

INTRODUCTION TO DATA MINING ASSOCIATION RULES

Luiza Antonie

WHO AM I?

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 - Website: <http://www.uoguelph.ca/~lantonie/>
 - PhD on Associative Classifiers, with Osmar Zaïane and Rob Holte, at University of Alberta
 - Research Interests:
 - Classification, Association Rules
 - Historical Data Linkage
 - Text Collections and Medical Images
 - Natural Language Processing, Health Informatics

WHY DATA MINING?

- The Explosive Growth of Data: from terabytes to petabytes
 - Data collection and data availability
 - Automated data collection tools, database systems, Web, computerized society
 - Major sources of abundant data
 - Business: Web, e-commerce, transactions, stocks, ...
 - Science: Remote sensing, bioinformatics, scientific simulation,
...
 - Society and everyone: news, digital cameras, YouTube
- We are drowning in data, but starving for knowledge!
- “Necessity is the mother of invention”—Data mining—Automated analysis of massive data sets

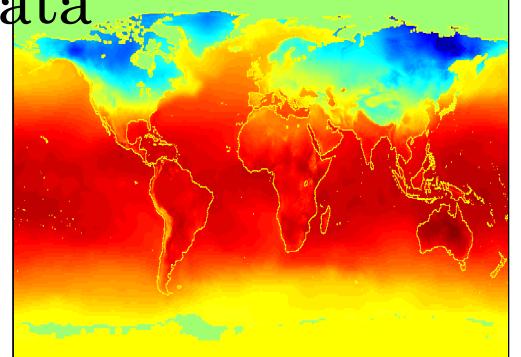
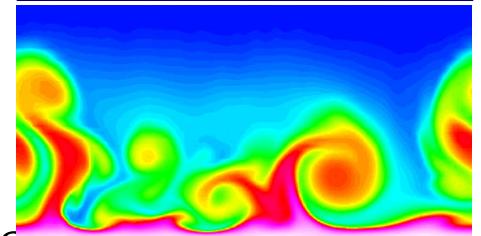
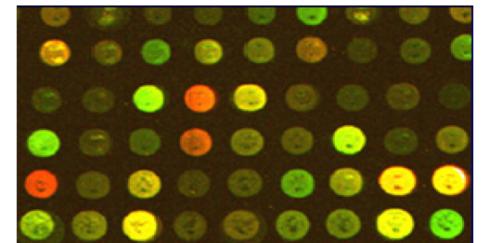
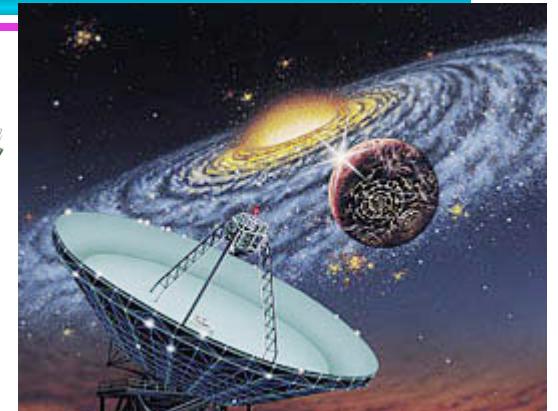
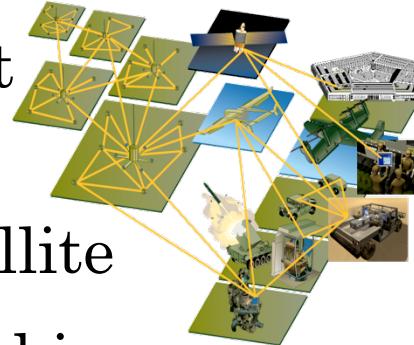
WHY MINE DATA? COMMERCIAL VIEWPOINT

- Lots of data is being collected and warehoused
 - Web data, e-commerce
 - purchases at department/grocery stores
 - Bank/Credit Card transactions
- Computers have become cheaper and more powerful
- Competitive Pressure is Strong
 - Provide better, customized services for an *edge* (e.g. in Customer Relationship Management)



WHY MINE DATA? SCIENTIFIC VIEWPOINT

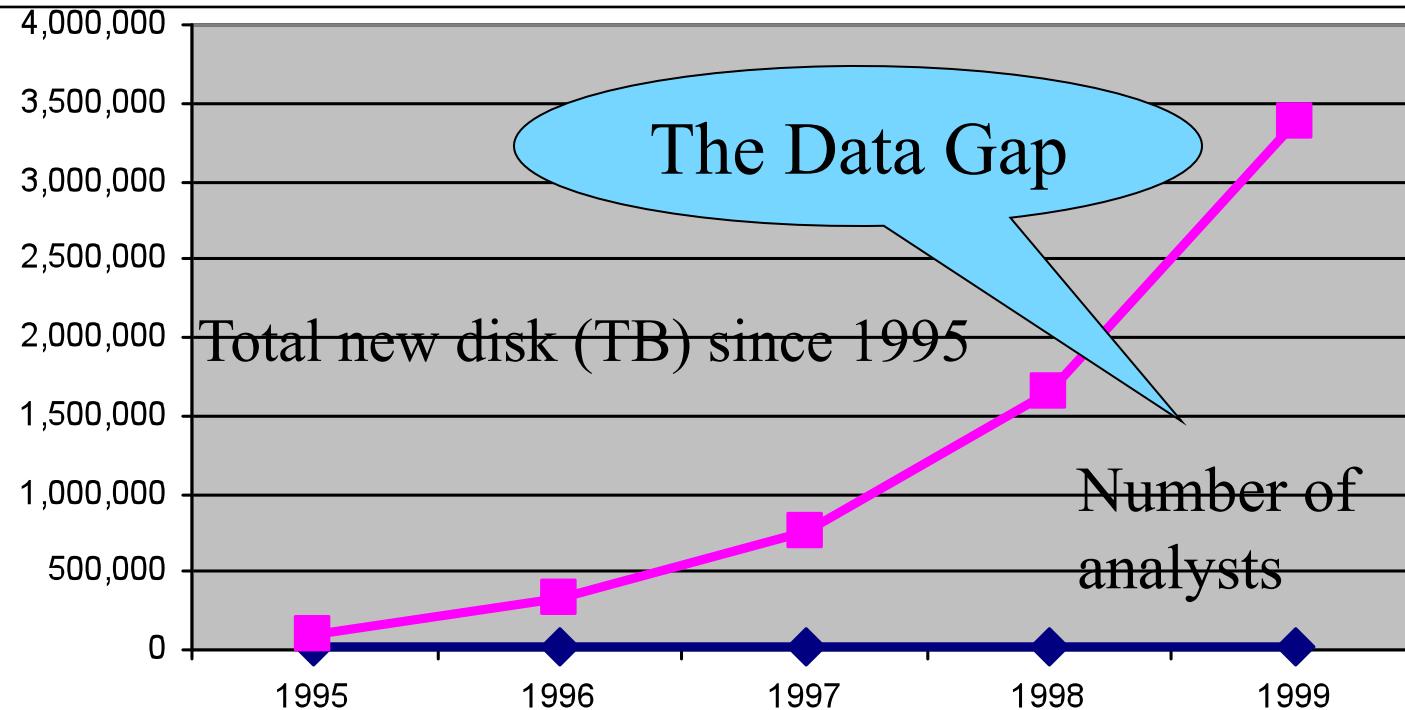
- Data collected and stored at enormous speeds (GB/hour)
 - remote sensors on a satellite
 - telescopes scanning the skies
 - microarrays generating gene expression data
 - scientific simulations generating terabytes of data
- Traditional techniques infeasible for raw data
- Data mining may help scientists
 - in classifying and segmenting data
 - in Hypothesis Formation



MINING LARGE DATA SETS - MOTIVATION

- There is often information “hidden” in the data that is not readily evident
- Human analysts may take weeks to discover useful information
- Much of the data is never analyzed at all

From: R. Grossman, C. Kamath, V. Kumar, “Data Mining for Scientific and Engineering Applications”





WHAT IS DATA MINING?

- Data mining (knowledge discovery from data)
 - Extraction of interesting (non-trivial, implicit, previously unknown and potentially useful) patterns or knowledge from huge amount of data
 - Data mining: a misnomer?
- Alternative names
 - Knowledge discovery (mining) in databases (KDD), knowledge extraction, data/pattern analysis, data archeology, data dredging, information harvesting, business intelligence, etc.
- Watch out: Is everything “data mining”?
 - Simple search and query processing
 - (Deductive) expert systems



WHAT IS (NOT) DATA MINING?

- What is not Data Mining?

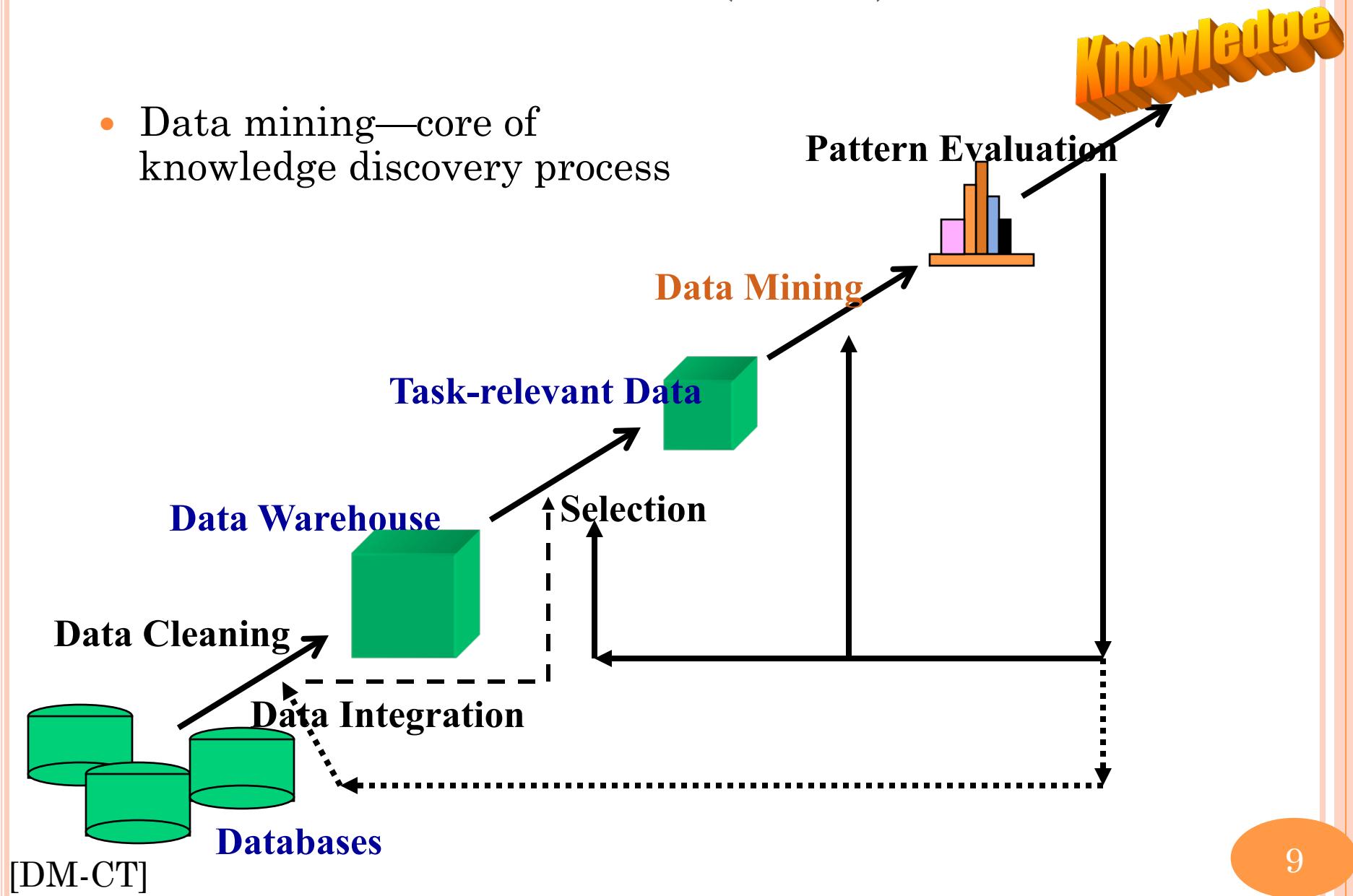
- Look up phone number in phone directory
 - Query a Web search engine for information about “Amazon”

- What is Data Mining?

