and will then combine these into a single image matrix of the original rectangular incidence matrix.

Once a block model, an image graph containing only 1 and 0 values in its cells, has been produced, the researcher must attempt to interpret it. Interpretations of block models produced from rectangular, incidence matrices are extremely difficult to make, and Breiger and his associates, the originators of block modelling, have published no detailed analyses of such models. In the earliest analysis of an incidence matrix, Breiger, Boorman and Arabie (1975) re-analysed the *Deep South* data collected by Davis and his colleagues (1941) on the participation of 18 women in 14 social events.[7] To analyse these data, they computed separate row and column solutions and combined them into the block model shown in Figure 7.4.

		Events	
		A	B
Women	1	0	1
	2	1	0

Figure 7.4 *A simple block model*

It can be seen that women in cluster 1 tend to meet in the events of cluster B and that women in cluster 2 tend to meet at the events of cluster A. The two clusters of women correspond closely to the two 'cliques' which had been identified by Homans (1951) in his commentary on the original data, but Breiger et al., did not go beyond this observation. Although they discuss the composition of the clusters, they give no attention to the pattern of block densities in the image matrix. In the same paper, they also re-analysed Levine's rectangular matrix of banks and corporations (Levine, 1972), but they again simply compare the separate row and column analyses with Levine's own analysis.

This failure on the part of the inventors of block modelling to analyse an incidence matrix in any detail suggests the existence of a fundamental difficulty in achieving the concurrent treatment of both rows and columns that was anticipated by Lorrain and White. A rectangular image matrix, if it is fairly simple, may give an initial and schematic overview of the network, but more detailed analyses can only be pursued by analysing the rows and the columns separately. An incidence matrix, then, must be analysed principally through the construction of separate block models for each of its constituent adjacency matrices. In these block models, those cells on the diagonal which contained a '1' would correspond to some kind of clique or social circle of the type discussed in the previous chapter. The other cells would show the presence or absence of connections between the various cliques and the other clusters which make up the graph.[8]

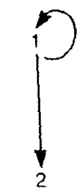
Breiger has shown how the CONCOR method can be applied in one of the central areas of social analysis. Using data on social mobility from Britain (Glass, 1954) and the United States (Blau and Duncan, 1967; Featherman and Hauser, 1978), he constructed a model of class structure in which the classes were defined as sets of occupations identifiable in a matrix of occupational mobility rates (Breiger, 1981, 1982). He sees this as an extension of Weber's (1920–1) claim that 'a structure of social classes exists only when the mobility chances of individuals within the classes cluster in such a way as to create a common nexus of social interchange' (Breiger, 1982: 18). CONCOR, he suggests, can be used to identify class boundaries. Breiger used inter-generational mobility matrices for adult males, the American matrices being 17 × 17 directed matrices of occupational categories and the British being 8 × 8 directed matrices. In each matrix, the cells contained the numbers of individuals moving from one category to another, the rows showed the 'origins' and the columns showed the 'destinations'. For the United States, Breiger (1981) concluded that there was a stable structure of eight classes over the period 1962–73, while for Britain he concluded that the earlier data (they related to 1949) could best be seen as reflecting a three-class structure. The central class boundaries in Britain were that which separated manual and non-manual and that which separated the salaried 'middle class' from lower-level clerical and administrative jobs.

By far the easiest of matrices to analyse through block modelling are adjacency matrices with directed data – matrices where, for example, the rows represent relations 'sent' and the columns represent relations 'received'. A useful aid to the interpretation of this kind of data is the construction of arrow diagrams which show the relations among the clusters. This can be illustrated with the matrices in Figure 7.5, which show some hypothetical data on power relations.[9] In these matrices, the power relations are directed from rows to columns. The row entries in the original matrix, for example, would show over which other agents a particular agent exercises power. Conversely, the column entries would show to which other agents a particular agent is subordinate in a power relation. In the block models, agents are clustered according to both their exercise of power and their subordination to power, and the 1 and 0 entries in the image matrices show the densities of the power relations among the clusters.

