Problem 5.1

1) $P(A \cup B)$?

$A \cap B = \emptyset$

$P(A \cup B) = P(A) + P(B) - \cancel{P(AB)}^{\,0}$

$= 0.2 + 0.3 = 0.5$

2) $P(A \cup B) = P(A) + P(B) - P(AB) =$

$= P(A) + P(B) - P(A|B) P(B) =$

$= P(A) + P(B) - P(A) P(B) =$

$= 0.2 + 0.3 - 0.06 = 0.44$

3) $P(ABC) = P[(AB) C] =$

$= P(A) P(B) P(C) = 0.024$

4) $P(ABC) = \emptyset$

If A, B, C are independent show that $A, B \cup C$ are independent

$$P[A(B \cup C)] = P(AB \cup AC) =$$

$$= P(AB) + P(AC) - P(AB \cap AC) =$$

$$= P(AB) + P(AC) - P(ABC) =$$

$$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) =$$

$$= P(A)\left[P(B) + P(C) - P(B)P(C)\right] =$$

$$= P(A)P(B \cup C)$$

Problem 5.2

$P(S) = 0.25$

$P(T) = 0.15$

$P(F \mid ST^c) = 0.05$

$P(F \mid STH^c) = 0.10$

$P(F \mid STH) = 1.00$

$S^cT = \emptyset$

$$P(F) = P(FSTH) + P(FSTH^c) + P(FST^c) + P(FS^cT^c)$$

$$= P(F \mid STH)\,P(STH) + P(F \mid STH^c)\,P(STH^c) + P(F \mid ST^c)\,P(ST^c) + P(F \mid S^cT^c)\,P(S^cT^c)$$

$$P(STH) = P(H|ST)\,P(ST) =$$

$$= P(H|ST)\,P(T|S)\,P(S) =$$

$$= 0.15 \times 0.25 \times 0.50 = 0.01875$$

$$P(STH^c) = P(H^c|ST)\,P(ST) =$$

$$= (1 - 0.15) \times 0.25 \times 0.50 = 0.10625$$

$$P(ST^c) = P(T^c|S)\,P(S) =$$

$$(1 - 0.25) \times 0.5 = 0.3750$$

$$P(S^cT^c) = 1 - 0.01875 - 0.10675 - 0.375$$

$$= 0.5000$$

$$P(F) = 1.0 \times 0.01875$$
$$+ 0.10 \times 0.10125$$
$$+ 0.05 \times 0.3750$$
$$+ 0 \times 0.5000 = 0.054$$

Problem 5.3

L: leakage

S_A, S_B: seam of sand from X to A, B

$P(L) = 0.01$

$P(S_A) = 0.02$

$P(S_B) = 0.03$

$P(S_B | S_A) = 0.2$

1) $P(L, S_A) = P(L|S_A)\ P(S_A) =$

$= P(L)\ P(S_A) = 0.01 \times 0.02 = 2 \times 10^{-4}$

2) $P(L S_A \cup L S_B) =$

$P(L S_A) + P(L S_B) - P(L S_A S_B) =$

$= P(L)\left[P(S_A) + P(S_B) - P(S_A S_B)\right] =$

$= P(L)\left[P(S_A) + P(S_B) - P(S_B / S_A) P(S_A)\right]$

$= 0.01\left(0.02 + 0.03 - 0.2 \times 0.02\right) = 0.00046$

Problem 5.4

$P(G) = 0.80$

$P(T|G) = 0.9$

$P(T|G^c) = 0.1$

1) $P(G|T) = \dfrac{P(T|G)\,P(G)}{P(T|G)\,P(G) + P(T|G^c)\,P(G^c)}$

$= \dfrac{0.9 \times 0.8}{0.9 \times 0.8 + 0.10 \times 0.2} = 0.973$

2) $P(G \mid T_1 T_2) = \dfrac{P(T_1 T_2 \mid G) P(G)}{P(T_1 T_2 \mid G) P(G) + P(T_1 T_2 \mid G^c) P(G^c)}$

$= \dfrac{0.9 \times 0.9 \times 0.8}{0.9 \times 0.9 \times 0.8 + 0.1 \times 0.1 \times 0.2} = 0.997$

3) $P(G \mid T_1 T_2^c) = \dfrac{P(T_1 T_2^c \mid G) P(G)}{P(T_1 T_2^c \mid G) P(G) + P(T_1 T_2^c \mid G^c) P(G^c)}$

$= \dfrac{0.9 \times 0.1 \times 0.8}{0.9 \times 0.1 \times 0.8 + 0.1 \times 0.9 \times 0.2} = 0.80$

Problem 5.5

$P(M_{100}) = 0.6 = P(M_1)$

$P(M_{200}) = 0.4 = P(M_2)$

$P(S|M_1) = 0.1$

$P(S|M_2) = 0.3$

1) $P(S) = P(S|M_1)P(M_1) + P(S|M_2)P(M_2)$

$= 0.1 \times 0.6 + 0.3 \times 0.4 = 0.18$

2) $P(M_1|S) = \dfrac{P(S|M_1)P(M_1)}{P(S)} =$

$= \dfrac{0.1 \times 0.6}{0.18} = \dfrac{1}{3}$