

## **Chap 8. Hashing (2)**

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## 8.3 Dynamic Hashing

### 8.3.1 Motivation for Dynamic Hashing

- It is necessary to increase *the size of a hash table* whenever its *loading density* exceeds a pre-specified threshold
  - Ex)  $b$  buckets with divisor  $D = b$ 
    - 버킷수 증가  
 $2b+1$  buckets with divisor  $D = 2b+1$
- But, rebuilding the hash table takes time  
➡ Reduce the rebuild time using *dynamic hashing*

- Two forms of dynamic hashing
  - *Using directory*
  - *Directoryless*
- For both forms,
  - $h$  maps keys into non-negative integers
  - The range of  $h$  is sufficiently large
  - $h(k, p)$  : the integer formed by the  $p$  LSBs of  $h(k)$ 
    - Ex)  $h(12346, 5) \Rightarrow 12346$
    - $h(12346, 3) \Rightarrow 346$

- Example

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

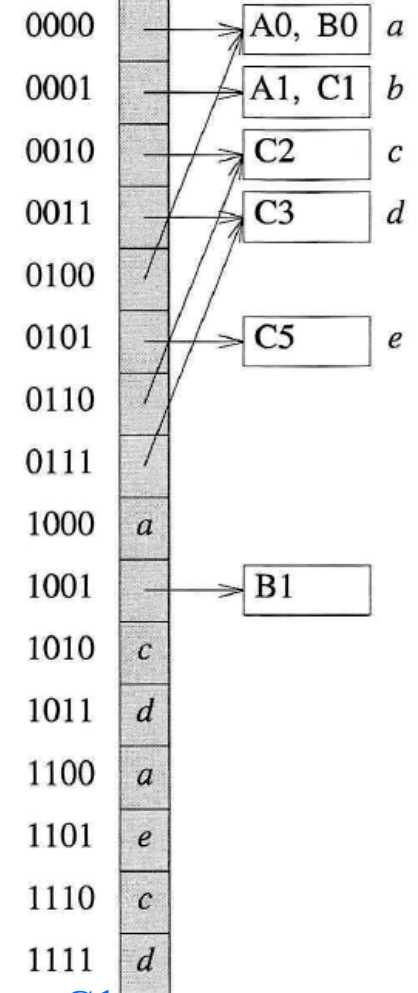
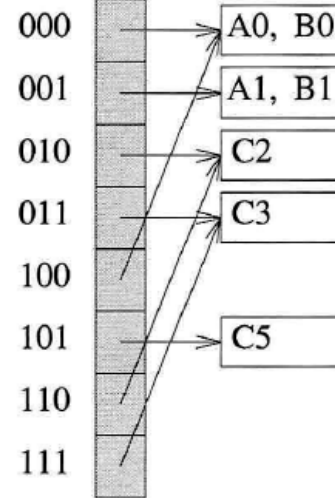
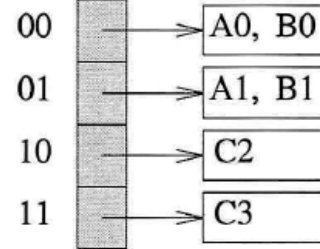
- keys composed of two characters
- Hash function  $h(k)$ 
  - transforms keys into 6-bit nonnegative integers
    - letters A~C  $\rightarrow$  100, 101, 110
    - digits 0~7  $\rightarrow$  000, 001, ..., 111
  - $h(A0,1) = 100\ 000$ ,  $1 = 1$
  - $h(A1,3) = 100\ 001$ ,  $1 = 1$
  - $h(B1,4) = 011\ 001 = 9$
  - $h(C1,6) = 110\ 001 = 49$

## 8.3.2 Dynamic Hashing Using Directories

- Use a directory  $d$  of pointers to buckets
- Directory depth
  - the number of bits of  $h(k)$  used to index the directory
  - $h(k, t) : t$  is a directory depth
- The size of directory depends on directory depth
  - $h(k, 2) \rightarrow \text{directory size} = 2^2 = 4$
  - $h(k, 5) \rightarrow \text{directory size} = 2^5 = 32$
- The number of buckets is at most equal to the directory size.

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

buckets with  
directory 2 slots



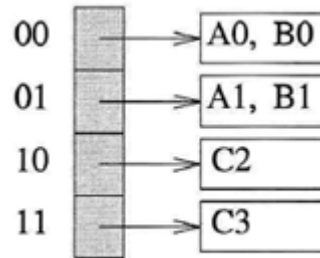
(a) depth = 2  $\xrightarrow{\text{insert C5}}$  (b) depth = 3  $\xrightarrow{\text{insert C1}}$  (c) depth = 4

**Figure 8.8:** Dynamic hash tables with directories

# < Case 1 >

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

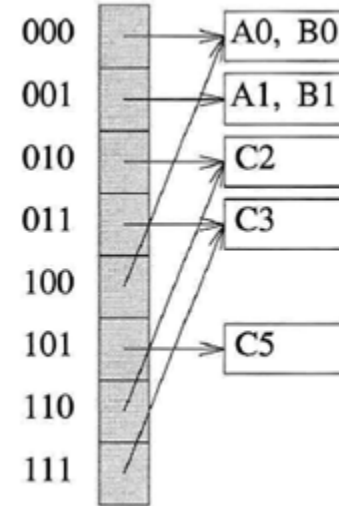
(a) depth = 2



insert C5



(b)



$$h(C5, 2) = 01$$

*bucket overflow*

the least  $u$  that  $h(k, u)$  is not the same for all keys in the overflowed bucket ?  $u = 3$