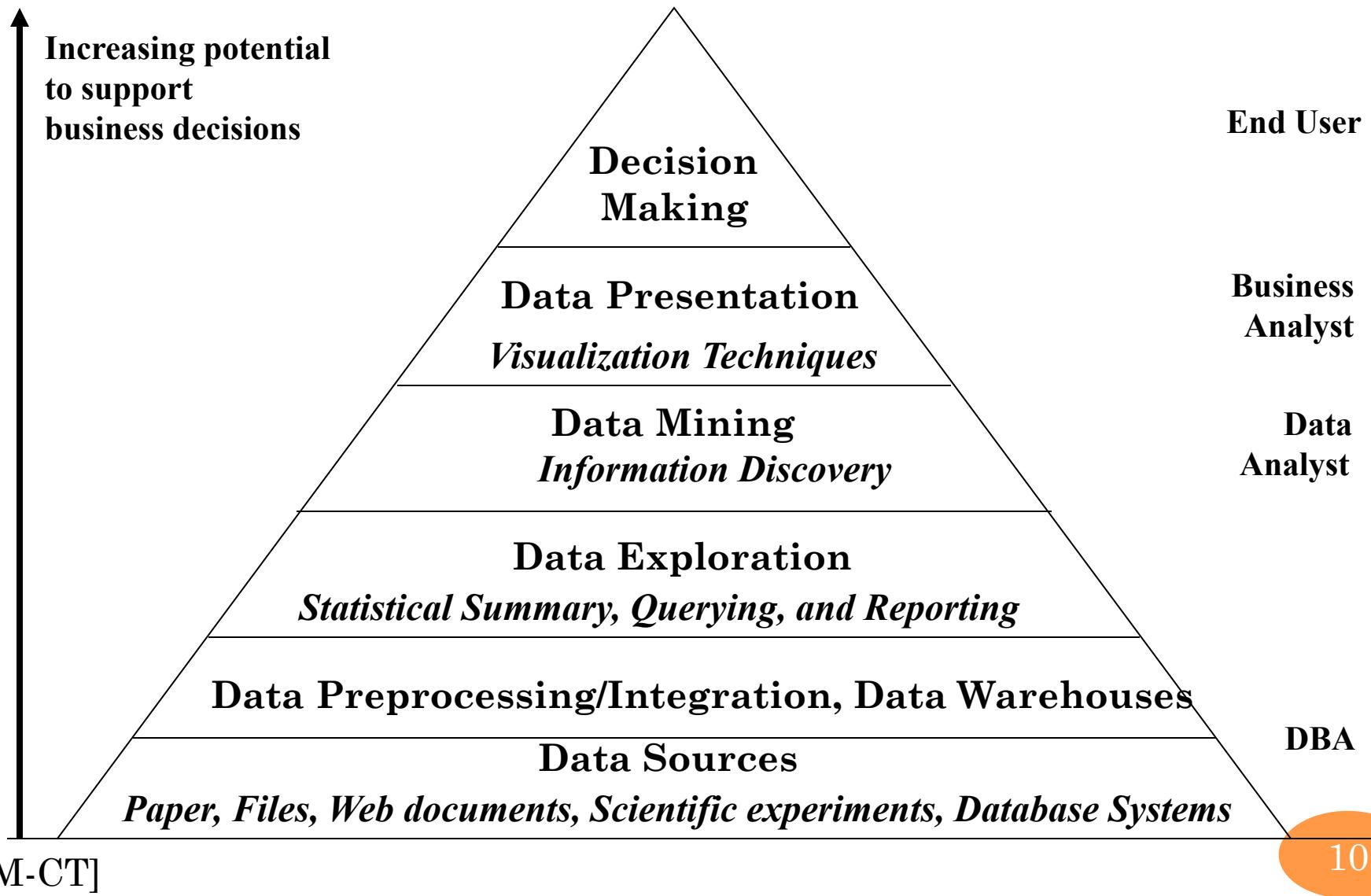
- Certain names are more prevalent in certain US locations (O'Brien, O'Rurke, O'Reilly... in Boston area)
 - Group together similar documents returned by search engine according to their context (e.g. Amazon rainforest, Amazon.com)

KNOWLEDGE DISCOVERY (KDD) PROCESS

- Data mining—core of knowledge discovery process

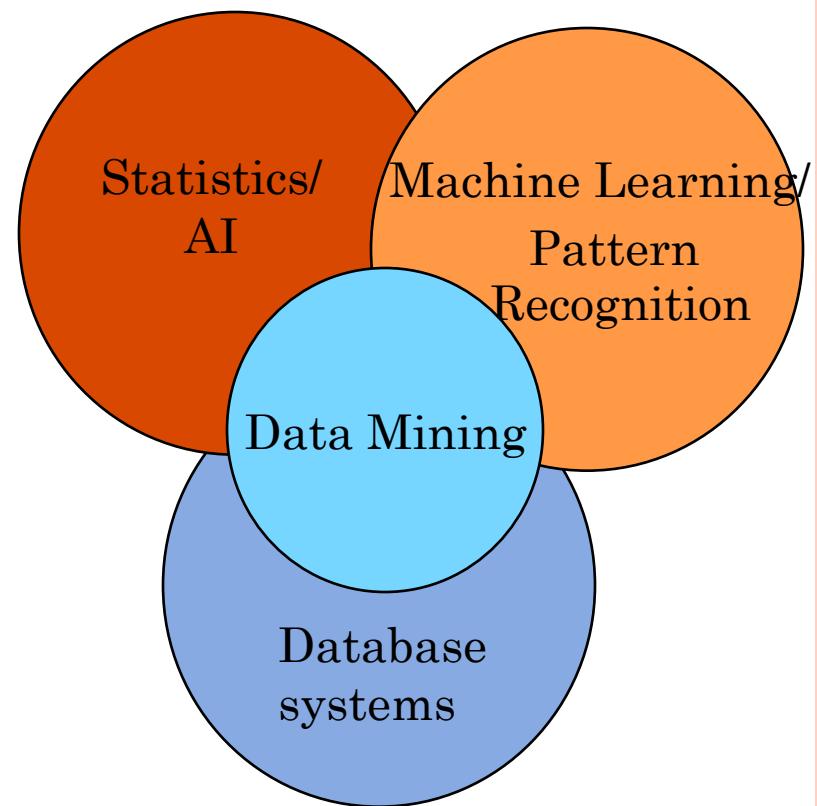


DATA MINING AND BUSINESS INTELLIGENCE



ORIGINS OF DATA MINING

- Draws ideas from machine learning/AI, pattern recognition, statistics, and database systems
- Traditional Techniques may be unsuitable due to
 - Enormity of data
 - High dimensionality of data
 - Heterogeneous, distributed nature of data



DATA MINING: CLASSIFICATION SCHEMES

- General functionality
 - Descriptive data mining
 - Predictive data mining
- Different views lead to different classifications
 - **Data** view: Kinds of data to be mined
 - **Knowledge** view: Kinds of knowledge to be discovered
 - **Method** view: Kinds of techniques utilized
 - **Application** view: Kinds of applications adapted

DATA MINING: ON WHAT KINDS OF DATA?

- Database-oriented data sets and applications
 - Relational database, data warehouse, transactional database
- Advanced data sets and advanced applications
 - Data streams and sensor data
 - Time-series data, temporal data, sequence data (incl. bio-sequences)
 - Structure data, graphs, social networks and multi-linked data
 - Object-relational databases
 - Heterogeneous databases and legacy databases
 - Spatial data and spatiotemporal data
 - Multimedia database
 - Text databases
 - The World-Wide Web

MAJOR ISSUES IN DATA MINING

- Mining methodology

- Mining different kinds of knowledge from diverse data types, e.g., bio, stream, Web
- Performance: efficiency, effectiveness, and scalability
- Pattern evaluation: the interestingness problem
- Incorporation of background knowledge
- Handling noise and incomplete data
- Parallel, distributed and incremental mining methods
- Integration of the discovered knowledge with existing one: knowledge fusion

- User interaction

- Data mining query languages and ad-hoc mining
- Expression and visualization of data mining results
- Interactive mining of knowledge at multiple levels of abstraction

- Applications and social impacts

- Domain-specific data mining & invisible data mining
- Protection of data security, integrity, and privacy

A BRIEF HISTORY OF DATA MINING SOCIETY

- 1989 IJCAI Workshop on Knowledge Discovery in Databases
 - Knowledge Discovery in Databases (G. Piatetsky-Shapiro and W. Frawley, 1991)
- 1991-1994 Workshops on Knowledge Discovery in Databases
 - Advances in Knowledge Discovery and Data Mining (U. Fayyad, G. Piatetsky-Shapiro, P. Smyth, and R. Uthurusamy, 1996)
- 1995-1998 International Conferences on Knowledge Discovery in Databases and Data Mining (KDD'95-98)
 - Journal of Data Mining and Knowledge Discovery (1997)
- ACM SIGKDD conferences since 1998 and SIGKDD Explorations
- More conferences on data mining
 - PAKDD (1997), PKDD (1997), SIAM-Data Mining (2001), (IEEE) ICDM (2001), etc.
- ACM Transactions on KDD starting in 2007

CONFERENCES AND JOURNALS ON DATA MINING

- KDD Conferences
 - ACM SIGKDD Int. Conf. on Knowledge Discovery in Databases and Data Mining (**KDD**)
 - SIAM Data Mining Conf. (**SDM**)
 - (IEEE) Int. Conf. on Data Mining (**ICDM**)
 - Conf. on Principles and practices of Knowledge Discovery and Data Mining (**PKDD**)
 - Pacific-Asia Conf. on Knowledge Discovery and Data Mining (**PAKDD**)
- Other related conferences
 - ACM SIGMOD
 - VLDB
 - (IEEE) ICDE
 - WWW, SIGIR
 - ICML, CVPR, NIPS
- Journals
 - Data Mining and Knowledge Discovery (DAMI or DMKD)
 - IEEE Trans. On Knowledge and Data Eng. (TKDE)
 - KDD Explorations
 - ACM Trans. on KDD

DATA MINING TASKS

- Prediction Methods

- Use some variables to predict unknown or future values of other variables. (Classification, Regression, Outlier Detection)

- Description Methods

- Find human-interpretable patterns that describe the data. (Clustering, Association Rule Mining, Sequential Pattern Discovery)

From [Fayyad, et.al.] Advances in Knowledge Discovery and Data Mining, 1996

CLASSIFICATION: APPLICATION 1

- Direct Marketing

- Goal: Reduce cost of mailing by *targeting* a set of consumers likely to buy a new cell-phone product.
- Approach:
 - Use the data for a similar product introduced before.
 - We know which customers decided to buy and which decided otherwise. This *{buy, don't buy}* decision forms the *class attribute*.
 - Collect various demographic, lifestyle, and company-interaction related information about all such customers.
 - Type of business, where they stay, how much they earn, etc.
 - Use this information as input attributes to learn a classifier model.

From [Berry & Linoff] Data Mining Techniques, 1997

CLASSIFICATION: APPLICATION 2

- Fraud Detection

- Goal: Predict fraudulent cases in credit card transactions.
- Approach:
 - Use credit card transactions and the information on its account-holder as attributes.
 - When does a customer buy, what does he buy, how often he pays on time, etc
 - Label past transactions as fraud or fair transactions. This forms the class attribute.
 - Learn a model for the class of the transactions.
 - Use this model to detect fraud by observing credit card transactions on an account.

CLASSIFICATION: APPLICATION 3

- Customer Attrition/Churn:
 - Goal: To predict whether a customer is likely to be lost to a competitor.
 - Approach:
 - Use detailed record of transactions with each of the past and present customers, to find attributes.
 - How often the customer calls, where he calls, what time-of-the day he calls most, his financial status, marital status, etc.
 - Label the customers as loyal or disloyal.
 - Find a model for loyalty.

From [Berry & Linoff] Data Mining Techniques, 1997

CLASSIFICATION: APPLICATION 4

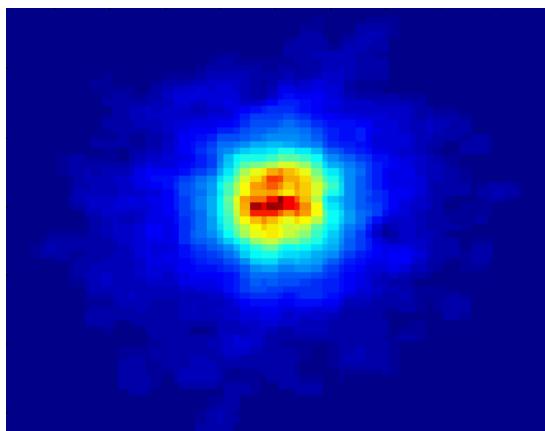
- Sky Survey Cataloging
 - Goal: To predict class (star or galaxy) of sky objects, especially visually faint ones, based on the telescopic survey images (from Palomar Observatory).
 - 3000 images with 23,040 x 23,040 pixels per image.
 - Approach:
 - Segment the image.
 - Measure image attributes (features) - 40 of them per object.
 - Model the class based on these features.
 - Success Story: Could find 16 new high red-shift quasars, some of the farthest objects that are difficult to find!

From [Fayyad, et.al.] Advances in Knowledge Discovery and Data Mining, 1996

CLASSIFYING GALAXIES

Courtesy: <http://aps.umn.edu>

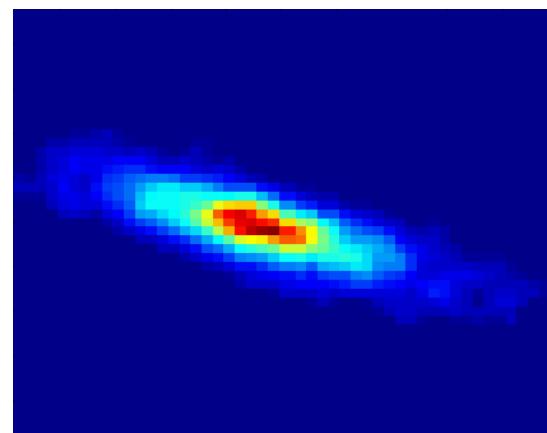
Early



Class:

- Stages of Formation

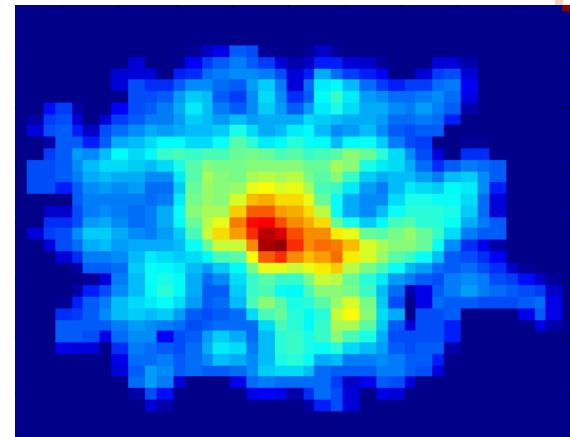
Intermediate



Attributes:

- Image features,
- Characteristics of light waves received, etc.

