HW4

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1 HW4

author: Luis Perez
email: luis0@stanford.edu

1.1 Imports

[47]:
```
import numpy as np
from scipy import linalg
import seaborn as sns
import math
```

1.2 Problem 1: Fitting a model with hourly temperature

1.2.1 Part (b)

[5]:
```
"""Loading y vector of temperatures."""
y = [
    1.244822e+001,
    1.056210e+001,
    1.033174e+001,
    9.606498e+000,
    8.556261e+000,
    9.959871e+000,
    1.077175e+001,
    1.146578e+001,
    1.328712e+001,
    1.496854e+001,
    1.644381e+001,
    1.826723e+001,
    1.856764e+001,
    2.046171e+001,
    1.939858e+001,
    1.930111e+001,
    1.934969e+001,
    1.831423e+001,
    1.766321e+001,
```

```
    1.669083e+001,
    1.658678e+001,
    1.498528e+001,
    1.505830e+001,
    1.438545e+001,
    1.197421e+001,
    1.148069e+001,
    1.055407e+001,
    8.324995e+000,
    8.069414e+000,
    9.311587e+000,
    9.640021e+000,
    1.149359e+001,
    1.319649e+001,
    1.490358e+001,
    1.684989e+001,
    1.790743e+001,
    1.912722e+001,
    1.844008e+001,
    1.912928e+001,
    1.883938e+001,
    1.768906e+001,
    1.808924e+001,
    1.686000e+001,
    1.749298e+001,
    1.571663e+001,
    1.559855e+001,
    1.449279e+001,
    1.279592e+001,
    1.091916e+001,
    1.070769e+001,
    9.108555e+000,
    9.117091e+000,
    8.732967e+000,
    9.562628e+000,
    9.827205e+000,
    1.051960e+001,
    1.267284e+001,
    1.373827e+001,
    1.589141e+001,
    1.724662e+001,
    1.823062e+001,
    1.858238e+001,
    1.938788e+001,
    1.768315e+001,
    1.840879e+001,
    1.811321e+001,
```

```
    1.745971e+001,
    1.678153e+001,
    1.584773e+001,
    1.538232e+001,
    1.437838e+001,
    1.284104e+001,
    1.152896e+001,
    1.030374e+001,
    8.591905e+000,
    8.409658e+000,
    8.256319e+000,
    8.593017e+000,
    9.973718e+000,
    1.038751e+001,
    1.251709e+001,
    1.436635e+001,
    1.609681e+001,
    1.650109e+001,
    1.806422e+001,
    1.858905e+001,
    1.806650e+001,
    1.798032e+001,
    1.846825e+001,
    1.733324e+001,
    1.702398e+001,
    1.627260e+001,
    1.524704e+001,
    1.450239e+001,
    1.405536e+001,
    1.223471e+001,
    1.185029e+001,
    1.047937e+001,
    8.663015e+000,
    8.139456e+000,
    7.350008e+000,
    7.082453e+000,
    9.494333e+000,
    1.005408e+001,
    1.212076e+001,
    1.383665e+001,
    1.539110e+001,
    1.625036e+001,
    1.724103e+001,
    1.804050e+001,
    1.773624e+001,
    1.758339e+001,
    1.842454e+001,
```

```
    1.718926e+001,
    1.598398e+001,
    1.597279e+001,
    1.476660e+001,
    1.387474e+001,
    1.347138e+001,
    1.295600e+001,
    1.129482e+001,
    1.029512e+001,
    8.263162e+000,
    7.816357e+000,
    7.588277e+000,
    7.353837e+000,
    8.119445e+000,
    1.055820e+001,
    1.174310e+001,
    1.317865e+001,
    1.556603e+001,
    1.665215e+001,
    1.700492e+001,
    1.868391e+001,
    1.826153e+001,
    1.863960e+001,
    1.804732e+001,
    1.661052e+001,
    1.573907e+001,
    1.575054e+001,
    1.496028e+001,
    1.392356e+001,
    1.364802e+001,
    1.302191e+001,
    1.075686e+001,
    9.344309e+000,
    8.525904e+000,
    8.181372e+000,
    7.096798e+000,
    7.165355e+000,
    8.526029e+000,
    9.867947e+000,
    1.178932e+001,
    1.301388e+001,
    1.489993e+001,
    1.613176e+001,
    1.727414e+001,
    1.859689e+001,
    1.762994e+001,
    1.704406e+001,
```

```
    1.753179e+001,
    1.632080e+001,
    1.622418e+001,
    1.529564e+001,
    1.521687e+001,
    1.444662e+001,
    1.312329e+001,
    1.107770e+001,
]
N = 168
```