In model (i) of Figure 7.5, members of cluster 1 exercise power over one another and also over members of cluster 2. This is shown by the entries of '1' in the relevant blocks. Members of cluster 2, however, exercise no power whatsoever, being completely subordinate to the power of those in cluster 1. This structure is summarized in the corresponding arrow diagram. In model (ii), on the other hand, there are two separate and self-regulating categories (clusters 1 and 3), and members of these clusters jointly exercise power over the members of cluster 2. Finally, in model (iii), cluster 1 dominates both cluster 2 and cluster 3, but there is little mutual exercise of

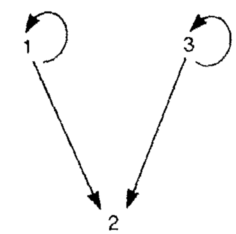
(i)

	1	2
1	1	1
2	0	0



(ii)

	1	2	3
1	1	1	0
2	0	0	0
3	0	1	1



(iii)

	1	2	3
1	0	1	1
2	0	0	0
3	0	0	0

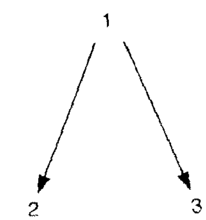


Figure 7.5 *Hierarchical block models*

power among the members of cluster 1 itself – the individual members of cluster 1 are each relatively autonomous agents.

Undirected matrices are, except in the most simple cases, rather more difficult to interpret, as the lack of any direction to the relations means that it is not possible to construct arrow diagrams to show their structures. Few such analyses have been published, and the application of block modelling to real and complex data sets of various kinds is essential if the value of the procedure is to be demonstrated.[10]

A fundamental problem with the CONCOR algorithm, as I have already suggested, is that it is not known exactly *why* it produces its solutions. The mathematical reasons for the convergence to a distribution of 1 and 0 entries are uncertain, and so an assessment of the validity of the results is difficult to make. This might seem to be a fairly damning criticism, but the fact that it does work and that it does seem to produce plausible models of small social networks helps to offset this criticism somewhat. There is, however, another difficulty, which suggests a further limitation on its applicability: CONCOR can identify structurally equivalent positions only *within* the components and sub-groups of a graph. If, for example, power relations were divided into distinct components within the network, CONCOR would not group together those who were dominant in the separate components as a single cluster of 'dominant' agents. Their equivalence as occupants of a position of dominance is masked, so far as CONCOR is concerned, by their sociometric division into separate components. Similarly, when a component is internally divided into relatively distinct cliques and circles, CONCOR will tend to identify only the dominant members within each of the sub-groups.

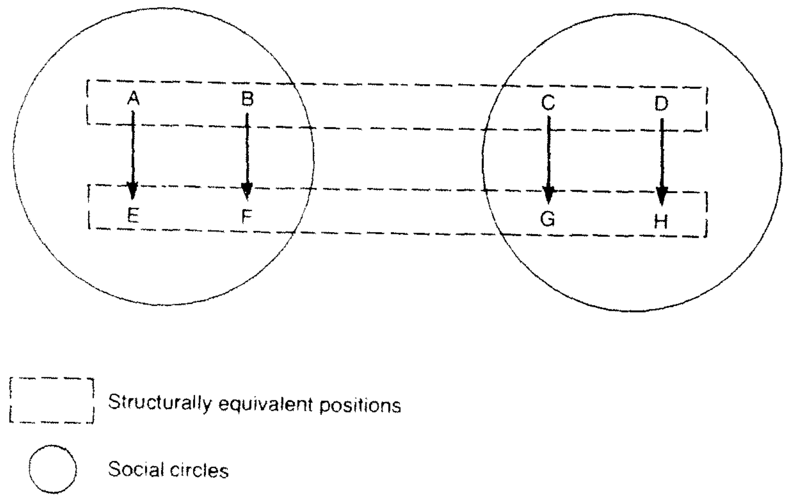


Figure 7.6 *Structural equivalence and social circles*

This can be illustrated through Figure 7.6, which shows a network in which A, B, C and D are structurally equivalent as 'dominant' agents, and E, F, G and H are structurally equivalent as subordinate agents. An adequate clustering of the network into structurally equivalent positions should identify two clusters {ABCD} and {EFGH}. If, however, the agents are divided into two distinct social circles, as shown, CONCOR will tend to identify four clusters instead: {AB}, {CD}, {EF} and {GH}. The conclusion to be drawn is that the CONCOR algorithm combines structural equivalence

with conventional sociometric measures and so fails to produce a thoroughgoing analysis of structural equivalence.

