TUTORIAL-2 NAME: SUBUAI GRG ROll NO:- 43 'A'

(1) show that the shortest distance betn the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+1}{2}=\frac{z-6}{4}$ is $3\sqrt{30}$ units. Also find the eqn of the line of the shortest distance.

→ Soln:

Given line is,

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \quad \text{—— (i)}$$

The point of line (i) is $(x_1,y_1,z_1)=(3,8,3)$ and the direction ratio of (i) are ~~(l₁ m₁ n₁)~~ $(l_1,m_1,n_1)=(3,-1,1)$

~~As~~ Another line is,

$$\frac{x+3}{-3}=\frac{y+1}{2}=\frac{z-6}{4} \quad \text{—— (ii)}$$

The point of line (ii) is, $(x_2,y_2,z_2)=(-3,-7,6)$ and the direction ratio of (ii) are $(l_2,m_2,n_2)=(-3,2,4)$

Let, l,m,n be the dr's then the line of shortest distance is, ⊥ar to both lines (i) & (ii), so,

$$3l-m+n=0$$

$$\text{or, } -3l+2m+4n=0$$

Solving by cross multiplication,

	l		m		n
-1		1		-3	-1
2		4		-3	2

$$\text{or, } \frac{l}{-4-2}=\frac{m}{-3-12}=\frac{n}{6-3}$$

$$\frac{l}{-6}=\frac{m}{-15}=\frac{n}{3}$$

$$\text{or, } \frac{l}{-2}=\frac{m}{-5}=\frac{n}{1}=k(\text{say})$$

This gives, $l=-2k,\ m=-5k,\ n=k$

we have, $l^2+m^2+n^2=1$

$\Rightarrow 4k^2+25k^2+k^2=1$

$\Rightarrow k=\frac{1}{\sqrt{30}}$ (Taking + sign)

Then, $l=-\frac{2}{\sqrt{30}},\ m=-\frac{5}{\sqrt{30}},\ n=\frac{1}{\sqrt{30}}$

Now the length of shortest distance is,

$$SD=(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n$$

$$=(-3-3)\left(-\frac{2}{\sqrt{30}}\right)+(-7-8)\left(-\frac{5}{\sqrt{30}}\right)+(6-3)\left(\frac{1}{\sqrt{30}}\right)$$

$$=\frac{12}{\sqrt{30}}+\frac{75}{\sqrt{30}}+\frac{3}{\sqrt{30}}=\frac{12+75+3}{\sqrt{30}}=\frac{90}{\sqrt{30}}=3\sqrt{30}$$

And the eqn ~~be~~ of shortest distance be,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix}=0=\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} x-3 & y-8 & z-3 \\ 3 & -1 & 1 \\ -\frac{2}{\sqrt{30}} & -\frac{5}{\sqrt{30}} & \frac{1}{\sqrt{30}} \end{vmatrix}=0=\begin{vmatrix} x+3 & y+7 & z-6 \\ -3 & 2 & 4 \\ -\frac{2}{\sqrt{30}} & -\frac{5}{\sqrt{30}} & \frac{1}{\sqrt{30}} \end{vmatrix}$$

$$\Rightarrow (x-3)(-1+5)-(y-8)(3+2)+(z-3)(-15-2)=0=(x+3)(2+20)-(y+7)(-3+8)+(z-6)(15+4)$$

$$\Rightarrow (x-3)4-(y-8)5+(z-3)(-17)=0=(x+3)(22)-(y+7)5+(z-6)19.$$

$$\Rightarrow 4x-5y-17z-12+40+51=0=22x-5y+19z+66-35-114$$

$$\Rightarrow 4x-5y-17z+79=0=22x-3y+19z-83.$$

$\therefore$ The magnitude of the shortest distance is $3\sqrt{30}$ & eqn is,

$$4x-5y-17z+79=0=22x-3y+19z-83$$

(2) Find the two points on the line $\frac{x-2}{1} = \frac{y+3}{2} = \frac{z+5}{2}$ either of $(2,-3,-5)$ and at a distance 3 from it.

→ Soln:

Here, Given that, $\frac{x-2}{1} = \frac{y+3}{2} = \frac{z+5}{2} = k$ (let)

$\Rightarrow x = k+2,\ y = 2k-3,\ z = 2k-5$

Now,

Distance from the point to given lay,

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\Rightarrow 3 = \sqrt{(k+2-2)^2 + (2k-3+3)^2 + (2k-5+5)^2}$

$\Rightarrow 3 = \sqrt{k^2 + 4k^2 + 4k^2}$

$\Rightarrow 9 = 9k^2$

$\therefore k = \pm 1$,,

$\therefore (3,-1,-3)$ and $(1,-5,-7)$ is the required ~~points~~. two points.

