

Ch 4

(1) Payback period (PBP)

- Simple payback period (Time value consider नहीं)
- Discounted payback period (consider सही method)

(2) Equivalent worth period

- Present worth (PW)
- Future worth (FW)
- Annual worth (AW)

(3) Rate of Return method

- Internal rate of return (IRR)
- External rate of return (ERR)

(4) Benefit cost ratio method

- Conventional method
- Modified method.

(5) Minimal Attractive rate of Return (MARR)

(6) Payback Period

Payback period Method

The payback period is the length of time required to recover the cost of an investment.

Simple payback period.

The time period required to break even on an investment without considering time value of money.

$$SPP(D) = \frac{\text{Initial Investment}}{\text{Net cash inflow per period.}}$$

Discounted period Method.

The time period required to recover the investment from discounting cash flow ie considering time value of money.

Ex 1

Determine the simple PBP and discounted PBP from the FF cashflow of the project if MARR is 20% per year

End of the year	Cashflow
0	-25000
1	8000
2	8000
3	8000
4	8000
5	13000

So PBP

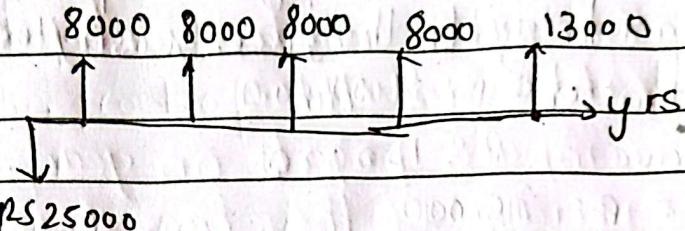
Year	Net CF (Rs)	Cumulative CF (Rs)
0	-25000	-25000
1	8000	-17000
2	8000	-9000
3	8000	-1000
4	8000	+7000
5	13000	+20000

$$\text{Now } \text{SPBP} = 3 + \frac{1000}{8000}$$

$$= 3.125 \text{ yrs}$$

i.e < 5 years. accepted

DPBP



$$P \cancel{F} (1+i)^{-N}$$

$$\text{Eq1 } P_1 = F_1 (1+i)^{-N} = 8000 (1+0.20)^{-1} = \text{Rs } 6667$$

$$\text{Eq2 } P_2 = F_2 (1+i)^{-N} = 8000 (1+0.20)^{-2} = \text{Rs } 5556$$

$$\text{Eq3 } P_3 = F_3 (1+i)^{-N} = 8000 (1+0.20)^{-3} = \text{Rs } 4630$$

$$\text{Eq4 } P_4 = F_4 (1+i)^{-N} = 8000 (1+0.20)^{-4} = \text{Rs } 3858$$

$$P_5 = F_5 (1+i)^{-N} = 13000 (1+0.20)^{-5} = \text{Rs } 5224$$

Year	Net cash	Discounted payback	Cumulative cash (Rs)
0	-25000	6667	-25000
1	8000	5556	-18333
2	8000	4630	-12777
3	8000	3858	-8147
4	8000	3858	-4289
5	13000	5224	+935

$$\text{DPBP} = 4 + \frac{4289}{5224} = 4.82 \text{ yrs} < 5 \text{ yrs, accepted}$$

But it is almost 5 yrs, may be rejected.

2015 Fall, 2017 Fall

Find simple and discounted payback period and justify investment with the given cash flow info.

Initial investment = Rs 4,00,000

Annual revenue = Rs 15,000

Annual cost = Rs 3,000

Annual life = 5 years

Annual MARR = 10%

Salvage value = Rs 1,00,000

So I/P

$S = Rs 1,00,000$

$$AR = Rs 15,000$$

$$MARR = 10\%$$

$$Rs 4,00,000 \quad Ae = 3,000$$

SPBP

Year	Net cash	Cumulative
0	-4,00,000	-4,00,000
1	12,000	-12,80,000
2	12,000	-16,000
3	12,000	-4,000
4	12,000	+8,000
5	22,000	+3,00,000

$$SPBP = 3 + \frac{4,00,000}{12,000} = 3.33 \text{ yrs} < 5 \text{ yrs}$$

Accepted.

i) Discounted payback period.

EoY	Net cashflow	Discounted payback	cumulative
0	-400000	-400000	-400000
1	120000	109090.90	-290909
2	120000	99174.99 99173.55379	-191735
3	120000	90158	-101577
4	120000	81961	-19,616
5	220000	136603	1,16,987

$$\text{Discounted payback period} = 4 + \frac{19616}{136603}$$

$$= 4.14 \text{ years}$$

Project is feasible

Equivalent worth Method

Future worth - compare which project gives more future wealth.

