Question2

1. Objective

Derive a simple analytical model that links power consumption to thrust, drag, climb rate, and UAV mass, and relate mechanical power, aerodynamic efficiency, and battery power .

2. Main Equation

$$P_{\text{mechanical}} = P_{\text{drag}} + P_{\text{climb}} + P_{\text{induced}} \tag{1}$$

This fundamental power decomposition is widely used in rotorcraft and UAV performance modeling [1, 2].

3. Definition of Each Term

- **Drag (Parasite) Power:**

$$P_{\text{drag}} = D\,V = \tfrac{1}{2}\rho C_D A\,V^3 \tag{2}$$

  This represents the power required to overcome aerodynamic drag [4, 5].

- **Climb Power:**

$$P_{\text{climb}} = mgV\sin\gamma = mgv_{\text{climb}} \tag{3}$$

  The rate of change of potential energy; directly proportional to the climb rate [6, 7].

- **Induced Power:**

$$P_{\text{induced}} = \kappa\frac{(mg)^{3/2}}{\sqrt{2\rho A}} \tag{4}$$

  Derived from actuator-disk momentum theory [2, 1, 3]. The factor κ accounts for non-ideal inflow and tip losses.

- **(Optional) Acceleration Power:**

$$P_{\text{acc}} = ma_v v_{\text{climb}} \tag{5}$$

  Additional power term when the UAV experiences vertical acceleration [2].

4. Combined Mechanical Power

$$P_{\text{mechanical}} = \tfrac{1}{2}\rho C_D A\,V^3 + mgv_{\text{climb}} + \kappa\frac{(mg)^{3/2}}{\sqrt{2\rho A}} \tag{6}$$

As applied to UAVs in [3, 8, 9].

5. Battery Power Relationship

$$P_{\text{battery}} = \frac{P_{\text{mechanical}}}{\eta_{\text{total}}} \tag{7}$$

where η_{total} represents the overall propulsion efficiency (motor, ESC, propeller, and electrical losses) [10, 3].

6. Compact Linear Fitting Form

$$P_{\text{battery}} = \alpha V^3 + \beta v_{\text{climb}} + \gamma \tag{8}$$

with parameter mapping:

$$\alpha = \frac{1}{2}\rho C_D A/\eta_{\text{total}}, \tag{9}$$

$$\beta = mg/\eta_{\text{total}}, \tag{10}$$

$$\gamma = \frac{\kappa(mg)^{3/2}}{\eta_{\text{total}}\sqrt{2\rho A}} \tag{11}$$

This linearized model form is used for regression fitting on flight-data segments [8, 3].

7. Parameter Identification from Flight Phases

- **Hover:** $V \approx 0, v_{\text{climb}} \approx 0 \Rightarrow$ identifies γ (induced term).
- **Cruise:** $v_{\text{climb}} \approx 0 \Rightarrow$ fit P_{batt} vs. V^3 to estimate α ($C_D A/\eta_{\text{tot}}$).
- **Climb/Descent:** $v_{\text{climb}} \neq 0 \Rightarrow$ fit P_{batt} vs. v_{climb} to get $\beta = mg/\eta_{\text{tot}}$.

References

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