(3) Find the eqn of the sphere having circle $x^2+y^2+z^2=9$, $x-2y+2z=5$ as a great circle. Also determine its radius and center.

→ Solution:

Let, the eqn of sphere through the circle $x^2+y^2+z^2=9$ and $x-2y+2z-5=0$ as a great circle be,

$$x^2+y^2+z^2-9+k(x-2y+2z-5)=0 \quad \text{—(1)}$$

The given circle is a great circle of (1); then the centre of (1) sphere lies on the plane $x-2y+2z-5=0$ —(2)

The centre of (1) is $\left[-\frac{k}{2}, k, -k\right]$ which lies on (2)

$\therefore -\frac{k}{2}-2k-2k-5=0$

or $-k-8k-10$

or $k=-\frac{10}{9}$

putting values in eqn (1),

$$x^2+y^2+z^2-9-\frac{10}{9}(x-2y+2z-5)=0$$

or $x^2+y^2+z^2-\frac{10}{9}x+\frac{20}{9}y+\frac{20}{9}z-9+\frac{50}{9}=0$

or $x^2+y^2+z^2-\frac{10}{9}x+\frac{20}{9}y-\frac{20}{9}z-\frac{31}{9}=0$

This gives centre of sphere is $\left(\frac{5}{9}, -\frac{10}{9}, \frac{10}{9}\right)$

and

Radius $=\sqrt{\frac{25}{81}+\frac{100}{81}+\frac{100}{81}+\frac{31}{9}}$

$=\sqrt{\frac{225+279}{81}}$

$=\sqrt{\frac{504}{81}}$

$\therefore$ Radius $=\frac{2}{3}\sqrt{14}$ //

(4) Find the eqn of plane through $(2,2,1)$, $(1,-2,3)$ and parallel to the line joining the points $(2,1,-3)$ and $(-1,5,-8)$.

Soln :-

The plane through the points $(2,2,1)$ is,

$$a(x-2)+b(y-2)+c(z-1)=0 \quad \text{—(i)}$$

As (i) passes through the points $(1,-2,3)$ then,

$$a(1-2)+b(-2-2)+c(3-1)=0$$

or $-a-4b+2c=0$

or $a+4b-2c=0$ —— (ii)

The eqn of line joining points $(2,1,-3)$ and $(-1,5,-8)$ is,

$$\frac{x-2}{2-(-1)}=\frac{y-1}{1-5}=\frac{z-(-3)}{-3-(-8)}$$

or $\dfrac{x-2}{3}=\dfrac{y-1}{-4}=\dfrac{z+3}{5}$ —— (iii)

Given that the plane (i) is parallel to line (iii) so; condn of parallelism,

$$3a-4b+5c=0 \quad \text{—— (iv)}$$

Solving eqn (ii) & (iv),

$$\begin{array}{cccc} & a & b & c \\ 4 & -2 & 1 & 4 \\ -4 & 5 & 3 & -4 \end{array}$$

$\Rightarrow \dfrac{a}{20-8}=\dfrac{b}{-6-5}=\dfrac{c}{-4-12}=k \text{ (say)}$

or $a=12k$, $b=-11k$, $c=-16k$

Now, from eqn (i),

$$12k(x-2)-11k(y-2)-16k(z-1)=0$$

or $12x-24-11y+22-16z+16=0$

or $12x-11y-16z+14=0$

$\therefore$ Eqn of req plane is $12x-11y-16z+14=0$ //

(5) Find the distance of the point $(1,-3,5)$ from the plane $3x-2y+6z=15$ along with the direction cosine proportional of to $(2,1,-2)$

Solution:

Eqn of line through $(1,-3,5)$ and dr's $(2,1,-2)$ is,

$$\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-5}{-2}$$

Now, $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-5}{-2}=k$ (let)

$\Rightarrow x=2k+1, y=k-3, z=-2k+5$

$\therefore (2k+1, k-3, -2k+5)$ is any point on the line,

which also lies on the plane $3x-2y+6z=15$

Now, $3(2k+1)-2(k-3)+6(-2k+5)=15$

or $6k+3-2k+6-12k+30=15$

or $-8k=-24$

$\therefore k=3$

$\therefore$ The point on a line lies on the plane is $(7,0,-1)$

Now,

Distance from a point to the plane is,

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$=\sqrt{(7-1)^2+(0+3)^2+(-1-5)^2}$$

$$=\sqrt{36+9+36}$$

$$=\sqrt{81}$$

$=9$ units

(6) Find the two points on the line $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z+5}{2}$ either side of $(2,-3,-5)$ and at a distance 3 from it.

→ Question No. 2

(7) Change the eqn $x+y+z+1=0=4x+y-2z+2$ in symmetrical form.