Late



Data Size:

- 72 million stars, 20 million galaxies
- Object Catalog: 9 GB
- Image Database: 150 GB

CLUSTERING: APPLICATION 1

- Market Segmentation:
 - Goal: subdivide a market into distinct subsets of customers where any subset may conceivably be selected as a market target to be reached with a distinct marketing mix.
 - Approach:
 - Collect different attributes of customers based on their geographical and lifestyle related information.
 - Find clusters of similar customers.
 - Measure the clustering quality by observing buying patterns of customers in same cluster vs. those from different clusters.

CLUSTERING: APPLICATION 2

- Document Clustering:

- Goal: To find groups of documents that are similar to each other based on the important terms appearing in them.
- Approach: To identify frequently occurring terms in each document. Form a similarity measure based on the frequencies of different terms. Use it to cluster.
- Gain: Information Retrieval can utilize the clusters to relate a new document or search term to clustered documents.

ILLUSTRATING DOCUMENT CLUSTERING

- Clustering Points: 3204 Articles of Los Angeles Times.
- Similarity Measure: How many words are common in these documents (after some word filtering).

| <i>Category</i> | <i>Total Articles</i> | <i>Correctly Placed</i> |
|-----------------------------|-----------------------|-------------------------|
| <i>Financial</i> | 555 | 364 |
| <i>Foreign</i> | 341 | 260 |
| <i>National</i> | 273 | 36 |
| <i>Metro</i> | 943 | 746 |
| <i>Sports</i> | 738 | 573 |
| <i>Entertainment</i> | 354 | 278 |

CLUSTERING OF S&P 500 STOCK DATA

- Observe Stock Movements every day.
- Clustering points: Stock-{UP/DOWN}
- Similarity Measure: Two points are more similar if the events described by them frequently happen together on the same day.
 - We used association rules to quantify a similarity measure.

| | <i>Discovered Clusters</i> | <i>Industry Group</i> |
|---|--|-----------------------|
| 1 | Applied-Mat1-DOWN,Bay-Network-Down,3-COM-DOWN,Cabletron-Sys-DOWN,CISCO-DOWN,HP-DOWN,DSC-Comm-DOWN,INTEL-DOWN,LSI-Logic-DOWN,Micron-Tech-DOWN,Texas-Inst-Down,Tellabs-Inc-Down,Natl-Semiconduct-DOWN,Orac1-DOWN,SGI-DOWN,Sun-DOWN | Technology1-DOWN |
| 2 | Apple-Comp-DOWN,Autodesk-DOWN,DEC-DOWN,ADV-Micro-Device-DOWN,Andrew-Corp-DOWN,Computer-Assoc-DOWN,Circuit-City-DOWN,Compaq-DOWN,EMC-Corp-DOWN,Gen-Inst-DOWN,Motorola-DOWN,Microsoft-DOWN,Scientific-Atl-DOWN | Technology2-DOWN |
| 3 | Fannie-Mae-DOWN,Fed-Home-Loan-DOWN,MBNA-Corp-DOWN,Morgan-Stanley-DOWN | Financial-DOWN |
| 4 | Baker-Hughes-UP,Dresser-Inds-UP,Halliburton-HLD-UP,Louisiana-Land-UP,Phillips-Petro-UP,Unocal-UP,Schlumberger-UP | Oil-UP |

ASSOCIATION RULE DISCOVERY: DEFINITION

- Given a set of records each of which contain some number of items from a given collection;
 - Produce dependency rules which will predict occurrence of an item based on occurrences of other items.

| <i>TID</i> | <i>Items</i> |
|------------|---------------------------|
| 1 | Bread, Coke, Milk |
| 2 | Beer, Bread |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Coke, Diaper, Milk |

Rules Discovered:

$\{\text{Milk}\} \rightarrow \{\text{Coke}\}$
 $\{\text{Diaper}, \text{Milk}\} \rightarrow \{\text{Beer}\}$

ASSOCIATION RULE DISCOVERY: APPLICATION 1

- Marketing and Sales Promotion:
 - Let the rule discovered be
$$\{Bagels, \dots\} \rightarrow \{Potato\ Chips\}$$
 - Potato Chips as consequent => Can be used to determine what should be done to boost its sales.
 - Bagels in the antecedent => Can be used to see which products would be affected if the store discontinues selling bagels.
 - Bagels in antecedent and Potato chips in consequent => Can be used to see what products should be sold with Bagels to promote sale of Potato chips!

ASSOCIATION RULE DISCOVERY: APPLICATION 2

- Supermarket shelf management.
 - Goal: To identify items that are bought together by sufficiently many customers.
 - Approach: Process the point-of-sale data collected with barcode scanners to find dependencies among items.
 - A classic rule --
 - If a customer buys diaper and milk, then he is very likely to buy beer.
 - So, don't be surprised if you find six-packs stacked next to diapers!

ASSOCIATION RULE DISCOVERY: APPLICATION 3

- Inventory Management:

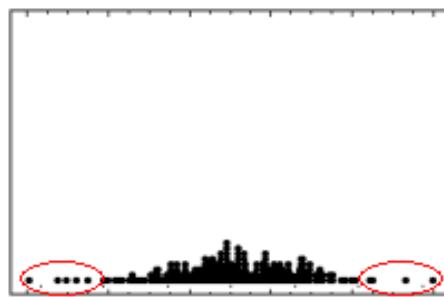
- Goal: A consumer appliance repair company wants to anticipate the nature of repairs on its consumer products and keep the service vehicles equipped with right parts to reduce on number of visits to consumer households.
- Approach: Process the data on tools and parts required in previous repairs at different consumer locations and discover the co-occurrence patterns.

OUTLIER DETECTION

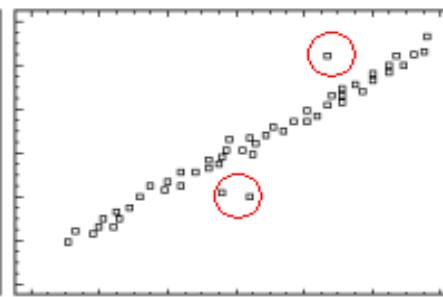
- To find exceptional data in various datasets and uncover the implicit patterns of rare cases
- Inherent variability - reflects the natural variation
- Measurement error (inaccuracy and mistakes)
- Long been studied in statistics
- An active area in data mining in the last decade
- Many applications
 - Detecting credit card fraud
 - Discovering criminal activities in E-commerce
 - Identifying network intrusion
 - Monitoring video surveillance
 - ...

OUTLIERS ARE EVERYWHERE

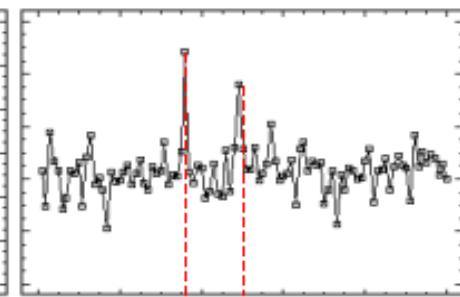
- Data values that appear inconsistent with the rest of the data.
- Some types of outliers



(a) Outliers w.r.t. a distribution



(b) Outliers w.r.t. a pattern

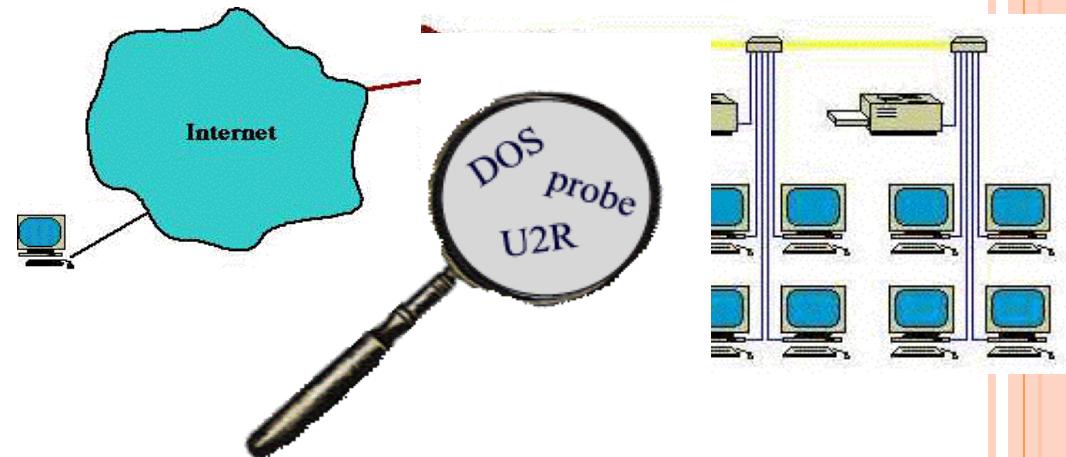


(c) time series outliers

- We will see:
 - Statistical methods;
 - distance-based methods;
 - density-based methods;
 - resolution-based methods, etc.

DEVIATION/ANOMALY DETECTION

- Detect significant deviations from normal behavior
- Applications:
 - Credit Card Fraud Detection
 - Network Intrusion Detection



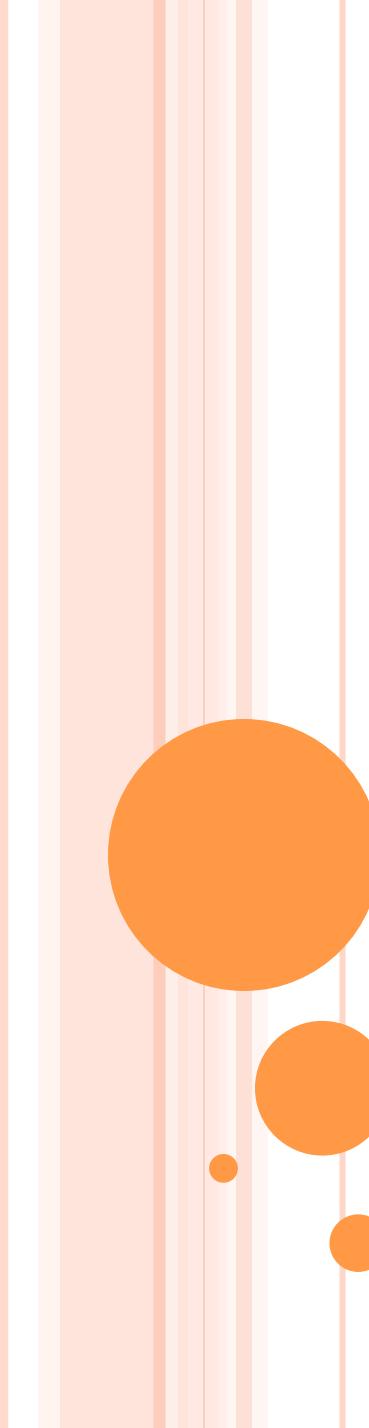
Typical network traffic at University level may reach over 100 million connections per day

CHALLENGES OF DATA MINING

- Scalability
- Dimensionality
- Complex and Heterogeneous Data
- Data Quality
- Data Ownership and Distribution
- Privacy Preservation
- Streaming Data

REFERENCES AND BOOKS

- [DM-CT]: Data Mining: Concepts and Techniques, by Jiawei Han and Micheline Kamber
- [IDM]: Introduction to Data Mining, by P.-N. Tan, M. Steinbach, and V. Kumar
- [Zaïane]: Principles of Knowledge Discovery in Data, Course Notes by O. Zaïane
- Data Mining: Practical Machine Learning Tools and Techniques with Java Implementations, by I. H. Witten and E. Frank



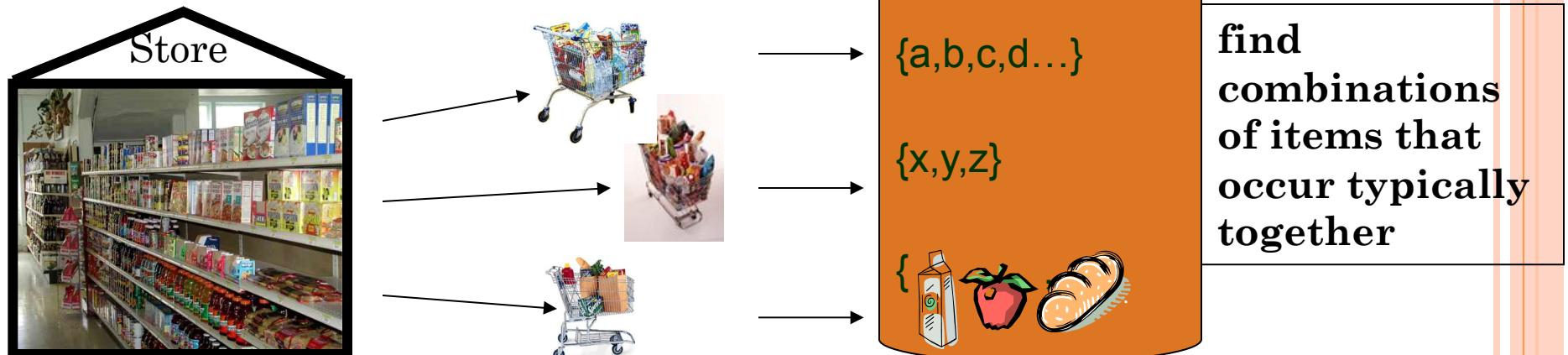
ASSOCIATION RULES

Luiza Antonie

WHAT IS ASSOCIATION RULE MINING?

