

# CHAPTER 1

## PATTERNS AND SEQUENCES

### 1.1 Patterns:

#### Recognizing and describing patterns:

Pattern is a certain arrangement of a certain numbers, objects or shapes that may repeat or vary.

**For example:**

a) 2, 4, 6, 8, ... is a set of even numbers.



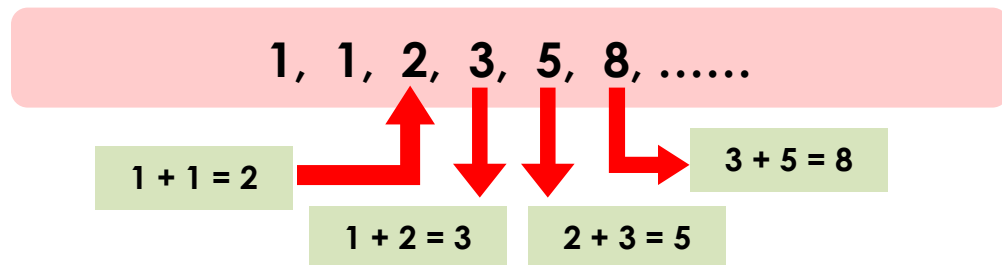
This set of even numbers start with 2 and the pattern is adding 2 to obtain each subsequent number.

b) 1, 3, 5, 7, ... is a set of odd numbers.



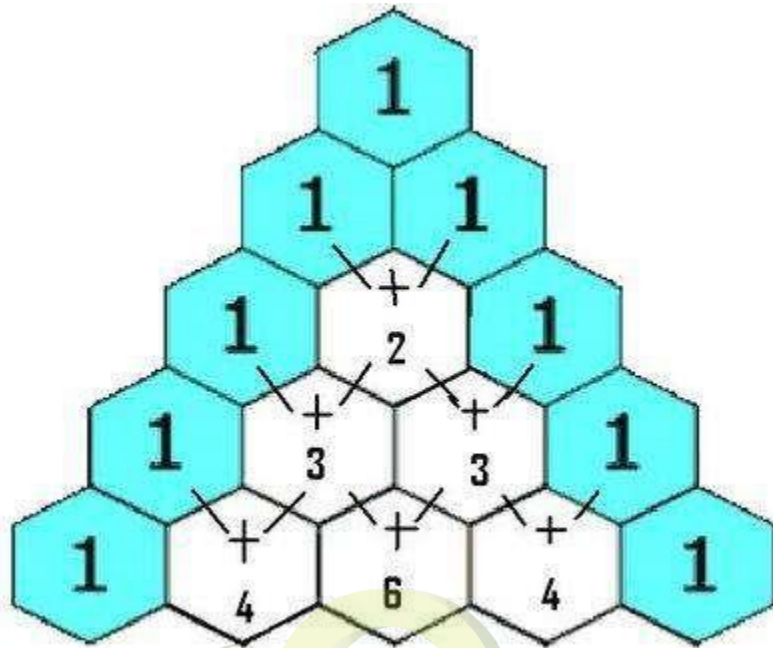
This set of odd numbers start with 1 and the pattern is adding 2 to obtain each subsequent number.

c) 1, 1, 2, 3, 5, 8, ..... is a set of Fibonacci numbers.



In a set of Fibonacci numbers, the first two numbers are 1. The pattern of each subsequent number is the sum of the two numbers before it.

d) Pascal's triangle



Based on the Pascal's triangle above, each of the number is the direct sum of the two numbers on the top where each row starts and end with 1.

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### Example 1:

#### Identify and describe the pattern for the following sets of numbers or objects.

The committee of Mathematics Society holds meetings in March 2017 on the following date:

- a. 1 March, 8 March, 15 March, 22 March, and 29 March.
- b. 3, 9, 27, 81,
- c. 22, 18, 14, 10, ...
- d. 3, 5, 8, 13, 21, ....

#### Solution:

- a. 1 March, 8 March, 15 March, 22 March, 29 March...



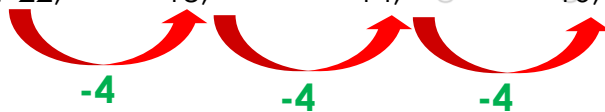
The first meeting was held on 1 March 2017, and the following meeting was held every 7 days in March or each Wednesday of the month.

- b. 3, 9, 27, 81, ...



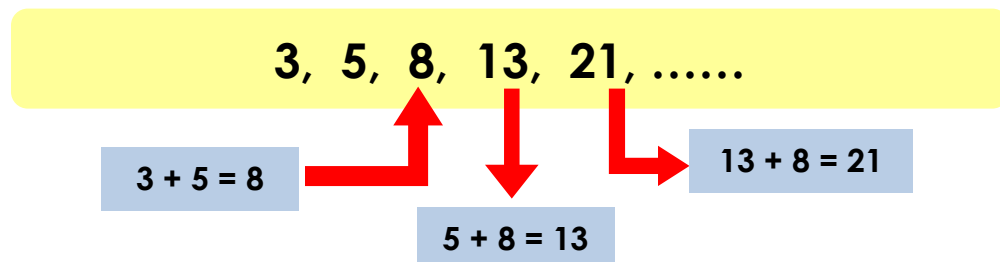
This set of the numbers starts with the number of 3. Then multiply the previous number with 3 to obtain the subsequent numbers.

- c. 22, 18, 14, 10, ...



This set of numbers start with the number of 22 . Then minus the previous number with 4 to obtain the subsequent numbers.

- d.