⇒ Soln:

Given eqn of st. line is,

$$x+y+z+1=0=4x+y-2z+2$$

Then,

$$x+y+z+1=0 \quad \text{—(i)}$$

$$4x+y-2z+2=0 \quad \text{—(ii)}$$

Put, $z=0$;

$$x+y+1=0$$

$$4x+y+2=0$$

Solving,

$$x+y+1=0$$

$$4x+y+2=0$$

$$-3x=1$$

$$x=-\frac{1}{3}$$

Then, $-\frac{1}{3}+y+1=0$

$$y=-\frac{2}{3}$$

∴ line through the point $\left(-\frac{1}{3},-\frac{2}{3},0\right)$

Let l,m,n be the dr's of line which is lies on the plane (i) & (ii)

So $l+m+n=0$

$4l+m-2n=0$

Solving these using cross multiplication method,

$$\Rightarrow \frac{l}{-2-1}=\frac{m}{4+2}=\frac{n}{1-4}$$

$$\frac{l}{1}=\frac{m}{-2}=\frac{n}{1}$$

Eqn of st line is, $\frac{x+\frac{1}{3}}{1}=\frac{y+\frac{2}{3}}{-2}=\frac{z}{1}$ //

(9) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find their common point and the plane containing them.

Soln;

Given eqn of lines is $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ & $\frac{x-8}{7} = y-4 = \frac{z-5}{3}$

$(x_1, y_1, z_1) = (5, 7, -3)$, $(x_2, y_2, z_2) = (8, 4, 5)$

$(l_1, m_1, n_1) = (4, 4, -5)$, $(l_2, m_2, n_2) = (7, 1, 3)$

Two lines are coplanar if, $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

Now $\begin{vmatrix} 8-5 & 4-3 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$

$= 3(12+5) - 1(12+35) + 8(4-28)$

$= 0$

$\therefore$ Two lines are coplanar.

Eqn of plane containing two lines i.e. coplanar plane is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\text{or } \begin{vmatrix} x-5 & y-7 & z+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = 0$$

or $(x-5)(12+5) - (y-7)(12+35) + (z+3)(4-28) = 0$

or $17x - 85 - 47y + 329 - 24z - 72 = 0$

or $17x - 47y - 24z + 172 = 0$

For common points,

Let, $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = k$ and $\frac{x-8}{7} = y-4 = \frac{z-5}{3} = k'$

$\therefore (4k+5, 4k+7, -5k-3)$ and $(7k'+8, k'+4, 3k'+5)$ are any points on the line.

For common points,

$$4k+5 = 7k'+8 \quad — (1)$$

$$4k+7 = k'+4 \quad — (2)$$

$$-5k-3 = 3k'+5 \quad — (3)$$

From (1) & (2), $4k+5-4k-7 = 7k'+8-k'-4$

$$-2 = 6k'+4$$

$$\therefore k' = -1$$

$\therefore (1,3,2)$ is a common points.

(10) Find the shortest distance betn the lines $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ and $\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5}$. Also find the eqn of the line of shortest distance.

→ Soln:

Given, $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ —(i)

The line passes through $(x_1, y_1, z_1) = (5, 7, 3)$ and dr's $(l_1, m_1, n_1) = (3, -16, 7)$.

Also the given line is,

$$\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5} \quad \text{—(ii)}$$

which passes through $(x_2, y_2, z_2) = (9, 13, 15)$ and dr's $(l_2, m_2, n_2) = (3, 8, -5)$

Let, l, m, n be the dr's of the line

Then,

$3l - 16m + 7n = 0$

$3l + 8m - 5n = 0$

Solving by cross multiplication,

$$\begin{array}{cccc} l & & m & n \\ -16 & 7 & 3 & -16 \\ 8 & -5 & 3 & 8 \end{array}$$

$\Rightarrow \frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48}$

$\Rightarrow \frac{l}{24} = \frac{m}{36} = \frac{n}{72} = k$ (say)

$\Rightarrow l = 24k, m = 36k, n = 72k$

We know,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow (24k)^2 + (36k)^2 + (72k)^2 = 1$

$\Rightarrow 576k^2 + 1296k^2 + 5184k^2 = 1$

$\therefore k = \frac{1}{84}$ (Taking +ve sign)

So, $l = 24 \times \frac{1}{84} = \frac{2}{7}$, $m = 36 \times \frac{1}{84} = \frac{3}{7}$, $n = 72 \times \frac{1}{84} = \frac{6}{7}$

Now, the shortest distance betn (i) & (ii) is,

$SD = (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$

$= (9-5) \times \frac{2}{7} + (13-7) \times \frac{3}{7} + (15-3) \times \frac{6}{7}$

$= \cancel{98}\ \frac{8+18+72}{7}$

$= \frac{98}{7}$

$= 14$

∴ The shortest distance is 14 units.

