CHAPTER 2
FACTORISATION AND ALGEBRAIC FRACTION

EXPANSION

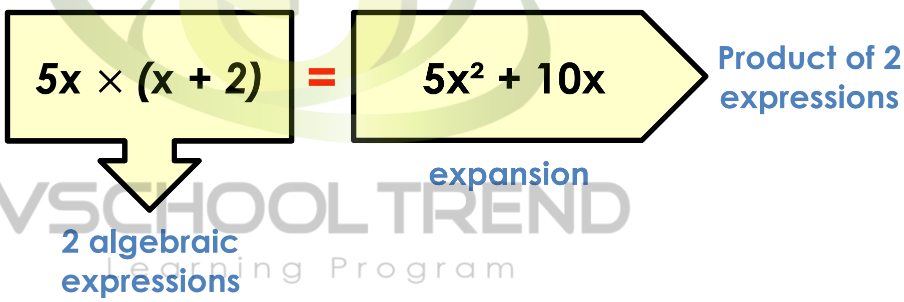
Expansion of Two Algebraic Expressions

Expansion of two algebraic expressions is the product of 2 algebraic expressions to form a single algebraic expression.

Example 1

Given $5x(x + 2) = 5x^2 + 10x$, state the expression of $5x(x + 2)$.

Solution:



Performing the expression of 2 algebraic expression

1. Expansion of two algebraic expansion can take place by multiplying each term in the 1st expression with every term in the second expression.

2. Generally,

a) $a(a+b) = a^2 + ab$

Each term in the bracket is multiplying by

b) $(a+b)(c+d) = ac + ad + bc + bd$

Each term in the 1st bracket is multiplied by each term in the 2nd bracket respectively

Example 2

Expand:

a) $p(2p+5)$
b) $(m-1)(m+5)$
c) $(x-8)^2$

Solution

a) $p(2p+5) = (p)(2p) + (p)(5)$
$= 2p^2 + 5p$

b) $(m-1)(m+5) = (m)(m) + 5(m) - m - 5$
$= m^2 + 5m - m - 5$
$= m^2 + 4m - 5$

c) $(x-8)^2 = (x-8)(x-8)$
$= x^2 - 16x + 64$

Simplifying Algebraic Expressions

The following flowchart shows the steps to simplifying an algebraic expansion.

Expansion of two expression is performed

↓

like term are grouped

↓

Addition and subtraction is performed on like term.

Example 3

Simplify each of the following:

a. $(y + 2x)(5y - x) - 4x(x - 7y)$

b. $3(2x - 5) - 4(x - 3)$

c. $20m + 19 - (2m + 5)^2$

Solution:

a. $(y + 2x)(5y - x) - 4x(x - 7y)$

$= 5y^2 - xy + 10xy - 2x^2 - 4x^2 + 28xy$

$= 5y^2 - xy + 10xy + 28xy - 2x^2 - 4x^2$

$= 5y^2 + 37xy - 6x^2$

Group the like term

b. $3(2x - 5) - 4(x - 3)$

$= 6x - 15 - 4x + 12$

$= 6x - 4x + 12 - 15$

$= 2x - 3$

Group the like term

(c) $20m + 19 - (2m + 5)^2$

$= 20m + 19 - (4m^2 + 25 + 20m)$

$= 20m + 19 - 4m^2 - 25 - 20m$

$= 20m - 20m + 19 - 25 - 4m^2$

$= -6 - 4m^2$

Group the like term

Problems involving expansion of Two Algebraic Expression

Example 4

Jane took part in a walkathon. She started her walk at a speed of $(x + 5)$ km/h for x hours. Then she walked at a speed of $(2x - 3)$ km/ h for $(x - 1)$ hours until she crossed the finish line. What is the distance of the walkathon?

Solution:

From question: Jane walked at speed of $(x + 5)$ km/h for x hours. Then she walked at a speed of $(2x - 3)$ km/h for $(x - 1)$ hours.

Distance of walkathon = ?