Annual worth: compare projects on yearly earnings, especially if they have diff life times.

Present worth: Decide if a project is worth starting.

Decision Rule

if $PW(i_Y) > 0$, accepted

if $PW(i_Y) = 0$, then remain ~~in some~~

if $PW(i_Y) < 0$, rejected.

Evaluate the XYZ project whether it is feasible or not to invest, whose cash flow details follows.

Initial investment = Rs 450000

Annual revenue = Rs 200000

Annual cash = Rs 50000

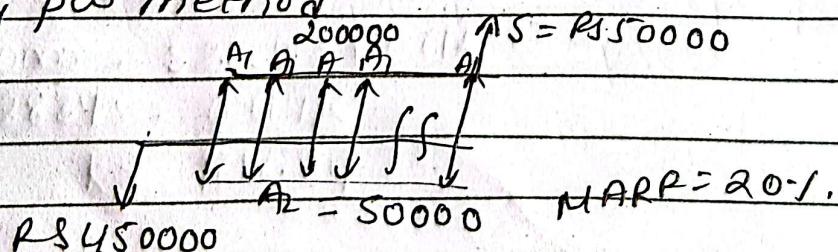
Salvage value = Rs 50000

Useful life = 8 yrs

MARR = 20%.

Use PW, FW, AW method.

By PW method



$$A_1 = 200000$$

$$A_2 = 80000$$

$$A = 150000$$

By PW

$$\begin{aligned}
 PW(i) &= A(P/A, i\%, N) + F(P/F, i\%, N) - P_0 \\
 &= 150000(P/A, 20\%, 8) + 50000(P/F, 20\%, 8) - 450000 \\
 &= \text{Rs } 1,37,210 \\
 i - e > 0 \quad \therefore & \text{ accepted}
 \end{aligned}$$

By FW

$$\begin{aligned}
 FW &= A(F/A, i\%, N) + F - P(F/P, i\%, N) \\
 &= 150000(F/A, 20\%, 8) + 50000 - 450000(F/P, 20\%, 8) \\
 &= \text{Rs } 58,995.5 \\
 i - e > 0 \quad \therefore & \text{ accepted}
 \end{aligned}$$

By AW method

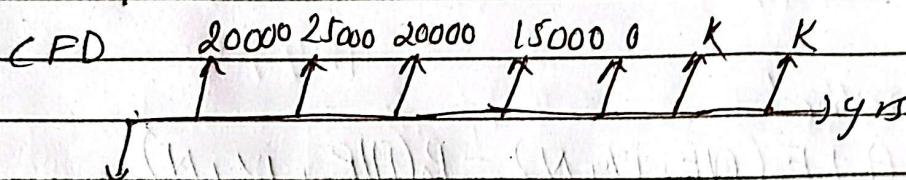
$$\begin{aligned}
 AW(i) &= A + F(A/F, i\%, N) - P_0(A/P, i\%, N) \\
 &= 150000 + 50000(A/F, 20\%, 8) - 450000(A/P, 20\%, 8) \\
 &= \text{Rs } 35760
 \end{aligned}$$

$i - e > 0$, accepted.

Ex 2

Find the value of k , from the FF : cashflow if
MARR is 20%.

EOY	Net cash flow
0	-50000
1	20000
2	25000
3	20000
4	15000
5	0
6	k
7	k



Eg:

$$PW = 20000(P/F, 20\%, 1) + 25000(P/F, 20\%, 2) + 20000(P/F, 20\%, 3) \\ + 15000(P/F, 20\%, 4) + k(P/F, 20\%, 6) + k(P/F, 20\%, 7)$$

$$50000 = 20000 \times 0.8337 + 25000 \times 0.6944 + 20000 \times 0.5787 + \\ 15000 \times 0.4823 + 0 + k \times 0.3349 + k \times 0.2791$$