[6]:
```python
def constructAMatrixFor1b(start:int, end:int):
    """We construct the matrix A as described in the homework assignment.

    Args:
        start: The starting timestep. Must be % 24 == 1, >= 1.
        end: The ending timestep. Must be a multiple of 24
        and must be >= 24.

    Returns:
        The matrix A.
    """
    assert start % 24 == 1 and start >= 1
    assert end % 24 == 0 and end >= 24
    identityMat = np.identity(24)
    onesPart = np.concatenate([identityMat.copy() for _ in range(start // 24,␣
 ↪end // 24)], axis=0)
    aColumn = np.flip(np.array(range(start, end + 1)))
    aColumn.shape = (aColumn.shape[0], 1)
    return np.append(onesPart, aColumn, axis=1)
```

[7]:
```python
A = constructAMatrixFor1b(start=1, end=N)
```

[8]:
```python
x = np.dot(np.linalg.inv(np.dot(A.T, A)), np.dot(A.T, np.flip(y)))
yhat = np.flip(np.dot(A, x))
```

[9]:
```python
print("The trend parameter is a=%s" % (x[-1]))
```

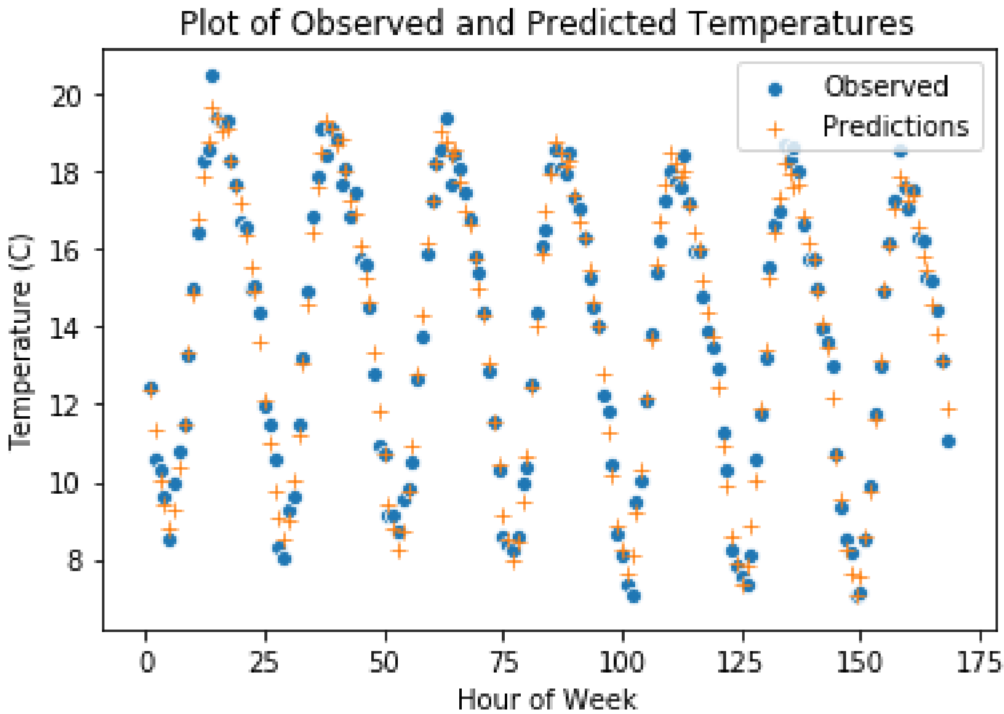
```
The trend parameter is a=-0.012075460503471858
```

[10]:
```python
RMSE = np.sqrt(np.sum((y-yhat)**2) / len(yhat))
print("The RMSE on training data is: %s" % (RMSE))
```

```
The RMSE on training data is: 0.4050510038014836
```

