Nonlinear Pricing

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Outline

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1. Pricing for Customer Segments
2. Bundling
3. Volume Discounts

Calculating Differentiated Prices

How to price our product to a segmented population?

$$w_1(p) = \begin{cases} w(p) & p \leq \hat{p} \\ 0 & \text{otherwise} \end{cases} \qquad w_2(p) = \begin{cases} w(p) & p \geq \hat{p} \\ 0 & \text{otherwise} \end{cases}$$

Calculating Differentiated Prices

How to price our product to a segmented population?

$$w_1(p) = \begin{cases} w(p) & p \leq \hat{p} \\ 0 & \text{otherwise} \end{cases}$$

and

$$d_1(p) = D \int_p^\infty w_1(x) \, \mathrm{d}x$$

$$w_2(p) = \begin{cases} w(p) & p \geq \hat{p} \\ 0 & \text{otherwise} \end{cases}$$

and

$$d_2(p) = D \int_p^\infty w_2(x) \, \mathrm{d}x$$

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We can then use these price-response functions to calculate the optimal prices for each segment.

Example

You sell a widget that exhibits a linear price-response function $d(p) = 100 - p$, and you succesfully segment your full population into two populations – one with willingness to pay $\leq$ \$40, and one with w.t.p $\geq$ \$40. The variable cost of each unit is \$10.

What price should you offer to the lower and upper segment respectively?

Example

First, calculate the w.t.p for the whole population

$$w(p) = \begin{cases} 1/100 & 0 \leq p \leq 100 \\ 0 & \text{otherwise} \end{cases}$$

The size of the population, D, is 100.

Example

Now calculate the w.t.p and price-response function for each segment.

$$w_1(p) = \begin{cases} 1/100 & 0 \leq p \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

And

$$\begin{aligned} d_1(p) &= 100 \int_p^\infty w_1(p) \\ &= \begin{cases} 100 \int_p^{40} (1/100)\ \mathrm{d}x & 0 \leq p \leq 40 \\ 0 & \text{otherwise} \end{cases} \\ &= 40 - p \end{aligned}$$

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$$w_2(p) = \begin{cases} 1/100 & 40 \leq p \leq 100 \\ 0 \text{otherwise} \end{cases}$$

And

$$\begin{aligned} d_2(p) &= 100 \int_p^{\infty} w_2(p) \\ &= \begin{cases} 100 \int_{40}^{100} (1/100)\, \mathrm{d}x & p \leq 40 \\ 100 \int_{p}^{100} (1/100)\, \mathrm{d}x & 40 \leq p \leq 100 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 60 & p \leq 40 \\ 100 - p & 40 \leq p \leq 100 \end{cases} \end{aligned}$$

Example

For segment 1, we recall that for a price-response function $A - Bp$, the optimal price is $(Bc + A)/2B$.

Thus, for segment 1, the optimal price is

$$p_2^\star = \frac{10 + 40}{2} = \$25$$

Example

For the second segment, we have two parts to our price-response function.

- If we're in the first part, where $p \leq 40$, the optimal price is clearly $p = 40$, and profit is $60 \cdot (40 - 10) = \$1,800$.
- If we're in the second part, the optimal price is at

$$\frac{10 + 100}{2} = \$55$$

  This is in range of the second part of the price-response function, and gives a profit of $(100 - 55)(55 - 10) = \$2,025$.

Thus

$$p_2^{\star} = \$55$$

Example

Without differentiation, the optimal price would've been

$$\frac{10 + 100}{2} = \$55$$

and so all the profit from the lower segment is what differentiation has allowed us to capture.

Calculating Differentiated Prices with Cannibalization

What happens if a proportion α of customers in the higher w.t.p group are able to purchase the item for the cheaper price?

Back to our example...

Before cannibalization

$$d_1(p) = 40 - p \qquad d_2(p) = \begin{cases} 60 & p \leq 40 \\ 100 - p & 40 \leq p \leq 100 \end{cases}$$

What are the new price-response functions with cannibalization ratio α?

Back to our example...

