Lecture 2

Pricing Fundamentals

Prof. Daniel Guetta

Outline

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1. What is Price Optimization, and why Should I Care?
2. Basic tools
   - Price response function
   - Willingness to pay
   - Elasticity
   - Common Price Response Functions

Why is Pricing Important

> "The single most important business decision in evaluating a business is pricing power."
>
> — Warren Buffet

A Brief History of Price

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"How much does the over-charging have to amount to in order that he who committed it shall be obligated to repay it? A sixth of the value of the article... constitutes fraud in which the transaction is valid but the defrauder has to pay the entire difference to the aggrieved party. If the overreaching amounts to anything less than that, the defrauder is not obligated to repay anything, because it is the general custom to waive the right to frauds amounting to less than a sixth. If the overreaching amounts to anything more than a sixth, the transaction is void and the aggrieved party may return the article and not buy it at all. The defrauder, however, may not retract if the aggrieved party wishes the transaction to stand..."

— Maimonides, Mishneh Torah, Laws of Sale 12:2-4

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Location	Price
74th and Broadway	$1.39
79th and Amsterdam	1.59
77th and Broadway	1.59
74th and Columbus	1.69
73rd and Columbus	1.79
74th and Amsterdam	1.79
75th and Broadway	1.89
71st and Columbus	1.99
78th and Amsterdam	2.00
AVERAGE	$1.75
STANDARD DEVIATION	0.20

Table: Retail prices for a half gallon of whole milk on the Upper West Side of Manhattan, May 2002.

What Defines a Commodity?

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- Large number of sellers and buyers.
- Arbitrage is cheap.
- All buyers and sellers are “very small” relative to the size of the market.
- Products are identical.
- All buyers and sellers have perfect information.
- No cost difference servicing different buyers.

Examples: zinc oxide, crude oil...

$\rightarrow$ no pricing decision.

What Drives Price Variations?

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- Product variation
- Brand strength/marketing effectiveness
- Customer service costs
- Channel costs (eg: online vs. on the phone)
- Strategic goals
- Market segmentation
- Changing costs
- Imperfect information sharing
- Changing competitive environment
- Cost differences among firms
- Product life cycle/obsolescence
- Organizational inconsistency
- Pricing errors
- Internal inertia

Dynamic pricing lies in these details – in the gap between commodities and items priced at their "intrinsic value".

Common Approaches to Pricing

Approach	Based on	Ignores	Liked by
Cost-plus	Costs	Competition, customers	Finance
Market based	Competition	Cost, customers	Sales
Value based	Customers	Cost, competition	Marketing

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- One per product/market segment/channel combination.
- Can result in thousands or even hundreds of thousands for any given situation.
- There may be interactions among products – then the price-response function for one product may be a function of the prices for other products.

The Demand Curve vs. the Price Response Function

The Demand Curve

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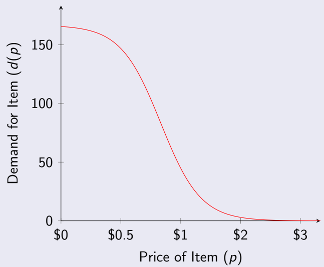
The Demand Curve

The Price Response Function



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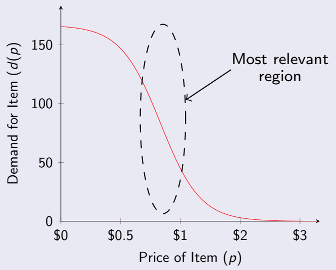


Properties of the Price Response Function

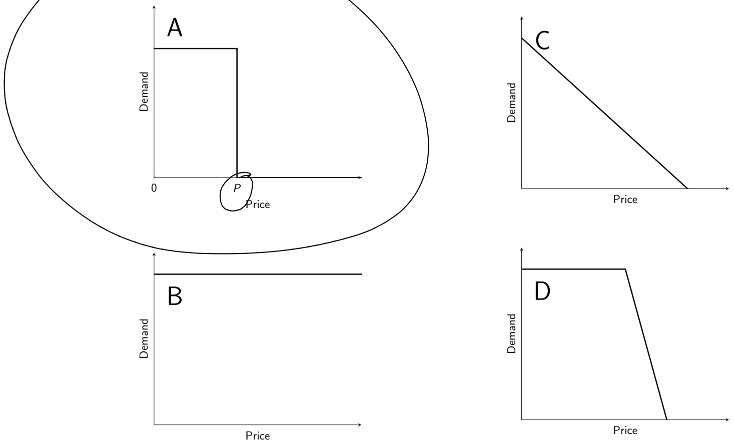
Nonnegative domain : We assume all prices are ≥ 0.