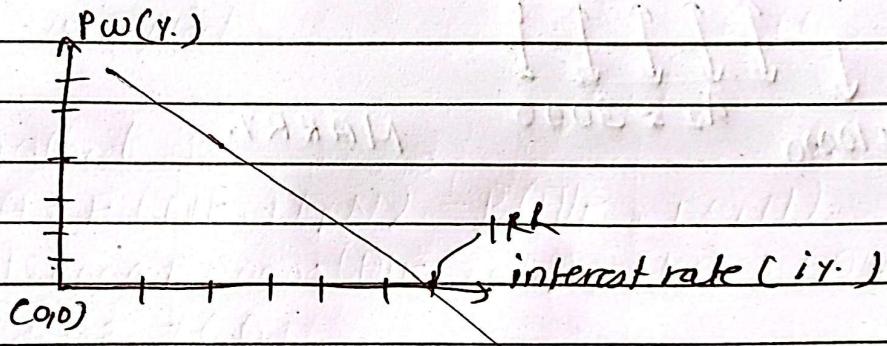
$$50000 = 52834.5 + 0.61k$$

$$-2834.5 = 0.61k$$

$$k = 4616.44$$

Internal Rate of Return (IRR)

- is the interest rate at which the PW of all cash flow become zero.
- It is the break-even rate of return for a project
 $\text{cash inflow} = \text{cash outflow}$
- The equivalent worth can be computed by different ways like PW, FW, AW formula.
- The resultant interest rate is called IRR.
- This method is also called hit and trial method or assumption method.

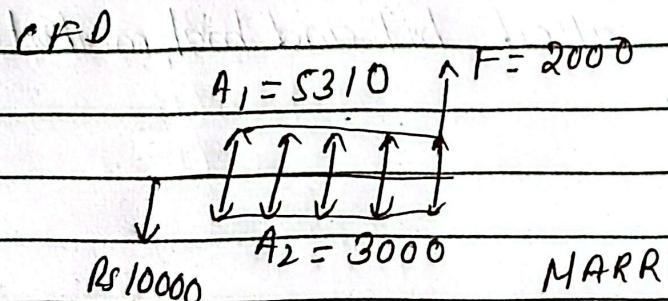


Decision Rule

- if $IRR > MARR$, then accepted
- if $IRR = MARR$, then remaining indifferent
- if $IRR < MARR$, then rejected.

Determine IRR of the FF : project whose cash flows are as follow:

- Initial investment = Rs 10,000
- Project life = 5 yrs
- Annual Revenue = Rs 5310
- Annual expenses = Rs 3000
- SV = Rs 2000
- MARR = 8%



$$A = 5310 - 3000 = 2310$$

$$\begin{aligned}
 PW(i^*) &= A(P/A, i^*, N) + F(P/F, i^*, N) - P \\
 &= 2310(P/A, 8\%, 5) + 2000(P/F, 8\%, 5) - 10000
 \end{aligned}$$

Assume $i^* = 7\%$

$$\begin{aligned}
 PW(i^*) &= 2310 \times 4.1002 + 2000 \times 0.7130 - 10000 \\
 &= 897.462
 \end{aligned}$$

Assume $i = 10\%$.

$$PW(i) = -1.452$$

formula for IRR

$$IRR(i\%) = \text{Interest of LR} + \frac{PWF_{LR}}{PWF_{IR}} (MARR - LR)$$
$$= 7\% + \frac{897.462}{897.462 + 1.452} (10\% - 7\%)$$
$$= 9.99\%$$

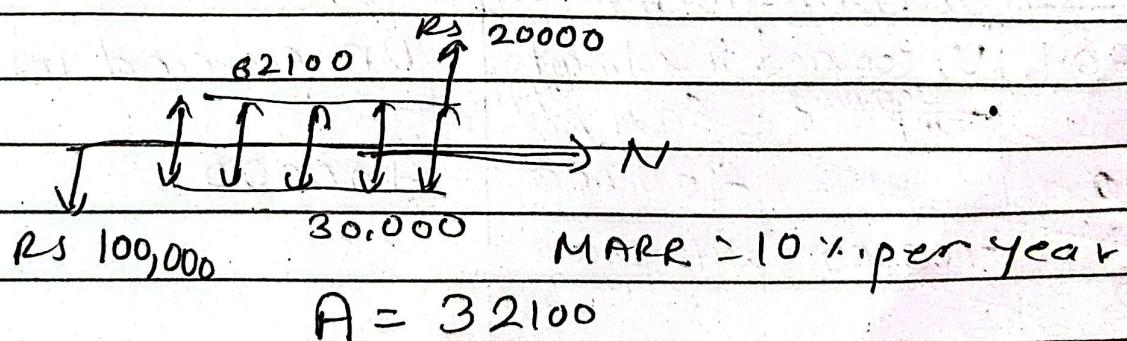
> MARR. accepted

Q2 2018 spring

An investment of RS 100,000 can be made in a project that will produce uniform annual revenue of RS 62100 for 5 years and then have a market salvage value of 20,000. Annual expenses will be RS 30,000 each year.

