Pilot wave simulation notes

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October 27, 2014

1 Discretization

According to [Bush's review paper], the field created by a walking droplet is of the form:

$$h(\mathbf{x},t) = A \sum_{n=-\infty}^{[t/T_F]} J_0\left(K_F\left|\mathbf{x}-\mathbf{x}_p(nT_F)\right|\right) e^{-(t-nT_F)/(MT_F)}, \tag{1}$$

and the position of the droplet is determined by

$$m\ddot{\mathbf{x}}_p + D\dot{\mathbf{x}}_p = -mg\nabla h(\mathbf{x}_p,t). \tag{2}$$

1.1 Field discretization

The field at time $t+T_F$ is

$$\begin{aligned}
h(\mathbf{x},t+T_F) &= A \sum_{n=-\infty}^{[t/T_F]+1} J_0\left(K_F\left|\mathbf{x}-\mathbf{x}_p(nT_F)\right|\right) e^{-(t+T_F-nT_F)/(MT_F)} \\
&= A \sum_{n=-\infty}^{[t/T_F]} J_0\left(K_F\left|\mathbf{x}-\mathbf{x}_p(nT_F)\right|\right) e^{-(t-nT_F)/(MT_F)} e^{-1/M} \\
&\quad + AJ_0\left(K_F\left|\mathbf{x}-\mathbf{x}_p(t+T_F)\right|\right) \\
h(\mathbf{x},t+T_F) &= AJ_0\left(K_F\left|\mathbf{x}-\mathbf{x}_p(t+T_F)\right|\right) + e^{-1/M} h(\mathbf{x},t) \qquad (3)
\end{aligned}$$

1.2 Discrete particle motion

Similarly we can express the position of the particle at time $t+T_F$ using the centered second-difference approximation for the second derivative

$$\ddot{\mathbf{x}}_p(t) \approx \frac{\mathbf{x}_p(t+T_F) - 2\mathbf{x}_p(t) + \mathbf{x}_p(t-T_F)}{T_F^2} \tag{4}$$

and Sterling's formula for centered differences for the first derivative

$$\dot{\mathbf{x}}_p(t) \approx \frac{\mathbf{x}_p(t+T_F) - \mathbf{x}_p(t-T_F)}{2T_F}. \tag{5}$$

Plugging these into Eq. 2 gives:

$$-g\nabla h(\mathbf{x}_p(t),t) = \frac{\mathbf{x}_p(t+T_F) - 2\mathbf{x}_p(t) + \mathbf{x}_p(t-T_F)}{T_F^2} + \frac{D}{m}\frac{\mathbf{x}_p(t+T_F) - \mathbf{x}_p(t-T_F)}{2T_F}. \tag{6}$$

Solving for $\mathbf{x}_p(t+T_F)$, we find:

$$\mathbf{x}_p(t+T_F) = \left(\frac{4m}{2m+DT_F}\right)\mathbf{x}_p(t) + \left(\frac{DT_F - 2m}{DT_F + 2m}\right)\mathbf{x}_p(t-T_F) - \left(\frac{2T_F^2 g}{DT_f + 2m}\right)\nabla h(\mathbf{x}_p(t),t). \tag{7}$$

1.3 Nondimensionalization

The expressions giving the discrete time evolution of the field (Eq. 3) and droplet (Eq. 7) can be simplified by introducing the following non-dimensional variables:

$$Q \equiv \frac{DT_F}{2M}, \text{ and} \tag{8}$$

$$S \equiv T_F^2 gAK_F. \tag{9}$$

Q encapsulates properties which could vary between droplets on a single field (in particular, mass and drag), and S accounts for general properties of the wave field (amplitide of bounces, gravitational force, faraday wave number).

In terms of these parameters, Eqs. 3 and 7 become:

$$h(\mathbf{x}, t+T_F) = SJ_0\left(K_F\left|\mathbf{x} - \mathbf{x}_p(t+T_F)\right|\right) + e^{-1/M}h(\mathbf{x},t) \tag{10}$$

$$\mathbf{x}_p(t+T_F) = \left(\frac{2}{Q+1}\right)\mathbf{x}_p(t) + \left(\frac{Q-1}{Q+1}\right)\mathbf{x}_p(t-T_F) - \left(\frac{1}{K_F(Q+1)}\right)\nabla h(\mathbf{x}_p(t),t) \tag{11}$$

Finally, we switch to more suggestive notation for discrete numerical computation by defining:

$$\mathbf{x}_p[n] = \mathbf{x}_p(nT_F), \text{ and} \tag{12}$$

$$h_n[\mathbf{x}] = h(\mathbf{x}, nT_F) \tag{13}$$

We then have:

$$h_{n+1}[\mathbf{x}] = e^{-1/M}h_n[\mathbf{x}] + SJ_0(|\mathbf{x} - \mathbf{x}_p[n+1]|), \text{ with} \tag{14}$$

$$x_p[n+1] = \left(\frac{2}{Q+1}\right)\mathbf{x}_p[n] + \left(\frac{Q-1}{Q+1}\right)\mathbf{x}_p[n-1] - \frac{1}{Q+1}\nabla h_n[x]. \tag{15}$$