

Contents

Chapter 1. Basic Concepts

Chapter 2. Arrays And Structures

Chapter 3. Stacks And Queues

Chapter 4. Linked Lists

Chapter 5. Trees (Midterm exam)

Chapter 6. Graphs

Chapter 7. Sorting

Chapter 8. Hashing (Final exam)

Contents

2.1 Arrays

2.2 Dynamically Allocated Array

2.3 Structures and Unions

2.4 Polynomials

2.5 Sparse Matrices

2.6 Representation of Multidimensional Arrays

2.1 Arrays – three perspectives

- A consecutive set of memory locations
 - emphasis on implementation issues
 - not always true
- A set of pairs, $\langle index, value \rangle$
 - set of *mappings* or *correspondence* between index and values
 - $array : i \rightarrow a_i$
- ADT
 - more concerned with the operations that can be performed on an array

2.1.1. The Abstract Data Type

ADT Array is

objects: A set of pairs $\langle index, value \rangle$ where for each value of $index$ there is a value from the set $item$. $Index$ is a finite ordered set of one or more dimensions, for example, $\{0, \dots, n-1\}$ for one dimension, $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)\}$ for two dimensions, etc.

functions:

for all $A \in \text{Array}, i \in index, x \in item, j, size \in \text{integer}$

Array Create($j, list$) ::= **return** an array of j dimensions where $list$ is a j -tuple whose i th element is the size of the i th dimension. $Items$ are undefined.

Item Retrieve(A, i) ::= **if** ($i \in index$) **return** the item associated with index value i in array A
else return error

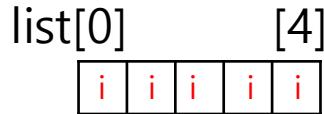
Array Store(A, i, x) ::= **if** ($i \in index$)
return an array that is identical to array A except the new pair $\langle i, x \rangle$ has been inserted **else return** error.

end Array

2.1.2 Arrays in C

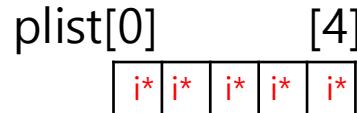
- one-dimensional array

int list[5];



※ i stands for int
i* stands for int pointer

int *plist[5];



cf) int (*ary)[5] , 배열포인터

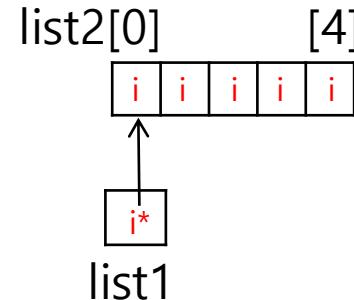
Variable	Memory address
list[0]	base address = α
list[1]	$\alpha + \text{sizeof(int)}$
list[2]	$\alpha + 2 \cdot \text{sizeof(int)}$
list[3]	$\alpha + 3 \cdot \text{sizeof(int)}$
list[4]	$\alpha + 4 \cdot \text{sizeof(int)}$

one-dimensional array & pointer

- interpretations of pointers: list1, list2

```
int *list1, list2[5];
```

```
list1 = list2;  
variable constant
```



```
list2 == &list2[0]  
list2 + i == &list2[i]  
*(list2+i) == list2[i]
```

```
list1 == &list2[0]  
list1 + i == &list2[i]  
*(list1+i) == list2[i]
```

```

#include <stdio.h>
#define MAX_SIZE 100
float sum(float [], int);
float input[MAX_SIZE], answer;
void main(void)
{
    int i;
    for (i = 0; i < MAX_SIZE; i++)
        input[i] = i;
    answer = sum(      , MAX_SIZE);
    printf("The sum is: %f\n", answer);
}

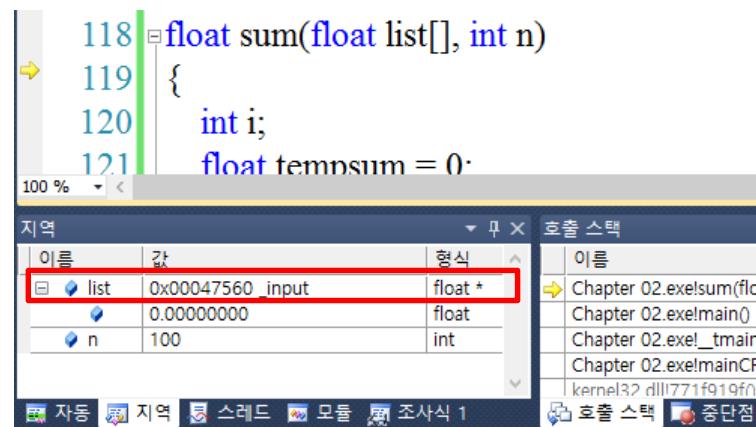
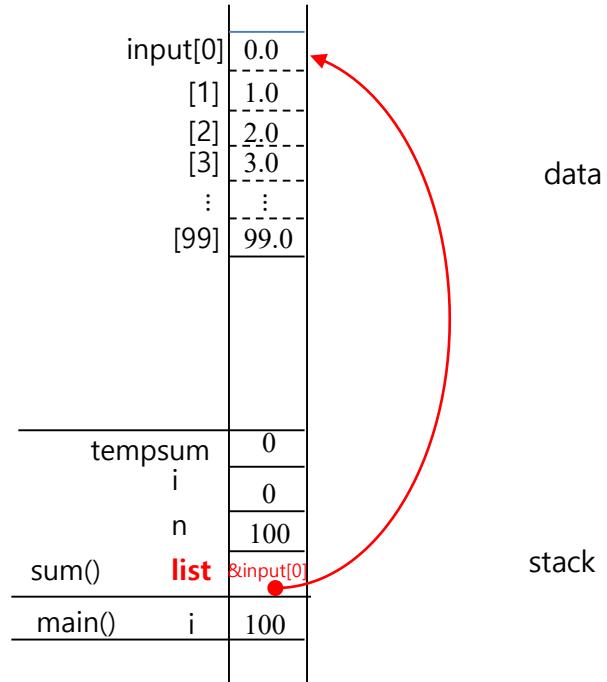
```