Company accepts project that earns 10% of more, evaluate IRR of this project & suggest whether the project is feasible or not? Also draw an investment balance diagram.

SOLN



using PW formulation

$$PW(i) = \cancel{62100} \cdot 32100 (PIA, i, 5) + 20000 (PIF, i, 5) - 100,000$$

if ~~i~~ $i = 10\%$.

$$PW = 34102.68$$

If $i = \cancel{18\%} 20\%$.

$$PW = 4036$$

If $i = \cancel{20\%} 22\%$.

$$PW = -686.86$$

$$IRR = 20 + \frac{4036}{4036 + 686.86} (22 - 20)$$

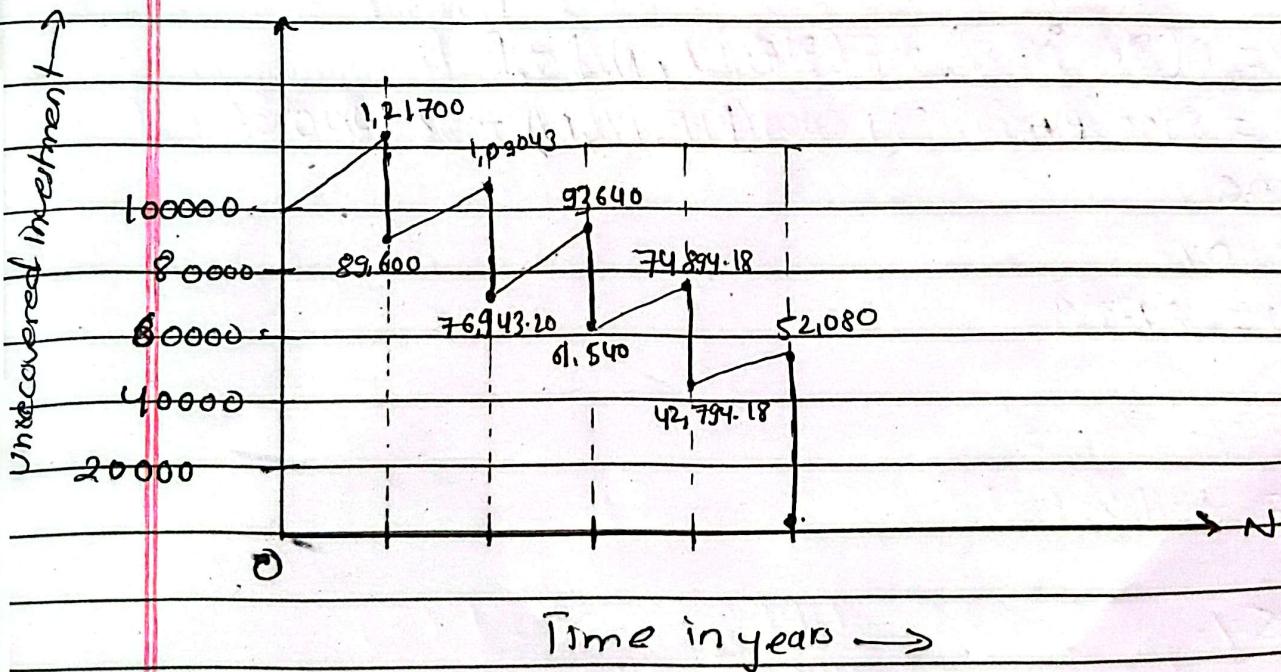
$$= 21.70$$

$> MARR$.

The project is feasible.

Unrecoverable investment calculation

Year	Cash	Unrecoverable Investment		Unrecoverable amount (Cash + investment end)
		Beginning	End (21.7.1) <small>PxI_t+P</small>	
0	100000	-	-100000	-100000
1	32100	-100000	-121700	-89600
2	32100	-89600	-109043	-76943.2
3	32100	-76943.20	-93640	-61540
4	32100	-61540	-74894.18	-42794.18
5	52100	-42794.18	-52080	-80



Q.

Compute the IRR for the FF : project and also show unrecov.
Investment balance in the graphical and tabular form.

