Problem 6.1

$P(E_1) = 0.1$ $\quad P(E_1 \mid E_2) = 0.4$

$P(E_2) = 0.2$ $\quad P(E_3 \mid E_1^c E_2^c) = 0.2$

1) $P(E_3)$?

$$P(E_3) = P(E_3 \mid E_1 E_2) P(E_1 E_2) +$$
$$P(E_3 \mid E_1^c E_2) P(E_1^c E_2) +$$
$$P(E_3 \mid E_1 E_2^c) P(E_1 E_2^c) +$$
$$P(E_3 \mid E_1^c E_2^c) P(E_1^c E_2^c)$$

$$P(E_1 E_2) = P(E_1 \mid E_2) P(E_2) = 0.4 \times 0.2 = 0.08$$

$$P(E_1^c E_2) = P(E_1^c \mid E_2) P(E_2) = 0.6 \times 0.2 = 0.12$$

$$P(E_1 E_2^c) = P(E_2^c \mid E_1) P(E_1) = \left[1 - P(E_2 \mid E_1)\right] P(E_1)$$

$$= \left[1 - \frac{P(E_1 \mid E_2) P(E_2)}{P(E_1)}\right] P(E_1)$$

$$= \left[1 - \frac{0.4 \times 0.2}{0.1}\right] 0.1 = 0.02$$

$$P(G^c G_1^c) = 1 - 0.02 - 0.08 - 0.12 = 0.78$$

$$P(E) = 1.0 \times 0.08 + 1.0 \times 0.12 +$$
$$1.0 \times 0.02 + 0.2 \times 0.78$$
$$= 0.376$$

Problem 6.2

C → $G \cap L \cup M$

C → $G^c \cap L \cap M$

$P(L) = 0.7$

$P(G) = 0.6$

L s/I M, G

$P(M|G) = 1.0$

$P(M|G^c) = 0.5$

1) $C = [G(L \cup M)] \cup [G^c L M]$

2) $G(L \cup M)$, $G^c L M$ mutually exclusive

$$P(C) = P[G(L \cup M)] + P(G^c L M)$$

$$= P(LG \cup GM) + P(L) P(M G^c) =$$

$$= P(LG) + P(GM) - P(LGM) + P(L) P(M G^c)$$

$$= P(L) P(G) + P(M|G) P(G) - P(L) P(M|G) P(G)$$

$$+ P(L) P(M|G^c) P(G^c) =$$

$$= 0.7 \times 0.6 + 1 \times 0.6 - 0.7 \times 1 \times 0.6$$

$$+ 0.7 \times 0.5 \times 0.4 = 0.74$$

3) $P(L^c|C)$?

$$P(L^c|C) = \frac{P(L^c C)}{P(C)} =$$

$$= \frac{P\{\overbrace{L^c([(L \cup M)G] \cup (LMG^c))}^{X}\}}{P(C)} \quad (1)$$

$X = L^c(L \cup M)G \cup L^c L M G^c =$ ($L^c L M G^c = \emptyset$)

$= (L^c L \cup L^c M)G = L^c M G$

$$(1) = \frac{P(L^c M G)}{P(C)} = \frac{P(L^c) P(M G)}{P(C)} =$$

$$= \frac{(1 - P(L)) P(M|G) P(G)}{P(C)} =$$

$$= \frac{(1-0.7) \times 1 \times 0.6}{0.74} \cong 0.243$$