array parameter

```

float sum(      , int n)
{
    float * list → pointer parameter
    int i;
    float tempsum = 0;
    for (i = 0; i < n; i++)
        tempsum += *list;
    return tempsum; *(list+i)
}

```



Program 2.1: Example array program

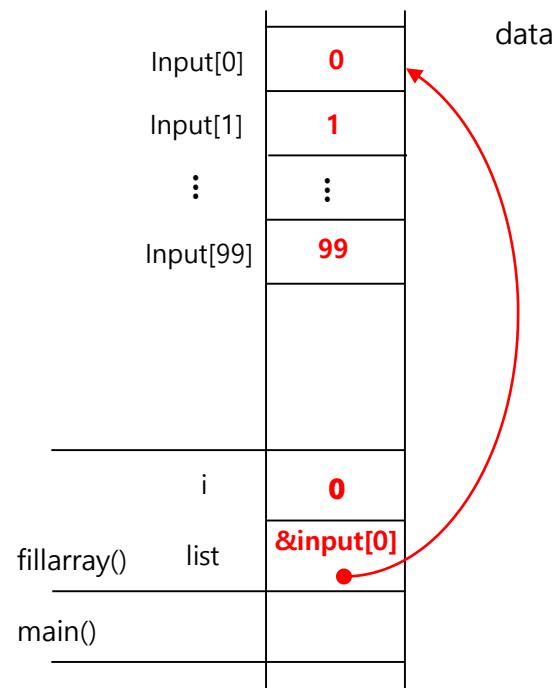
```

1 #include <stdio.h>
2 #define MAX_SIZE 100
3 float sum(float[], int);
4 void fillarray(float[], int);
5 float input[MAX_SIZE], answer;
6 void main(void)
7 {
8     fillarray(input, MAX_SIZE);
9     answer = sum(input, MAX_SIZE);
10    printf("The sum is: %f\n", answer);
11 }
12 void fillarray(float list[], int n)
13 {
14     int i;
15     for (i = 0; i < n; i++)
16         list[i] = i;
17 }
18 float sum(float list[], int n)
19 {
20     int i;
21     float tempsum = 0;
22     for (i = 0; i < n; i++)
23         tempsum += list[i];
24     return tempsum;
25 }

```

■ left-side of equal sign

- the value produced on the right-hand side is stored in the location (*list+i*)