The set of numbers start with 3 and 5. The following number is the sum of two numbers before it.

## Sequence:

### Recognizing and describing patterns:

1. Sequence is a list of succession of numbers created from a set of geometric shapes, numbers or objects.
2. Each number or object in a sequence is the term of the sequence. The value of a term in a sequence depends on the position of the term in the sequence.


### **Example 2:**

Describe the pattern for the following sequence and hence, find the next two terms.

a. 56, 50, 44, 38, ...


b. 2, 6, 18, 54, ...

### Solution:

a. 56, 50, 44, 38, ...  


The pattern of the sequence is subtracting 6 from the previous term to obtain the subsequent term.

Next two term is 32 ( $38 - 6$ ) and 26 ( $32 - 6$ ).

b. 2, 6, 18, 54, ...  


The pattern of the sequence is multiply the previous number by 3 to obtain the subsequent term

Next two term is 162 ( $54 \times 3$ ) and 486 ( $162 \times 3$ ).

### Example 3:

Complete the following sequence.

a. 9, 13,  $x$ , 21, 25,  $y$

b.  $x$ , 18, 6,  $y$ ,  $\frac{2}{3}$ ,  $\frac{2}{9}$

#### Solution:

a) 9, 13,  $x$ , 21, 25,  $y$ ,  
 $+4$   $+4$

The pattern of the sequence is adding 4 to obtain subsequent number.

Hence,  $x = 13 + 4 = 17$

$y = 25 + 4 = 29$

Check the pattern again by inserting the value of  $x$ ,  $y$ .

9, 13, 17, 21, 25, 29, 33  
 $+4$   $+4$   $+4$   $+4$   $+4$   $+4$

So,  $x = 17$ ,  $y = 29$ .

b)  $x$ , 18, 6,  $y$ ,  $\frac{2}{3}$ ,  $\frac{2}{9}$   
 $\div 3$   $\div 3$

The pattern of the sequence is dividing 3 to obtain subsequent value.

Hence  $x = 18 \times 3 = 54$ ,  $y = \frac{6}{3} = 2$

Check the pattern again by inserting  $x$  and  $y$

54, 18, 6, 2,  $\frac{2}{3}$ ,  $\frac{2}{9}$   
 $\div 3$   $\div 3$   $\div 3$   $\div 3$   $\div 3$

So,  $x = 54$ ,  $y = 2$ .



## 1.2 Patterns and Sequences

### Making Generalisation about Patterns of a sequence

1. Generalization about the pattern of a sequence can be written in the form of algebraic expression for the  $n$ -th term,  $T_n$  of the sequence.
2. The value of a term can be determined by substituting the value of  $n$  into the algebraic expression.

#### Example 4:

It is given a sequence of perfect square : 1,4,9,16,25,....

State the  $n$ -th term,  $T_n$  of the sequence in the form of algebraic expression.

#### Solution:

$$T_1 = 1^2, T_2 = 2^2, T_3 = 3^2, T_4 = 4^2, T_5 = 5^2$$

$$\text{Hence, } T_n = n^2.$$

#### Example 5:

It is given that  $n$ -th term of a sequence is  $T_n = 3n - 7$ . Find

- a. The first three term
- b. The 18<sup>th</sup> term

#### Solution:

a.  $T_n = 3n - 7$

$$T_1 = 3(1) - 7 = -4$$

$$T_2 = 3(2) - 7 = -1$$

$$T_3 = 3(3) - 7 = 2$$

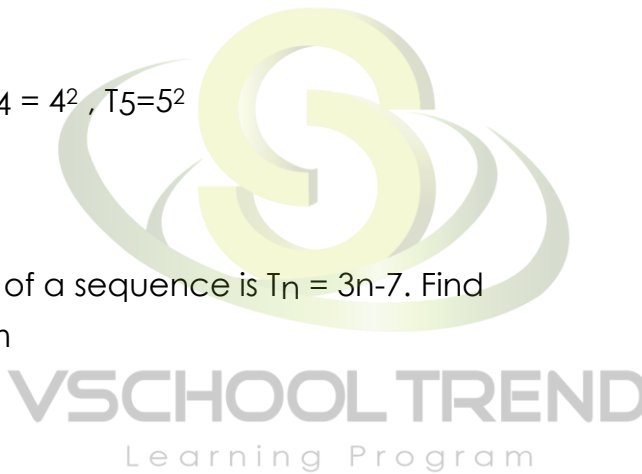
b. The 18<sup>th</sup> term

$$= T_{18}$$

$$= 3(18) - 7$$

$$= 54 - 7$$

$$= 47$$



### Problems involving sequences:

#### Example 6:

Fariz walks along a road towards a hilltop. There are 12 lamp posts with equal distance from one another along the road. Fariz take 10 seconds to walk from the first lamp post to the second lamp post, and 11 second to walk from the second lamp post to the third lamp post. He takes 1 second longer to reach the next lamp post. What is the total time taken by Fariz to walk from the first lamp post to the last lamp post?

#### Solution:

Given: 12 lamppost;  $T_1 = 10$  and  $T_2 = 11$ ; pattern: +1 second.

Total time taken from the 1<sup>st</sup> lamp post to the last lamp post = ?

12 lamp posts are separated by 11 equal distance.

The sequence of time with 11 terms:

**10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20**

Total time taken by Fariz to walk from the first lamp post to the last lamp post  
= sum of term in the sequence of time

$$= 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20$$

$$= 165 \text{ seconds}$$

#### **Answer:**

**Total time taken by Fariz to walk from 1<sup>st</sup> lamp post to last lamp post is 165 seconds.**