- Association rule mining searches for relationships between items in a dataset:

- aims at discovering associations between items in a transactional database.



- Rule form: “**Body → Head [support, confidence]**”.

$\text{buys}(x, \text{"bread"}) \rightarrow \text{buys}(x, \text{"milk"}) [0.6\%, 65\%]$

$\text{major}(x, \text{"CS"}) \wedge \text{takes}(x, \text{"DB"}) \rightarrow \text{grade}(x, \text{"A"}) [1\%, 75\%]$

TRANSACTIONAL DATABASES

Transaction

Frequent itemset

Rule

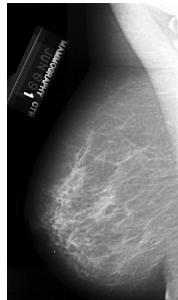


{bread, milk, Pop,...} → (Bread, milk) → Bread → milk

Automatic diagnostic
Background, Motivation and General
Outline of the Proposed Project

We have been collecting tremendous
amounts of information counting on the
power of computers to help efficiently sort
through this amalgam of information.
Unfortunately, most of the collections of
data stored on disparate dispersed media
very rapidly become over whelming.
Regrettably, most of the collected large
datasets remain unanalyzed due to lack of
appropriate, effective and scalable
techniques.

{term₁, term₂,...,term_n} → (term₂, term₂₅) → term2 → term25



{f₁, f₂,...,f_a} → (f₃, f₅, f_a) → f₃^f₅ → f_a

ASSOCIATION RULE MINING

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

| <i>TID</i> | <i>Items</i> |
|------------|----------------------------------|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

Example of Association Rules

$\{\text{Diaper}\} \rightarrow \{\text{Beer}\}$,
 $\{\text{Milk, Bread}\} \rightarrow \{\text{Eggs, Coke}\}$,
 $\{\text{Beer, Bread}\} \rightarrow \{\text{Milk}\}$,

Implication means co-occurrence, not causality!

DEFINITION: FREQUENT ITEMSET

- **Itemset**

- A collection of one or more items
 - Example: {Milk, Bread, Diaper}
- k-itemset
 - An itemset that contains k items

- **Support count (σ)**

- Frequency of occurrence of an itemset
- E.g. $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$

- **Support**

- Fraction of transactions that contain an itemset
- E.g. $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$

- **Frequent Itemset**

- An itemset whose support is greater than or equal to a $minsup$ threshold

| TID | Items |
|-----|---------------------------|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

DEFINITION: ASSOCIATION RULE

- Association Rule

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
- Example:
 $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

| TID | Items |
|-----|---------------------------|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
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- Rule Evaluation Metrics

- Support (s)
 - ◆ Fraction of transactions that contain both X and Y
- Confidence (c)
 - ◆ Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

ASSOCIATION RULE MINING TASK

- Given a set of transactions T , the goal of association rule mining is to find all rules having
 - support $\geq \text{minsup}$ threshold
 - confidence $\geq \text{minconf}$ threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the minsup and minconf thresholds

⇒ Computationally prohibitive!

MINING ASSOCIATION RULES

| TID | Items |
|-----|---------------------------|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

Example of Rules:

$\{\text{Milk}, \text{Diaper}\} \rightarrow \{\text{Beer}\}$ ($s=0.4, c=0.67$)
 $\{\text{Milk}, \text{Beer}\} \rightarrow \{\text{Diaper}\}$ ($s=0.4, c=1.0$)
 $\{\text{Diaper}, \text{Beer}\} \rightarrow \{\text{Milk}\}$ ($s=0.4, c=0.67$)
 $\{\text{Beer}\} \rightarrow \{\text{Milk}, \text{Diaper}\}$ ($s=0.4, c=0.67$)
 $\{\text{Diaper}\} \rightarrow \{\text{Milk}, \text{Beer}\}$ ($s=0.4, c=0.5$)
 $\{\text{Milk}\} \rightarrow \{\text{Diaper}, \text{Beer}\}$ ($s=0.4, c=0.5$)

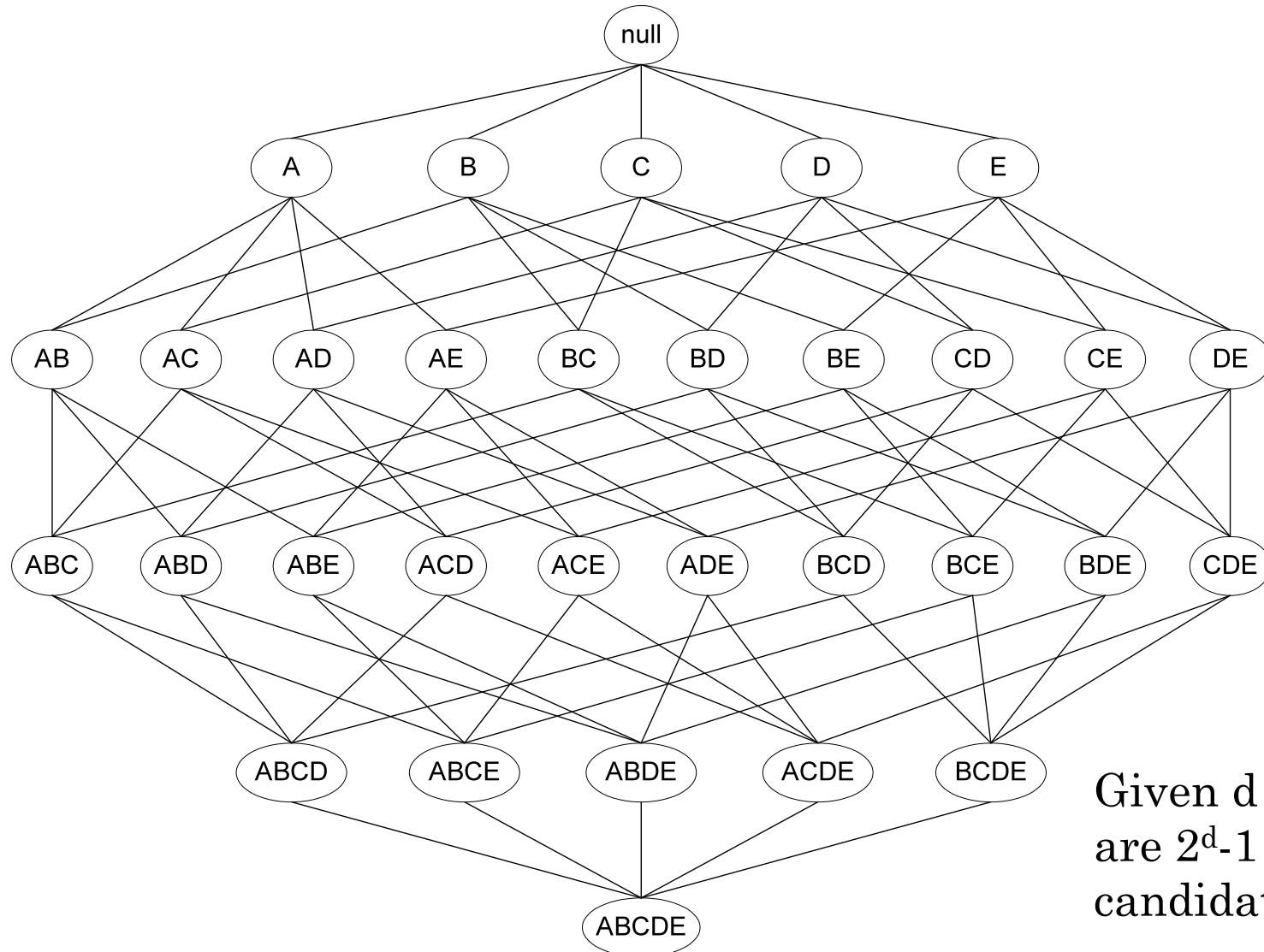
Observations:

- All the above rules are binary partitions of the same itemset:
 $\{\text{Milk}, \text{Diaper}, \text{Beer}\}$
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

MINING ASSOCIATION RULES

- Two-step approach:
 1. Frequent Itemset Generation
 - Generate all itemsets whose support $\geq \text{minsup}$
 2. Rule Generation
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

FREQUENT ITEMSET GENERATION

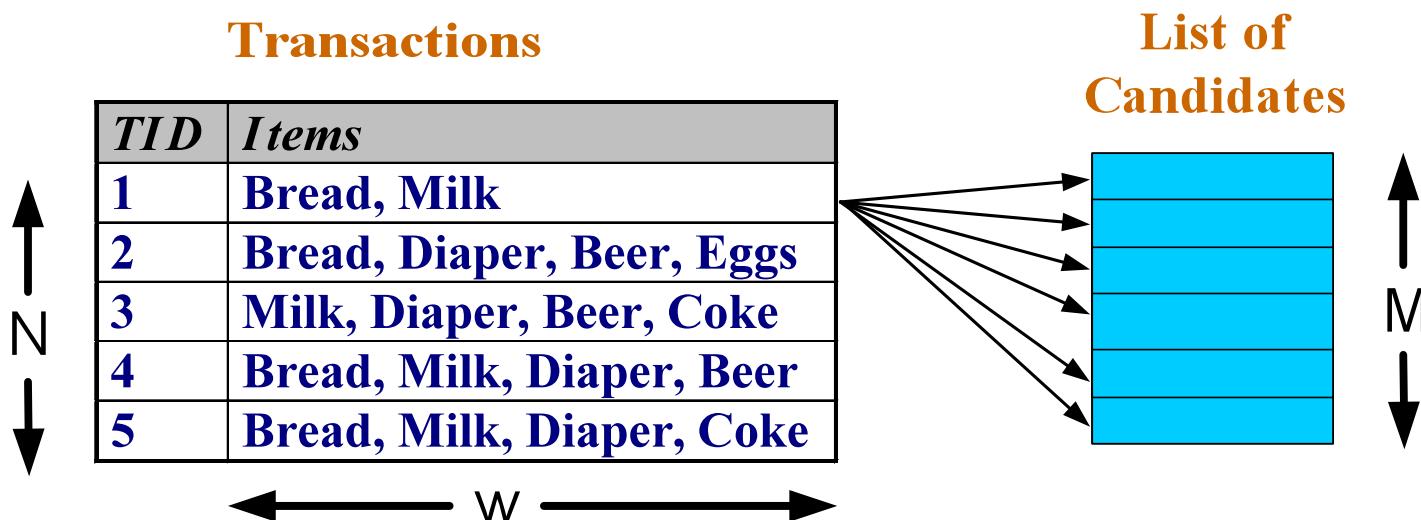


Given d items, there
are $2^d - 1$ possible
candidate itemsets

FREQUENT ITEMSET GENERATION

- Brute-force approach:

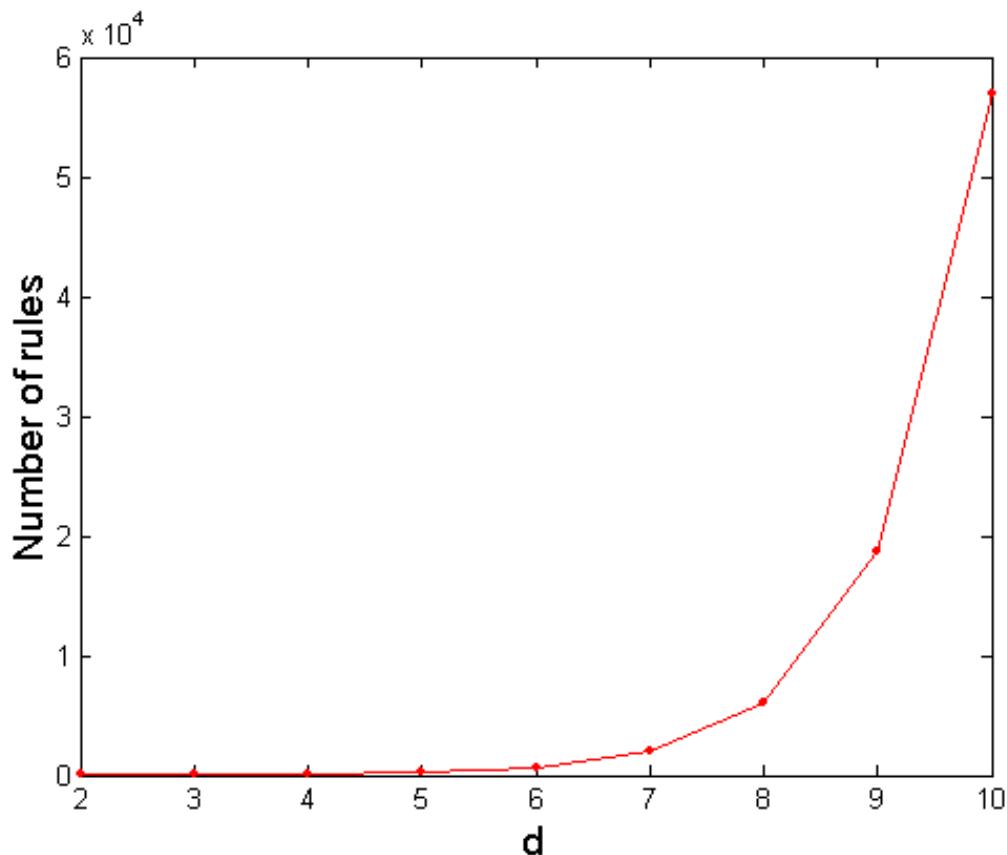
- Each itemset in the lattice is a **candidate** frequent itemset
- Count the support of each candidate by scanning the database