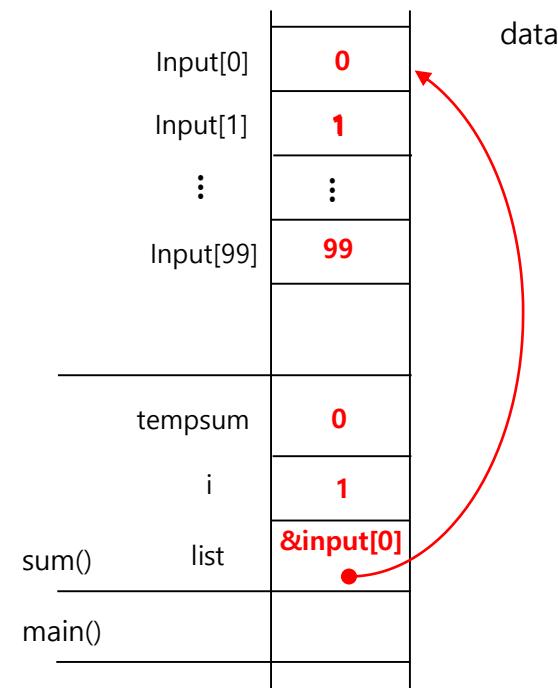
```

1 #include <stdio.h>
2 #define MAX_SIZE 100
3 float sum(float[], int);
4 void fillarray(float[], int);
5 float input[MAX_SIZE], answer;
6 void main(void)
7 {
8     fillarray(input, MAX_SIZE);
9     answer = sum(input, MAX_SIZE);
10    printf("The sum is: %f\n", answer);
11 }
12 void fillarray(float list[], int n)
13 {
14     int i;
15     for (i = 0; i < n; i++)
16         list[i] = i;
17 }
18 float sum(float list[], int n)
19 {
20     int i;
21     float tempsum = 0;
22     for (i = 0; i < n; i++)
23         tempsum += list[i];
24     return tempsum;
25 }

```

■ right-side of equal sign

- a dereference takes place
- the value pointed at by $(list+i)$ is returned



```

1 #include <stdio.h>
2 #define MAX_SIZE 100
3 float sum(float[], int);
4 void fillarray(float[], int);
5 float input[MAX_SIZE], answer;
6 void main(void)
7 {
8     fillarray(input, MAX_SIZE);
9     answer = sum(input, MAX_SIZE);
10    printf("The sum is: %f\n", answer);
11 }
12 void fillarray(float list[], int n)
13 {
14     int i;
15     for (i = 0; i < n; i++)
16         list[i] = i;
17 }
18 float sum(float list[], int n)
19 {
20     int i;
21     float tempsum = 0;
22     for (i = 0; i < n; i++)
23         tempsum += list[i];
24     return tempsum;
25 }

```

■ left-side of equal sign

- the value produced on the right-hand side is stored in the location (*list+i*)

■ right-side of equal sign

- a dereference takes place
- the value pointed at by (*list+i*) is returned

In C, array parameters have their values altered, despite the fact that the parameter passing is done using *call-by-value*.

```
int one[] = {0, 1, 2, 3, 4};  
print1(&one[0], 5);
```

```
void print1(int *ptr, int rows)  
{ /* print out a one-dimensional array using a pointer */  
    int i;  
    printf("Address Contents\n");  
    for (i = 0; i < rows; i++)  
        printf("%8u%5d\n", ptr + i, *(ptr + i));  
    printf("\n");  
}
```

Program 2.2: One-dimensional array accessed by address

Address	Contents
12244868	0
12344872	1
12344876	2
12344880	3
12344884	4

Assumption: `sizeof(int) == 4`

2.2 Dynamically Allocated Arrays

2.2.1 One-dimensional Arrays

```
pf = (float *) malloc(sizeof(float));
```

can be replaced by

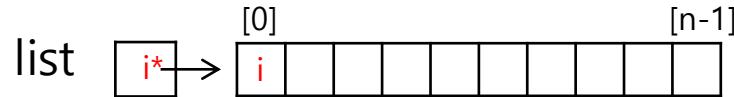
```
#define MALLOC(p, s) \
    if (!(p = malloc(s))) {\ \
        fprintf(stderr, "Insufficient memory"); \
        exit(EXIT_FAILURE); \
    } \
MALLOC(pf, sizeof(float));
```

- Change the first few lines of *main* of Program 1.4 to:

```

int i, n, *list;
printf("Enter the number of numbers to generate: ");
scanf("%d", &n);
if( n < 1 ) {
    fprintf(stderr, "Improper value of n\n");
    exit(EXIT_FAILURE);
}
MALLOC(list, n * sizeof(int));

```



2.2.2 Two-Dimensional Arrays

- A multidimensional array in C
 - *Array-of-arrays* representation

```
int x[3][5];
```