[11]:
```python
sns.scatterplot(x=range(1,N+1), y=y, label='Observed')
ax = sns.scatterplot(x=range(1,N+1), y=yhat, marker='+', label='Predictions')
ax.set_title('Plot of Observed and Predicted Temperatures')
```

```python
ax.set(xlabel='Hour of Week', ylabel='Temperature (C)')
ax.get_figure().savefig("temps_on_train")
```



1.2.2 Part (c)

[12]:
```python
# The measured values for the next 24 hours.
ytom = [
    1.092478e+001,
    1.039376e+001,
    8.989078e+000,
    6.891658e+000,
    7.317766e+000,
    7.259584e+000,
    7.909287e+000,
    9.120217e+000,
    1.152908e+001,
    1.293617e+001,
    1.484307e+001,
    1.600680e+001,
    1.692412e+001,
    1.785879e+001,
    1.814252e+001,
    1.862022e+001,
```

```
    1.646605e+001,
    1.611008e+001,
    1.657515e+001,
    1.526571e+001,
    1.411094e+001,
    1.418155e+001,
    1.385655e+001,
    1.231481e+001,
]
```

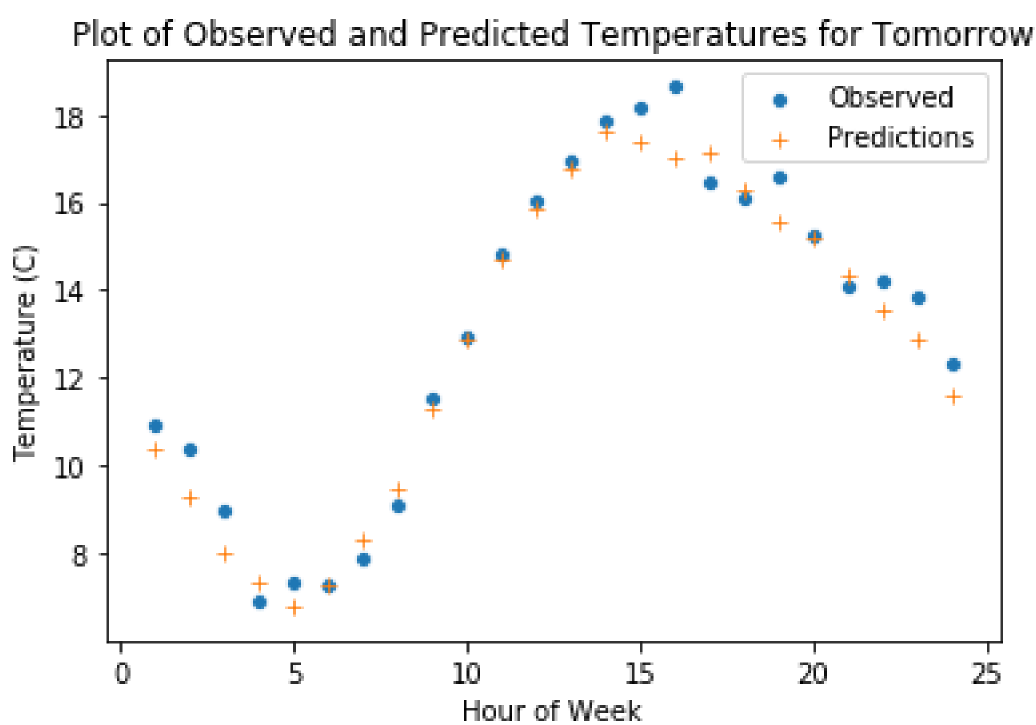
[13]:
```
APred = constructAMatrixFor1b(start=169, end=192)
```

[14]:
```
yhattom = np.flip(np.dot(APred, x))
```

[15]:
```
RMSETest = np.sqrt(np.sum((ytom-yhattom)**2) / len(ytom))
print("The RMSE of the prediction error for tomorrow's temperatures is: %s" %␣
 ↪(RMSETest))
```

```
The RMSE of the prediction error for tomorrow's temperatures is:
0.6521628280735887
```