SD eqn, $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0 = \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix}$

$\Rightarrow \begin{vmatrix} x-5 & y-7 & z-3 \\ 3 & -16 & 7 \\ \frac{2}{7} & 3/7 & 6/7 \end{vmatrix} = 0 = \begin{vmatrix} x-9 & y-13 & z-15 \\ 3 & 8 & -5 \\ 2/7 & 3/7 & 6/7 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} x-5 & y-7 & z-3 \\ 3 & -16 & 7 \\ 2 & 3 & 6 \end{vmatrix} = 0 = \begin{vmatrix} x-9 & y-13 & z-15 \\ 3 & 8 & -5 \\ 2 & 3 & 6 \end{vmatrix}$

$\Rightarrow (x-5)(-96-21) - (y-7)(18-14) + (z-3)(9+32) = 0 = (x-9)(48+15)$
$- (y-13)(18+10) + (z-15)(9-16)$

$\Rightarrow (x-5)(-117) - (y-7)4 + (z-3)41 = 0 = (x-9)9 - (y-13)4 + (z-15)(-1)$

$\Rightarrow -117(x-5) - 4(y-7) + 41(z-3) = 0 = (x-9)9 - (y-13)4 - (z-15)$

$\Rightarrow -117x - 4y + 41z + 490 = 0 = 9x - 4y - z - 14$

(11) Find the image of the point $P(1,2,3)$ in the plane $2x-y+z+3=0$

Soln:

The eqn of line passing through $(1,2,3)$ and perpendicular to the plane $2x-y+z+3=0$ is,

$$\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{1}=k\text{ (say)} \quad \text{—(1)}$$

Then $x=2k+1$, $y=-k+2$, $z=k+3$

Here the coordinate of R is,

$$R\left(\frac{2k+1+1}{2}, \frac{2-k+2}{2}, \frac{k+3+3}{2}\right) \text{ or } R\left((k+1), 2-\frac{k}{2}, 3+\frac{k}{2}\right)$$

So,

$\cancel{2(k+1)-2+\frac{k}{2}}$

The point lies on the plane $2x-y+z+3=0$

or $2(k+1)-2+\frac{k}{2}+3+\frac{k}{2}+3=0$

or $2k+2+k+4=0$

or $k=-2$

Then the coordinate is,

$(2k+1, 2-k, k+3)=(-4+1, 2+2, -2+3)$

$=(-3, 4, 1)$

$\therefore$ The image of the point $P(1,2,3)$ in the plane is $(-3,4,1)$

(13) Find the eqn of sphere having the circle $x^2+y^2+z^2=9, x-2y+2z=5$ as a great circle, determine its center & radius.

→ Question no. 3

(14) Find the eqn of sphere one of whose great circle is $x^2+y^2+z^2=4$, $x^2+y^2+z^2-2x+4y-6z=11$. Also find the center & radius.

→ Soln:

Given eqn of circle is,

$$x^2+y^2+z^2=4,\ x^2+y^2+z^2-2x+4y-6z=11$$

From these two eqn, eqn of plane is,

$$2x-4y+6z+7=0 \quad \text{--- (i)}$$

Let, eqn of sphere through the circle be,

$$x^2+y^2+z^2-4+\lambda(2x-4y+6z+7)=0 \quad \text{--- (ii)}$$

or $x^2+y^2+z^2-4+2\lambda x-4\lambda y+6\lambda z+7\lambda=0$

$\therefore u=\lambda,\ v=-2\lambda,\ w=3\lambda$

Center of sphere $=(-u,-v,-w)=(-\lambda, 2\lambda, -3\lambda)$

For great circle, center sphere lies on the plane (i),

$2\times(-\lambda)-4\times 2\lambda+6\lambda-3\lambda+7=0$

or $-2\lambda-8\lambda-18\lambda+7=0$

$\therefore \lambda=\frac{1}{4}$

From eqn (ii), eqn of sphere is,

$$x^2+y^2+z^2-4+\frac{1}{4}(2x-4y+6z+7)=0$$

or $4(x^2+y^2+z^2-4)+2x-4y+6z+7=0$

or $4(x^2+y^2+z^2)+2x-4y+6z-9=0$ is req equation.

$\therefore$ Center of sphere is, $(-2, \frac{1}{2}, -\frac{3}{4})$

& Radius $=\sqrt{4+\frac{1}{4}+\frac{9}{16}+9}$

$=\frac{\sqrt{221}}{4}$

$\therefore$ Radius $=\frac{\sqrt{221}}{4}$ //

(18) Find the eqn of sphere through the circle $x^2+y^2=4, z=0$ and cut by the plane $x+2y+2z=0$ in a circle of radius 3.

