Quantity-Based Revenue Management

Prof. Daniel Guetta

Outline

1. Introduction
2. Two-class problems
3. Multi-class Problems
   - Heuristics

Outline

1. Introduction
2. Two-class problems
3. Multi-class Problems
   - Heuristics

Quantity-Based RM

- Most of our work thus far has been around price-based RM; changing prices to control demand.
- We will now focus instead of revenue-based RM – situations in which retailers control *capacity* to manage demand.

Quantity-Based RM

The cases we'll consider

- Inventory is limited and perishable
- Inventory is sold to a number of segments, at a number of different prices set ahead of time
- The inventory is interchangeable between classes
- Demand arrives over time, and we need to decide how much to 'reserve' for higher classes

Question

Which of the following situations meet the conditions on the previous slides

- A party that sells ‘standard’ tickets, and ‘VIP’ tickets that allow early admission into the venue.
- A theatre that sells ‘standard’ tickets, and ‘premium’ tickets for seats closer to the stage.
- A restaurant that only accepts reservations for groups of 6 or more.

Three Separate Problems

Quantity-based RM involves three separate, but related, problems

- Capacity allocation
- Overbooking
- Network capacity allocation

Quantity-Based RM in Different Industries

Industry	Capacity Unit	Capacity Types	Capacity Fixed?	Network
Passenger airlines	Seat on leg	1–10	mostly	Origin and destination
Hotel	Room/night	1 – 5	Yes	Length of stay
Rental car	Rental/day	1 – 10	No	Length of rental
Events	Seat	1 – 10	Yes	N/A
Cruise line	Berth	1 – 15	Yes	N/A
Freight	Weight, volume	1 – 3	Mostly	Origin and destination

The Basic Question

Suppose a customer is ‘on the phone’ requesting a particular fare. Do we. . .

The Basic Question

Suppose a customer is 'on the phone' requesting a particular fare. Do we. . .

Say yes , sell them the requested fare and get their revenue (what if we're already fully booked? Should we overbook?)

The Basic Question

Suppose a customer is 'on the phone' requesting a particular fare. Do we...

Say yes, sell them the requested fare and get their revenue (what if we're already fully booked? Should we overbook?)

Say no, to ensure we don't displace a future, more profitable bookings (or because we decide not to overbook).

The Basic Question

Suppose a customer is 'on the phone' requesting a particular fare. Do we. . .

Say yes , sell them the requested fare and get their revenue (what if we're already fully booked? Should we overbook?)

Say no , to ensure we don't displace a future, more profitable bookings (or because we decide not to overbook).

How is it Managed in Practice?

- Bookings limits
- Protection levels
- Bid prices

Outline

1 Introduction

2 Two-class problems

3 Multi-class Problems
- Heuristics

A Simple Two-Class Capacity Allocation Problem

- Suppose we have a fixed capacity C. There are two fare classes – a full class fare f_f and a discount fare f_d, here $f_f > f_d > 0$.

A Simple Two-Class Capacity Allocation Problem

- Suppose we have a fixed capacity C. There are two fare classes – a full class fare f_f and a discount fare f_d, here $f_f > f_d > 0$.
- Marginal costs are zero (for example, once a flight is scheduled, it doesn't cost any more to take an extra passenger onto the plane, assuming fuel costs aren't too high).

A Simple Two-Class Capacity Allocation Problem

- Suppose we have a fixed capacity C. There are two fare classes – a full class fare f_f and a discount fare f_d, here $f_f > f_d > 0$.
- Marginal costs are zero (for example, once a flight is scheduled, it doesn't cost any more to take an extra passenger onto the plane, assuming fuel costs aren't too high).
- Customers willing to pay full-fare don't arrive until later; earlier customers have lower WTPs.