The cash flow are as follow

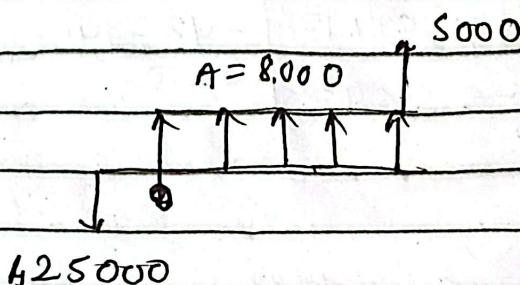
Initial Investment = Rs 25000

Net annual Revenue = Rs 8000

$$SV = 5000$$

$$\text{useful life} = 5 \text{ yrs}$$

$$MARR = 20\%$$



$$PW = A(PIA, i^*, N) + F(PIF, i^*, N) - P$$

$$= 8000(PIA, 10\%, 5) + 5000(PIF, 10\%, 5) - 25000$$

Assume

$$i = 10\%$$

$$PW = 8430.9$$

$$i = 20\%$$

$$PW = 934.3$$

$$i = 25\%$$

$$PW = -1347.10$$

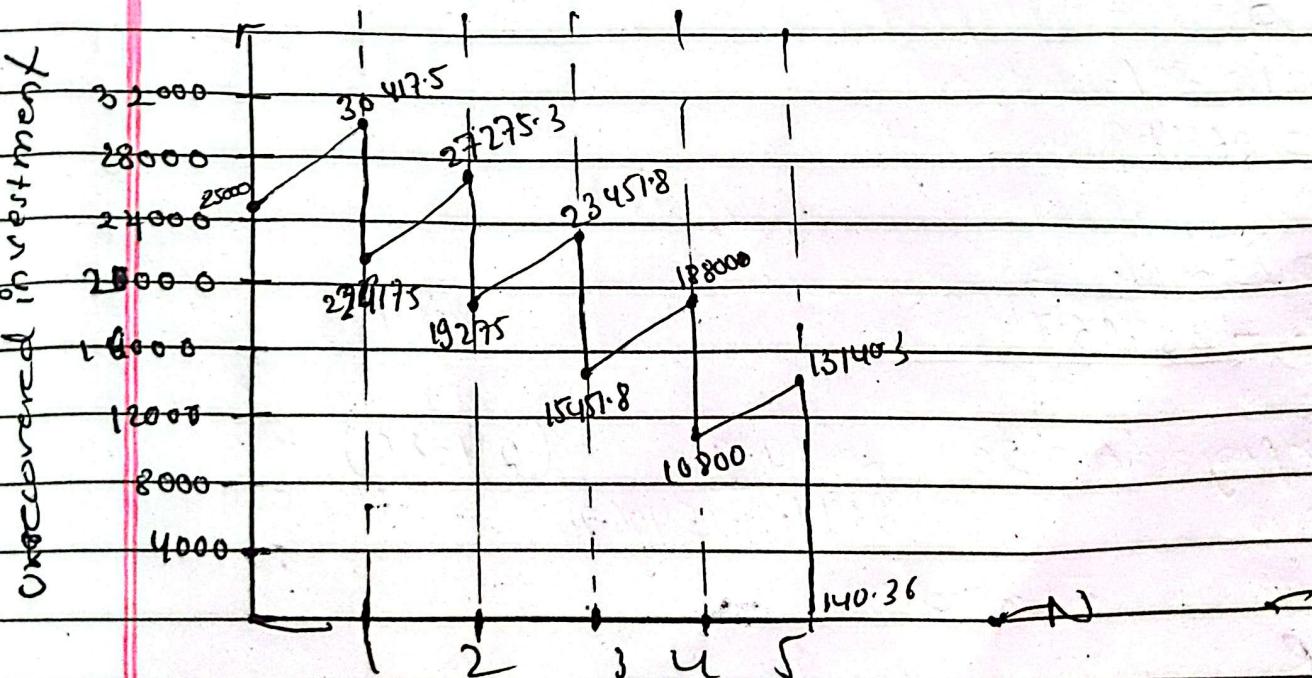
$$IRR = 20 + \frac{934.3}{934.3 + 1847.10} (25 - 20)$$

$$= 21.67\%$$

$\gamma MARR$ accepted

Unrecovered Investment table.

cash	EOY	Unrecovred investment		Unrecovred amount
		Beginning	End at 21.67%	
25000	0	-	-25000	-25000
8000	1	-25000	-30417.5	-30417.5 -22417.5
8000	2	-22417.5	-27275.3	-19275
8000	3	-19275	-23451.8	-15451.8
8000	4	-15451.8	-188000	-10800
13000	5	-10800	-13140.3	-140.36



Q. 2020 Fall

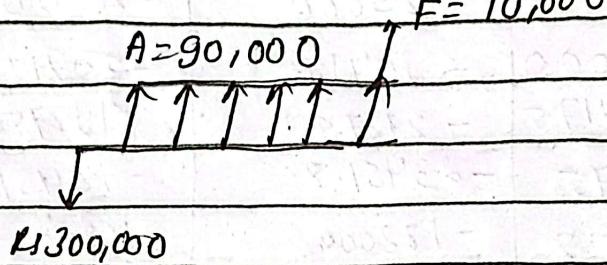
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Select the best project by using IRR method when MARR is 8%.