[16]:
```
sns.scatterplot(x=range(1,25), y=ytom, label='Observed')
ax = sns.scatterplot(x=range(1,25), y=yhattom, marker='+', label='Predictions')
ax.set_title('Plot of Observed and Predicted Temperatures for Tomorrow')
ax.set(xlabel='Hour of Week', ylabel='Temperature (C)')
ax.get_figure().savefig("temps_on_test")
```



1.3 Problem 2: Identifying a system from input/output data

1.3.1 Part (b)

[17]:
```
# data for EE263 problem on system identification
n=3
m=10
N=100
X=np.array([
    5.8248,
    3.1342,
    0.3754,
    1.7580,
   -3.4826,
    8.4807,
    0.2953,
    8.9854,
    1.3203,
    4.3584,
   -7.2309,
   -3.5058,
    6.2299,
   -3.1949,
    2.8868,
   -1.8001,
   -0.6779,
   -6.7467,
   -6.3522,
    4.9229,
   -0.2244,
   -3.9947,
   -3.8259,
    4.3087,
   -0.2811,
    2.5674,
    1.9834,
    3.7811,
    2.0024,
   -6.7069,
    1.8752,
    5.6258,
    3.6432,
  -11.8873,
   -1.3689,
   -1.6147,
```

```
   1.5899,
  -2.5559,
  -0.0102,
   8.0326,
   4.2382,
   1.3405,
  -4.6174,
  -0.3525,
   0.7395,
  -2.7855,
  -1.6835,
   2.0761,
   7.7891,
 -12.2215,
  -5.4910,
   5.6132,
   2.9083,
  -1.3568,
   2.0710,
  -4.8891,
  -5.1073,
   1.5884,
   7.5805,
   3.7472,
  -2.5385,
   4.4265,
  -1.2405,
  -3.6312,
  -2.2252,
  -3.0646,
  -1.0457,
   2.8107,
  -5.3196,
   1.7579,
   5.6650,
   0.7500,
   3.5157,
  -0.2621,
  10.0925,
   4.6208,
  -9.0706,
   0.1749,
  -9.0393,
   5.1410,
   1.9730,
   3.1970,
   4.3711,
```

```
  8.7620,
 -1.6003,
 -0.6871,
  3.0788,
  4.8895,
 -5.5767,
 -2.7501,
  0.1994,
-12.4142,
  5.7933,
 -5.1314,
  5.7674,
 -3.9323,
  3.1740,
  4.1020,
 -0.8801,
  2.8124,
 -0.6372,
  2.7709,
 -5.4867,
 -3.6565,
  7.0237,
 -3.1011,
  1.1857,
 -7.9342,
 -2.0074,
 -3.8535,
 -1.3134,
  4.8824,
  4.8891,
  5.8501,
  0.7966,
  2.4976,
 -5.2769,
 -2.2537,
  6.3519,
  4.4935,
  2.1935,
 -6.2367,
  1.6233,
  1.9504,
 -2.0257,
  1.4616,
 12.8296,
 -2.2891,
 -8.0541,
-13.3476,
```

```
  -3.7985,
  -3.3736,
  -5.8584,
  10.1647,
   4.8424,
   3.3515,
   2.1007,
 -14.3638,
   8.4294,
   0.1396,
  -4.5102,
 -10.2663,
   0.4454,
  10.4355,
   1.8256,
   4.2305,
  -0.9227,
   5.1536,
  -7.6381,
   4.8247,
   2.6308,
  -0.9223,
   0.9939,
   7.9521,
   0.1610,
   4.4458,
  -6.4958,
   5.9129,
   9.0874,
  -2.9215,
  -5.0534,
  -4.8025,
   3.4558,
  -3.7931,
  -0.4849,
  -7.0347,
   5.1541,
  -3.7994,
   4.3706,
   3.8056,
  -0.8296,
   1.5045,
  -1.6123,
  -1.8421,
   5.7395,
   0.2072,
  -5.4902,
```

```
    7.8336,
   -5.2421,
    2.1136,
   -4.2221,
   -1.5581,
    1.9891,
    5.2489,
   -1.7040,
    1.6815,
   -1.1068,
    0.0832,
   -5.9618,
   -0.6582,
    7.4376,
   -4.1841,
   -6.5049,
    7.8707,
    5.8302,
    3.9321,
   -7.3082,
    7.7723,
   -2.9877,
   -6.0528,
   -3.5133,
    1.7821,
    3.2632,
    1.0784,
   -1.3195,
    9.0122,
   -3.2149,
    0.5478,
   -3.5952,
    2.1031,
   -9.6557,
    3.3015,
   -5.5125,
   -0.5149,
   -5.2990,
   -6.1928,
   -9.4462,
   -4.8679,
    1.0606,
    2.4672,
    7.7359,
    3.2247,
  -10.7418,
   -5.1442,
```

```
 -0.7079,
-12.6335,
 -1.5649,
 -2.9681,
  1.6616,
  2.7943,
  4.4994,
 -1.0045,
 -1.1687,
  7.2495,
  9.1807,
 -1.9146,
  0.7754,
 -4.8232,
  0.1938,
  3.8273,
 -2.9726,
  0.6512,
  0.1751,
 -3.1234,
 -2.6989,
  9.3998,
 -5.0192,
 -2.4872,
 -7.5220,
 -0.4772,
  1.9836,
 -2.6356,
  1.7229,
 -3.6165,
  6.3410,
 -0.1562,
  3.8911,
 10.9024,
  2.1891,
  6.6666,
  1.2554,
 -1.5524,
 -4.6150,
 -1.9239,