Continuous : We assume there are no "jumps" in the market response to our price. A corollary of this is that the price response function is *invertible*.

Downward sloping : we assume that – at any given time – if price increases, demand decreases.

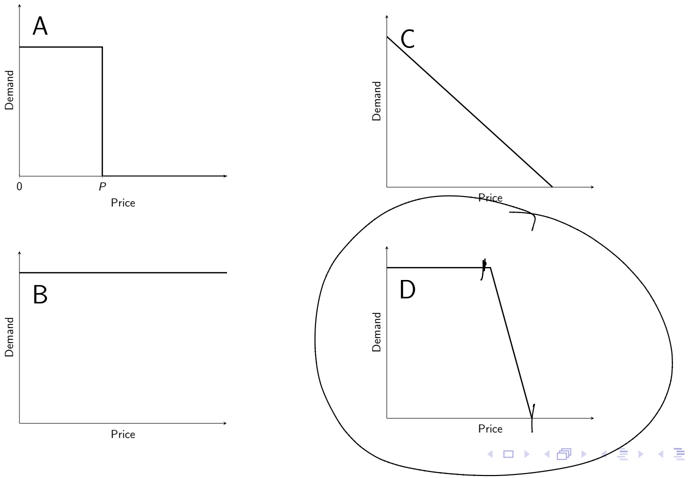
Question

Which of the following is most likely to represent the price response function for a commodity (eg: shares in the S&P 500)?



Question

Which of the following is most likely to represent the price response function for a luxury good (eg: a diamond ring)?



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The Willingness to Pay

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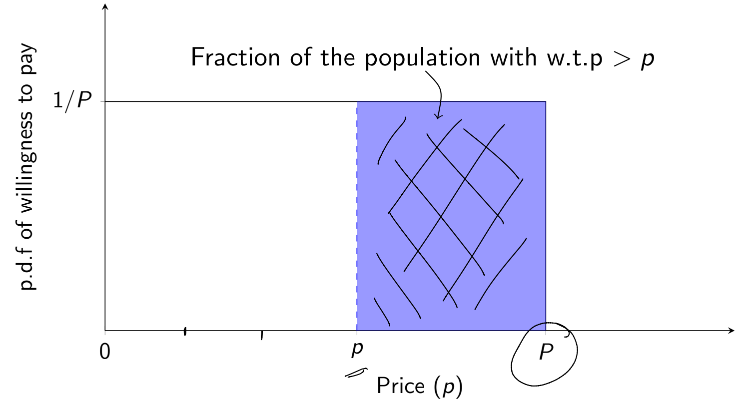
Model of consumer behavior with three assumptions

Assumption 1 : Potential customers each have a maximum *willingness to pay* for a product, which we denote w. Each customer will buy if the price $p \leq w$, and will not buy if $p > w$.

Assumption 2 : There is a fixed population of potential customers of size D. The size of this population is independent of the price.

Assumption 3 : Willingness-to-pay has a known distribution across the distribution of customers, which we shall call $w(p)$.

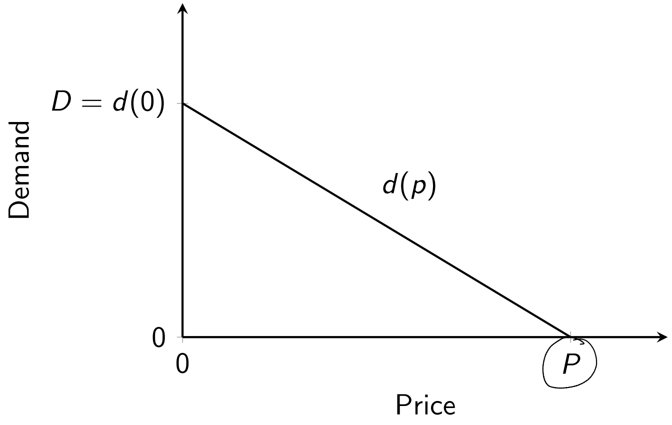
Example: the Uniform w.t.p Distribution



$d(p)$ = Size of population willing to pay more than p

$$= D \times \text{Blue area above} = D\left(\frac{P-p}{P}\right) = D - \frac{D}{P}p$$

Example: Uniform w.t.p $\rightarrow$ Linear Price Response Function



More Generally...