	Project A	Project B
Initial Investment	3,00,000	5,00,000
Annual Revenue	90,000	175,000
Life year	6	6
Salvage year	10,000	1,00,000
SOLN		

$$MARR = 8\%$$

for Project A



$$\begin{aligned} PW(i) &= A(P/A, i, N) + F(P/F, i, N) - P_0 \\ &= 90000(P/A, i, 6) + 10000(P/F, i, 6) - 300000 \end{aligned}$$

when $i = 20\%$.

$$PW = 2644.9$$

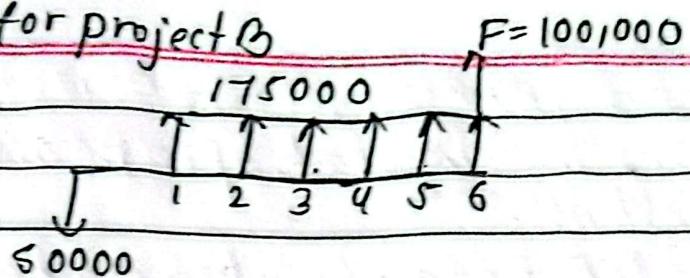
when $i = 24\%$.

$$PW = -25406.71$$

$$\begin{aligned} IRR \text{ for Project A} &= 20 + \frac{2644.9}{2644.9 + 25406.71} (24 - 20) \\ &= 20.38\% \end{aligned}$$

$\gamma MARR$

for project B



$$PW = A(PIA, i\% ; N) + F(PIF, i\%, N) - P_0 \\ = 175000(PIA, i\%, 6) + 100000(PIF, i\%, 6) - 50000$$

When $i = 28\%$

$$PW = 5628.82$$

when $i = 32\%$

$$PW = -37602.78$$

IRR for project B

$$= 28 + \frac{5628.82}{5628.82 + 37602.78} \times (32 - 28)$$

$$= 28.52\%$$

Here IRR of project B is greater than Project A and both IRR are greater than MARR. Hence Project B should be selected.

External Rate of Return (ERR)

- IRR method eliminates the drawback of reinvestment assumption to some extent -
 - The reinvestment of IRR may not be always practical

Benefit cost ratio Method (B/C R)

- Special tool of benefit cost analysis
 - It is used to evaluate public sector projects as well as big projects of private sector.
 - It is defined as the ratio of the equivalent worth of benefit to the equivalent worth of costs.
 - Also known as saving investment ratio
 - It is actually a ratio of discounted benefit to the discounted cost.

Has two types

1. Conventional method

Modified method

Conventional method.

By PW formula $\frac{\text{B/C ratio}}{\text{Benefit}} \geq \text{PW}(B)$

$$pw(I) = pw(s) + pw(o \oplus m)$$

Investment Salvage Operating & Maintenance

By F W

$$\text{B/C ratio} = \frac{\text{FW(B)}}{\text{FW(T)} - \text{FW(S)} + \text{FW(OEM)}}$$

~~Block cost salvage & evn~~

~~O~~ ~~evn~~

useful life year assumption (fixed)



By AW

BC ratio = $\frac{AW(B)}{AW(I)}$

$$AW(I) = Aw(s) + Aw(OSM)$$

By modified method

By Pw method

$$BC(R) = \frac{Pw(B) - Pw(OSM)}{Pw(I) - Pw(s)}$$

Decision rules

if $BC(R) > 1.0$ Accepted

if $BC(R) = 0$ i Remain indifferent

if $BC(R) < 1.0$ i Rejected

Ex 1

Find both types of Bioratio by using PW, FW & AI.

Initial Investment = Rs 100000

Annual Investment = Rs 40000

Annual Cost = Rs 19000

SV = Rs 20000

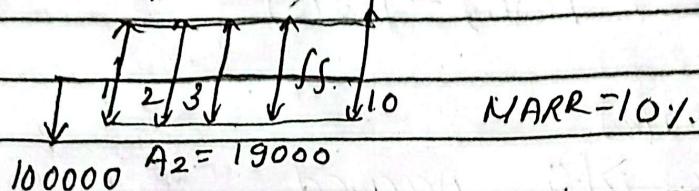
Useful life = 10 yrs

MARR = 10%.