- Match each transaction against every candidate
- Complexity $\sim O(NMw)$ => **Expensive since $M = 2^d$!!!**

COMPUTATIONAL COMPLEXITY

- Given d unique items:
 - Total number of itemsets = 2^d
 - Total number of possible association rules:



$$\begin{aligned} R &= \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right] \\ &= 3^d - 2^{d+1} + 1 \end{aligned}$$

If $d=6$, $R = 602$ rules

FREQUENT ITEMSET GENERATION STRATEGIES

- Reduce the **number of candidates** (M)
 - Complete search: $M=2^d$
 - Use pruning techniques to reduce M
- Reduce the **number of transactions** (N)
 - Reduce size of N as the size of itemset increases
 - Used by DHP and vertical-based mining algorithms
- Reduce the **number of comparisons** (NM)
 - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction

REDUCING NUMBER OF CANDIDATES

- **Apriori principle:**

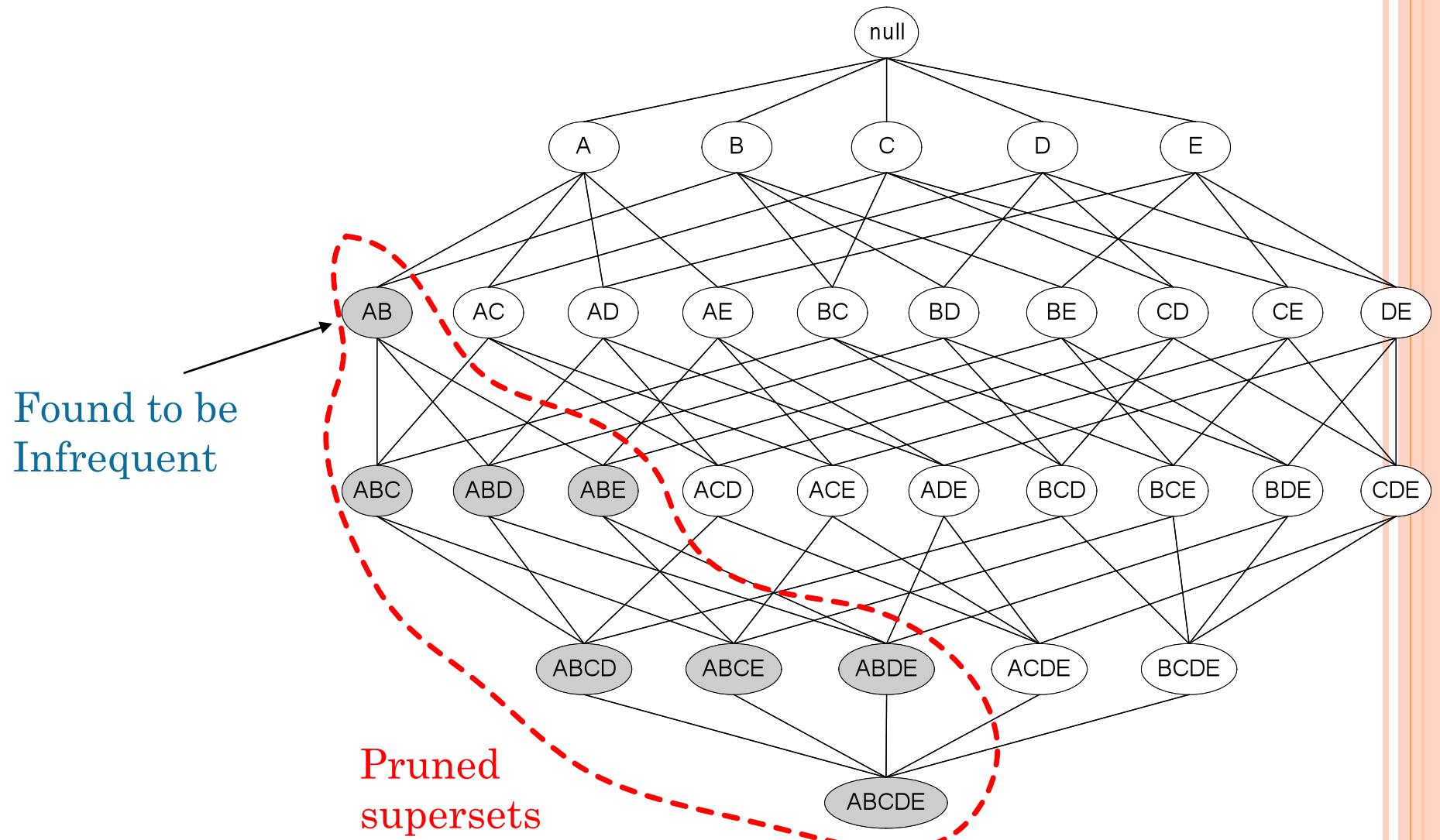
- If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the **anti-monotone** property of support

ILLUSTRATING APRIORI PRINCIPLE



ILLUSTRATING APRIORI PRINCIPLE

| Item | Count |
|--------|-------|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Minimum Support = 3

If every subset is considered,
 ${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$
 With support-based pruning,
 $6 + 6 + 1 = 13$

Items (1-itemsets)



| Itemset | Count |
|----------------|-------|
| {Bread,Milk} | 3 |
| {Bread,Beer} | 2 |
| {Bread,Diaper} | 3 |
| {Milk,Beer} | 2 |
| {Milk,Diaper} | 3 |
| {Beer,Diaper} | 3 |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)



Triplets (3-itemsets)

| Itemset | Count |
|---------------------|-------|
| {Bread,Milk,Diaper} | 3 |

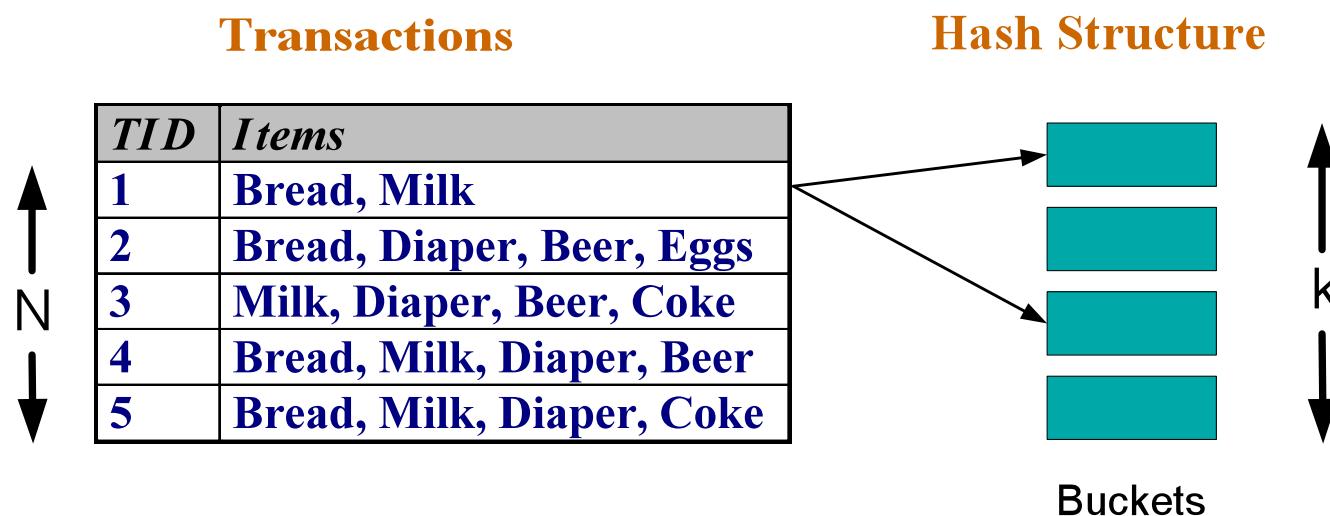
APRIORI ALGORITHM

- Method:

- Let $k=1$
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
 - Generate length $(k+1)$ candidate itemsets from length k frequent itemsets
 - Prune candidate itemsets containing subsets of length k that are infrequent
 - Count the support of each candidate by scanning the DB
 - Eliminate candidates that are infrequent, leaving only those that are frequent

REDUCING NUMBER OF COMPARISONS

- Candidate counting:
 - Scan the database of transactions to determine the support of each candidate itemset
 - To reduce the number of comparisons, store the candidates in a hash structure
 - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets



FACTORS AFFECTING COMPLEXITY

- Choice of minimum support threshold
 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
 - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
 - transaction width increases with denser data sets
 - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

COMPACT REPRESENTATION OF FREQUENT ITEMSETS

- Some itemsets are redundant because they have identical support as their supersets

| TID | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 | A9 | A10 | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|-----|----|----|----|----|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |

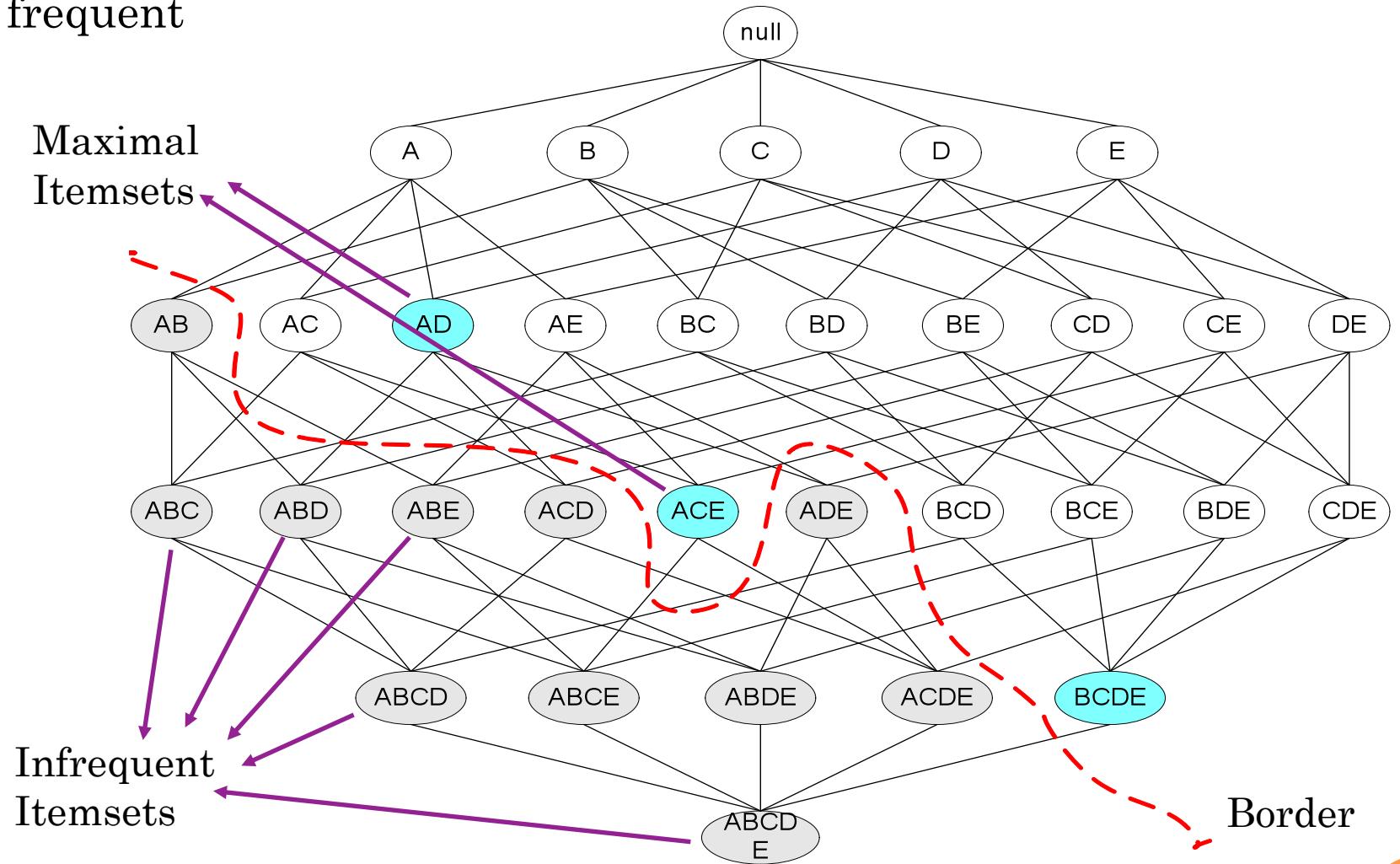
- Number of frequent itemsets

$$= 3 \times \sum_{k=1}^{10} \binom{10}{k}$$

- Need a compact representation

MAXIMAL FREQUENT ITEMSET

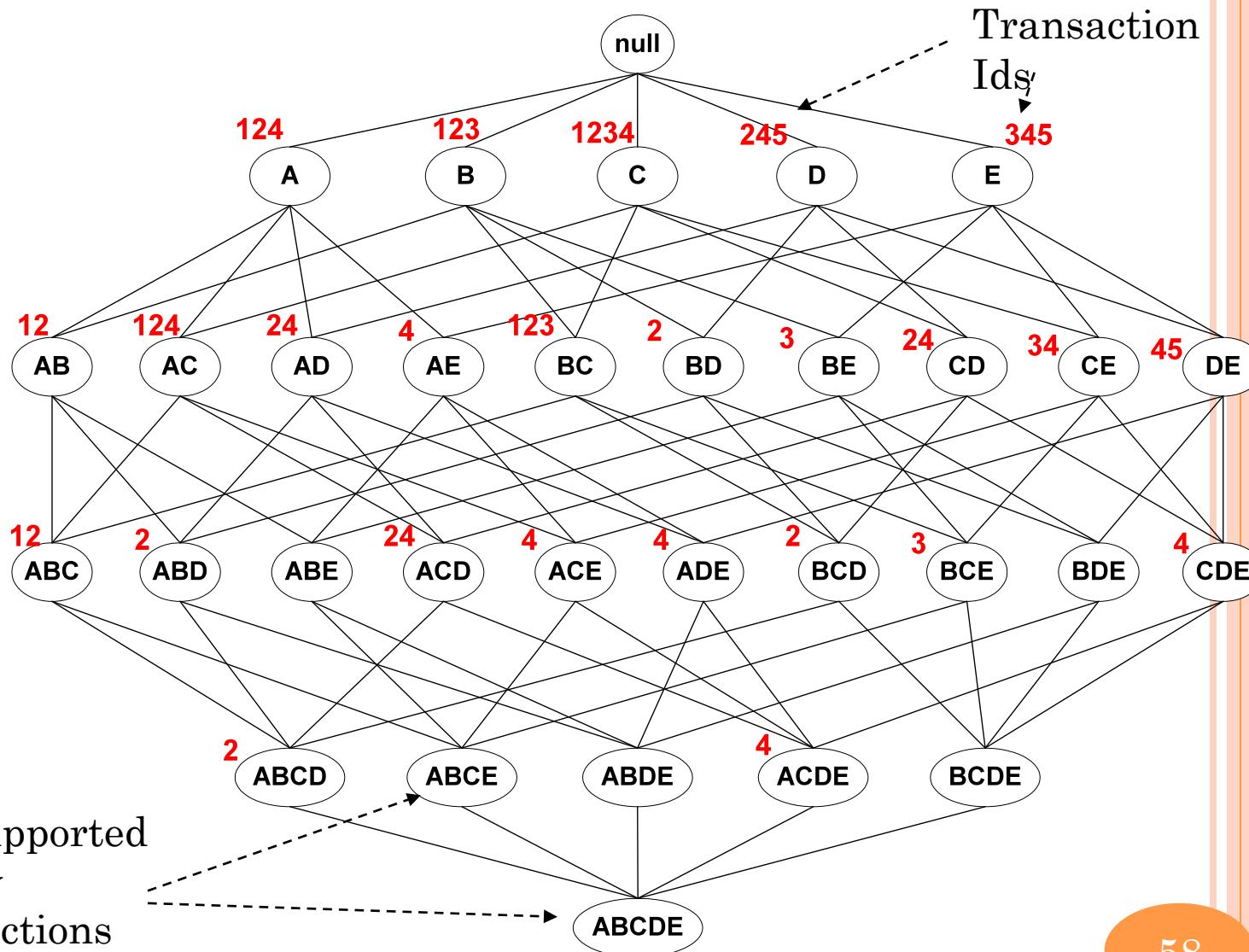
An itemset is maximal frequent if none of its immediate supersets is frequent



MAXIMAL VS CLOSED ITEMSETS

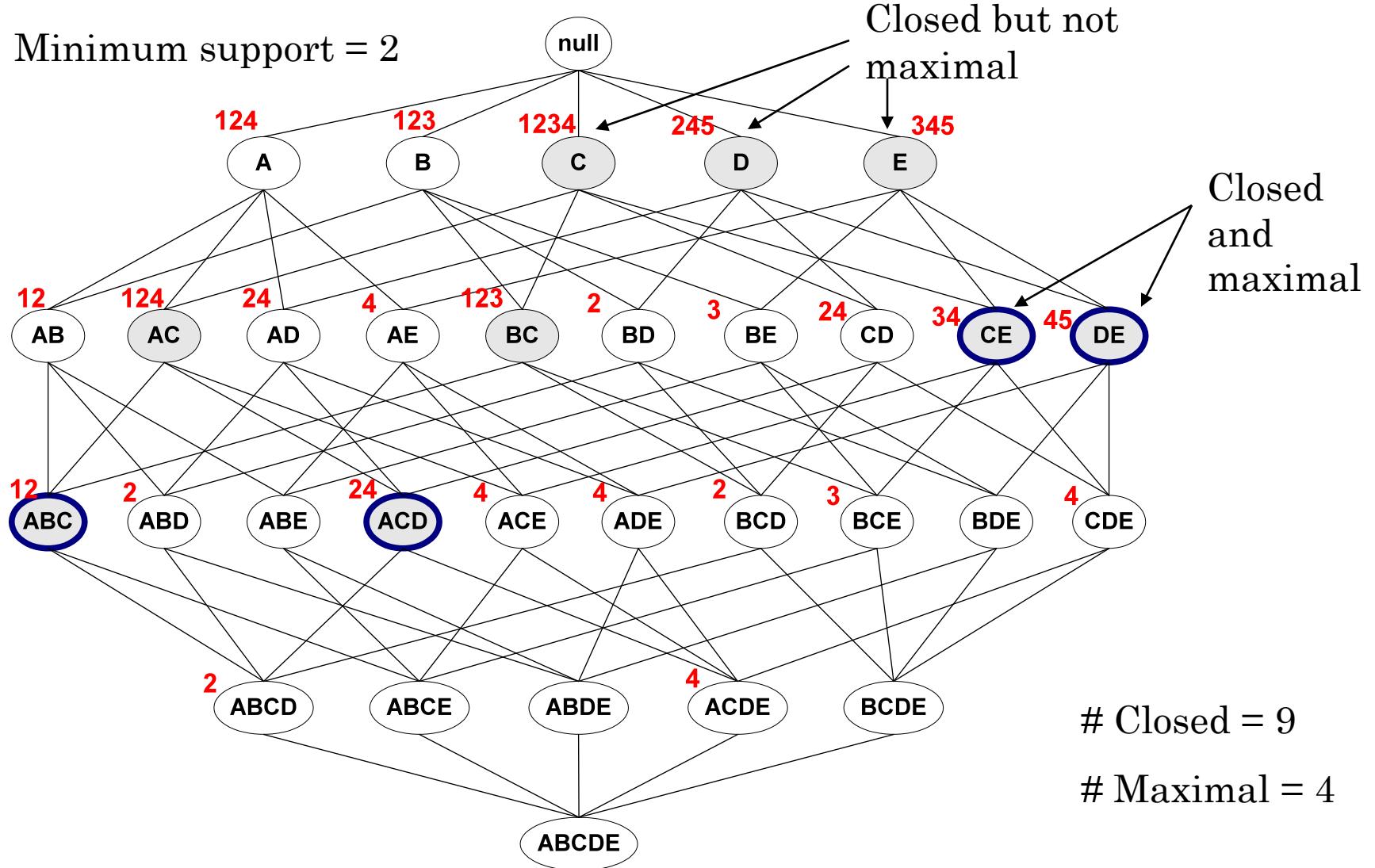
An itemset is closed if none of its immediate supersets has exactly the same support.

| TID | Items |
|-----|-------|
| 1 | ABC |
| 2 | ABCD |
| 3 | BCE |
| 4 | ACDE |
| 5 | DE |

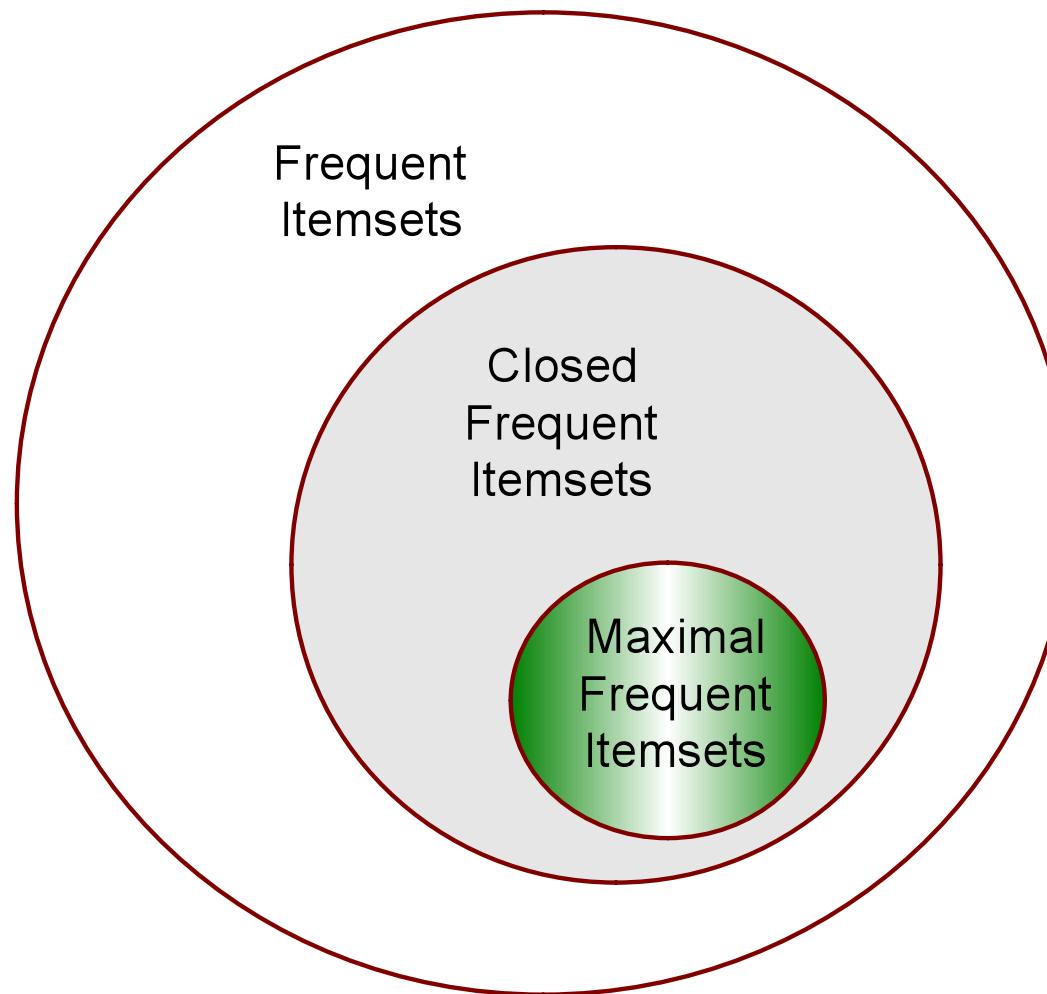


MAXIMAL VS CLOSED FREQUENT ITEMSETS

Minimum support = 2



MAXIMAL VS CLOSED ITEMSETS



PROBLEMS WITH APRIORI

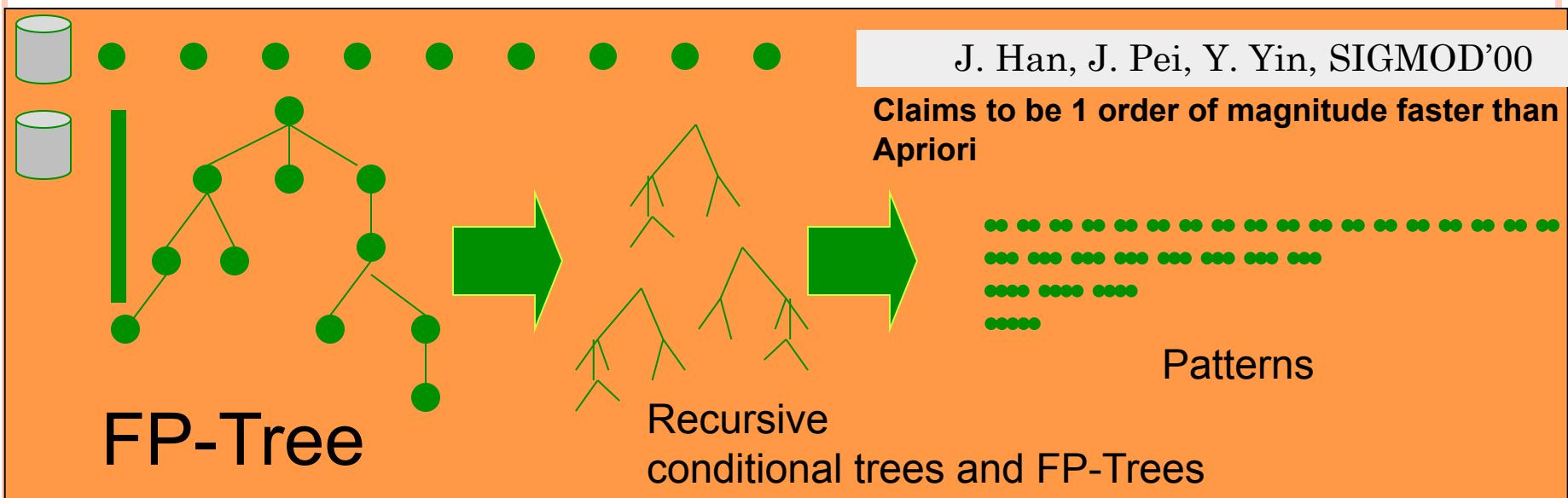
- Generation of candidate itemsets are expensive (Huge candidate sets)
 - 10^4 frequent 1-itemset will generate 10^7 candidate 2-itemsets
 - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \dots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
- High number of data scans