A Simple Two-Class Capacity Allocation Problem

- Suppose we have a fixed capacity C. There are two fare classes – a full class fare f_f and a discount fare f_d, here $f_f > f_d > 0$.
- Marginal costs are zero (for example, once a flight is scheduled, it doesn't cost any more to take an extra passenger onto the plane, assuming fuel costs aren't too high).
- Customers willing to pay full-fare don't arrive until later; earlier customers have lower WTPs.
- No cancellations or no-shows.

A Simple Two-Class Capacity Allocation Problem

- Suppose we have a fixed capacity C. There are two fare classes – a full class fare f_f and a discount fare f_d, here $f_f > f_d > 0$.
- Marginal costs are zero (for example, once a flight is scheduled, it doesn't cost any more to take an extra passenger onto the plane, assuming fuel costs aren't too high).
- Customers willing to pay full-fare don't arrive until later; earlier customers have lower WTPs.
- No cancellations or no-shows.
- The demands at each fare are random variables d_f and d_d. We let

$$F_i(x) = \mathbb{P}(d_i \leq x) \text{ for } i = d, f$$

The Decision

Set a *booking limit* b while balancing
balance

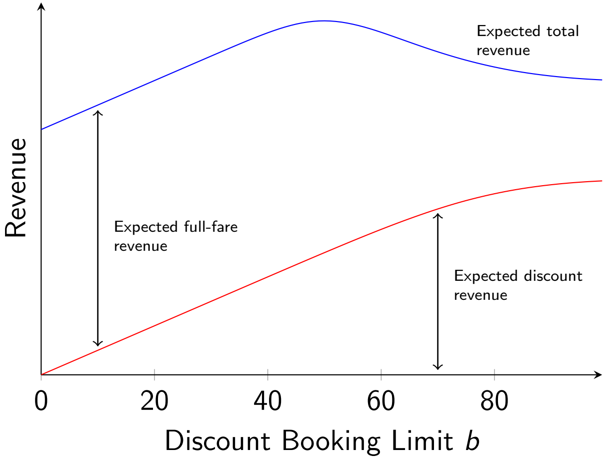
The Decision

Set a *booking limit b* while balancing
balance

Cannibalization : selling seats at the discount rate and having to turn away full-fare customers later

Spoilage : discount customers turned away, but seats remained unfilled

The Trade-Off



Setting the Booking Limit

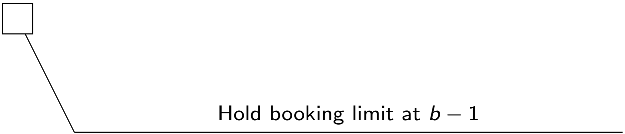
What is the effect of increasing the booking limit from $b-1$ to b?

Relative Impact

Setting the Booking Limit

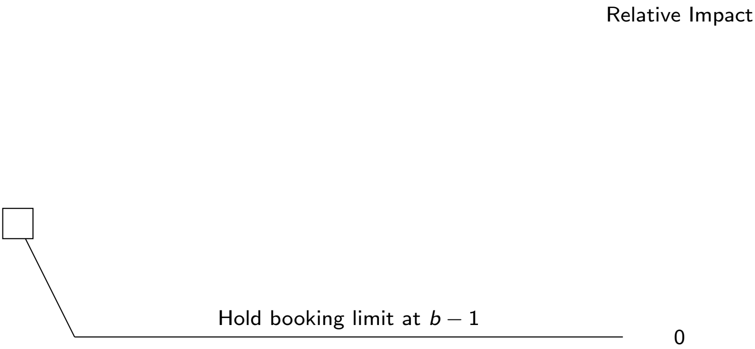
What is the effect of increasing the booking limit from $b-1$ to b?