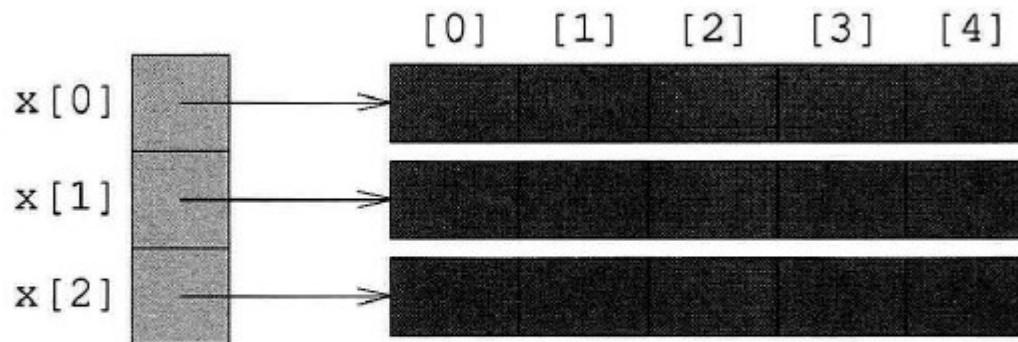
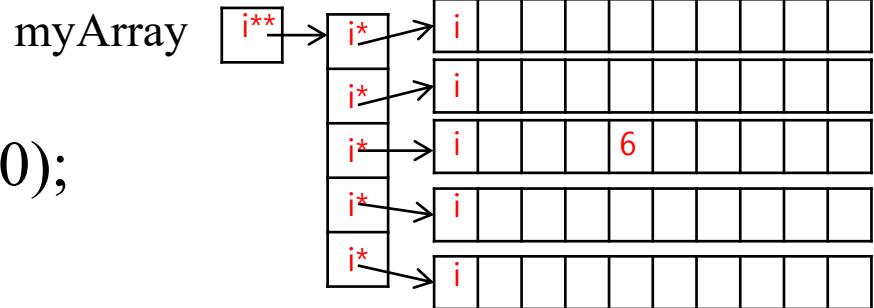


Figure 2.2: Array-of-arrays representation

x[i] : a pointer to zeroth element of row i of the array

x[i][j] : an element accessed by the address, `x[i]+j*sizeof(int)`

```
int **myArray;  
myArray = make2dArray(5,10);  
myArray[2][4] = 6;
```



```
int** make2dArray(int rows, int cols)  
{ /* create a two dimensional rows × cols array */  
    int **x, i;  
  
    /* get memory for row pointers */  
    MALLOC(x, rows * sizeof (*x));  
  
    주소 포인터 배열  
    /* get memory for each row */  
    for (i = 0; i < rows; i++)  
        MALLOC(x[i], cols * sizeof (**x));  
    return x;  
}
```

Program 2.3: Dynamically create a two-dimensional array

정적 arry 와 동적 array

정적 Array

int sary[3][4]

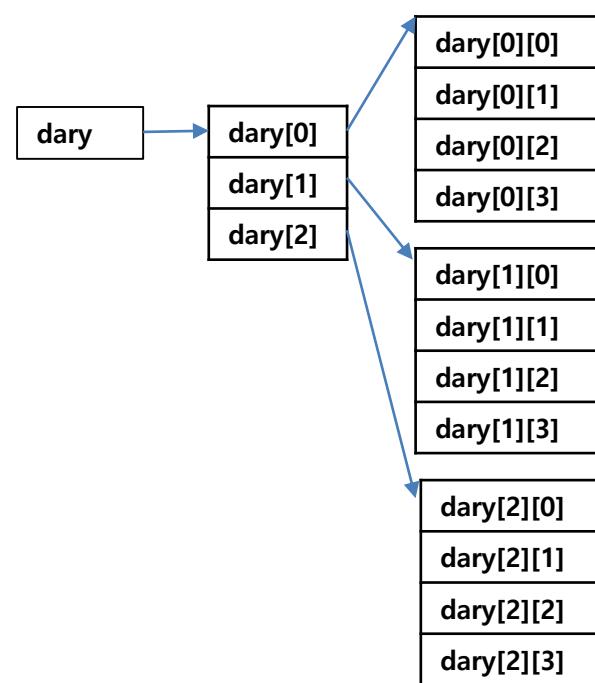


$\text{sary} == \&\text{sary}[0][0]$
 $= \&\text{sary}[0]$

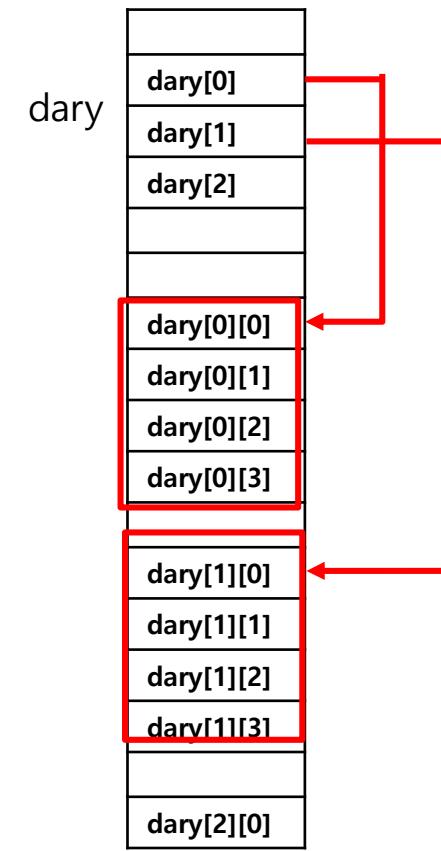
동적 Array

int **dary

dary는 동적으로 할당 받은 2차원 배열[3][4]의 시작 주소를 가지고 있는 포인터임



$\text{dary} != \&\text{dary}[0][0]$
 $\text{dary} == \&\text{dary}[0]$
 $\text{dary}[0] == \&\text{dary}[0][0]$