Tips:
Distance = speed × time

Distance

= distance that Jane walked at speed of $(x+5)$ km/h for x hours + distance that she walked at a speed of $(2x - 3)$km/h for $(x - 1)$ hours until the end of walkathon

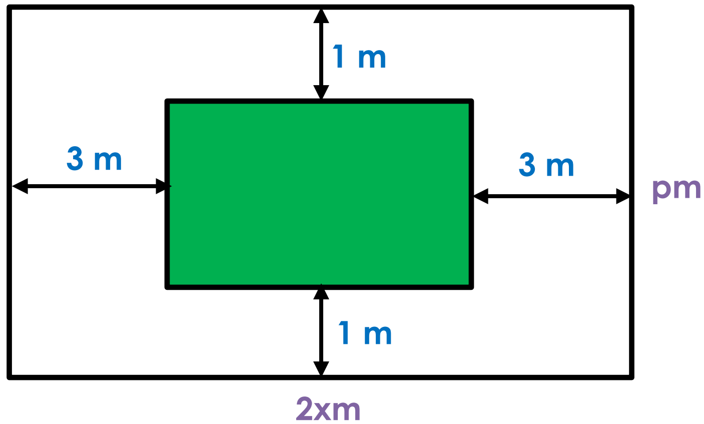
$= x(x + 5)$ km $+ (x - 1)(2x - 3)$ km

$= x^2 + 5x + 2x^2 - 3x - 2x + 3$

$= x^2 + 2x^2 - 3x - 2x + 5x + 3$

$= (3x^2 + 3)$ km

Example 5



Meng wants to place a rectangular carpet in the middle of her room as shown in the diagram above. Calculate the area, in m^2 of the carpet.

Solution:

The length of the carpet = $(2x - 6)$ m

The width of the carpet = $(p - 2)$ m

The area of the carpet = $(2x - 6)(p - 2) = 2px - 4x - 6p + 12$

Area of the rectangle
= length $\times$ width

2.2 Factorisation

1. Factorisation is a process to find factor of an algebraic expression.
2. Factorisation is the reverse of expansion.

Factorisation

$$x(3x + 4) \quad \rightleftarrows \quad 3x^2 + 4x$$

Expansion

3. Highest common factor (HCF) can be used to factorise an algebraic expression with two terms
   a. Identify the HCF of two terms
   b. Divide the two terms with the HCF
   c. Multiply the HCF with the quotient.

Example 6

Given $(k - 3)(k + 3) = k^2 - 9$, state the factorization of $k^2 - 9$.
Hence, list the factors of $k^2 - 9$.

Solution:

Factorization = $(k - 3)\ (k + 3)$

$k^2 - 9 = (k - 3)\ (k + 3)$

$\quad = 1 \times (k - 3) \times (k + 3)$

Hence, factors of $k^2 - 9$ is 1, $(k - 3)$ and $(k + 3)$

Example 7

Factorise $18ab - 27bc$.

Solution:

3	18ab	27bc
3	6ab	9bc
b	2ab	3bc
	2a	3c

$18ab = 9b \times 2a$ $\quad$ $27bc = 9b \times 3c$

HCF = $3 \times 3 \times b = 9b$

$18ab - 27\ bc$

$= 9b \times 2a - 9b \times 3c$

$= 9b\ (2a - 3c)$

4. Factorization of algebraic expression can also be presented in the following forms:
   a. $a^2 + b^2 + 2ab = (a + b)^2$
   b. $a^2 - b^2 = (a+b)(a - b)$

Example 8

Factorise $x^2 - 16$

Solution:

