Tutorial Sheet-4

Sol 1 ⇒ $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant and $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here.

$T(n) = \theta(n^2)$

Sol 2 ⇒ $T(n) = 4T(n/2) + n^2$.

$a = 4 \quad b = 2 \quad f(n) = n^2$

$\because$ a and b are const. and $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\Rightarrow n^c = n^2$

which is $n^2 = f(n)$

$\therefore$ case 2 is applied here

$T(n) = \theta(n^2 \log n)$

Sol 3 ⇒ $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1, b = 2, f(n) = 2^n$

∴ a & b are const. and f(n) is a +ve function

∴ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

∴ $f(n) > n^c$

∴ Case 3 is applied here

$\Rightarrow T(n) = \theta(2^n)$

Sol 4 ⇒ $T(n) = 2^n \, T\left(\frac{n}{2}\right) + n^n$

$a = 2^n \; b = 2 \; f(n) = n^n$

∴ a is not const, its value depends on n

∴ Master's theorem is not applicable here.

Sol 5 ⇒ $T(n) = 16 \, T\left(\frac{n}{4}\right) + n$

$a = 16 \quad b = 4 \quad f(n) = n$

∴ a and b are const., and f(n) is a +ve function.

∴ Master's theorem is applicable here.

$c = \log_b a = \log_4 16 = \log_4 (4)^2$

$= 2 \log_4 4 = 2.$

$\Rightarrow n^c = n^2$

∴ $f(n) < n^c$

∴ Case 1 is applied here.

$T(n) = \theta(n^2)$

Sol 6 ⇒ $T(n) = 2T(n/2) + n \log n$

$a = 2$ $b = 2$ $f(n) = n \log n$

∴ a and b are const. & $f(n)$ is a +ve function

∴ $C = \log_b a$

$\log_2 2 = 1$

$n^c = n$

$n < n \log n \Rightarrow f(n) > n^c$

∴ Case 3 is applied.

$\Rightarrow T(n) = \theta(n \log n)$.

Sol 7 ⇒ $T(n) = 2T(\frac{n}{2}) + n/\log n$.

$a = 2$ $b = 2$ $f(n) = n/\log n$

∴ a and b are const. and $f(n)$ is a +ve function $C = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

∴ non- polynomial difference. b/w $f(n)$ & n^c

∴ Master's theorem is not applicable.

Sol 8 ⇒ $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

$a = 2$ $b = 4$ $f(n) = n^{0.51}$

∴ a and b are const. and $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applicable.

$T(n) = \Theta(n^{0.50})$.

Sol 9 ⇒ $T(n) = 0.5 \quad T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5 \quad b = 2 \quad f(n) = \frac{1}{n}$

$\therefore a < 1$

$\therefore$ Master's theorem is not applicable.

Sol 10 ⇒ $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$\therefore$ a and b are const. and f(n) is a +ve function.

Master's theorem is applicable.

$C = \log_b a$.

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applied here

$T(n) = \Theta(n!)$.

Sol 11 ⇒ $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

∵ a and b are constant and f(n) is a +ve f^n.

∴ Master's theorem is applicable.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$.

$n^c = n^2$

∵ $f(n) < n^c$

∴ case 1 is applied.

$T(n) = \Theta(n^2)$.

Sol 12 ⇒ $\sqrt{n}\ T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{n},\ b = 2,\ f(n) = \log n$

∵ a is not constant.

∴ Master's theorem is not applicable.

Sol 13 ⇒ $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3 \quad b = 2 \quad f(n) = n$

∵ a & b are const & f(n) is a +ve function.

∴ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 01.58$.

$n^c = n^{1.58}$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here.

$T(n) = \theta(n^{1.58})$.

Sol 14 $\Rightarrow$ $T(n) = 3T(n/3) + \sqrt{n}$.

$\because$ a and b are const. and f(n) is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ case 1 is appliable.

$T(n) = \theta(n)$

Sol 15 $\Rightarrow$ $T(n) = 4T\left(\frac{n}{2}\right) + C.n$.

$a = 4,\ b = 2\ f(n) = C.n$

$\because$ a and b are constant and f(n) is a +ve f^n.

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here

$\Rightarrow T(n) = \theta(n^2)$

Sol 16 ⇒ $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3, b = 4 \quad f(n) = n\log n$

∴ a and b are constant & f(n) is a +ve function.

∴ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

∴ $f(n) > n^c$

∴ case 3 is applicable here.

⇒ $T(n) = \theta(n\log n)$.

Sol 17 ⇒ $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3, \; b = 3, \; f(n) = \frac{n}{2}$.

∴ a & b are const. and f(n) is a +ve fn.

∴ Master's theorem is applicable here.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

∴ $f(n) = n^c$.

∴ case 2 is applied here

⇒ $T(n) = n\log n$

Sol 18 ⇒

$T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

$a = 6 \quad b = 3 \quad f(n) = n^2\log n$

∴ a & b are const. and f(n) is a +ve function.

∴ Master's theorem is applicable here.

$$c = \log_b a = \log_3 6 = 1.63$$

$$n^c = n^{1.63}$$

$$\therefore f(n) > n^c$$

$\Rightarrow$ case 3 is applied here.

$$\Rightarrow T(n) = \Theta(n^2 \log n).$$

Sol 19 $\Rightarrow$ $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

$a = 4$, $b = 2$, $f(n) = n/\log n$.

∴ a & b are const. and $f(n)$ is a +ve f^n.

∴ master's theorem is applicable here

$$c = \log_b a$$
$$= \log_2 4$$
$$= \log_2 2^2$$
$$= 2\log_2 2 = 2$$

$$n^c = n^2$$

$$\therefore f(n) < n^2$$

$$\therefore f(n) < n^c$$

∴ case 1 is applied here

$$\Rightarrow T(n) = \Theta(n^2).$$

Sol 20 ⇒ $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$ (9)

∴ a and b are const. but f(n) is a -ve function

∴ Master's theorem is not applicable.

Sol 21 ⇒ $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

∴ a, b are const. & f(n) is a +ve f n.

∴ Master's theorem is applied here

⇒ $c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

∴ $f(n) \geq n^c$

∴ Case 3 is applied here.

$T(n) = \Theta(n^2)$

Sol 22 ⇒ $T(n) = T(n/2) + n(2 - \cos n)$

∴ f(n) is not regular function.

∴ Master's theorem can not be applied here.