  5.7909,
  4.3125,
 -5.1735,
 -0.9634,
 -6.4986,
  1.5330,
  4.8450,
```

```
   -3.7366,
  -13.9801,
    3.4837,
   16.0345,
    2.6800,
    1.4923,
    1.4202,
    4.7983,
   10.4380,
    7.6234,
   -0.9763,
    0.0863,
    1.2317,
   -4.2724,
    5.7889,
    0.8095,
    7.7853,
   -0.9677,
    8.2565,
   -9.4939,
    9.1126,
   -7.5921,
   -5.2554,
    0.2497,
   -7.2737,
    2.3327,
    2.7272,
    6.6016,
   -2.0225])
X=np.reshape(X,(n,N), order='F')

Y=np.array([
   30.0345,
   22.8095,
   44.8094,
   56.0756,
   56.2002,
   42.7479,
   53.3022,
   64.7067,
   26.2156,
   35.9164,
   28.1879,
  -10.4892,
   71.2742,
   32.0160,
   72.5273,
```

```
  67.0702,
  51.7024,
  15.4489,
 -29.2450,
  33.8679,
  49.6661,
  64.3571,
  10.5165,
  46.0675,
  23.6143,
  50.3220,
  75.9194,
  61.8370,
  79.3337,
  82.1494,
 -45.7625,
 -52.3933,
   5.6125,
  -8.6894,
   7.7985,
 -40.4989,
 -56.9415,
 -22.0384,
 -64.3470,
 -78.4266,
   8.1397,
 -19.0188,
  64.3590,
  41.6441,
  75.8355,
  36.5258,
  30.0059,
  39.3254,
 -27.9618,
  -0.1393,
 -41.2597,
 -13.6561,
 -55.7996,
 -38.4113,
 -67.1491,
 -66.7537,
 -69.5111,
 -39.3169,
  -4.9904,
 -55.0985,
  13.4654,
  36.6802,
```

```
-49.2285,
-27.8074,
-55.3381,
 -6.8952,
  0.0367,
-20.5416,
 44.7614,
 29.4258,
 -5.2957,
-23.9685,
  0.9397,
-23.2421,
-12.9130,
  3.4658,
-11.3018,
-45.0969,
-34.7140,
 -3.9510,
 21.6568,
 18.8146,
 11.5414,
 18.6524,
 12.6434,
 28.2068,
 31.1844,
 20.2500,
 21.6577,
 36.5262,
-15.5867,
 10.5000,
-19.8380,
  9.3835,
 -8.3324,
-36.3594,
-21.5052,
 21.9175,
 16.0932,
-31.2623,
 50.3955,
 44.1079,
 39.1377,
 49.5880,
 47.1709,
 65.0665,
 78.6850,
 62.8964,
 52.2841,
```

```
  75.3249,
 -41.4504,
 -11.7174,
 -92.5243,
 -91.4773,
-123.0891,
 -67.3832,
 -78.0250,
-107.8328,
 -13.5776,
 -37.0191,
  -9.7837,
 -17.1545,
  11.4928,
   0.5064,
  11.7335,
  -2.6714,
  -8.1768,
  -5.7565,
 -21.6756,
 -20.1474,
  45.0893,
  29.7866,
  68.0736,
  80.5315,
  84.3078,
  59.9653,
  83.9376,
  95.6181,
  34.2841,
  52.3477,
  -6.5127,
  -3.5389,
 -29.6949,
 -30.0812,
 -39.1238,
 -11.8903,
 -19.7746,
 -40.3075,
  -1.5598,
  -4.0908,
  -4.6359,
  -5.9160,
  -5.4718,
 -18.2769,
 -10.2622,
   3.3884,
```

```
  -10.1641,
  -27.5424,
  -16.8714,
   -0.6627,
  -73.4399,
  -90.2879,
   18.5297,
  -14.1298,
   17.7372,
  -68.9569,
  -89.1356,
  -26.5242,
 -108.3701,
 -127.7640,
   15.8105,
   17.1600,
   28.5667,
   44.8296,
   44.0908,
   21.7802,
   36.3409,
   58.7663,
   26.4251,
   18.9353,
  -46.6222,
  -42.0949,
  -19.1020,
  -22.9500,
  -21.0272,
  -59.1512,
  -63.8237,
  -30.5963,
  -43.4888,
  -71.9505,
   61.9001,
   59.2292,
   38.9111,
   54.0701,
   50.1095,
   68.3506,
   88.8696,
   75.2077,
   67.6894,
   87.4858,
   11.6920,
   28.2595,
  -25.3070,
```

```
 -5.3128,
-27.4170,
 -4.8357,
  8.5700,
  0.3038,
 41.7528,
 23.0344,
-34.7236,
-20.6072,
-47.7450,
-43.2963,
-55.2976,
-55.2515,
-59.9498,
-52.4430,
-22.0860,
-43.5828,
-15.9823,
 16.3353,
-45.7119,
-19.9126,
-43.2753,
-43.5170,
-27.9246,
 -4.6798,
 25.6032,
-14.1943,
 35.9961,
 39.5620,
 19.9932,
 34.4625,
 32.6274,
 36.8137,
 55.7788,
 49.4435,
 48.6918,
 48.5811,
 58.9997,
  5.3052,
 93.6387,
 63.5379,
 99.7714,
104.2068,
 96.5440,
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   36.7386,
   15.1112,
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   43.7402,
   54.1967,
   58.1558,
   41.7045])
Y=np.reshape(Y,(m,N), order='F')
```