These limitations of the CONCOR procedure have led Ronald Burt to advocate a different approach to structural equivalence, one which aims to avoid CONCOR's reliance on uncertain mathematical procedures. His approach differs from that of CONCOR both in the measure of similarity which is used and in the method of clustering. Having examined CONCOR in some detail, it will be fairly easy to understand how Burt's procedure operates and in what respects it is an improvement on CONCOR. Burt's procedure is implemented in his program STRUCTURE, and in order to distinguish it from the other procedures available in that program I shall term it 'BURT'.[11]

BURT uses a similarity measure based on path distances between pairs of points. Where CONCOR looked only at similarity in terms of direct contacts, BURT takes account also of indirect connections through paths of distance 2 or more in order to arrive at calculations of the minimum path distances between all pairs of points. The Burt measure of path distance also assumes that the strength of a relation declines with both the path distance and the significance of the path for the agent's overall pattern of contacts. This measure is based on the assumption that agents with large numbers of contacts are able to give less attention to their more distant ones.[12] Thus, the similarity measure used in BURT is a weighted distance measure.[13]

Structurally equivalent agents, in the strong sense, are those whose points are separated by zero distance. They are perfectly similar and substitutable. Burt has recognized that this strong criterion cannot be applied to most real data, and so he argued for the identification of weak structural equivalence through the use of a cut-off threshold of distance below which points would be regarded as structurally equivalent (Burt, 1980: 101ff.). While CONCOR's arbitrariness derives from an obscure mathematical procedure, BURT's arbitrariness has the virtue of being grounded in the informed judgement of the specialist researcher. BURT performs a hierarchical clustering of the distance matrix, using Johnson's (1967) aggregative single linkage method, and the researcher reads off the clusters that are found, if any, at the chosen cut-off level of distance.

Once a clustering has been produced by the BURT procedure, the investigation can proceed with the construction of a block model. If the densities in the image matrix are replaced by entries of 1 or 0, using the density of the whole network as a cut-off threshold, the resulting block model can be analysed in the same way as those derived from a CONCOR analysis. An image matrix which shows the density of connections among the clusters is a simplified mapping, a 'homomorphic reduction', of the concrete pattern of relations between agents, and Burt terms this a 'social topology'.

Burt argues that the departure from a strict measure of strong structural equivalence means that any analysis must be treated merely as a hypothetical model. Without some kind of statistical test of significance, he argues, researchers would be free to choose whichever cut-off threshold will produce the results that correspond most closely to their preconceptions. A significance test helps to introduce a degree of impartiality and objectivity to the assessment of block models. Burt's recommended test involves an examination of each cluster in order to measure how closely associated each of its members is with the other occupants of the cluster. The best solution, he argues, is that which optimizes this measure of association.[14]

Towards Regular Structural Equivalence

The CONCOR and BURT procedures are probably the most widely used methods for identifying structural equivalence, but a number of alternatives have been suggested. Although some of these have become relatively easily available, they have rarely been applied to real data, and their long-term value still remains to be assessed.

Where CONCOR takes account only of path distances of length 1 and BURT takes account of all connections, regardless of their path length, Sailer (1978) has proposed a procedure in which the researcher is given the choice of a path length to use. A path distance is chosen, and the similarity of connections between pairs of points is calculated at that chosen level. Sailer's measure, then, which he terms simply 'substitutability', is based on the 'neighbourhood' of points. The degree of similarity between two points is measured in proportional rather than absolute terms, the number of contacts which they have in common at the specified path length being standardized by each point's adjacency. That is, the overlap between contacts is measured by the number of common connections expressed as a proportion of each point's total number of connections at that distance. Each point, therefore, can be given a standardized measure of its similarity to each of the other points. Complete overlap in contacts produces a standardized score of 1, while complete absence of overlap gives a score of 0.[15] As in the CONCOR procedure, Sailer sees this as simply a first step in an iterative process. The matrix of similarities is treated as an initial estimate of the 'substitutability' of points, and the continued repetition of the method on each new set of estimates results in a convergence to a

solution in which all values are either '1' or '0'. In this way, then, a block model can be produced for analysis. Sailer's procedure, however, fails to overcome the principal limitation of the CONCOR and BURT methods, which is that they cannot adequately handle networks which are divided into components or tight sub-groups of the kind illustrated in Figure 7.6 (see also Carrington and Heil, 1981; Wu, 1984).