Suppose we have a population with willingness-to-pay distribution $w(p)$. We can derive the price response function $d(p)$ as follows

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&= D \times \text{Fraction of population with w.t.p} > p \\
&= D \int_p^{\infty} w(x)\,\mathrm{d}x
\end{aligned}
$$

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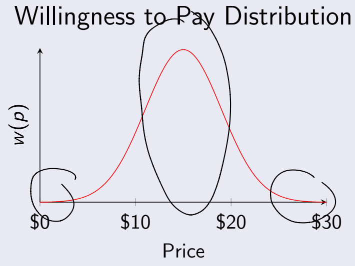
- Customers are myopic $\rightarrow$ game theory models.
- Customers only purchase a single item $\rightarrow$ bulk discount models.
- Infinite population model $\rightarrow$ segmentation and skimming.
- No competition $\rightarrow$ auction models.

Normal Willingness to Pay Distribution

Normal w.t.p. distribution



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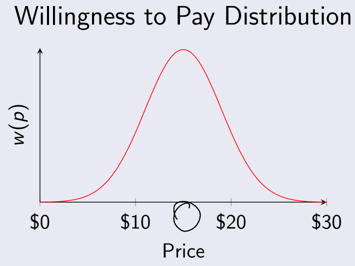

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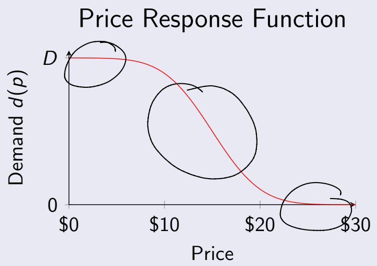
Probit price response curve



$$\begin{aligned} d(p) &= D \int_{-\infty}^{p} w(x)\,\mathrm{d}x \\ &= D\left[1 - \Phi\left(\frac{p-15}{4}\right)\right] \end{aligned}$$

Question

There are 100 students in the first year of the MS&E program, of which 30 are enrolled in Demand and Supply Analytics. Those enrolled in the class have a willingness to pay for the Phillips book that is uniformly distributed between \$100 and \$250. Those who are not have a willingness to pay uniformly distributed between \$0 and \$140.

What is the price-response function for the Phillips book among the population of first year students in the MS&E program?

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A *large* elasticity at price p means that increasing the price slightly will result in a *large* decrease in demand.

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- It strongly depends on the 'level of measurement'.

How would you rank the elasticity of demand for these products?

Salt
(10 lb. bulk)

Morton's Salt
1 lb. container

Le Saunier de Camargue Fleur de Sel
4.4 oz. container

Estimated Elasticities

Inelastic		$\epsilon(p) \approx 1$		Elastic	
Product	$\epsilon(p)$	Product	$\epsilon(p)$	Product	$\epsilon(p)$
Salt	0.1	Movies	0.9	Restaurant Dining	2.3
Matches	0.1	Tires (s.r.)	1.1	Airline travel (l.r.)	2.4
Gasoline (s.r.)	0.2	Tires (l.r.)	0.9	Fresh peas	2.8
Gasoline (l.r.)	0.7	Televisions	1.2	Autos (s.r.)	1.2
Coffee	0.25			Chevrolets (s.r.)	4
Cigarettes (s.r.)	0.45			Fresh tomatoes	4.6
Doctor services	0.6			Morton's Salt	6.5
Taxis (s.r.)	0.6				
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Table: Estimated elasticities for various items. (s.r. = short run, l.r. = long run). *Source: Gwatney and Stroup*.

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Question

Consider a product with the following price-response function:

$$d(p) = \begin{cases} 100 - 50p & p < 1 \\ 50e^{1-p} & 1 < p < 10 \\ -50e^{-9}p + 550e^{-9} & 10 < p < 11 \end{cases}$$

What is the size of the population, the willingness-to-pay distribution for that population, and the price-elasticity of demand? Plot each of these functions

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Whilst this price response function can be useful in a local sense, it often a poor estimator of *global* price response.

Sigmoidal Price Response Curves

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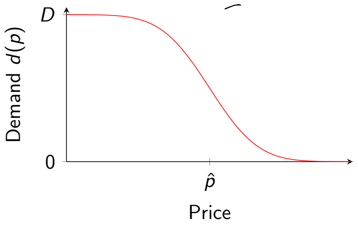
- Describe a reality in which customers are price sensitive near market price, but more insensitive far from it.
- We saw an example of a sigmoidal price response function above – the probit response curve:

$$d(p) = D\left[1 - \Phi\left(\frac{p-\hat{p}}{\sigma}\right)\right]$$



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More popular is the *logit* price response-function

$$d(p) = \frac{Ce^{-(a+bp)}}{1 + e^{-(a+bp)}}$$

What do the constants C, a, and b represent?