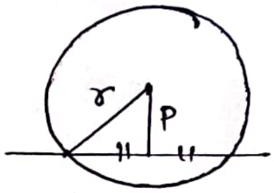
→ Soln

Given, Eqn of circle is, $x^2+y^2=4, z=0$ (XY plane)

Let, Eqn of sphere through the circle,

$$z^2+x^2+y^2-4+\lambda z=0$$

$$u=0, v=0, w=\frac{\lambda}{2}$$



center of sphere $=(0,0,-\frac{\lambda}{2})$

radius of sphere $=\sqrt{\frac{\lambda^2}{4}+4}$

perpendicular distance from center of sphere to plane,

$$x+2y+2z=0$$

$$\left|\frac{0+0+2(-\frac{\lambda}{2})}{\sqrt{1+4+4}}\right|=\left|-\frac{\lambda}{3}\right|=\frac{\lambda}{3}$$

we know, $\left(\sqrt{\frac{\lambda^2}{4}+4}\right)^2=\left(\frac{\lambda}{3}\right)^2+3^2$

$$\frac{\lambda^2}{4}+4=\frac{\lambda^2}{9}+9$$

$$\lambda=\pm 6$$

$\left.\begin{array}{l} x^2+y^2+z^2-4+6z=0 \\ x^2+y^2+z^2-6z-4=0 \end{array}\right\}$ are eqn of sphere

//

(17) Find the cosine angle betn $x+2y+3z=4$ and $3x+4y+5z=10$

Soln;

Given eqn,

$$x+2y+3z=4 \quad \text{---(i)}$$

$$3x+4y+5z=10 \quad \text{---(ii)}$$

Let, θ be the angle betn the lines then,

Angle between them is given by,

$$\cos\theta = \frac{3.1+2.4+3.5}{\sqrt{1^2+2^2+5^2}\sqrt{3^2+4^2+5^2}}$$

$$\cos\theta = \frac{3+8+15}{\sqrt{30}\sqrt{50}}$$

$$\cos\theta = \frac{26}{10\sqrt{15}}$$

$\therefore$ ~~$\theta = 47.83°$~~

$$\therefore \theta = \cos^{-1}\left(\frac{13}{5\sqrt{15}}\right)$$

(8) Show that $x = ay+b, z = cy+d$ and $x = a'y+b', z = c'y+d'$ are perpendicular if $aa'+cc'+1=0$.

⇒ Soln

Given eqn of lines are,

$x = ay+b, z = cy+d$

$y = \frac{x-b}{a}, y = \frac{z-d}{c}$

$\Rightarrow \frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$

And, $x = a'y+b', z = c'y+d'$

$\Rightarrow y = \frac{x-b'}{a'}, y = \frac{z-d'}{c'}$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

These two lines are perpr if,

$aa'+1\times1+cc' = 0$

$\boxed{\Rightarrow aa'+cc'+1=0}$ //

(19) Find the center & radius of the sphere $x^2+y^2+z^2+4x-6y+8z=10$

$\rightarrow$ ~~for~~ Soln:-

Given Eqn of sphere is, $x^2+y^2+z^2+4x-6y+8z=0$

a Comparing with general form of eqn of sphere,

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0,$$

$u=2$, ~~$v=-3$~~, $v=-3$, $w=4$

Then

Center of sphere is $(-u,-v,-w)=(-2,3,-4)$

And radius $=\sqrt{u^2+v^2+w^2-d}$

$=\sqrt{2^2+(-3)^2+(4)^2}$

$=\sqrt{4+9+16}$

$=\sqrt{29}$

$\therefore$ Radius $=\sqrt{29}$ //

(21) Find the eqn of plane containing the point $(1,-1,2)$ and is perpendicular to the planes $2x+3y-2z=5$ & $x+2y-3z=8$

⇒ Soln

Let, eqn of plane containing $(1,-1,2)$ is,

$$a(x-1)+b(y+1)+c(z-2)=0 \quad \text{— (1)}$$

Eqn is perpendicular to the planes $2x+3y-2z=5$ & $x+2y-3z=8$

So, $2a+3b-2c=0$

$a+2b-3c=8$

Solving by cross multiplication method,

a		b		c
3	−2	2	3	
2	−3	1	2	

$$\Rightarrow \frac{a}{-9+4} = \frac{b}{-2-6} = \frac{c}{4-3}$$

$$\Rightarrow \frac{a}{-5} = \frac{b}{-8} = \frac{c}{1} = k \text{(say)}$$

$\Rightarrow a = -5k,\ b = -8k,\ c = k$

putting value of a, b, c in eqn (1),

$$-5k(x-1) + -8k(y+1) + k(z-2) = 0$$

$$\Rightarrow -5x+5-8y-8+z-2=0$$

$$\Rightarrow -5x-8y+z-5=0$$

$$\Rightarrow 5x+8y-z+5=0$$ is the req eqn.

