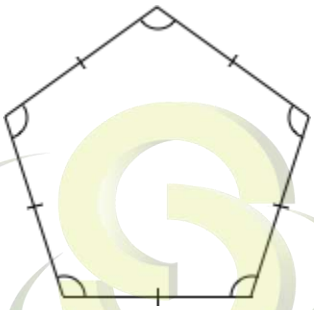
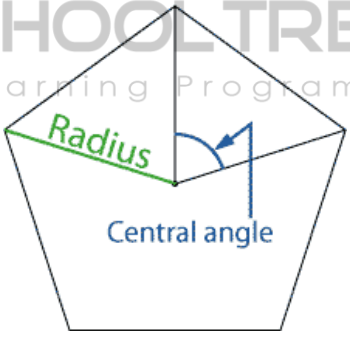
CHAPTER 4
POLYGON

4.1 Regular Polygon

1. Regular polygon is a polygon with all side having the same length and all interior angles are of same size.



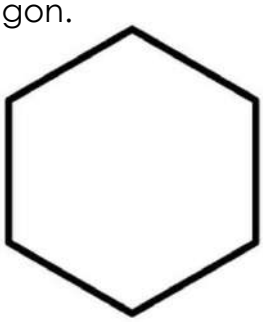
2. The number of axes of symmetry of a regular polygon is equal to the number of side of the polygon.
3. The point that regular polygon meet the axes of symmetry is known as centre.



4. For a regular polygon,
   a. all the vertices are of the same distance from the centre of the polygon
   b. every side attaches the same angle at the centre of angle
5. Irregular polygon is a polygon with unequal length of side and unequal size of interior angles.

Example 1

Diagram below shows a hexagon.



a. Write down the geometric properties of hexagon if hexagon is a regular hexagon.
b. Mark on given diagram to show the geometric properties of the regular hexagon
c. How many axes of symmetry the regular hexagon have?

Solution:

a. all the interior angle are same and all the sides are same of length
b.

c. 6 axes of symmetry

Constructing a regular polygon

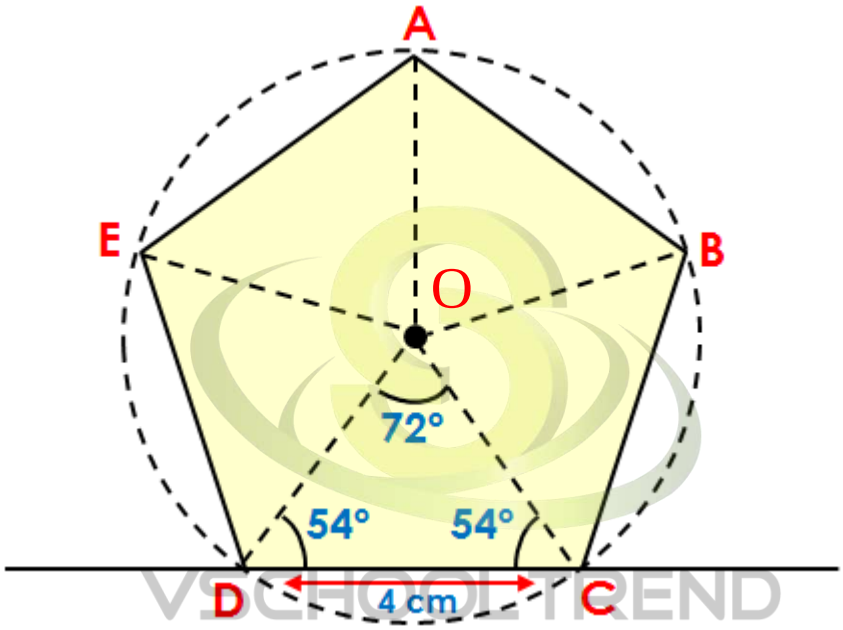
1. $\frac{360^0}{n}$ is the angle subtended by each side of the centre of n-sided regular polygon?
2. A n-sided regular polygon is formed from n congruent isosceles triangle.

Example 2

Construct a regular pentagon ABCDE with side of 4 cm length.

Solution:

i. A line segment of 4cm length is constructed.
ii. Centre of polygon O is construct so that OAB is an isosceles triangle for which the angle at A and B is $\frac{180^o-72^o}{2}$ = 54°
iii. Draw an arc passing through A and B through a complete turn by the point of compasses at O.
iv. Point C, D and E is marked, so that ∠AOB = ∠BOC = ∠COD = ∠DOE = ∠EOA = 72°
v. A line is drawn to complete the regular polygon.



4.2 Interior Angles and Exterior Angle of Polygons

Sum of Interior Angle

Notice that the sum of interior angle of triangle is 180°.

Therefore, the **sum of interior angle** of n-sided triangle is **(n - 2)** × **180°**.

Sum of Exterior Angle

Sum of exterior angle of polygon is always 360°.