Relative Impact



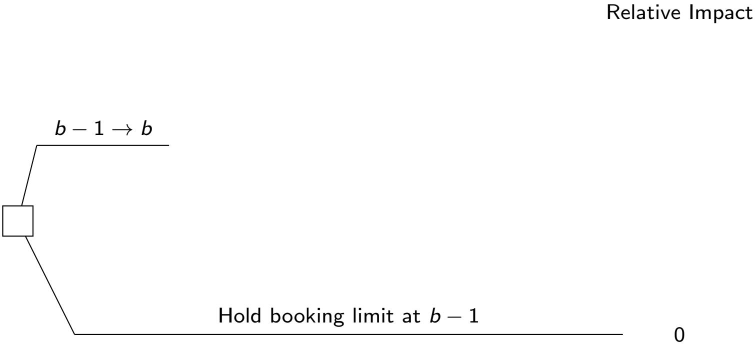
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



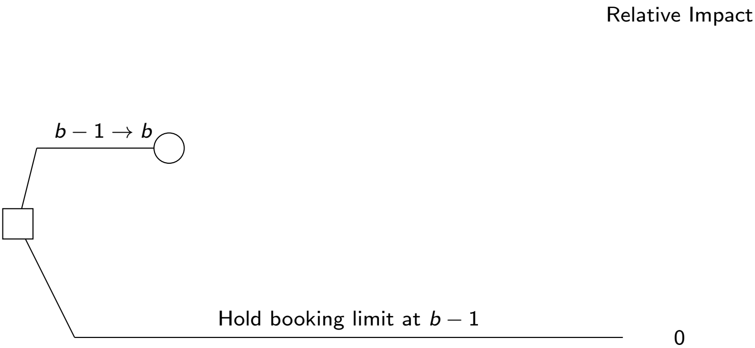
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



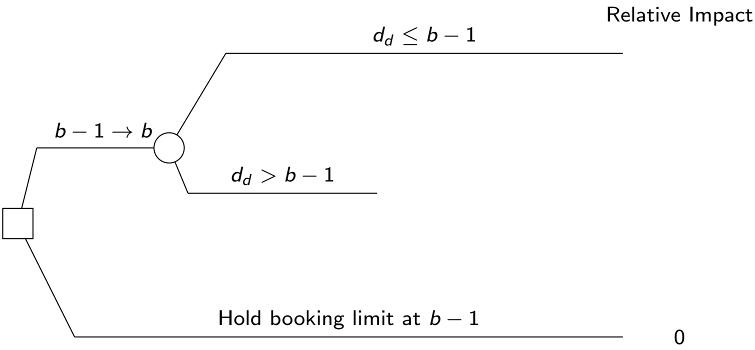
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



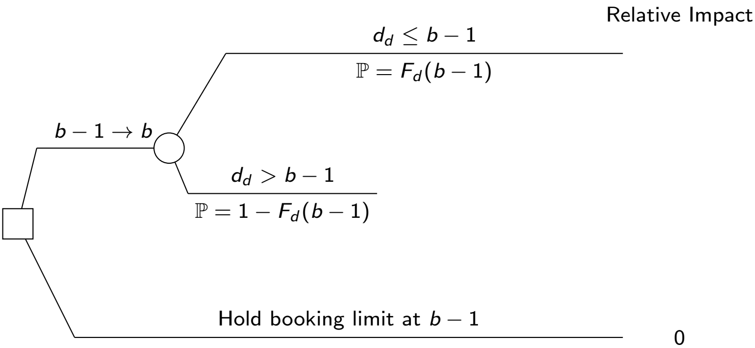
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



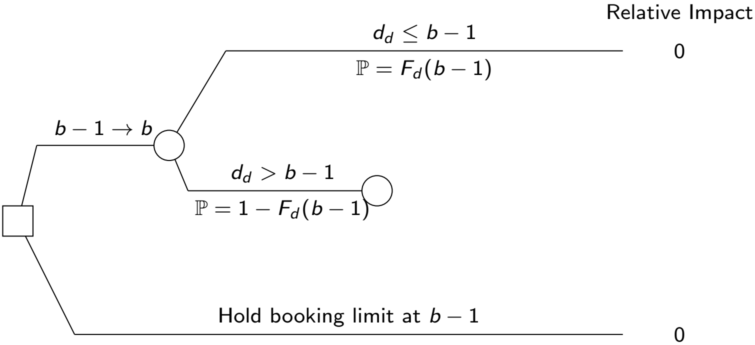
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