(22) Find the eq^n of plane through $Q(1,4,-2)$ at right angle to OQ.

⇒ Sol^n :

Let, eq^n of plane through $Q(1,4,-2)$ be,

$$a(x-1)+b(y-4)+c(z+2)=0 \quad \text{—(1)}$$

Here, plane (1) is right angle to OQ.

Then a, b, c are proportional to dr's of OQ.

Direction cosine of OQ $= x_2-x_1, y_2-y_1, z_2-z_1$

$= 1-0, 4-0, -2-0$

$= (1, 4, -2)$

From (1),

$$1(x-1)+4(y-4)-2(z+2)=0$$

$$\Rightarrow x-1+4y-16-2z-4=0$$

$$\Rightarrow x+4y-2z-21=0 \text{ is required eq}^n.$$

(23) Find the eqn of plane through the point $(-1,1,-1)$, $(6,2,1)$ & normal to the plane $2x+y+z=5$.

Soln:-

Let eqn of plane through $(-1,1,-1)$ be

$$a(x+1)+b(y-1)+c(z+1)=0 \quad \text{---(1)}$$

which is also through $(6,2,1)$

$$a(6+1)+b(2-1)+c(1+1)=0$$

$$\Rightarrow 7a+b+2c=0 \quad \text{---(2)}$$

which is also normal to $2x+y+z=5$

$$2a+b+c=0 \quad \text{---(3)}$$

Solving (2) & (3),

a		b	c
1	2	7	1
1	1	2	1

$$\Rightarrow \frac{a}{1-2}=\frac{b}{4-7}=\frac{c}{7-2}=k \text{ (let)}$$

$$\Rightarrow \frac{a}{-1}=\frac{b}{-3}=\frac{c}{5}=k$$

$$\Rightarrow a=-k,\ b=-3k,\ c=5k$$

From eqn (1),

$$-1(x+1)+(-3)(y-1)+5(z+1)=0$$

$$\Rightarrow -x-1-3y+3+5z+5=0$$

$$\Rightarrow -x-3y+5z+7=0$$

$$\Rightarrow x+3y-5z-7=0$$ is req equation.

(24) Find the eqn of plane, which passes through the point $(-1,3,2)$ and is perpendicular to each of the two planes $x+2y+2z=5$ and $3x+3y+2z=8$.

→ Soln

Let, eqn of plane through $(-1,3,2)$ be,

$$a(x+1)+b(y-3)+c(z-2)=0 \quad \text{—①}$$

and is perpn to the,

$$x+2y+2z-5=0 \quad \text{—②}$$

$$\& \ 3x+3y+2z-8=0 \quad \text{—③}$$

Now solving ② & ③,

x		y	z
2	2	1	2
3	2	3	3

$$\Rightarrow \frac{a}{4-6}=\frac{b}{6-2}=\frac{c}{3-6}=k \text{ (say)}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}=k$$

$$\Rightarrow a=-2k,\ b=4k,\ c=-3k$$

From eqn ①,

$$-2x+2+4y-12-3z+6=0$$

$$\Rightarrow -2x+4y-3z-4=0$$

$$\Rightarrow 2x-4y+3z+4=0 \text{ // is the req eqn.}$$

(25) Find the eqn of plane through the intersection of the planes $x+2y-3z=5$ & $5x+7y+3z=10$ through $(-1,2,-3)$.

⇒ Soln :-

Given eqn of planes are

$$x+2y-3z-5=0$$
$$5x+7y+3z-10=0$$

Let, eqn of plane through the intersection of planes is given by,

$$(x+2y-3z-5)+\lambda(5x+7y+3z-10)=0 \quad \text{--- (1)}$$

which passes through $(-1,2,-3)$ so,

$$(-1+2(2)-3(-3)-5)+\lambda(5(-1)+7(2)+3(-3)-10)=0$$

$$\text{or } -1+4+9-5-5\lambda+14\lambda-9\lambda-10\lambda=0$$

$$\text{or } -10\lambda=-7$$

$$\therefore \lambda=\frac{7}{10}$$

Now, from eqn (1),

$$x+2y-3z-5+\frac{7}{10}(5x+7y+3z-10)=0$$

$$\text{or } 10x+20y-30z-50+35x+49y+21z-70=0$$

or $45x+69y-9z-120=0$ is the required eqn of plane.

(26) Find the eqn of bisector of the acute angle betn the planes $3x+4y-5z+1=0$ and $5x+12y-13z=0$.