$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$

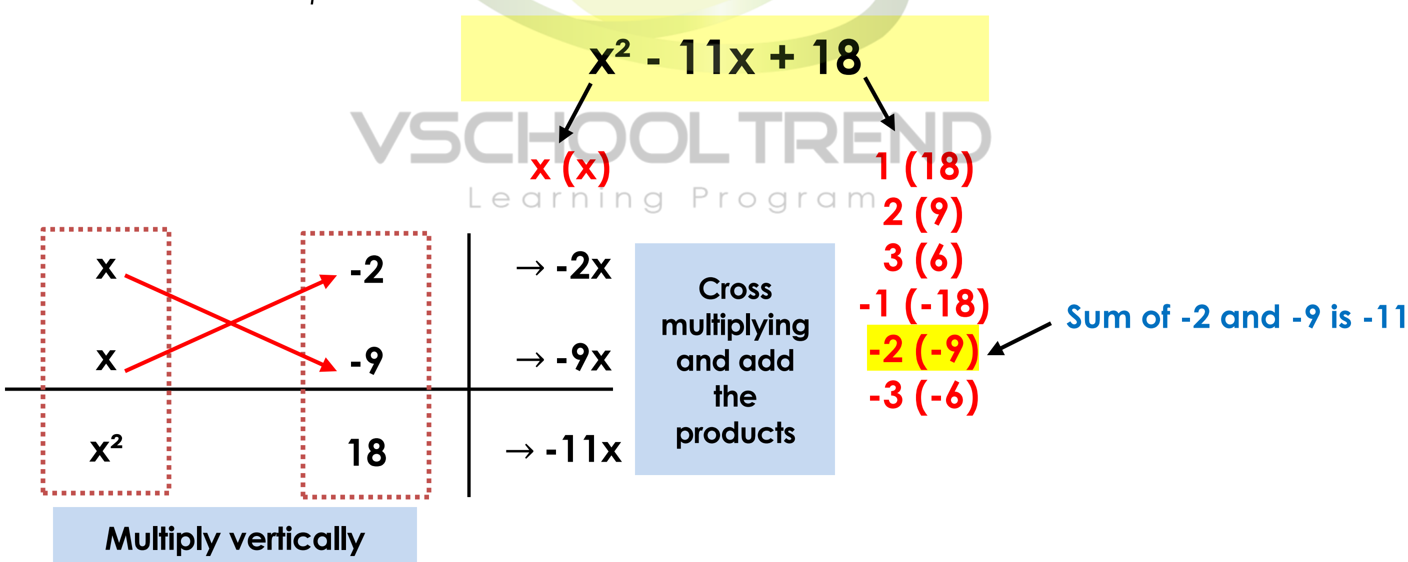
5. Cross multiplication can be used to factorise the algebraic expression with three terms.

Example 9

Factorise $x^2 - 11x + 18$

Solution:

Cross multiplication method:

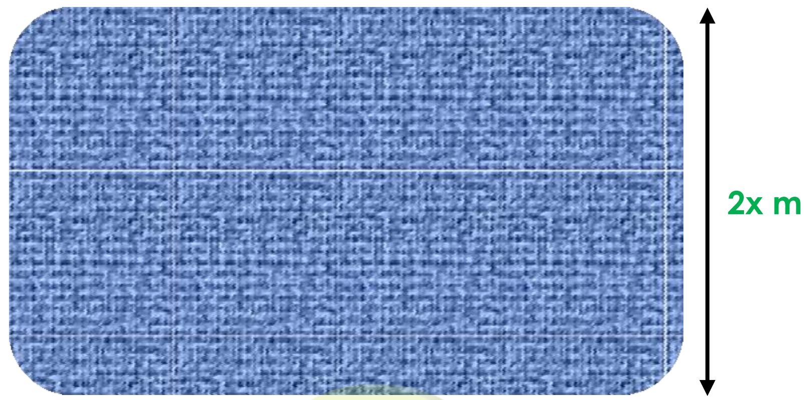


therefore, $x^2 - 11x + 18 = (x - 2)(x - 9)$

Problem Involving Factorisation:

Example 10

Stella bought a rectangular carpet with the width of 2x m. The area of the carpet is $(8x^2+18x)$ m^2. What is the length of the carpet in terms of x?



Area = length × width

Solution:

From the question, width = 2x m, area = $(8x^2 + 18x)$ m^2.
Length of the carpet is ?
Area
$= (8x^2 + 18x)\ m^2.$
$= 2x(4x+9)$

Given width = 2x m

So, length of carpet = $(4x + 9)$ m.

Example 11

In an examination hall, the position of candidates is arranged so that each row consist of $(n + 4)$ candidates. If there are $(n^2 + 19n + 60)$ candidates, how many rows are there in the hall?

Solution:

Total number of candidates
= (number of rows) × (number of candidates in each rows)

$n^2 + 19n + 60 = (?) \times (n + 4)$

n	4	$4n$
n	15	$15n$
n^2	60	$19n$

$n^2 + 19n + 60 = (n + 4)(n + 15)$

Therefore, the number of row is $n + 15$.