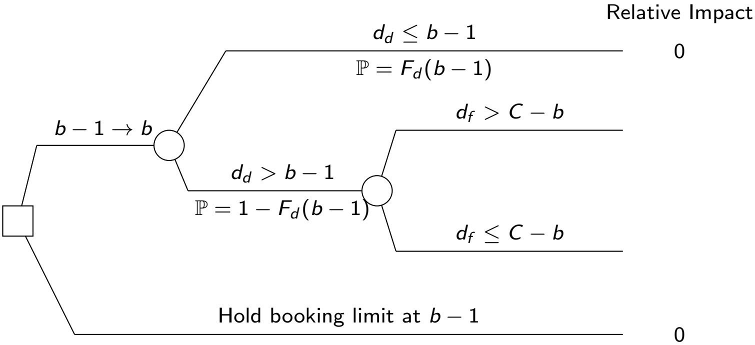
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



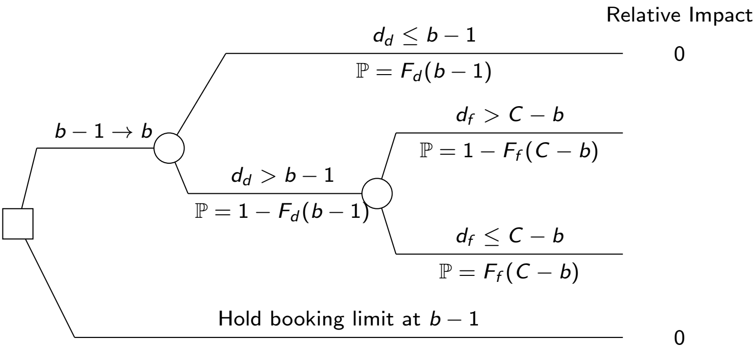
Setting the Booking Limit

What is the effect of increasing the booking limit from $b - 1$ to b?



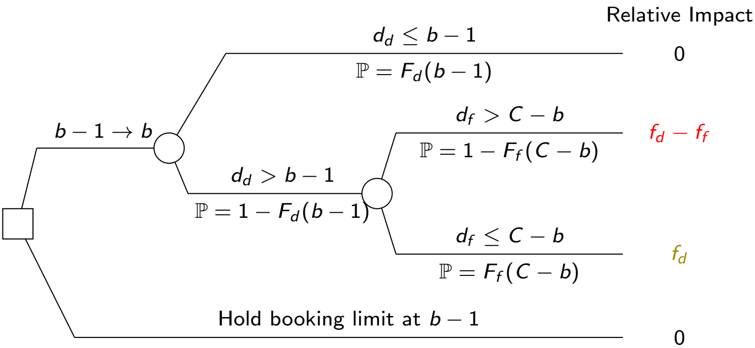
Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



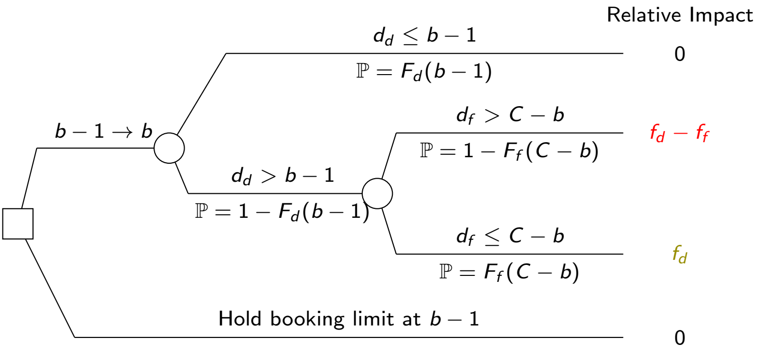
Setting the Booking Limit

What is the effect of increasing the booking limit from $b - 1$ to b?