SOLN

$$A_1 = 40000$$

$$c = 20000$$



$$A = A_1 - A_2 = 21000$$

By conventional method.

$$\begin{aligned} PW(B) &= A_1(P/A, i\%, N) \\ &= 40000(P/A, 10\%, 10) \\ &= 245784 \end{aligned}$$

$$PW(I) = \text{Rs } 100000$$

$$\begin{aligned} PW(S) &= F(P/F, i\%, N) \\ &= 20000(P/F, 10\%, 10) \\ &= \text{Rs } 7710 \end{aligned}$$

$$\begin{aligned} PW(OM) &= A_2(P/A, i\%, N) \\ &= 19000(P/A, 10\%, 10) \\ &= 11674740 \end{aligned}$$

BCR ratio

$$= \frac{100000}{100000} = 1.00$$

$$100000 - 7710 + 116747.40$$

$$= 1.17 \text{ i.e. } > 1 \text{ accepted.}$$

By PW

$$FW(B) = A_1(F/A, i\%, N)$$

$$= 637496$$

$$FW(I) = 100000(F/P, i\%, N)$$

$$= 259370$$

$$FW(S) = Rs 20000$$

$$FW(0\%M) = A_2(F/A, i\%, N)$$

$$= 302810.60$$

For conventional method

BCR ratio - FW(B)

$$FW(I) - FW(S) + FW(0\%M)$$

$$= 1.17 > 1 \text{ accepted}$$

Modified method

$$BCR = \frac{FW(B) - FW(0\%M)}{FW(I) - FW(S)}$$

$$= \frac{637496 - 302810.60}{259370 - 20000}$$

$$= 1.39 > 1 \text{ accepted}$$

2023 Fall

Calculate both types of SC ratio from the following information and also give your decision whether the project should be selected or not.

Initial Investment = 800000

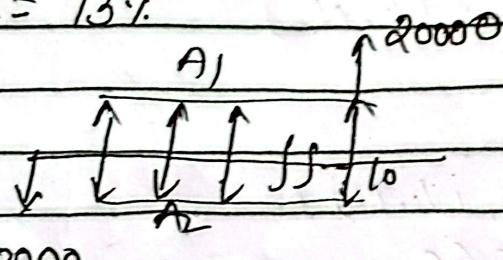
Annual Revenue = 150000

Operating and maintenance = 25000

Life = 10 years

Salvage value = 20000

MARR = 13%



$$\begin{aligned} Aw(A) &= P(A/F, i\%, N) \\ &= 800000(A/F, 13\%, 10) \\ &= 147440 \end{aligned}$$

$$Aw(B) = 150000$$

$$Aw(Obm) = 25000$$

$$\begin{aligned} Aw(C) &= F(A/F, i\%, N) \\ &= 20000(A/F, 13\%, 10) \\ &= 1086 \end{aligned}$$

By conventional method

$$= 150000$$

$$\begin{aligned} &\frac{147440 - 1086 + 25000}{150000} \\ &= 0.875 \leq 1 \end{aligned}$$

$$\text{Modified BIC} = \frac{150000 - 25000}{147440 - 2086} \\ = 0.854 < 1$$

since BIC ratio is both conventional & modified < 1
then reject the project.