⇒ Solution:

The eqn of two planes bisecting the angles betn the given planes are;

$$\frac{3x+4y-5z+1}{\sqrt{3^2+4^2+5^2}} = \pm \frac{5x+12y-13z}{\sqrt{5^2+12^2+13^2}}$$

or $14x-8y+13=0$ —— (i)

and, $64x+112y-130z+13=0$ —— (ii)

Let, θ be the angle betn (i) & then,

$$\cos\theta = \frac{14\times3-8\times4}{\sqrt{14^2+8^2}\sqrt{3^2+4^2+5^2}}$$

$$= \frac{1}{\sqrt{130}}$$

$\therefore \tan\theta = \sqrt{129}$ which is greater than 1.

$\therefore \theta > 45^\circ$.

Hence, the bisector of acute angle is

$64x+112y-130z+13=0$ //

(27) Show $6x^2+4y^2-10z^2+3yz+4zx-11xy=0$ represents a pair of perpr planes, find their eqn.

→ Given eqn of pair of planes is,

$$6x^2+4y^2-10z^2+3yz+4zx-11xy=0$$

Comparing with, $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

$$a=6,\ b=4,\ c=-10$$

Here, $a+b+c=6+4-10=0$

∴ Two planes are perpr.

For separate eqn of plane.

$$6x^2+(4z-11y)x+(4y^2-10z^2+3yz)=0$$

which is quadratic in x. i.e. in the form,

$$ax^2+bx+c=0$$

$$\therefore x = \frac{-(4z-11y)\pm\sqrt{(4z-11y)^2-4.6.(4y^2-10z^2+3yz)}}{2\times6}$$

$$\Rightarrow 12x = 11y-4z \pm\sqrt{16z^2-88yz+121y^2-96y^2+240z^2-72yz}$$

$$\Rightarrow 12x = 11y-4z\pm\sqrt{25y^2-160yz+256z^2}$$

$$\Rightarrow 12x = 11y-4z\pm(5y-16z)$$

Taking +ve sign,

$$12x=11y-4z+5y-16z$$

$$\Rightarrow 12x-16y+20z=0$$

Taking -ve sign,

$$\Rightarrow 12x=11y-4z-5y+16z$$

$$\Rightarrow 12x-6y-12z=0$$

is the req two separate equations.

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(28) Find the eqn to the line through $(-1,3,2)$ and perpendicular to the plane $x+2y+2z=3$, the length of perpr and the co-ordinates of its foot..

→ Soln :-

Given eqn of plane, $x+2y+2z=3$

$(1,2,2)$ be the dr's of the normal to the plane. Therefore, the dr's of reqd line is $(1,2,2)$.

Eqn of line through point $(-1,3,2)$ is,

$$\frac{x+1}{6} = \frac{y-3}{2} = \frac{z-2}{2}$$

Now, $\frac{x+1}{1} = \frac{y-3}{2} = \frac{z-2}{2} = k$ (let)

$\Rightarrow x = k-1, y = 2k+3, z = 2k+2$

$\therefore (k-1, 2k+3, 2k+2)$ be any points on the line which lies on the plane $x+2y+2z=3$ so,

$$(k-1)+2(2k+3)+2(2k+2)=3$$

$\Rightarrow k-1+4k+6+4k+4=3$

$\Rightarrow 9k+9=3$

$\Rightarrow 9k=-6$

$\therefore k = -\frac{2}{3}$ //

$\therefore$ The eqn of line is $\frac{x+1}{1} = \frac{y-3}{2} = \frac{z-2}{2}$ //

(30) Find the eqn of the sphere which passes through the circle $x^2+y^2+z^2-6x-2z+5=0,\ y=0$ & touch the plane $3y+4z+5=0$

$\Rightarrow$ Given circle is,

$$x^2+y^2+z^2-6x-2z+5=0,\ y=0 \longrightarrow (1)$$

so the eqn of sphere that passes through (1) is,

$$x^2+y^2+z^2-6x-2z+5+ly=0 \longrightarrow (11)$$

The centre of sphere (11) is,

$$P\left(\frac{-6}{-6}, \frac{l}{-2}, \frac{-2}{-2}\right) = P\left(3, -\frac{l}{2}, 1\right)$$