Setting the Booking Limit

What is the effect of increasing the booking limit from $b-1$ to b?



$$\Rightarrow (1 - F_d(b-1))\left\{(f_d - f_f)\cdot[1 - F_f(C-b)] + f_d \cdot F_f(C-b)\right\}$$

Setting the Booking Limit

We have thus seen that increasing the booking limit from $b-1$ to b will result in the following expected impact to the profit

$$(1-F_d(b-1))\left\{(f_d-f_f)\cdot[1-F_f(C-b)]+f_d\cdot F_f(C-b)\right\}$$

Setting the Booking Limit

We have thus seen that increasing the booking limit from $b-1$ to b will result in the following expected impact to the profit

$$(1-F_d(b-1))\left\{(f_d - f_f)\cdot[1-F_f(C-b)] + f_d\cdot F_f(C-b)\right\}$$

Dividing by f_f and $(1-F_d(b-1))$ throughout, we can write this as

$$\frac{f_d}{f_f} - [1-F_f(C-b)]$$

Setting the Booking Limit

$$\frac{f_d}{f_f} - [1 - F_f(C - b)]$$

Setting the Booking Limit

$$\frac{f_d}{f_f} - [1 - F_f(C - b)]$$

Is this an increasing or decreasing function of b?

How can we use this to find the optimal booking limit?

Setting the Booking Limit

Thus, the optimal booking limit $b^\star$ satisfies *Littlewood's Rule*

$$b^\star = C - F_f^{-1}\left(1 - \frac{f_d}{f_f}\right)$$

Two interesting notes:

Setting the Booking Limit

Thus, the optimal booking limit $b^\star$ satisfies *Littlewood's Rule*

$$b^\star = C - F_f^{-1}\left(1 - \frac{f_d}{f_f}\right)$$

Two interesting notes:

- Protection level is independent of C.
- Booking limit is independent of F_d.

Example

You are in charge of revenue management at a scholastic publisher. Your company is about to publish a new book. The initial print run comprises 10,000 books.

You intend to sell some books in the USA at \$150 each, and some internationally, at \$75 each. Due to shipping time, demand first arrives from international booksellers and then from USA booksellers.

Market research on similar books has indicated that on average, demand for books in the USA is 8,000 and demand for books abroad is 23,000.

How many books should you reserve for USA booksellers?

Outline

Multi-class Problems

When there are more than two classes, things get a little more hairy...

- n classes $1, \cdots, n$, with prices $p_1 > p_2 > \cdots > p_n$.
- Demand for each class arrives sequentially in n non-overlapping stages – demand of the least expensive class (n) arrives first, followed by that for $n - 1$, etc... We denote demand for class j by D_j
- The demands for different classes are independent random variables
- Demand for a given class does not depend on the availability of other classes; our segmentation is perfect
- There are no group bookings – all bookings can be accepted one by one.

Multi-Class Problems – A Dynamic Program

Suppose we are at the beginning of stage j ($D_n, \cdots, D_{j+1}$ observed, but not $D_j, D_{j-1}, \cdots, D_1$)

Multi-Class Problems – A Dynamic Program

Suppose we are at the beginning of stage j ($D_n, \cdots, D_{j+1}$ observed, but not $D_j, D_{j-1}, \cdots, D_1$)

Our state is fully defined by the number of units of inventory remaining – we'll call it x.

Multi-Class Problems – A Dynamic Program

Suppose we are at the beginning of stage j ($D_n, \cdots, D_{j+1}$ observed, but not $D_j, D_{j-1}, \cdots, D_1$)

Our state is fully defined by the number of units of inventory remaining – we'll call it x.