An interesting attempt to overcome this sociometric limitation is REGE, an algorithm which is intended to detect 'regular' structural equivalence. This is defined as those equivalences which are regular across all the various subgroups of a network (White and Reitz, 1983; Winship and Mandel, 1984). The concept of regular equivalence is closer to the idea of the substitutability of agents by role or by function within a social system. Where CONCOR and BURT see points as being structurally equivalent when they have *identical* links to all the other points in the graph, REGE sees points as equivalent if they have *similar* links to points which are themselves structurally equivalent. Two points are regularly equivalent in relation to another set of points if the relation of one point to the points in that set is similar to the relation between the other point and the set. Each point has an identical relation with a counterpart in the same set, though this relation need not be with the same point or points. This can be illustrated by the obvious fact that all fathers are related to children, but they are not all related to the *same* children. White and Reitz argue, therefore, that the block models produced by REGE are *homomorphic* reductions, but not necessarily *isomorphic* reductions of their corresponding graphs.

The way in which REGE works can best be understood through the case of a directed matrix, although it is very difficult to understand the details of the procedure. The algorithm uses a partitioning method which looks at direct connections and also at the contacts of points adjacent to each pair. It begins by making estimates of the equivalence values between all pairs of connected points. These estimates are all initially set at 1, and they are revised with each round of calculation, which involves computing revised estimates of equivalences from the smallest in- and outdegrees for each pair of points. At the end of each round, therefore, there is a new matrix of estimated equivalences between pairs of points. The procedure is, ideally, continued until the revised estimates of equivalence no longer alter; that is, the computations are no longer resulting in any greater precision for the estimates. In practice, the researcher can choose to stop when it appears that further calculations will make little difference to the estimates. The UCINET manual holds that the version of REGE implemented in UCINET produces optimum estimates after three rounds of calculation (see also Borgatti and Everett, 1989; Borgatti et al., 1989).

This approach can be used only on directed data, though Doreian (1987) has suggested an adaptation which allows undirected data to be analysed. With a symmetrical matrix, as Doreian shows, the initial estimates are not altered by the calculation: the algorithm simply identifies all connected points as being regularly equivalent. Such matrices can be analysed only if they are divided into two asymmetrical matrices, which can then be jointly analysed by REGE. Doreian suggests using centrality scores to make this division, though Everett and Borgatti (1990) have suggested that any graph theoretical attribute could be used. If centrality is used, for example, one matrix would consist of the relations directed from more to less central points, while the other matrix contains the relations directed from less to more central points. At present, the standard social network analysis packages are unable to make such divisions of the matrix, and researchers have to prepare their undirected data manually before using REGE.

Despite its limitations, REGE is the first structural equivalence procedure to offer a true approximation to the regular structural equivalence described by Lorrain and White (1971). The substantive assumptions that it makes about the data are, however, obscured by complex mathematics, and it is difficult for a non-mathematician to assess whether these assumptions are valid and realistic. As with CONCOR, the fact that it *does* appear to work as expected on small-scale data is a powerful argument in its favour, but researchers must be aware that they are taking a certain amount on trust.

The aspiration of writers such as Nadel, it will be recalled, was to build a framework of sociological analysis in which positional analysis would complement more traditional sociometric concerns with cliques and components. The approaches to structural equivalence which have culminated in REGE have eschewed graph theory and so remain at one remove from these sociometric concerns. The approach of 'graph role analysis', on the other hand, tries to use the structural position of points as measured in graph theory as a basis for a measure of structural similarity (Zegers and ten Berghe, 1985). The procedure uses local dependency or geodesic matrices to calculate correlations between pairs of actors.[16] Structural equivalence is assessed in terms of how similar these measures are for the various points. A pair of points with, for example, similarly high betweenness scores might be recognized as being structurally equivalent in certain important respects. In order to avoid the obvious problem of regarding points as structurally equivalent only if they lie between the *same* points, the algorithm can compute whether they lie between points which are themselves similar in their betweenness scores. The particularly interesting feature of this procedure is that it begins to build a bridge between the relatively well understood concepts of graph theory and the rather less well understood measures of structural equivalence. Instead of conflating the approaches, as in CONCOR, it aims to theorize and to articulate their interdependence.[17]

Interlocks and Participations

Burt has pursued a long-standing interest in the question of interlocking directorships in the business world, but he has eschewed conventional clique-based approaches to their investigation. His earliest paper on this question (Burt, 1979) set out his aspiration to discover the linkages which occur between profitability and the structural location of enterprises in the corporate system, and his development of the idea of structural equivalence was specifically geared towards this issue of structural location.