Before cannibalization

$$d_1(p) = 40 - p \qquad d_2(p) = \begin{cases} 60 & p \leq 40 \\ 100 - p & 40 \leq p \leq 100 \end{cases}$$

What are the new price-response functions with cannibalization ratio α?

$$d_1(p) = 60\alpha + (40 - p)^+ \qquad d_2(p) = (1 - \alpha) \begin{cases} 60 & p \leq 40 \\ 100 - p & 40 \leq p \leq 100 \end{cases}$$

An Example

Working through the same logic, optimal prices are

Lower segment : $\min(25 + 30\alpha, 40)$

Upper segment : \$ 55

Picking the Optimal Segments

What if you are fortunate enough to be able to pick the segmentation point?

In other words, we are able to choose a price ν such that

- The first segment contains customers with w.t.p $\leq \nu$
- The second segment contains customers with w.t.p $\geq \nu$

Picking the Optimal Segments

Let $d_1(p_1; \nu)$ denote the price-response function for the lower segment with split ν, and $d_2(p_2; \nu)$ the upper segment's. We let

$$m^\star(\nu) = \max_{p_1, p_2} [(p_1 - c)d_1(p_1; \nu) + (p_2 - c)d_2(p_2; \nu)]$$

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$$m^\star(\nu) = \max_{p_1,p_2}\left[(p_1 - c)d_1(p_1;\nu) + (p_2 - c)d_2(p_2;\nu)\right]$$

We then need to solve the following problem

$$\max_{\nu}\{m^\star(\nu)\}$$

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Bundling

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Works best when w.t.p of products being bundles are negatively correlated across segments.

Bundling vs. Tying

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Bundling example

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For simplicity, assume every member of each segment has the same willingness-to-pay, as follows

	IB	Trader	Lawyer	Consultant
Population	20,000	20,000	20,000	20,000
Excel	\$400	\$450	\$50	\$200
Word	\$200	\$50	\$450	\$400
Total	\$600	\$500	\$500	\$600

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Total	$600	$500	$500	$600

(Notice the inverse correlation between Excel and Word).

Separate products – Excel

Price at →	\$50	\$200	\$400	\$450
IB	✓	✓	✓	
Trader	✓	✓	✓	✓
Lawyer	✓			
Consultant	✓	✓		
Total revenue	\$ 4M	\$ 12 M	\$ 16 M	\$ 9M

We therefore price Excel at \$400.

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Trader	✓	✓	✓	✓
Lawyer	✓			
Consultant	✓	✓		
Total revenue	\$ 4M	\$ 12 M	\$ 16 M	\$ 9M

We therefore price Excel at \$400. Using similar logic, we price Word at \$400, with the same revenue.

The total revenue is : \$ 32 M

Bundle Only

Price at $\rightarrow$	\$500	\$600
IB	✓	✓
Trader	✓	
Lawyer	✓	
Consultant	✓	✓
Total revenue	\$ 40M	\$ 24 M

We therefore price the bundle at \$ 500.

The total revenue is : \$ 40 M

Bundled and Individual Products

- If we price the bundle at \$500, everyone buys it

$$\text{The total revenue is : \$ 40 M}$$

- If we price the bundle at \$600, IBs and Consultants buy it for a total profit of \$24M. Using the same methods as above, we optimally price Excel and Word at \$450 individually. Traders buy Excel, and Lawyers buy Word, at a total revenue of \$18M. Thus

$$\text{The total revenue is : \$ 42 M}$$

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Volume Discounts

Extending the Concept of Willingness to Pay

Instead of a willingness-to-pay, each customer gets a *utility function* $u(q)$

$u(q)$ is the utility derived by a customer from buying q units

(Could be defined only over the integers or continuous).

Customers as surplus maximizers

Suppose the cost to the consumer of buying q units is $p(q)$. We assume that consumers will pick the quantity that maximizes

$$u(q) - p(q)$$

Pricing Volume Discounts – An Extended Example

You are an executive at a major energy company in the North-East of America, and are tasked with pricing electricity contract for an important customer segment.

Extensive market research has revealed that members of this market segment value q MWH (Megawatt-hour) of energy per day at

$$u(q) = \begin{cases} 500\sqrt{q} & q \leq 9 \\ 1,500 & q > 9 \end{cases}$$

Each MWH of energy costs your company \$100 to produce and deliver to this segment of customers. Due to regulation, you are prohibited from charging any more than \$1000 per MWH.

A first question...

Suppose you price electricity at a fixed rate of \$$p$ per MWH. How much will each consumer buy?

A first question...