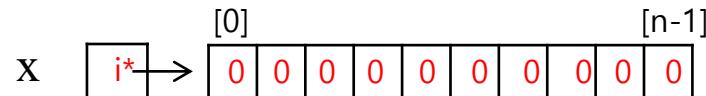


- **calloc**

```
int *x, n;
```

```
x = (int *) calloc(n, sizeof(int));
```

/* allocated bits are set to 0 */



```
#define CALLOC(p, n, s) \
    if (!((p) = calloc(n, s))) { \
        fprintf(stderr, "Insufficient memory"); \
        exit(EXIT_FAILURE); \
    } \
CALLOC( x, n, sizeof(int));
```

• realloc

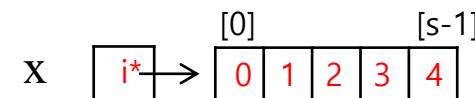
```

int *old, *x, s;
...
old = x;
/* changes the size of memory block
pointed by x to s*sizeof(int) */
if ( (x = (int *)realloc(x, s*sizeof(int))) == NULL ){
    free(old);
    exit(EXIT_FAILURE);
}
...
free(x);

```



재 할당이 적을 경우



재 할당이 클 경우

• realloc(cont')

errata

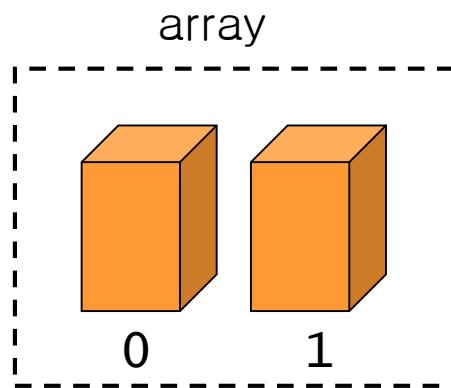
```
#define REALLOC(p, s) \
    if (!((p) = realloc(p, s))) { \
        fprintf(stderr, "Insufficient memory"); \
        exit(EXIT_FAILURE); \
    } \
... \
REALLOC(x, s*sizeof(int));
```

```
#define REALLOC(o, p, s) \
    if (!((p) = realloc(o, s))) { \
        free(o); \
        fprintf(stderr, "Insufficient memory"); \
        exit(EXIT_FAILURE); \
    } \
... \
REALLOC(old, x, s*sizeof(int)); //x = realloc(x, s*sizeof(int))
```

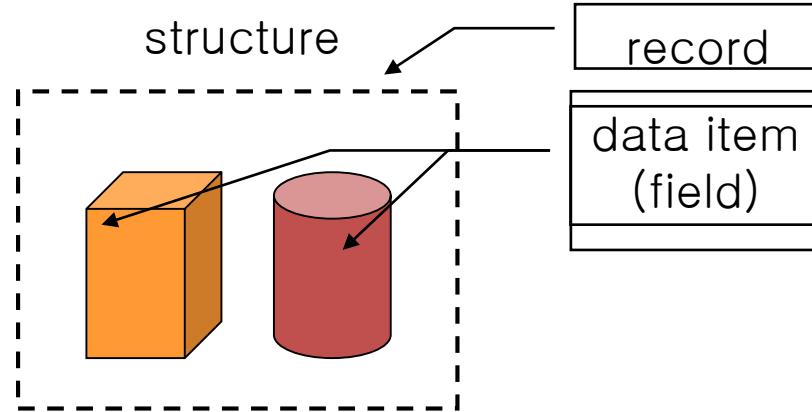
※ realloc() 실패 시 종료

2.3 Structures and Union

2.3.1 Structures



```
char array[100];
```



```
struct example {  
    char cfield;  
    int ifield;  
    float ffield;  
    double dfield;  
};  
struct example s1;
```

2.3 Structures and Union

2.3.1 Structures

- Called a *record*
- Collection of data items
 - Each item is identified as to its type and name

```
struct {  
    char name[10];  
    int age;  
    float salary;  
} person;           person is a variable.
```

- Structure member operator : dot(.)

```
strcpy(person.name, "james");  
person.age = 10;  
person.salary = 35000;
```

- Using the `typedef` statement

```
typedef struct {  
    char name[10];  
    int age;  
    float salary;  
} humanBeing;
```

humanBeing is a data type.