His starting point was the hypothesis that many interlocks can be understood as 'cooptive mechanisms' through which enterprises absorb into their own leadership those people from other enterprises who might threaten their continued operations. Thus, the suppliers who create market 'uncertainty' are objects of 'cooptive interlocks' by those to whom they supply goods or capital. Financial institutions, for this reason, are of particular importance in corporate interlocking: 'The use of money as a general resource makes the actions of financial corporations a source of significant uncertainty, so that firms would be expected to establish cooptive interlocks with financial corporations so as to secure access to money when it is needed' (Burt, 1979: 416).

Drawing on his earlier discussions of 'positional' concepts (Burt, 1976, 1977a and b), Burt saw the firms which operated in each sector of the economy as structurally equivalent to one another – the economic sectors comprised positions in a social topology. Using input–output data at the sectoral level for the United States in 1967, Burt attempted to show in which inter-sector exchanges there existed the degree of uncertainty that would make cooptive interlocking a rational strategy. That is to say, he was interested in seeing whether the structure of constraining economic transactions was reflected in a parallel structure of interlocks. The idea of 'constraint' between sectors was operationalized in terms of competitive pressures: enterprises were more constrained by their transactions with oligopolistic sectors than they were by those with competitive sectors. Market constraint reduced the structural autonomy of enterprises, and interlocking reduces the effects of this constraint and so transforms the economic environment in which enterprises operate. Burt holds that 'structure in the two networks is a symbiotic phenomena [sic]: market structure patterning interlock structure and interlock structure repatterning market structure' (Burt, 1979: 433; see also Burt et al., 1980; Burt, 1982: Chapters 4 and 8).

Burt's data comprised two parallel directed adjacency matrices, in which the rows and columns corresponded to economic sectors. One matrix contained information on the economic transactions between sectors, while the other showed their patterns of interlocking. The results of a block modelling of these data have not been directly reported, but Burt concluded that the two networks did mirror one another and that it was possible to identify a 'directorate tie market' – a structure of interlocks which provided a 'non-market' context for the regulating of commercial transactions (Burt, 1983a, b).[18]

Both the power and the limitations of the CONCOR procedure are apparent in an investigation of corporate shareholding which I undertook (Scott, 1986). The 250 largest British companies in 1976 were selected for study, and their largest shareholders were identified from their share registers. This allowed the construction of a 250 × 250 incidence matrix of cross-shareholdings among the companies. In this matrix, the rows showed the companies as 'shareholders' and the columns showed them as the targets of shareholding relations: shareholdings were directed from rows to columns. It was found that only 69 of the companies held controlling blocks of shares in other large companies, and that only 140 of them were targets of shareholdings by these 69. Thus, the effective data set was a 69 × 140 matrix. Centrality analysis showed that the Prudential Assurance was the most central shareholding participant, it having shareholdings in 88 of the 140 target enterprises. Similarly, Boots was found to be the most 'blue chip' of the targets – 18 of its 20 largest shareholders were among the 69 leading companies.

The main purpose of the analysis was to uncover some of the global features of the intercorporate network, using the CONCOR algorithm. The controlling companies were regarded as the major agents in the economy, and the research aimed to uncover whether they formed a unified group or were divided into rival and solidaristic coalitions. Analysis of components gave little indication that the enterprises were organized into coalitions, and the conclusion was drawn that the network was not fragmented into distinct

corporate groupings. CONCOR, however, disclosed the existence of a number of structural positions in the network, among which hierarchical relations could be identified. A joint row and column analysis suggested the existence of five sets of enterprises, which are shown in the arrow diagram of Figure 7.7.[19]

