Tutorial Sheet - 2

Sol 1 ⇒

```
void fun (int n){
    int j = 1, i = 0;
    while (i < n){
        i = i + j;
        j++;
    }
}
```

Series = 0, 1, 3, 6, 10, 15 - - -

$$n = 0 + 1 + 2 + 3 + \dots + k$$

$$n = \frac{k(k+1)}{2}$$

$$n = \frac{k^2 + k}{2}$$

$$n \cong k^2$$

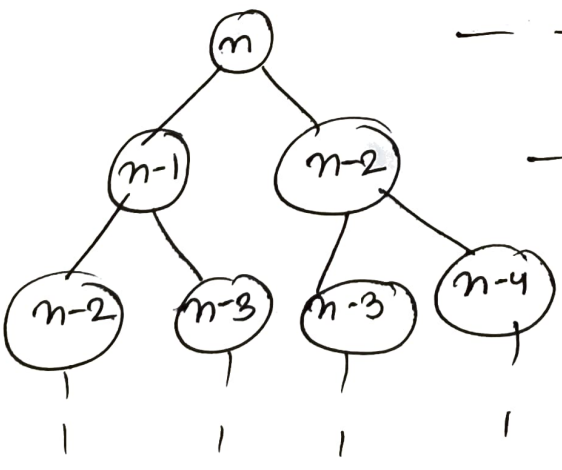
$$n \cong \sqrt{n}$$

Time Complexity $= O(\sqrt{n})$

Sol 2 ⇒ Recurrence relation for fibonacci series

$$T(n) = T(n-1) + T(n-2) + 1$$

using Recurrence ~~tree~~ tree method



— — 1

— — 2

— — 2^2

— — 2^n

Time complexity $= 1 + 2 + 4 + \cdots + 2^n$.

$$= \frac{1(2^{n+1} - 1)}{2 - 1} = 2^{n+1} - 1$$

or time complexity $= O(2^n)$.

Space complexity = space complexity of fibonacci series using recursion is proportional to height of binary tree.

so, space complexity $= O(n)$.

Sol 3 ⇒ Write code for complexity :-

(i) $n \log n$

```
for (i to n)
    { for (j = 1; j <= n; j *= 2)
          O(1) statements;
    }.
```

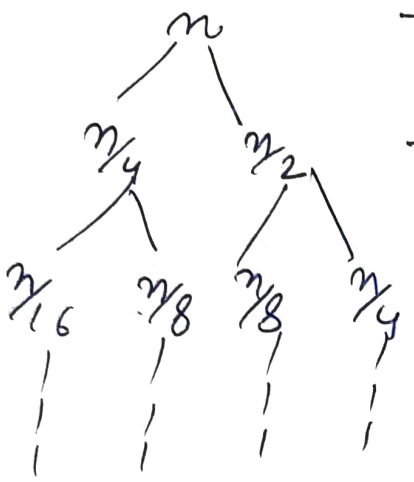
(ii) n^3

```
for (i to n)
    for (j = 1 to n)
        for (k to n)
            O(1) statements.
```

(iii) $\log(\log n)$.

```
int i = n;
while (i > 0)
    { i = √i;
    }
```

Sol 4 => $T(n) = T(n/4) + T(n/2) + cn^2$



$-cn^2$

$-\frac{cn^2}{16} + \frac{cn^2}{4} = \frac{5cn^2}{16}$

$-\frac{cn^2}{256} + \frac{cn^2}{64} + \frac{cn^2}{64} + \frac{cn^2}{16} =$

$\frac{25}{256} cn^2$

So, $T(n) = c\left(n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \cdots\right)$

Here, $r = \frac{5}{16}$ so, $S_n = \frac{1}{1-r}$

$T(n) = cn^2\left(1 + \frac{5}{16} + \frac{25}{256} + \cdots\right)$

$= cn^2\left(\frac{1}{1-5/16}\right)$

$= cn^2 \cdot \frac{16}{11}$

Time Complexity $= O(n^2)$

Sol 5 =>

```
int fun(int n){
    for (int i=1; i<=n; i++){
        for (int j=1; j<n; j+=i){
            // some O(1) task
        }
    }
}
```

i	j	time
1	1 to n	n-1
2	1 to n	(n-1)/3
3	1 to n	(n-1)/3
⋮	⋮	⋮
n	1 to n	(n-1)/n

$n \log n$

Time complexity = $O(n \log n)$

Sol 6 ⇒

```
for (int i = 0; i <= n; i = pow(i, k))
{
    // some O(1) expressions or
       statements
}
```

$i = 2, 2^k, 2^{k^2}, 2^{k^3}, \ldots 2^{k^x}$

$n = 2^{k^x}$

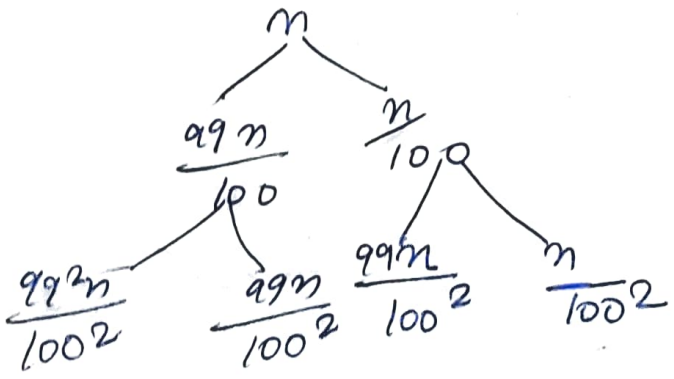
$\log n = k^x \log 2$

$\frac{\log \log n}{\log 2} = x \log k$

$x = \frac{\log \log n}{\log 2 * \log k}$

Time complexity = $O(\log \log n)$

Sol 7 ⇒



Taking longer branch that is $\frac{99n}{100}$

Time complexity $= \log_{\frac{100}{99}} n$

$\cong \log n$

$n = \left(\frac{99}{100}\right)^k$

or $k = \log\left(\frac{100n}{99}\right)$

$T(n) = n\left(\log \frac{100}{99}\right)^{n/100}$

Time Complexity $= O(n \log_{\frac{99}{99}} n)$

Sol 8 $\Rightarrow$ Increasing order of rate of growth:-

(a) $n, n!, \log n, \log\log n, \text{root}(n), \log(n!), n\log n, \log^2 n, \log 2^n, 2^{2^n}, 4^n, n^2, 100$

$100 < \log\log n < \log n < \sqrt{n}\ (\text{root}(n)) < n < n\log n < n^2 < 2^n < 2^{2^n} < 4^n < n!$

(b) $1 < \log\log n < \sqrt{\log(n)} < \log n < \log 2n < \log n < n < 2n < 4n < n\log n < n^2 < \log(n!) < 2^{2^n} < n!$

(c) $96 < \log 8^n < \log_2 n < 5n < n\log(n) < n\log_2 n < 8n^2 < 7n^3 < \log(n!) < 8^{2n} < n!$