Radius is, $r=\sqrt{9+\left(\frac{l}{-2}\right)^2+1^2-5}=\sqrt{9+\frac{l^2}{4}+1-5}=\sqrt{5+\frac{l^2}{4}}$

since, the length of perpendicular from centre P to the plane $3y+4z+5=0$ is equal to the radius of the sphere. So,

$$\frac{(0)(3)+(3)\left(-\frac{l}{2}\right)+(4)(1)+5}{\sqrt{3^2+4^2}}=\sqrt{\frac{l^2}{4}+5}$$

$$\Rightarrow \frac{\left(\left(-\frac{3l}{2}\right)+9\right)^2}{25}=\frac{l^2}{4}+5$$

$$\Rightarrow \frac{(-3l+18)^2}{100}=\frac{l^2}{4}+5$$

$$\Rightarrow 9l^2-108l+324=\frac{100l^2}{4}+500$$

$$\Rightarrow 36l^2-432l+1296=100l^2+2000$$

$$\Rightarrow 64l^2+432l+704=0$$

$$\Rightarrow l^2+6.75l+11=0$$

$$\Rightarrow l=\frac{-6.75-\sqrt{(6.75)^2-(4)(1)(11)}}{2(1)}$$

$$\Rightarrow l=\frac{-6.75-1.25}{2}$$

$$\Rightarrow l=-4, -\frac{11}{4}$$

Now, (1) becomes,

$$x^2+y^2+z^2-6x-2z+5-4y=0$$

and $x^2+y^2+z^2-6x-2z+5-\frac{11}{4}y=0$

$\Rightarrow 4(x^2+y^2+z^2)-24x-8z+20-11y=0$ is req eqn //

(31) Find the eqn of the sphere through the circle $x^2+y^2+z^2=1$, $2x+4y+5z=6$ and touching the plane $z=0$.

Soln:

Eqn of sphere through the given circle $x^2+y^2+z^2=1, 2x+4y+5z=6$ be,

$$(x^2+y^2+z^2-1)+k(2x+4y+5z-6)=0$$

$$\Rightarrow x^2+y^2+z^2+2kx+4ky+5kz-(1+6k)=0 \quad \text{--- (1)}$$

Comparing with $x^2+y^2+z^2+2ux+2vy+2wz+d=0$,

$u=k$, $v=2k$, $w=\frac{5k}{2}$ & $d=-(1+6k)$

Centre of (1) $(-u,-v,-w)=(-k,-2k,-\frac{5k}{2})$

and radius $(r)=\sqrt{u^2+v^2+w^2-d}$

$$=\sqrt{k^2\left(1+4+\frac{25}{4}\right)+1+6k}$$

$$=\frac{1}{2}\sqrt{45k^2+24k+4}$$

Since, the sphere (1) touches the plane $z=0$. which means the length of z-coordinate is equal to the radius of the sphere.

i.e. $\frac{5k}{2}=\frac{1}{2}\sqrt{45k^2+24k+4}$

or $25k^2=45k^2+24k+4$

$\Rightarrow 20k^2+24k+4=0$

$\Rightarrow 5k^2+6k+1=0$

$\Rightarrow (5k+1)(k+1)=0$

or, $k=-1,-\frac{1}{5}$

from eqn (1),

$$x^2+y^2+z^2-2x-4y-5z+5=0$$

and, $5(x^2+y^2+z^2)-2x-4y-5z+1=0$

which are required eqn of sphere.

(37) Show that the plane $2x-2y+z+12=0$ touches sphere $x^2+y^2+z^2-2x-4y+2z=3$ and find the point of contact.

→ Soln :-

Given, $x^2+y^2+z^2-2x-4y+2z=3$ — (i)

Comparing with $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then we get,

$u=-1, v=-2, w=1$ and $d=-3$.

Then the centre of (i) is,

$(-u,-v,-w)=(1,2,-1)$

Radius is, $r=\sqrt{u^2+v^2+w^2-d}$

$=\sqrt{1+4+1+3}$ Also given plane is,

$=\sqrt{9}=3$ $\qquad 2x-2y+z+12=0$ — (ii)

Then the perpendicular distance from the centre of sphere $(1,2,-1)$ to (ii) is

$$d=\left|\pm\frac{2\times1-2\times2-1+12}{\sqrt{4+4+1}}\right|=\left|\pm\frac{2-4-1+12}{\sqrt{9}}\right|=\left|\pm\frac{9}{3}\right|=3.$$

The dr's of the line perpr to (ii) are $(2,-2,1)$.

The eqn of line through the centre $(1,2,-1)$ and having ~~dr's~~ direction cosines $(2,-2,-1)$ be,

$$\frac{x-1}{2}=\frac{y-2}{-2}=\frac{z+1}{1}=r\text{ (let)} \quad \text{— (iii)}$$

Then, $x=2r+1, y=2-2r$ and $z=r-1$

So, $(2r+1,2-2r,r-1)$ be general point of (iii),

i.e.

$2(2r+1)-2(2-2r)+(r-1)+12=0$

$\Rightarrow 4r+2-4+4r+r-1+12=0$

$\Rightarrow 9r+9=0$

$\Rightarrow r=-1$

Then, $x=-1, y=4, z=-2$

$\therefore (-1,4,-2)$ be the point of contact.