Frequent Pattern Growth

- First algorithm that allows frequent pattern mining without generating candidate sets
- Requires Frequent Pattern Tree

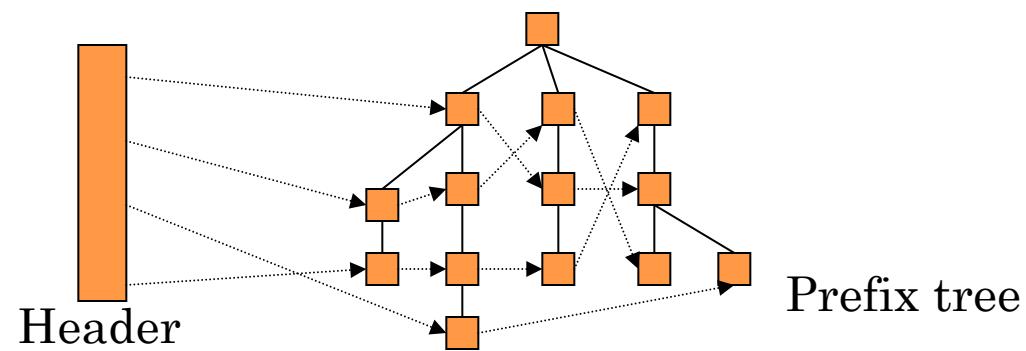
FP-GROWTH

- Grow long patterns from short ones using local frequent items
 - “abc” is a frequent pattern
 - Get all transactions having “abc”: DB | abc
 - “d” is a local frequent item in DB | abc → abcd is a frequent pattern

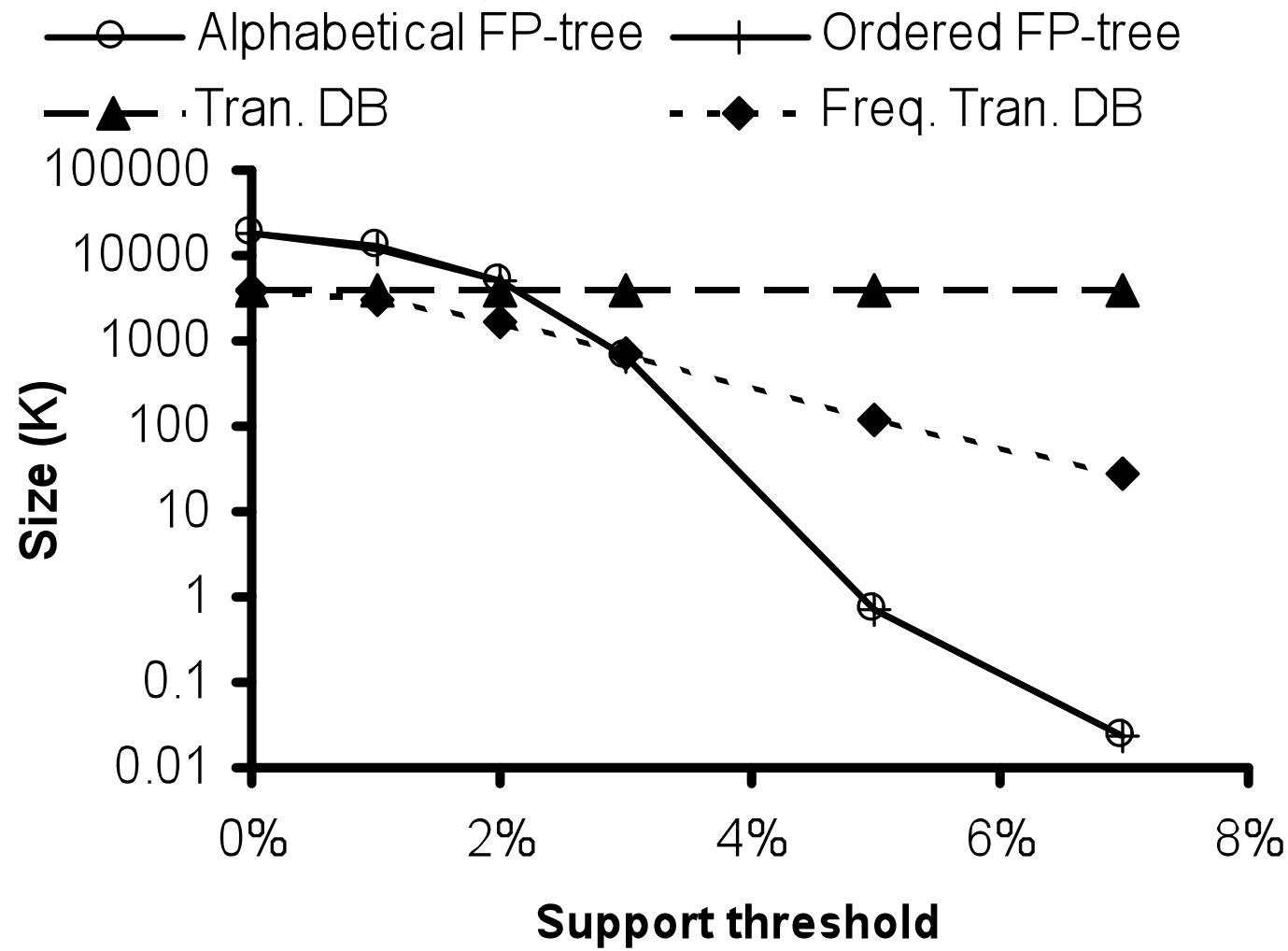


FREQUENT PATTERN TREE

- Prefix tree.
- Each node contains the item name, frequency and pointer to another node of the same kind.
- Frequent item header that contains item names and pointer to the first node in FP tree.



DATABASE COMPRESSION USING FP-TREE (ON T10I4D100K)



DISCUSSION (1/2)

- Association rules are typically sought for very large databases → efficient algorithms are needed
- The Apriori algorithm makes 1 pass through the dataset for each different itemset size
 - The maximum number of database scans is $k+1$, where k is the cardinality of the largest large itemset (4 in the clothing ex.)
 - potentially large number of scans – weakness of Apriori
- Sometimes the database is too big to be kept in memory and must be kept on disk
- The amount of computation also depends on the min.support; the confidence has less impact as it does not affect the number of passes
- Variations
 - Using sampling of the database
 - Using partitioning of the database
 - Generation of incremental rules

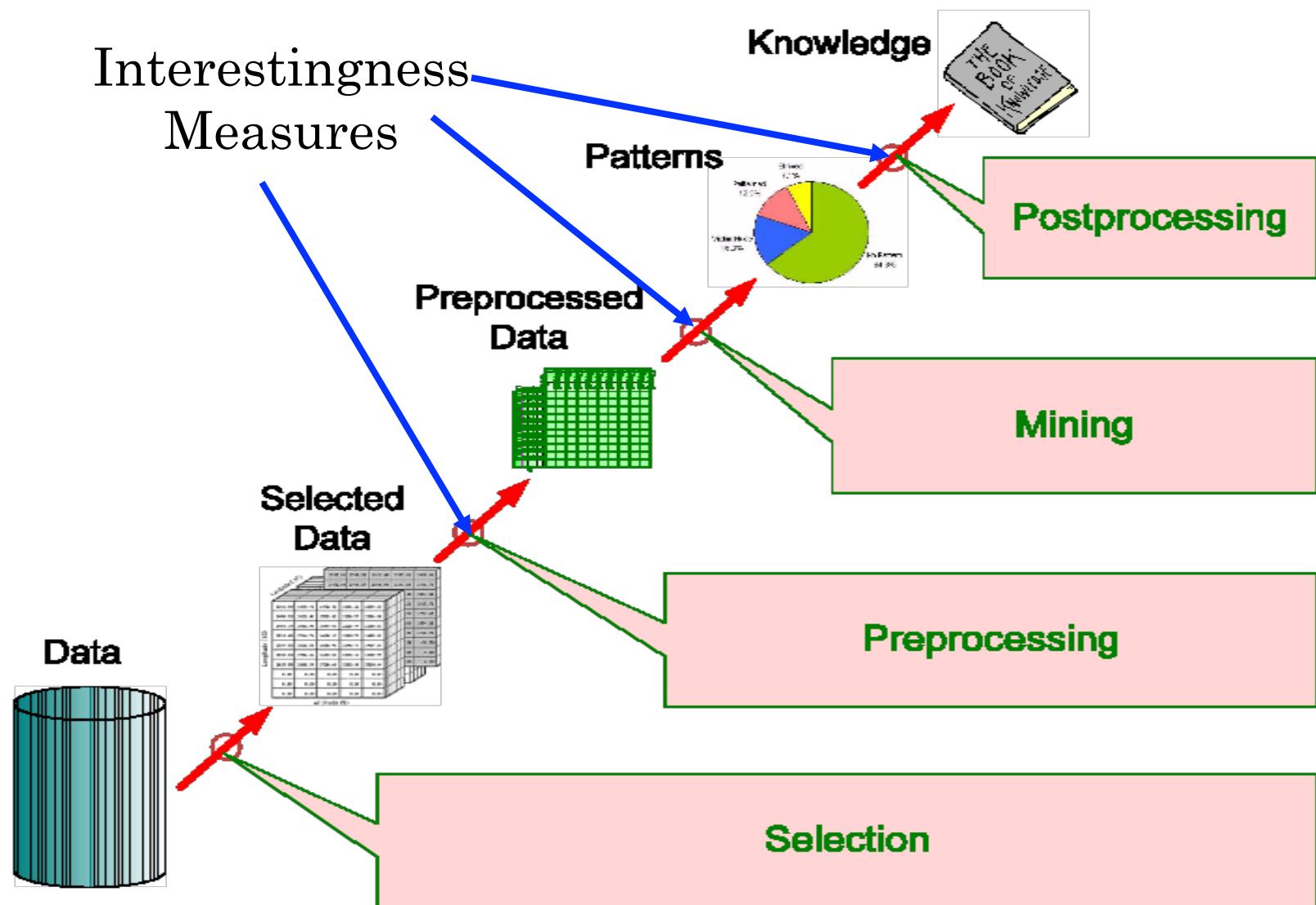
DISCUSSION (2/2)

- Choice of minimum support threshold
 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
 - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
 - transaction width increases with denser data sets
 - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

PATTERN EVALUATION

- Association rule algorithms tend to produce too many rules
 - many of them are uninteresting or redundant
 - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence
- Interestingness measures can be used to prune/rank the derived patterns
- In the original formulation of association rules, support & confidence are the only measures used

APPLICATION OF INTERESTINGNESS MEASURE



COMPUTING INTERESTINGNESS MEASURE

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

| | Y | \bar{Y} | |
|-----------|----------|-----------|----------|
| X | f_{11} | f_{10} | f_{1+} |
| \bar{X} | f_{01} | f_{00} | f_{0+} |
| | f_{+1} | f_{+0} | $ T $ |

f_{11} : support of X and Y

f_{10} : support of X and \bar{Y}

f_{01} : support of \bar{X} and \bar{Y}

f_{00} : support of \bar{X} and Y

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

DRAWBACK OF CONFIDENCE

| | Coffee | $\overline{\text{Coffee}}$ | |
|-------------------------|--------|----------------------------|-----|
| $\overline{\text{Tea}}$ | 15 | 5 | 20 |
| Tea | 75 | 5 | 80 |
| | 90 | 10 | 100 |

Association Rule: Tea \rightarrow Coffee

Confidence= $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

\Rightarrow Although confidence is high, rule is misleading

$\Rightarrow P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

STATISTICAL INDEPENDENCE

- Population of 1000 students
 - 600 students know how to swim (S)
 - 700 students know how to bike (B)
 - 420 students know how to swim and bike (S,B)

- $P(S \wedge B) = 420/1000 = 0.42$
- $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

- $P(S \wedge B) = P(S) \times P(B) \Rightarrow$ Statistical independence
- $P(S \wedge B) > P(S) \times P(B) \Rightarrow$ Positively correlated
- $P(S \wedge B) < P(S) \times P(B) \Rightarrow$ Negatively correlated

