

## Problem 6.1

$$P(E_1) = 0.1$$

$$P(E_1 | E_2) = 0.4$$

$$P(E_2) = 0.2$$

$$P(E_3 | E_1^c E_2^c) = 0.2$$

1)  $P(E_3)$ ?

$$\begin{aligned} P(E_3) &= P(E_3 | E_1 E_2) P(E_1 E_2) + \\ &\quad P(E_3 | E_1^c E_2) P(E_1^c E_2) + \\ &\quad P(E_3 | E_1 E_2^c) P(E_1 E_2^c) + \\ &\quad P(E_3 | E_1^c E_2^c) P(E_1^c E_2^c) \end{aligned}$$

$$P(E_1 E_2) = P(E_1 | E_2) P(E_2) = 0.4 \times 0.2 = 0.08$$

$$P(E_1^c E_2) = P(E_1^c | E_2) P(E_2) = 0.6 \times 0.2 = 0.12$$

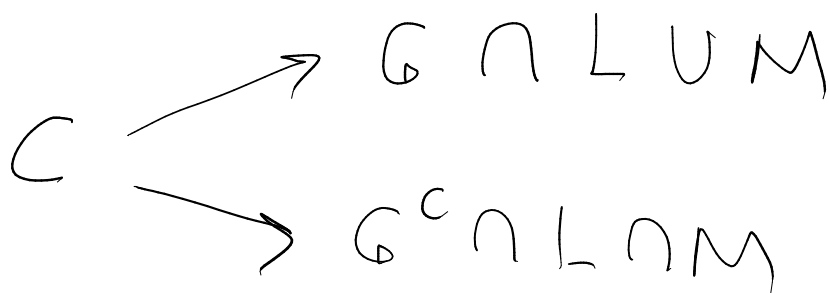
$$\begin{aligned} P(E_1 E_2^c) &= P(E_2^c | E_1) P(E_1) = [1 - P(E_2 | E_1)] P(E_1) \\ &= \left[ 1 - \frac{P(E_1 | E_2) P(E_2)}{P(E_1)} \right] P(E_1) \end{aligned}$$

$$= \left[ 1 - \frac{0.4 \times 0.2}{0.1} \right] 0.1 = 0.02$$

$$P(G^c G_2^c) = 1 - 0.02 - 0.08 - 0.12 = 0.78$$

$$\begin{aligned} P(E) &= 1.0 \times 0.08 + 1.0 \times 0.12 + \\ &\quad 1.0 \times 0.02 + 0.2 \times 0.78 \\ &= 0.376 \end{aligned}$$

Problem 6.2



$$P(L) = 0.7$$

$$P(G) = 0.6$$

$L$  is  $\perp$   $M, G$

$$P(M|G) = 1.0$$

$$P(M|G^c) = 0.5$$

$$1) C = [G(L \cup M)] \cup [G^c L M]$$

$$2) G(L \cup M), G^c L M \text{ mutually exclusive}$$

$$P(C) = P[G(L \cup M)] + P(G^c L M)$$

$$= P(LG \cup GM) + P(L)P(MG^c) =$$

$$= P(LG) + P(GM) - P(LGM) + P(L)P(MG^c)$$

$$= P(L)P(G) + P(M|G)P(G) - P(L)P(M|G)P(G) + P(L)P(M|G^c)P(G^c) =$$

$$= 0.7 \times 0.6 + 1 \times 0.6 - 0.7 \times 1 \times 0.6$$

$$+ 0.7 \times 0.5 \times 0.4 = 0.74$$

$$3) P(L^c | C) ?$$

$$P(L^c | C) = \frac{P(L^c C)}{P(C)} =$$

$$= \frac{P\{L^c([C \cup M)G] \cup (LMG^c))\}}{P(C)} \quad (1)$$

$$X = L^c(C \cup M)G \cup L^cLMG^c =$$

$$= (L^cL \cup L^cM)G = L^cMG$$

$$\begin{aligned} (1) &= \frac{P(L^cMG)}{P(C)} = \frac{P(L^c)P(MG)}{P(C)} = \\ &= \frac{(1 - P(L))P(M)P(G)}{P(C)} = \end{aligned}$$

$$= \frac{(1 - 0.7) \times 1 \times 0.6}{0.74} = 0.243$$