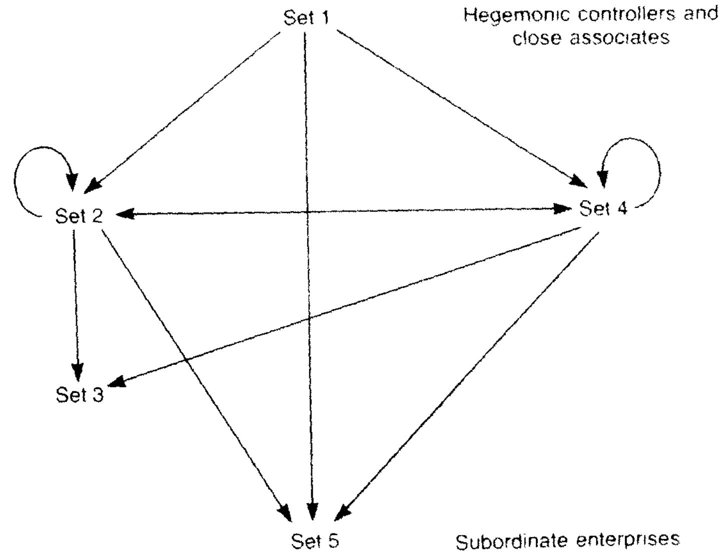


Figure 7.7 *The structure of financial hegemony in Britain (1976)*

In Figure 7.7 the arrows indicate the direction of the shareholding links between the various sets which comprise the positions in the network. Sets 1, 2 and 4 together comprise the 'hegemonic controllers' of the economy, with set 1 being the dominant element in this grouping. Set 1 contained 20 enterprises and included large public sector corporations and merchant banks, and its members were the major shareholding participants in three of the other sets. It owed its position at the top of the corporate hierarchy to the fact that its members were controlled by wealthy families and by the state rather than by other companies. Set 2 contained 11 enterprises which were involved in one another's capital (indicated by the self-referencing arrows) and which were important participants in controlling sets 3, 4 and 5. Set 4 was rather similar to set 2 (comprising clearing banks, insurance companies and large private sector industrials), but it was distinguished by the fact that its members were less likely to be involved in joint control of the companies and consortiums which made up set 3. Set 5, containing

91 enterprises, comprised the subordinate enterprises which had virtually no role in the control of other enterprises.

As the British network was not internally fragmented, CONCOR was very effective in disclosing the structurally equivalent positions occupied by corporate enterprises. Using similar data for Japan, however, it was found to be less useful. The Japanese economy was strongly divided into discrete components, each of which operated as cohesive business groupings. These are the familiar *kigyoshudan* of the Japanese business system (Scott, 1991). Although there were structurally equivalent categories of dominant and subordinate enterprises, CONCOR divided these along the lines of the business groups (Scott, 1986: 186ff.). No single set of structurally equivalent hegemonic controllers was identified by CONCOR. Seven sets were identified in the network, three of them corresponding to the well-known Sumitomo, Mitsui and Mitsubishi business groups, and within each set could be seen a hierarchical division into hegemonic and subordinate enterprises. Thus, the Japanese economy looked very much like Figure 7.6, with the structurally equivalent positions being cross-cut by the social circles which represented the major business groupings.

8

Dimensions and Displays

The sociogram – the network diagram – was one of the earliest of techniques for formalizing social network analysis, and the drawing of sociograms has remained a crucial means for the development and illustration of social network concepts. Sociograms have been used extensively throughout this book for just that purpose. Centrality, for example, can be illustrated by sociograms in which a central point is the 'hub' of a series of radiating 'spokes' which connect it to the more peripheral points. But the conventional sociogram has certain limitations as a method of representing and displaying relational data. Principal among these is that its use is limited by the difficulties of drawing large graphs on a sheet of paper. With more than ten or 20 points, even with networks having a relatively low density, the number of cross-cutting connections results in an un-interpretable thicket of lines.

In an attempt to overcome this limitation, various *ad hoc* extensions to the idea of the sociogram have been used as researchers have sought to complement their mathematical measures with some kind of diagramatic representation. One common technique has been to construct the sociogram around the circumference of a circle, so that the pattern of lines becomes more visible, as in Figure 4.1 (see Grieco, 1987: 30). Figure 8.1 shows one example of this method from Scott and Hughes (1980). The circle is used simply as an arbitrary visual framework for organizing the data, and the order in which the points are arranged around the circle is determined only by the attempt to ensure a minimum of overlap among the lines which connect them. The researcher engages in a trial-and-error process of drafting and re-drafting until an aesthetically satisfactory solution is achieved.[1]

Procedures such as those of computer-aided design (CAD) programs can be of considerable help in the construction of such maps, but they do not, in themselves, help the researcher to decide *where* each point is to be placed. Nevertheless, some progress has been made in this area, and a specialist plotting package for social network analysis – VIEWNET – has been devised (Klovdahl, 1981, 1986). The package allows sociograms to be constructed and