2.3 Algebraic Expression and Laws of Basic Arithmetic Operation

Addition and Subtracting algebraic expression

1. Algebraic function is a fraction where the numerator or denominator or both consist of algebraic expression
2. The steps used to simplify algebraic function are:
   a. expanding the algebraic expression
   b. arrange and group the like term
   c. simplify it by using adding or subtracting the like terms
   d. if needed, factorise algebraic function

Example 12

Simplify each of the following

a. (p – 6)(2q + 5) – 4q(p – 3)

b. $\frac{3n+12}{6n} + \frac{5n^2-10n}{n^2-3n+2}$

c. 2(y^2 –– y + 1) + 5y – 2

Solution:

a. (p – 6)(2q + 5) – 4q (p – 3) **Expand**

= 2pq + 5p – 12q – 30 – 4pq + 12q

= 2pq – 4pq + 5p – 12q + 12q – 30 **Group the like term**

= **-2pq + 5p - 30**

b. $\frac{3n+12}{6n} + \frac{5n^2-10n}{n^2-3n+2}$

$= \frac{\cancel{3}(n+4)}{2\,\cancel{6}n} + \frac{5n\cancel{(n-2)}}{(n-1)\cancel{(n-2)}}$ **factorise**

$= \frac{(n+4)}{2n} + \frac{5n}{(n-1)}$ **Equate the denominator**

$= \frac{(n+4)(n-1)}{2n(n-1)} + \frac{5n(2n)}{(2n)(n-1)}$ **Add the numerator**

$= \frac{n^2+3n-4+10n^2}{2n(n-1)}$ **Expand**

$= \boldsymbol{\frac{11n^2+3n-4}{2n(n-1)}}$

c. 2(y^2 – y +1) + 5y – 2

= $2y^2$ – 2y + 2 + 5y – 2

= $2y^2$ – 2y + 5y – 2 + 2

= $2y^2$ +3y

= **y (2y + 3)**

Multiplying and Dividing Algebraic expression

Example 13

Simplify each of the following:

a. (3 + 2x)(3 – 2x) × 5y(y + 7)

b. 8(a – 5) × $\frac{1}{4}$ (2a – 7)

c. $\frac{4m-8}{3m+3n} \div \frac{4}{5m+5n}$

Solution:

a. (3 + 2x)(3 – 2x) × 5y(y + 7)

= $(9 - 4x^2)(5y^2 + 35y)$

= $45y^2 + 315y - 20x^2y^2 - 140x^2y$

b. 8(a – 5) × $\frac{1}{4}$ (2a – 7)

= $(8a - 40)(\frac{1}{2}a - \frac{7}{4})$

= $8a(\frac{1}{2}a - \frac{7}{4}) - 40\,(\frac{1}{2}a - \frac{7}{4})$

= $4a^2 - 14a - 20a + 70$

= $4a^2 - 34a + 70$

c. $\frac{4m-8}{3m+3n} \div \frac{4}{5m+5n}$

$= \frac{4m-8}{3m+3n} \times \frac{5m+5n}{4}$

$= \frac{4(m-2)}{3\cancel{(m+n)}} \times \frac{5\cancel{(m+n)}}{4}$

$= \frac{(m-2)}{3} \times 5$

$= \frac{5(m-2)}{3}$

Performing combined operations of algebraic expression

Example 14

Simplify $(2y^2 - 6y) \div (y^2 - 9) - (2 - y)$

Solution:

$(2y^2 - 6y) \div (y^2 - 9) - (2 - y)$

$= \frac{2y^2-6y}{y^2-9} - (2 - y)$

$= \frac{2y\cancel{(y-3)}}{\cancel{(y-3)}(y+3)} - (2 - y)$

$= \frac{2y}{(y+3)} - \frac{(2-y)(y+3)}{(y+3)}$

$= \frac{2y-(2-y)(y+3)}{(y+3)}$

$= \frac{2y-2y-6+y^2+3y}{(y+3)}$

$= \frac{-6+y^2+3y}{(y+3)}$