We write

$$V_j(x) = \mathbb{E}\left[\max_{\text{Demand to fulfill}} \{\text{Revenue today} + \text{Revenue in the future}\}\right]$$

Multi-Class Problems – A Dynamic Program

$$V_j(x) = \mathbb{E}\left[\max_{\text{Demand to fulfill}} \{\text{Revenue today} + \text{Revenue in the future}\}\right]$$

Multi-Class Problems – A Dynamic Program

$$V_j(x) = \mathbb{E}\left[\max_{\text{Demand to fulfill}} \{\text{Revenue today} + \text{Revenue in the future}\}\right]$$

Mathematically

$$V_j(x) = \mathbb{E}\left[\max_{0 \le u \le \min(D_j, x)} \{p_j u + V_{j-1}(x-u)\}\right]$$

This is called the *Bellman Equation* of this dynamic program.

Multi-Class Problems – A Dynamic Program

We define the following notation

$$\Delta V_j(x) \equiv V_j(x) - V_j(x-1)$$

Multi-Class Problems – A Dynamic Program

We define the following notation

$$\Delta V_j(x) \equiv V_j(x) - V_j(x-1)$$

Two important properties

Multi-Class Problems – A Dynamic Program

We define the following notation

$$\Delta V_j(x) \equiv V_j(x) - V_j(x-1)$$

Two important properties

1. Every extra unit of inventory brings in less revenue than the previous one

$$\Delta V_j(x+1) \leq \Delta V_j(x)$$

Multi-Class Problems – A Dynamic Program

We define the following notation

$$\Delta V_j(x) \equiv V_j(x) - V_j(x-1)$$

Two important properties

1. Every extra unit of inventory brings in less revenue than the previous one
$$\Delta V_j(x+1) \leq \Delta V_j(x)$$

2. Extra inventory is more valuable the earlier you get it
$$\Delta V_{j+1}(x) \geq \Delta V_j(x)$$

Multi-Class Problems – A Dynamic Program

Using this new notation, we can take the Bellman Equation

$$V_j(x) = \mathbb{E}\left[\max_{0\leq u\leq \min(D_j,x)} \{p_j u + V_{j-1}(x-u)\}\right]$$

Multi-Class Problems – A Dynamic Program

Using this new notation, we can take the Bellman Equation

$$V_j(x) = \mathbb{E}\left[\max_{0 \le u \le \min(D_j, x)} \{p_j u + V_{j-1}(x-u)\}\right]$$

and re-write it as follows

$$V_j(x) = V_{j-1}(x) + \mathbb{E}\left[\max_{0 \le u \le \min(D_j, x)} \left\{\sum_{z=1}^{u} p_j - \Delta V_{j-1}(x+1-z)\right\}\right]$$

Multi-Class Problems – A Dynamic Program

Using this new notation, we can take the Bellman Equation

$$V_j(x) = \mathbb{E}\left[\max_{0 \le u \le \min(D_j, x)} \{p_j u + V_{j-1}(x-u)\}\right]$$

and re-write it as follows

$$V_j(x) = V_{j-1}(x) + \mathbb{E}\left[\max_{0 \le u \le \min(D_j, x)} \left\{\sum_{z=1}^{u} p_j - \Delta V_{j-1}(x+1-z)\right\}\right]$$

Recall $\Delta V_{j-1}(x)$ is decreasing in x. What does this mean about our optimal strategy?

Multi-Class Problems – A Dynamic Program

Optimal solution: pick the *largest* u so that $p_j \geq \Delta V_{j-1}(x + 1 - u)$.

Why does this make sense?

Multi-Class Problems – A Dynamic Program

Optimal solution: pick the *largest* u so that $p_j \geq \Delta V_{j-1}(x + 1 - u)$.

Why does this make sense?

More convenient to express this in terms of the *protection level* y_j – the amount of inventory that should be reserved in each period for future bookings of class $j, j-1, \cdots, 1$. The optimal protection level in period j is

$$y_j^\star = \max\{\psi : p_{j+1} < \Delta V_j(\psi)\}$$

Multi-Class Problems – A Dynamic Program

$$y_j^\star = \max\{\psi : p_{j+1} < \Delta V_j(\psi)\}$$

- Intuitive – accept orders until the revenue they bring becomes smaller than the marginal revenue we would get from having one extra unit of inventory in future periods.