Determining interior angle, exterior angle and number of side of a polygon

1. Formula that can used to calculate the interior angle of n-sided regular polygon is

Interior angle

$= \frac{sum\ of\ interior\ angles}{number\ of\ side}$

$= \frac{(n-2)\times 180^o}{n}$

OR

Interior angle

= 180° - exterior angle

$= 180^o - \frac{360^o}{n}$

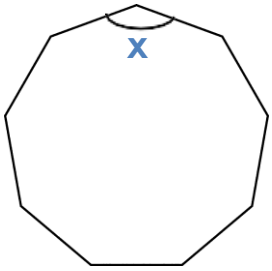
2. Formula that can used to calculated exterior angles of an n-sided regular polygon is

Exterior angle

$= \frac{sum\ of\ exterior\ angles}{number\ of\ side}$

$= \frac{360^o}{n}$

Example 3



A regular nonagon is shown in the diagram above.
What is the value of x?

Solution:

Method 1

Sum of all interior angles

$= (n - 2) \times 180°$

$= (9 - 2) \times 180°$

$= 7 \times 180°$

$= 1260°$

Thus, $x = \frac{1260^0}{9} = 140°$

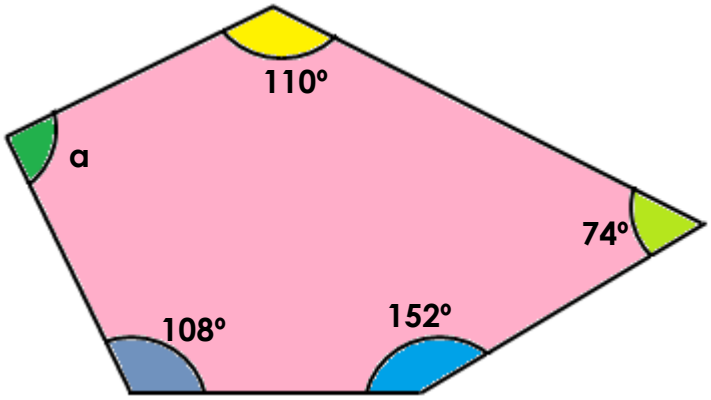
Method 2

Exterior angle $= \frac{360^o}{9} = 40°$

Thus, interior angle $= 180° - 40° = 140°$

Example 4

Diagram below shows an irregular pentagon.
Find the value of a.



Solution:

Sum of all interior angles = (5 – 2) × 180° = 540°

Thus,

a + 108° + 110° + 152° + 74° = 540°

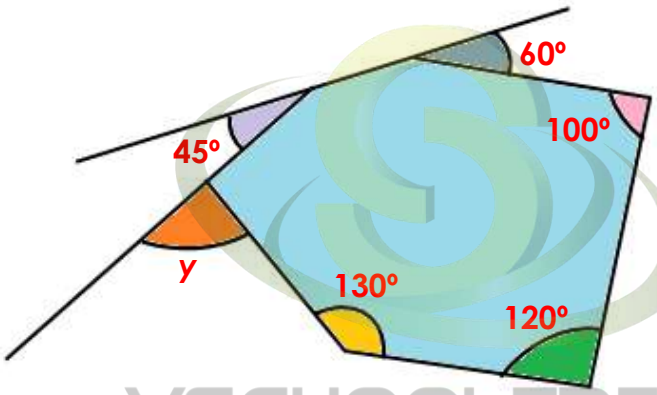
a = 540 – (108° + 110° + 152° + 74°)

a = 540° – 444°

a = 96°

Example 5

Find the value of y.



Solution:

Sum of the exterior angle = 360°

y + 45° + 60° + (180° - 100°) + (180° - 120°) + (180° - 130°) = 360°

y + 45° + 60° + 80° + 60° + 50° = 360°

y + 295° = 360°

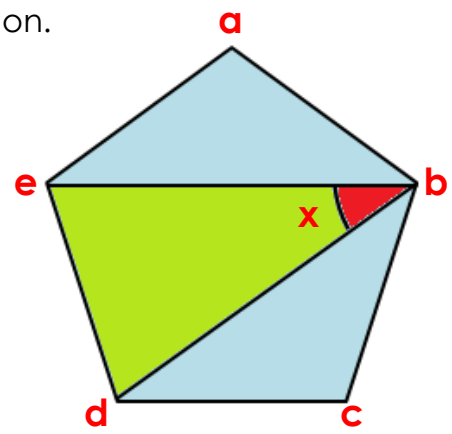
y = 360° - 295°

y = 65°

Solving the problem involving polygon

Example 6

Abcd is a regular pentagon.
Find the value of x.



Solution:

Interior angle = $\frac{(5-2)\times 180^o}{5}$ = 108°

$\angle$ abe = $\angle$ cbd = $\frac{180^o-108^o}{2}$ = 36°

x = 108° - 36° - 36° = 36°

Example 7

In a signage design competition, Michael had created a signage in the shape of a regular polygon with side of length 5cm and its perimeter is 30cm. What is the value of interior angle of the signage?

Solution:

From the question, length of side of the polygon = 5 cm, perimeter = 30 cm

Interior angle = ?

Number of sides, n = $\frac{30}{5}$ = 6

Number of interior angles = 6;

Sum of the interior angle = (6 - 2) × 180° = 720°

Interior value of the signage = $\frac{720^o}{6}$ = 120°