Problem 4.1

$$P(S) = 1 \Rightarrow P(A \cup A^c) = 1 \Rightarrow$$

$$P(A) + P(A^c) - P(A \cap A^c) = 1 \Rightarrow$$

$$P(A) + P(A^c) - 0 = 1 \Rightarrow$$

$$\boxed{P(A^c) = 1 - P(A)}$$

Problem 4.2

$P(A) = 0.03$

$P(B) = 0.05$

$P(C) = 0.05$

$P(AB) = P(A)\,P(B)$

$P(\text{Labor strike})$?

$$P(A \cup B \cup C) = P[(A \cup B) \cup C] =$$

$$= P(A \cup B) + P(C) - P[(A \cup B)C] =$$

$$= P(A) + P(B) - P(AB) + P(C) + P(AC \cup BC)$$

$$P(A)+P(B)+P(C)-P(AB)+P(AC)+P(BC)$$
$$-P(ACBC) =$$

$$= P(A)+P(B)+P(C)-P(A)P(B)+P(A)P(C)$$
$$+P(B)P(C)-P(A)P(B)P(C) =$$

$$= 0.03+0.05+0.05-0.015+0.015+0.025$$
$$-0.0075 = 0.132425 \simeq 0.13$$

Problem 4.3

$$P(EF) = P(E)+P(F)-P(E \cup F)$$

$$P(EF) \geq P(E)+P(F)-1 \Rightarrow$$

$$\cancel{P(E)}+\cancel{P(F)}-P(E \cup F) \geq \cancel{P(E)}+\cancel{P(F)}-1 \Rightarrow$$
$$P(E \cup F) \leq 1 \quad \text{True!}$$

$$P(E) = P(EF \cup EF^c) =$$

$$= P(EF) + P(EF^c) - P(\cancel{EFEF^c})^{\to 0} =$$

$$= P(EF) + P(EF^c) \Rightarrow$$

$$P(EF^c) = P(E) - P(EF)$$

Problem 4.4

$E_{1,2}$: route i is open

$P(E_1) = 0.75 \quad P(E_2) = 0.50 \quad P(E_1 \cap E_2) = 0.40$

1) $$P(E_1 \mid E_2) = \frac{P(E_1 E_2)}{P(E_2)} = \frac{0.40}{0.50} = 0.8$$

2) $$P(E_1^c \mid E_2^c) = \frac{P(E_1^c E_2^c)}{P(E_2^c)} =$$

$$= \frac{P[(E_1 \cup E_2)^c]}{1 - P(E_2)} = \frac{1 - P(E_1 \cup E_2)}{1 - P(E_2)} =$$

$$= \frac{1 - P(E_1) - P(E_2) + P(E_1 E_2)}{1 - P(E_2)}$$

$$= \frac{1 - 0.75 - 0.50 + 0.40}{1 - 0.50} = \frac{0.15}{0.5} = 0.3$$

Problem 4.5

$P(E_1) = P(\text{No rain}) = 0.8$

$P(E_2) = P(\text{@ site feasible}) = 0.7$

$P(E_3) = P(\text{concrete available}) = 0.95$

1) A : casting can be performed

$$A = (E_2 \cup E_3)\, E_1$$

B: casting cannot be performed

$$B = [(E_2 \cup E_3) E_1]^c = E_2^c E_3^c \cup E_1^c$$

2) $P(B) = P(E_2^c E_3^c \cup E_1^c) =$

$$P(E_1^c) + P(E_2^c E_3^c) - P(E_1^c E_2^c E_3^c) =$$

$$1 - P(E_1) + P(E_3^c \mid E_2^c)\, P(E_2^c) - P(E_1^c)\, P(E_2^c E_3^c) =$$

$$= 1 - 0.8 + (1-0.6)(1-0.7) - (1-0.8)(1-0.6)(1-0.7)$$

$$= 0.2 + 0.12 - 0.024 = 0.296$$

3) $P(\text{casting} \mid @\text{site not feasible}) =$

$$= P\left[(E_2 \cup E_3)\, E_1 \mid E_2^c\right] =$$

$$= \frac{P\left[(E_2 \cup E_3)\, E_1\, E_2^c\right]}{P(E_2^c)} =$$

$$= \frac{P\left(E_2 E_1 E_2^c \cup E_1 E_2^c E_3\right)}{1 - P(E_2)} =$$

$$= \frac{P(E_1 E_2^c E_3)}{1 - P(E_2)} = \frac{P(E_1)\, P(E_3 \mid E_2^c)\, \cancel{P(E_2^c)}}{\cancel{1 - P(E_2)}}$$

$$= (0.8)(0.6) = 0.48$$