Multi-Class Problems – A Dynamic Program

$$y_j^\star = \max\{\psi : p_{j+1} < \Delta V_j(\psi)\}$$

- Intuitive – accept orders until the revenue they bring becomes smaller than the marginal revenue we would get from having one extra unit of inventory in future periods.
- Because $p_1 > \cdots > p_n$ and $\Delta V_{j+1}(x) \geq \Delta V_j(x)$ (property 2 above), this result implies that $y_1^* \leq y_2^* \leq \cdots \leq y_N^*$. In other words, the optimal solution implies a *nested protection structure*.
- This can be interpreted in terms of *bid prices*.

Outline

EMSR-a

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.

EMSR-a

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.
- EMSR-a individually calculates the protection levels for each of these classes, and then simply adds them to find the total amount of inventory to protect for later classes.

EMSR-a

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.
- EMSR-a individually calculates the protection levels for each of these classes, and then simply adds them to find the total amount of inventory to protect for later classes.
- Thus, the total number of units to protect for classes $j, \cdots, 1$ under EMSR-a is given by

$$y_{j,\cdots,1} = \sum_{k=1}^{j} F_k^{-1}\left(1 - \frac{p_{j+1}}{p_k}\right)$$

EMSR-b

EMSR-b is also based on an approximation that reduces the problem to two classes at each step – but it is based on aggregating *demands* rather than *protection levels*.

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.

EMSR-b

EMSR-b is also based on an approximation that reduces the problem to two classes at each step – but it is based on aggregating *demands* rather than *protection levels*.

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.
- We let S_j denote the total demand in stage $j, \cdots, 1$, and $\bar{p}_j$ denote the *weighted average revenue* for these periods:

$$S_j = \sum_{k=1}^{j} D_k \qquad \bar{p}_j = \frac{\sum_{k=1}^{j} p_k \mathbb{E}[D_k]}{\sum_{k=1}^{j} \mathbb{E}[D_k]}$$

EMSR-b

EMSR-b is also based on an approximation that reduces the problem to two classes at each step – but it is based on aggregating *demands* rather than *protection levels*.

- Consider a period in which demand for class $j+1$ arrives – we need to figure out how much capacity to reserve for classes $j, \cdots, 1$.
- We let S_j denote the total demand in stage $j, \cdots, 1$, and $\bar{p}_j$ denote the *weighted average revenue* for these periods:

$$S_j = \sum_{k=1}^{j} D_k \qquad\qquad \bar{p}_j = \frac{\sum_{k=1}^{j} p_k \mathbb{E}[D_k]}{\sum_{k=1}^{j} \mathbb{E}[D_k]}$$

- The total number of units to protect for classes $j, \cdots, 1$ is then chosen using Littlewood's rule

$$y_{j,\cdots,1} = F_{S_j}^{-1}\left(1 - \frac{p_{j+1}}{\bar{p}_j}\right)$$

An Example

You have 1,000 tickets to sell for the quiddtich world cup final. The final is in three weeks. Die-hard supporters of a particular team are more likely to want to buy tickets *closer* to the final, when they know if their teams will be playing, whereas more casual fans won't be willing to pay as much for tickets, but will be willing to buy them earlier.

As such, you decide to price tickets as follows:

- This week, you'll price tickets at $100.
- Next week, you'll price tickets at $200.
- The week after (the week of the match), you'll price tickets at $250.

An Example

Your research team has carried out analyses based on the strength and weaknesses of each team, which have revealed that demands for tickets each week will be normally distributed and uncorrelated, with the following parameters

	μ	σ
Week 1 ($100 tickets)	1,000	300
Week 2 ($200 tickets)	525	50
Week 3 ($250 tickets)	275	75

Calculate the optimal pricing strategy using EMSR-a and EMSR-b.