Determine both type BIC ratio by FW

Initial Investment = 250000

Annual revenue = 50000 at end of year & increased by 30000
each for 5 years

Annual O&M cost = 30000

Salvage value = 50000

Useful life = 5

MARR = 15%

SOP

$$F(A) @ FW(1) = 250000 (FIP, 15\%, 5) = 502839$$

$$FW(S) = 50000$$

$$FW(B) = 50000 (FIA, 15\%, 5) + 30000 (FIA, 15\%, 5) \\ = 337120 + 348476$$

$$= 685595$$

Conventional ratio = 1.0475 > 1

Modified ratio = 1.0662 > 1

Find BIC ratio using PW and AW method

Initial Investment = Rs 600000

Annual benefit = Rs 30000 250000

Annual cost = Rs 20000

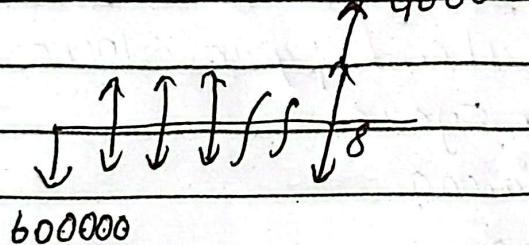
Salvage value = Rs 40,000

MARR = 10% per year

Useful life = 8 years

Repair and maintenance cost at 5th year = Rs 5000

By PW method



$$PW(B) = A_1(P/A, 10\%, 8)$$

$$= 1333725$$

$$PW(I) = 600000 + 5000(P/F, 10\%, 5)$$

$$= 603104.5$$

$$PW(S) = 40000(P/F, 10\%, 8)$$

$$= 18660$$

$$PW(OCM) = A_2(P/A, 10\%, 18)$$

$$= 160097$$

$$B/C = 1.7971 \text{ accepted}$$

Evaluate IRR of the following

Initial Investment = 600000

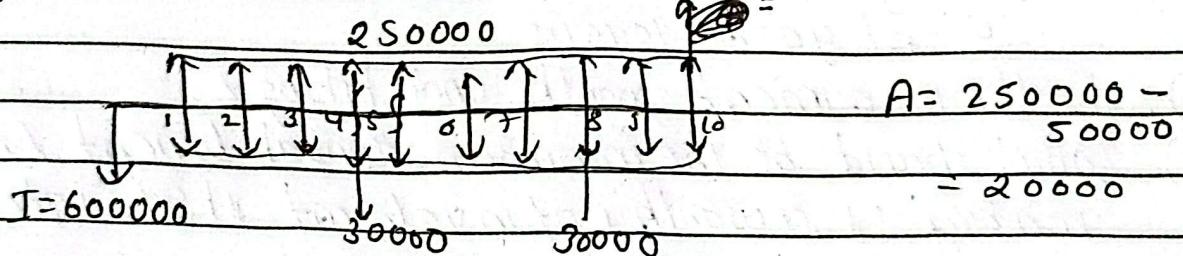
Annual revenue = 250000

Annual cost = 50000

Useful life = 10 years

MARR = 10% per year

Repair and maintenance at 4th and 8th = 30000



$$\begin{aligned}
 PW &= A(P/A, i\%, N) - 30000(P/F, i\%, N) \\
 &\quad - 30000(P/F, i\%, N) - 600000 \\
 &= 20000(P/A, i\%, 10) - 30000(P/F, i\%, 8) \\
 &\quad - 30000(P/F, i\%, 8)
 \end{aligned}$$

if $i = 30\%$.

$$PW(i=30\%) = 4119$$

If $i = 35$

$$PW = -68751$$

$$IRR = 30 + \frac{4119}{4119 + 68751} (35 - 30)$$

$$= 30.283\%$$

> MARR

A company is considering the purchase of new equipment. Interest rate is 9% per year. The cash flow for the equipment is as follows:

Initial Investment = \$0,000

Annual income = 9000

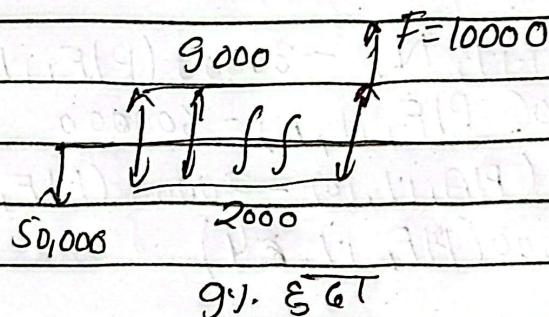
Annual O&M = 2000

Salvage value = 10,000

Useful year = 10 years

i) Is the investment worth undertaking?

ii) What should be the minimum annual benefit for making it a worthy of investment if 9% rate of return.



$$PW = 7000(P/A, i\%, 10) + 10000(P/F, i\%, 10) - 50,000$$

$$= -852.10$$

If $i = 10\%$ since $PW(9\%) < 0$, this is not worth increasing.

$$PW(10\%) = -RS 3132.80$$

If $i = 8\%$.

$$PW = 1602.7$$

IRR

$$= 8 + \frac{1602.7}{1602.7 + 3132.80} (10-8)$$

$$= 8.653\%$$

IRR < MARR, Project is not feasible



ii) for minimum benefit annually (a)

$$PW(9\%) = -50000 + B(P/A, 9\%, 10) + 10000 (P/F, 9\%, 10)$$

$$0 = -50000 + B \times 6.4177 + 10,000 \times 0.4224$$

$$45776 = B \times 6.4177$$

$$B = \text{Rs } 713277.35$$