- Declaration of variables

```
humanBeing person1, person2;
```

```
if (strcmp(person1.name, person2.name))  
    printf("The two people do not have the same name\n");  
else  
    printf("The two people have the same name\n");
```

- Structure assignment : `person1=person2;`
 - in ANSI C, OK!
 - However, don't use the assignment operation when the structure has a pointer to a memory space. Why?
 - in older versions of C, NOT OK!

```
strcpy(person1.name, person2.name);  
person1.age = person2.age;  
person1.salary = person2.salary;
```
- Check of equality or inequality :
`if(person1==person2)`
 - cannot be checked directly

- Check of equality or inequality(cont')

```
#define FALSE 0
#define TRUE 1

if (humansEqual(person1, person2))
    printf("The two human beings are the same\n");
else
    printf("The two human beings are not the same\n");
```

```
int humansEqual(humanBeing person1,
                  humanBeing person2)
/* return TRUE if person1 and person2 are the same human
   being otherwise return FALSE */
if (strcmp(person1.name, person2.name))
    return FALSE;
if (person1.age != person2.age)
    return FALSE;
if (person1.salary != person2.salary)
    return FALSE;
return TRUE;
```

- A structure within a structure

```
typedef struct {  
    int month;  
    int day;  
    int year;  
} date;
```

```
typedef struct {  
    char name[10];  
    int age;  
    float salary;  
    date dob;  
} humanBeing ;
```

```
humanBeing person1;  
person1.dob.month = 2;  
person1.dob.day = 11;  
person1.dob.year = 1944;
```

2.3.2 Unions

- The fields share their memory space
- Only one field is “active” at any given time.

```
typedef struct {
    enum tagField {female, male} sex;
union {
    int children;
    int beard ;
} u;
} sexType;

typedef struct {
    char name[10];
    int age;
    float salary;
    date dob;
    sexType sexInfo;
} humanBeing;

humanBeing person1, person2;
person1.sexInfo.sex = male;
person1.sexInfo.u.beard = FALSE;

person2.sexInfo.sex = female;
person2.sexInfo.u.children = 4;
```

2.3.4 Self-Referential Structures

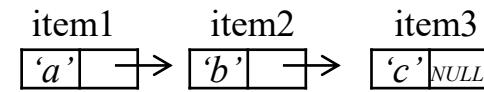
- A structure in which one or more of its components is a pointer to itself.

```
typedef struct list {  
    char data;  
    struct list *link ;  
} list ;
```

```
typedef struct list {  
    int data;  
    struct list* link;  
} t_list;
```

```
list item1, item2, item3; → t_list item1, item2, item3;  
item1.data = 'a';  
item2.data = 'b';  
item3.data = 'c';  
item1.link = item2.link = item3.link = NULL;
```

```
item1.link = &item2;  
item2.link = &item3;
```



2.4 Polynomials

2.4.1 The Abstract Data Type

- *Ordered list or linear list*

- an ordered set of data items

ex) Days-of-week

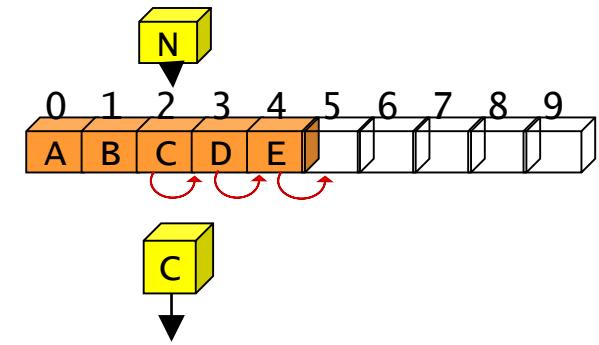
(Sun, Mon, Tue, Wed, Thu, Fri, Sat) : *list*
1st 2nd 3rd 4th 5th 6th 7th : *order*

- denote as $(item_0, item_1, \dots, item_{n-1})$
- empty list : ()
- operations on ordered list

- i. find the length n
- ii. read the items in a list from right to left (or left to right)
- iii. retrieve i th item, $0 \leq i < n$
- iv. replace i th item's value, $0 \leq i < n$
- v. insert i th position, $0 \leq i < n$: $i, i+1, \dots, n-1 \rightarrow i+1, i+2, \dots, n$
- vi. delete i th item, $0 \leq i < n$: $i+1, \dots, n-1 \rightarrow i, i+1, \dots, n-2$

Implementation of Ordered List

- Array
 - associate the list element, $item_i$, with the array index i
 - *sequential mapping*
 - retrieve, replace an item, or find the length of a list, in constant time
 - **problems in insertion and deletion**
 - sequential mapping forces us to move items