[18]:
```
def solveForA(X, Y):
    """
    X is (n x N)
    Y is (m x N)

    We are looking to solve for A, which will be m x n.
    """
    (n, Nx) = X.shape
    (m, N) = Y.shape
    assert Nx == N
    # m x n -- formed from X and Y.
    xMatrix = np.zeros((n, n))
    yMatrix = np.zeros((m,n))
    for i in range(N):
        x = X[:, i]
        y = Y[:, i]
        x.shape = (n, 1)
        y.shape = (m, 1)
        xMatrix += np.dot(x, x.T)
        yMatrix += np.dot(y, x.T)
    assert len(linalg.null_space(xMatrix)[0]) == 0
    xInv = np.linalg.inv(xMatrix)
    return np.dot(yMatrix, xInv)
```

[19]:
```
minimizerA = solveForA(X, Y)
```

[20]:
```
minimizerA
```

[20]:
```
array([[ 2.02992454e+00,  5.02077879e+00,  5.01040266e+00],
       [ 1.14300076e-02,  6.99991043e+00,  1.01061265e+00],
       [ 7.04239020e+00, -2.54352520e-03,  6.94476335e+00],
       [ 6.99765743e+00,  3.97592792e+00,  4.00242122e+00],
       [ 9.01295285e+00,  1.04493868e+00,  6.99800225e+00],
       [ 4.01187599e+00,  3.96488792e+00,  9.02674982e+00],
       [ 4.98710794e+00,  6.97233996e+00,  8.03363399e+00],
       [ 7.94249406e+00,  6.08754514e+00,  3.01735388e+00],
       [ 9.43583486e-03,  8.97218370e+00, -3.85465462e-02],
       [ 1.06123427e+00,  8.02076138e+00,  7.02847693e+00]])
```