The utility the consumer will derive from buying q units at a price p will be

$$u(q) = \begin{cases} 500\sqrt{q} - pq & q \leq 9 \\ 1,500 - pq & q > 9 \end{cases}$$

Differentiating and setting to 0, we find that the maximum of the first part of the function occurs at $250^2/p^2$.

Thus,

$$q^\star(p) = \min\left(\frac{250^2}{p^2}, 9\right)$$

Single-Price Optimization

The producer will want to set the price p to maximize his profit,

$$(p - c) \cdot q^{\star}(p) = (p - 100) \cdot \min\left(\frac{250^2}{p^2}, 9\right)$$

Working through the algebra, we find the seller will set $p^{\star} = \$200$.

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The resulting profit for the producer is given by $q(p^{\star})(p^{\star} - c)$, and the resulting utility for the consumer is given by $u(q(p^{\star})) - p^{\star} q(p^{\star})$. Working through the algebra, we find

$$u^{p}_{\text{producer}} = 78 \qquad u^{p}_{\text{consumer}} = 156$$

Two-Price Optimization

Now, suppose the producer is able to set a triplet ξ, p_1, p_2 (with $p_1 > p_2$), such that the consumer pays p_1 per unit up to ξ units, and p_2 per unit thereafter.

Two-Price Optimization

Now, suppose the producer is able to set a triplet ξ, p_1, p_2 (with $p_1 > p_2$), such that the consumer pays p_1 per unit up to ξ units, and p_2 per unit thereafter.

The total profit to the producer is then given by

$$\Pi = \int_{q=0}^{\xi} (p_1 - c)\mathbb{I}\left[q^\star(p_1) \geq q\right] \mathrm{d}q + \int_{q=\xi}^{\infty} (p_2 - c)\mathbb{I}\left[q^\star(p_2) \geq q\right] \mathrm{d}q$$

Two-Price Optimization

$$\Pi = (p_1 - c) \min\left(\xi, 9, \frac{250^2}{p_1^2}\right) + (p_2 - c)\left[\min\left(9, \frac{250^2}{p_2^2}\right) - \min\left(\xi, 9, \frac{250^2}{p_2^2}\right)\right]$$

Need to maximize this with respect to ξ, p_1, and p_2.

First, it is clear that provided $p_1 > p_2 > c$ we should set

$$\xi^\star = \frac{250^2}{p_1^2}$$

Two-Price Optimization

We therefore have

$$\Pi = (p_1 - c)\min\left(9, \frac{250^2}{p_1^2}\right) + (p_2 - c)\left[\min\left(9, \frac{250^2}{p_2^2}\right) - \min\left(9, \frac{250^2}{p_1^2}\right)\right]$$

Assuming no pathological prices

$$\Pi = (p_1 - c)\frac{250^2}{p_1^2} + (p_2 - c)\left[\frac{250^2}{p_2^2} - \frac{250^2}{p_1^2}\right]$$

Two-Price Optimization

The algebra is too ghastly for words, but we end with

$$p_1^\star = \$320 \qquad\qquad p_2^\star = 160$$

Recall the single optimal price was $200.

Calculating the producer's and consumer's profit, we find that

$$u^{p_1,p_2}_{\mathrm{producer}} = 1.6u^{p}_{\mathrm{producer}} \qquad\qquad u^{p_1,p_2}_{\mathrm{consumer}} = 0.94u^{p}_{\mathrm{consumer}}$$

Infinite-Price Optimization

Suppose that the producer is actually able to set fully-customized prices for every incremental quantity q.

Infinite-Price Optimization

Suppose that the producer is actually able to set fully-customized prices for every incremental quantity q.

We would then set a price exactly equal to the customer's marginal utility everywhere:

$$p(q) = \begin{cases} \$1000 & q < 0.25^2 \\ 250/\sqrt{q} & 0.25^2 < q < 2.5^2 \\ \$100 & \text{otherwise} \end{cases}$$

Infinite-Price Optimization

Total profit is then

$$\Pi(p(q)) = \int_{q=0}^{0.25^2} (\$1000 - c)\,\mathrm{d}q + \int_{q=0.25^2}^{2.5^2} \left(\frac{250}{\sqrt{q}} - c \right)\,\mathrm{d}q$$

Working through the algebra, we find that

$$u^{\infty}_{\text{producer}} = 7.2 u^{p}_{\text{producer}} = 4.5 u^{p_1,p_2}_{\text{producer}}$$