- Linked List
 - *Non-sequential mapping*
 - Chapter 4

A Problem Requiring Ordered Lists

- Manipulation of symbolic polynomials

$$A(x) = 3x^{20} + 2x^5 + 4, \quad B(x) = x^4 + 10x^3 + 3x^2 + 1$$

- degree : the largest exponent of a polynomial

When $A(x) = \sum a_i x^i$ and $B(x) = \sum b_i x^i$,

$$A(x) + B(x) = \sum (a_i + b_i)x^i$$

$$A(x) B(x) = \sum (a_i x^i \sum (b_j x^j))$$

- assumption: unique exponents arranged in decreasing order

ADT Polynomial is

objects: $p(x) = a_1x^{e_1} + \cdots + a_nx^{e_n}$; a set of ordered pairs of $\langle e_i, a_i \rangle$ where a_i in Coefficients and e_i in Exponents, e_i are integers ≥ 0

functions:

for all $poly, poly1, poly2 \in Polynomial, coef \in Coefficients, expon \in Exponents$

<i>Polynomial</i> Zero()	::= return the polynomial, $p(x) = 0$
<i>Boolean</i> IsZero(<i>poly</i>)	::= if (<i>poly</i>) return FALSE else return TRUE
<i>Coefficient</i> Coef(<i>poly,expon</i>)	::= if (<i>expon</i> \in <i>poly</i>) return its coefficient else return zero
<i>Exponent</i> LeadExp(<i>poly</i>)	::= return the largest exponent in <i>poly</i>
<i>Polynomial</i> Attach(<i>poly, coef, expon</i>)	::= if (<i>expon</i> \in <i>poly</i>) return error else return the polynomial <i>poly</i> with the term $\langle coef, expon \rangle$ inserted
<i>Polynomial</i> Remove(<i>poly, expon</i>)	::= if (<i>expon</i> \in <i>poly</i>) return the polynomial <i>poly</i> with the term whose exponent is <i>expon</i> deleted else return error
<i>Polynomial</i> SingleMult(<i>poly, coef, expon</i>)	::= return the polynomial $poly \cdot coef \cdot x^{expon}$
<i>Polynomial</i> Add(<i>poly1, poly2</i>)	::= return the polynomial $poly1 + poly2$
<i>Polynomial</i> Mult(<i>poly1, poly2</i>)	::= return the polynomial $poly1 \cdot poly2$

end *Polynomial*

2.4.2 Polynomial Representation

```
#define COMPARE(x, y) ( ((x) < (y)) ? -1 : ((x) == (y)) ? 0: 1 )  ※ p.12
```

```
/* d = a + b, where a, b, and d are polynomials */
d = Zero()
while (! IsZero(a) && ! IsZero(b)) do {
    switch COMPARE(LeadExp(a), LeadExp(b)) {
        case -1: d =
            Attach(d, Coef(b, LeadExp(b)), LeadExp(b));
            b = Remove(b, LeadExp(b));
            break;
        case 0: sum = Coef(a, LeadExp(a))
                  + Coef(b, LeadExp(b));
            if (sum) {
                Attach(d, sum, LeadExp(a));
                a = Remove(a, LeadExp(a));
                b = Remove(b, LeadExp(b));
            }
            break;
        case 1: d =
            Attach(d, Coef(a, LeadExp(a)), LeadExp(a));
            a = Remove(a, LeadExp(a));
    }
}
insert any remaining terms of a or b into d
```

Initial version of *pad2* function (cont')

$$D(x) = 0$$

$$A(x) = 2x^{1000} + 2x^3$$

$$B(x) = x^4 + 10x^3 + 3x^2 + 1$$

(step1)

$$D(x) = 2x^{1000}$$

$$A(x) = 2x^3$$

$$B(x) = x^4 + 10x^3 + 3x^2 + 1$$

(step2)

$$D(x) = 2x^{1000} + x^4$$

$$A(x) = 2x^3$$

$$B(x) = 10x^3 + 3x^2 + 1$$

(step3)

$$D(x) = 2x^{1000} + x^4 + 12x^3$$

$$A(x) = 0$$

$$B(x) = 3x^2 + 1$$

(step4)

$$D(x) = 2x^{1000} + x^4 + 12x^3 + 3x^2 + 1$$

$$A(x) = 0$$

$$B(x) = 0$$

Representation of polynomials in C

```
(1) #define MAX_DEGREE 101 /*Max degree of polynomial+1*/  
typedef struct {  
    int degree;  
    float coef[MAX_DEGREE];  
} polynomial;  
  
polynomial a;
```

$A(x) = \sum_{i=0}^n a_i x^i$ would be represented as :

```
a.degree = n  
a.coef[i] = a_{n-i}, 0 ≤ i ≤ n , n < MAX_DEGREE
```