STATISTICAL-BASED MEASURES

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y | X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

EXAMPLE: LIFT/INTEREST

| | Coffee | <u>Coffee</u> | |
|------------|--------|---------------|-----|
| Tea | 15 | 5 | 20 |
| <u>Tea</u> | 75 | 5 | 80 |
| | 90 | 10 | 100 |

Association Rule: Tea → Coffee

Confidence= $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

⇒ Lift = $0.75/0.9 = 0.8333 (< 1, \text{ therefore is negatively associated})$

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

| # | Measure | Formula |
|----|---------------------------------|--|
| 1 | ϕ -coefficient | $\frac{P(A, B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$ |
| 2 | Goodman-Kruskal's (λ) | $\frac{\sum_j \max_k P(A_j, B_k) + \sum_k \max_j P(A_j, B_k) - \max_j P(A_j) - \max_k P(B_k)}{2 - \max_j P(A_j) - \max_k P(B_k)}$ |
| 3 | Odds ratio (α) | $\frac{P(A, B)P(\bar{A}, \bar{B})}{P(\bar{A}, B)P(A, \bar{B})}$ |
| 4 | Yule's Q | $\frac{P(A, B)P(\bar{A}B) - P(A, \bar{B})P(\bar{A}, B)}{P(A, B)P(\bar{A}B) + P(A, \bar{B})P(\bar{A}, B)} = \frac{\alpha - 1}{\alpha + 1}$ |
| 5 | Yule's Y | $\frac{\sqrt{P(A, B)P(\bar{A}B)} - \sqrt{P(A, \bar{B})P(\bar{A}, B)}}{\sqrt{P(A, B)P(\bar{A}B)} + \sqrt{P(A, \bar{B})P(\bar{A}, B)}} = \frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}$ |
| 6 | Kappa (κ) | $\frac{P(A, B) + P(\bar{A}, \bar{B}) - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}$ |
| 7 | Mutual Information (M) | $\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))$ |
| 8 | J-Measure (J) | $\max \left(P(A, B) \log \left(\frac{P(B A)}{P(B)} \right) + P(\bar{A}B) \log \left(\frac{P(\bar{B} A)}{P(\bar{B})} \right), P(A, \bar{B}) \log \left(\frac{P(\bar{A} B)}{P(\bar{A})} \right) + P(\bar{A}\bar{B}) \log \left(\frac{P(\bar{\bar{A}} B)}{P(\bar{A})} \right) \right)$ |
| 9 | Gini index (G) | $\max \left(P(A)[P(B A)^2 + P(\bar{B} A)^2] + P(\bar{A})[P(B \bar{A})^2 + P(\bar{B} \bar{A})^2] - P(B)^2 - P(\bar{B})^2, P(B)[P(A B)^2 + P(\bar{A} B)^2] + P(\bar{B})[P(A \bar{B})^2 + P(\bar{A} \bar{B})^2] - P(A)^2 - P(\bar{A})^2 \right)$ |
| 10 | Support (s) | $P(A, B)$ |
| 11 | Confidence (c) | $\max(P(B A), P(A B))$ |
| 12 | Laplace (L) | $\max \left(\frac{NP(A, B) + 1}{NP(A) + 2}, \frac{NP(A, B) + 1}{NP(B) + 2} \right)$ |
| 13 | Conviction (V) | $\max \left(\frac{P(A)P(\bar{B})}{P(\bar{A}B)}, \frac{P(B)P(\bar{A})}{P(B\bar{A})} \right)$ |
| 14 | Interest (I) | $\frac{P(A, B)}{P(A)P(\bar{B})}$ |
| 15 | cosine (IS) | $\frac{P(A, B)}{\sqrt{P(A)P(B)}}$ |
| 16 | Piatetsky-Shapiro's (PS) | $P(A, B) - P(A)P(B)$ |
| 17 | Certainty factor (F) | $\max \left(\frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)} \right)$ |
| 18 | Added Value (AV) | $\max(P(B A) - P(B), P(A B) - P(A))$ |
| 19 | Collective strength (S) | $\frac{P(A, B) + P(\bar{A}B)}{P(A)P(B) + P(\bar{A})P(\bar{B})} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A, B) - P(\bar{A}B)}$ |
| 20 | Jaccard (ζ) | $\frac{P(A, B)}{P(A) + P(B) - P(A, B)}$ |
| 21 | Klosgen (K) | $\sqrt{P(A, B)} \max(P(B A) - P(B), P(A B) - P(A))$ |

SUBJECTIVE INTERESTINGNESS MEASURE

- Objective measure:

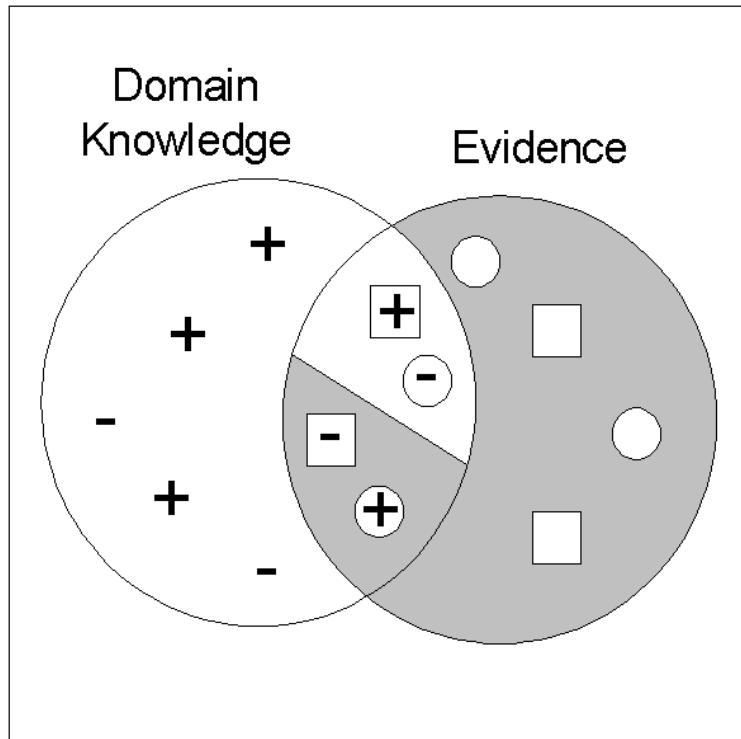
- Rank patterns based on statistics computed from data
- e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:

- Rank patterns according to user's interpretation
 - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
 - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

INTERESTINGNESS VIA UNEXPECTEDNESS

- Need to model expectation of users (domain knowledge)



- + Pattern expected to be frequent
- Pattern expected to be infrequent
- Pattern found to be frequent
- Pattern found to be infrequent
- ⊕ ⊖ Expected Patterns
- + Unexpected Patterns

- Need to combine expectation of users with evidence from data (i.e., extracted patterns)

CONTINUOUS AND CATEGORICAL ATTRIBUTES

How to apply association analysis formulation to non-asymmetric binary variables?

| Session Id | Country | Session Length (sec) | Number of Web Pages viewed | Gender | Browser Type | Buy |
|------------|-----------|----------------------|----------------------------|--------|--------------|-----|
| 1 | USA | 982 | 8 | Male | IE | No |
| 2 | China | 811 | 10 | Female | Netscape | No |
| 3 | USA | 2125 | 45 | Female | Mozilla | Yes |
| 4 | Germany | 596 | 4 | Male | IE | Yes |
| 5 | Australia | 123 | 9 | Male | Mozilla | No |
| ... | ... | ... | ... | ... | ... | ... |

Example of Association Rule:

$$\{\text{Number of Pages} \in [5,10] \wedge (\text{Browser}=\text{Mozilla})\} \rightarrow \{\text{Buy} = \text{No}\}$$

HANDLING CATEGORICAL ATTRIBUTES

- Transform categorical attribute into asymmetric binary variables
- Introduce a new “item” for each distinct attribute-value pair
 - Example: replace Browser Type attribute with
 - Browser Type = Internet Explorer
 - Browser Type = Mozilla
 - Browser Type = Mozilla

HANDLING CATEGORICAL ATTRIBUTES

○ Potential Issues

- What if attribute has many possible values
 - Example: attribute country has more than 200 possible values
 - Many of the attribute values may have very low support
 - Potential solution: Aggregate the low-support attribute values
- What if distribution of attribute values is highly skewed
 - Example: 95% of the visitors have Buy = No
 - Most of the items will be associated with (Buy=No) item
 - Potential solution: drop the highly frequent items

HANDLING CONTINUOUS ATTRIBUTES

- Different kinds of rules:

- $\text{Age} \in [21, 35] \wedge \text{Salary} \in [70k, 120k] \rightarrow \text{Buy}$
- $\text{Salary} \in [70k, 120k] \wedge \text{Buy} \rightarrow \text{Age: } \mu=28, \sigma=4$

- Different methods:

- Discretization-based
- Statistics-based
- Non-discretization based
 - minApriori

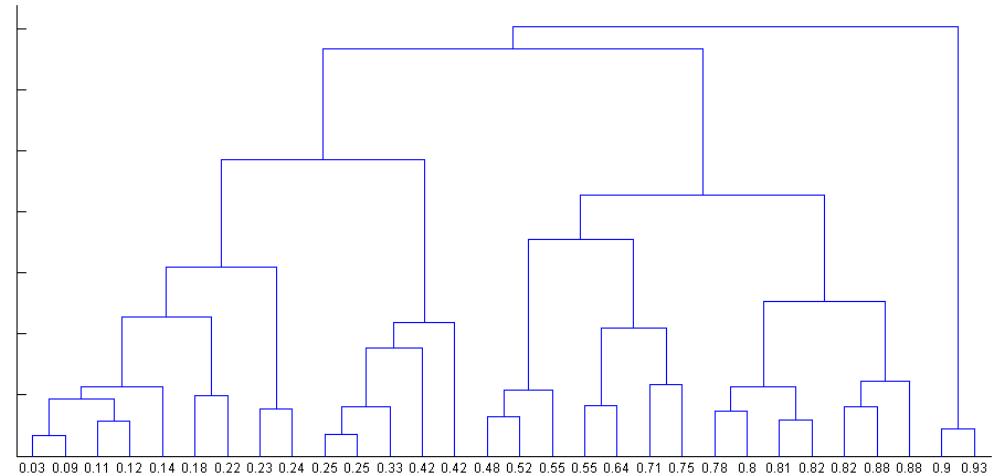
DISCRETIZATION ISSUES

- Size of the discretized intervals affect support & confidence
 - {Refund = No, (Income = \$51,250)} → {Cheat = No}
 - {Refund = No, (60K ≤ Income ≤ 80K)} → {Cheat = No}
 - {Refund = No, (0K ≤ Income ≤ 1B)} → {Cheat = No}
- If intervals too small
 - may not have enough support
- If intervals too large
 - may not have enough confidence
- Potential solution: use all possible intervals

DISCRETIZATION ISSUES

- Execution time
 - If intervals contain n values, there are on average $O(n^2)$ possible ranges

- Too many rules



$\{\text{Refund} = \text{No}, (\text{Income} = \$51,250)\} \rightarrow \{\text{Cheat} = \text{No}\}$

$\{\text{Refund} = \text{No}, (51\text{K} \leq \text{Income} \leq 52\text{K})\} \rightarrow \{\text{Cheat} = \text{No}\}$

$\{\text{Refund} = \text{No}, (50\text{K} \leq \text{Income} \leq 60\text{K})\} \rightarrow \{\text{Cheat} = \text{No}\}$

STATISTICS-BASED METHODS

- Example:

Browser=Mozilla \wedge Buy=Yes \rightarrow Age: $\mu=23$

- Rule consequent consists of a continuous variable, characterized by their statistics

- mean, median, standard deviation, etc.

- Approach:

- Withhold the target variable from the rest of the data
 - Apply existing frequent itemset generation on the rest of the data
 - For each frequent itemset, compute the descriptive statistics for the corresponding target variable
 - Frequent itemset becomes a rule by introducing the target variable as rule consequent
 - Apply statistical test to determine interestingness of the rule

STATISTICS-BASED METHODS

- How to determine whether an association rule interesting?

- Compare the statistics for segment of population covered by the rule vs segment of population not covered by the rule:
 -

$A \Rightarrow B: \mu$ versus $A \Rightarrow B: \mu'$

- Statistical hypothesis testing:
 - Null hypothesis: $H_0: \mu' = \mu + \Delta$
 - Alternative hypothesis: $H_1: \mu' > \mu + \Delta$
 - Z has zero mean and variance 1 under null hypothesis

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

STATISTICS-BASED METHODS

- Example:

$r: \text{Browser}=\text{Mozilla} \wedge \text{Buy}=\text{Yes} \rightarrow \text{Age}: \mu=23$

- Rule is interesting if difference between μ and μ' is greater than 5 years (i.e., $\Delta = 5$)
- For r , suppose $n_1 = 50, s_1 = 3.5$
- For r' (complement): $n_2 = 250, s_2 = 6.5$

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{30 - 23 - 5}{\sqrt{\frac{3.5^2}{50} + \frac{6.5^2}{250}}} = 3.11$$

- For 1-sided test at 95% confidence level, critical Z-value for rejecting null hypothesis is 1.64.
- Since Z is greater than 1.64, r is an interesting rule

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