[21]:
```
def getRelApproxErr(mat, X, Y):
    (n, N) = X.shape
    error = 0.0
    for i in range(N):
        error += 1 / N * (linalg.norm(np.dot(mat, X[:, i]) - Y[:, i]) / linalg.
↪norm(Y[:, i]))
    return error
```

[22]:
```
print("The relative error is: %s" % (getRelApproxErr(minimizerA, X, Y)))
```

```
The relative error is: 0.05814323689487761
```

1.4 Proble 3: Robus regression using the Huber penalty function

1.4.1 Part (b)

[62]:
```
def huber(x, delta = 1):
    if abs(x) <= delta:
        return 0.5 * x**2
    else:
        return delta * (abs(x) - 0.5 * delta)
```

[63]:
```
N = 25
t = np.array([
        [0.0115],
        [0.3576],
        [0.9111],
        [1.5272],
        [1.7587],
        [1.8866],
        [1.9175],
        [2.4285],
        [2.6906],
        [2.8750],
        [3.2247],
        [3.4112],
        [4.2435],
        [4.6092],
        [4.6245],
        [4.7136],
        [4.7349],
        [6.0739],
        [7.2176],
        [7.3843],
        [7.6550],
        [7.7016],
        [7.8474],
        [9.1599],
        [9.1742],
])
x = np.array([
        [23.5187],
        [0.0000],
        [18.9909],
        [15.7897],
        [14.8507],
        [14.2145],
        [14.0918],
        [11.4933],
        [10.4921],
```

```
        [8.8340],
        [7.5857],
        [6.1800],
        [1.9019],
        [0.5615],
        [0.3277],
        [-0.1091],
        [-0.0124],
        [-6.5992],
        [-12.7377],
        [-13.2593],
        [-14.8588],
        [-14.6661],
        [-15.4040],
        [0.0000],
        [-22.2578],
])
```

[64]:
```
# Solve the sytem using least squares.
A = np.concatenate((t, np.ones((N,1))), axis=1)
```

[65]:
```
# Least squares solution is:
paramsLeastSquares = np.dot(np.linalg.inv(np.dot(A.T, A)), np.dot(A.T, x))
```

[131]:
```
def iterativeLeastSquares(A, x, startParams, delta=1,
                          maxIterations=1000,eps=1e-4):
    N, _ = A.shape
    prevParams = startParams
    huberV = np.vectorize(huber)
    for _ in range(maxIterations):
        W = huberV(np.diag((np.dot(A, prevParams) - x).flatten()))
        inverse = np.linalg.inv(np.dot(np.dot(A.T, W), A))
        newParams =  np.dot(inverse, np.dot(np.dot(A.T, W), x))
        huberLoss = np.sum(np.dot(A, newParams) - x)
        if linalg.norm(newParams - prevParams) < eps:
            return newParams, huberLoss
        prevParams = newParams
    return newParams, huberLoss
```

[132]:
```
huberParams, loss = iterativeLeastSquares(A, x, paramsLeastSquares)
```

[133]:
```
paramsLeastSquares
```

[133]:
```
array([[-3.94855818],
       [19.03697567]])
```

[134]:
```
 huberParams
```

[134]:
```
array([[-4.9920376],
       [23.5669247]])
```

[135]:
```
loss
```

[135]: 0.40706964531961987

[121]:
```
# Plot the data
sns.scatterplot(x=t.flatten(), y=x.flatten(), label="Data")
sns.lineplot(x=t.flatten(), y=np.dot(A, paramsLeastSquares).flatten(),␣
 ↪color='green', label='Least Squares Fit')
ax = sns.lineplot(x=t.flatten(), y=np.dot(A, huberParams).flatten(),␣
 ↪color='red', label='Huber Fit')
ax.set_title('Fit Lines Plot')
ax.get_figure().savefig('line_plots_for_huber_fit')
```