* *a.coef[i]* is the coefficient of x^{n-i}

$$A(x) = 2x^{1000} + 1 \text{ and } B(x) = x^4 + 10x^3 + 3x^2 + 1$$

1000
2 0 0 0 0 0 0 0 0 0 0 0 0 1

4
1 10 3 0 1

Representation of polynomials in C(cont')

```
(2) #define MAX_TERMS 100 /*size of terms array*/  
typedef struct {  
    float coef;  
    int expon;  
} term;  
term terms[MAX_TERMS];  
int avail = 0;
```

$$A(x) = 2x^{1000} + 1 \text{ and } B(x) = x^4 + 10x^3 + 3x^2 + 1$$

<i>coef</i>	2	1
<i>exp</i>	1000	0
	0	.1

Using one array to represent two polynomials

$$A(x) = 2x^{1000} + 1 \text{ and } B(x) = x^4 + 10x^3 + 3x^2 + 1$$

	<i>startA</i>	<i>finishA</i>	<i>startB</i>		<i>finishB</i>	<i>avail</i>
<i>coef</i>	↓	↓	↓		↓	↓
<i>exp</i>	2	1	1	10	3	1
	1000	0	4	3	2	0
	0	.1	2	3	4	5
						6

iterations
 $\leq m+n-1$

$\leq m$
 $\leq n$

```
void padd(int startA,int finishA,int startB, int finishB,
          int *startD,int *finishD)
{ /* add A(x) and B(x) to obtain D(x) */
    float coefficient;
    *startD = avail;
    while (startA <= finishA && startB <= finishB)
        switch(COMPARE(terms[startA].expon,
                       terms[startB].expon)) {
            case -1: /* a expon < b expon */
                attach(terms[startB].coef,terms[startB].expon);
                startB++;
                break;
            case 0: /* equal exponents */
                coefficient = terms[startA].coef +
                               terms[startB].coef;
                if (coefficient)
                    attach(coefficient,terms[startA].expon);
                startA++;
                startB++;
                break;
            case 1: /* a expon > b expon */
                attach(terms[startA].coef,terms[startA].expon);
                startA++;
        }
    /* add in remaining terms of A(x) */
    for(; startA <= finishA; startA++)
        attach(terms[startA].coef,terms[startA].expon);
    /* add in remaining terms of B(x) */
    for( ; startB <= finishB; startB++)
        attach(terms[startB].coef, terms[startB].expon);
    *finishD = avail-1;
}
```

```
void attach(float coefficient, int exponent)
{ /* add a new term to the polynomial */
    if (avail >= MAX_TERMS) {
        fprintf(stderr,"Too many terms in the polynomial\n");
        exit(EXIT_FAILURE);
    }
    terms[avail].coef = coefficient;
    terms[avail++].expon = exponent;
}
```

Program 2.7: Function to add a new term

	<i>sA</i>	<i>fA</i>	<i>sB</i>	<i>fB</i>	<i>avail</i>						
<i>coeff</i>											
<i>exp</i>											

	<i>sA</i>	<i>fA</i>	<i>sB</i>	<i>fB</i>	<i>sD</i>	<i>avail</i>					
<i>coeff</i>	2										
<i>exp</i>	1000										

	<i>sA</i>	<i>fA</i>		<i>sB</i>		<i>fB</i>	<i>sD</i>		<i>avail</i>		
<i>coeff</i>	2		1								
<i>exp</i>	1000		4								

	<i>sA</i>	<i>fA</i>		<i>sB</i>	<i>fB</i>	<i>sD</i>		<i>avail</i>			
<i>coeff</i>	2		1	10							
<i>exp</i>	1000		4	3							

	<i>fA</i>	<i>sA</i>		<i>sB</i>	<i>fB</i>	<i>sD</i>		<i>avail</i>			
<i>coeff</i>	2	1	1	10	3	1					
<i>exp</i>	1000	0	4	3	2	0					

	<i>fA</i>	<i>sA</i>		<i>fB</i>	<i>sB</i>	<i>sD</i>		<i>fD</i>	<i>avail</i>		
<i>coeff</i>	2	1	1	10	3	1					
<i>exp</i>	1000	0	4	3	2	0					

- **Analysis of *padd***
 - Let m and n be the number of nonzero terms in A and B, respectively.
 - ① If $m>0$ and $n>0$, **while loop**
 - each iteration : $O(1)$
 - The iteration terminates when either $startA$ or $startB$ exceeds $finishA$ or $finishB$, respectively
 - The number of iterations is bounded by $m+n-1$
 - the worst case : ex) $a(x) = x^6+x^4+x^2+x^0$, $b(x) = x^7+x^5+x^3+x^1$
 - ② The remaining **two for loops** → $O(m+n)$
 - ①&② → **$O(m+n)$**