In the case least  $u$  is *greater than* the directory depth

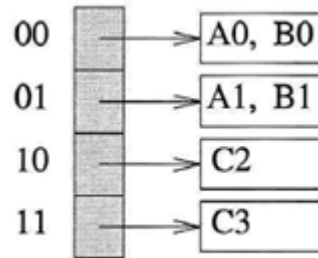
- ① increase directory depth to  $u = 3$
- ② *double* the directory size
- ③ the pointers in the original directory are duplicated  
 $d[i+4]=d[i], 0 \leq i < 4$
- ④ split the overflowed bucket using  $h(k, u)$   
 001: A1, B1, 101: C5



## < Case 2 >

(a) depth = 2

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101



insert C1  
→

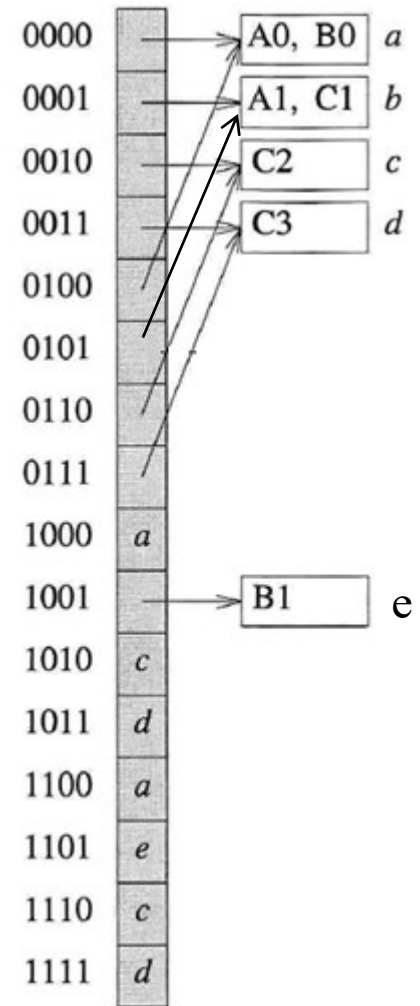
$h(C1,2) = 01$

*bucket overflow*

the least  $u$  that  $h(k,u)$  is not the same for all keys in the overflowed bucket ?  $u = 4$

In the case least  $u$  is *greater than* the directory depth

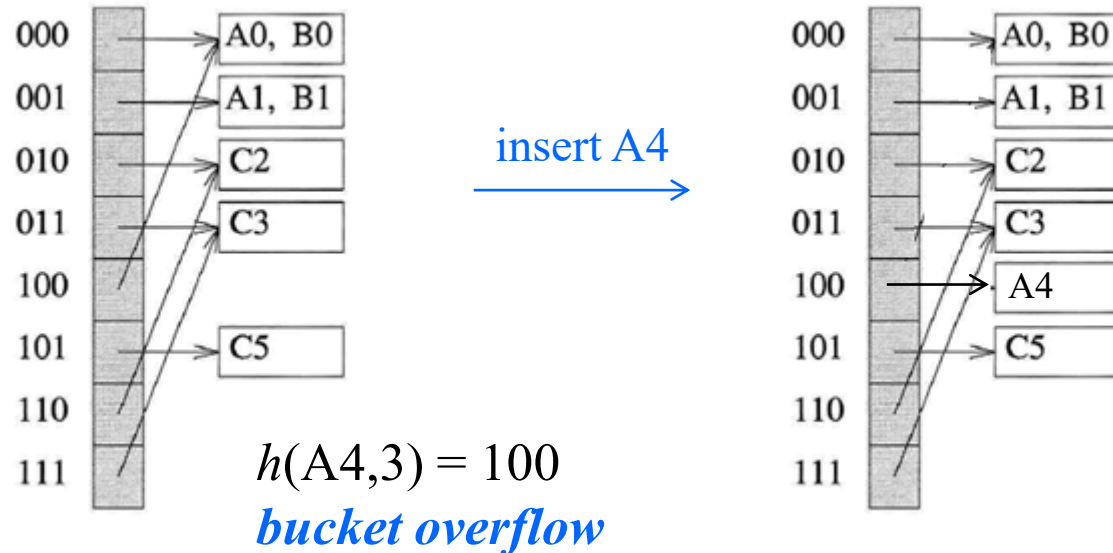
- ① increase directory depth to  $u = 4$
- ② *quadruple* the directory size
- ③ the pointers in the original directory are replicated 3 times,
- ④ split the overflowed bucket using  $h(k, u)$   
0001: A1, C1, 1001:B1



## < Case 3 >

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

(b) depth = 3



the least  $u$  that  $h(k,u)$  is not the same for all keys in the overflowed bucket ?  **$u = 3$**

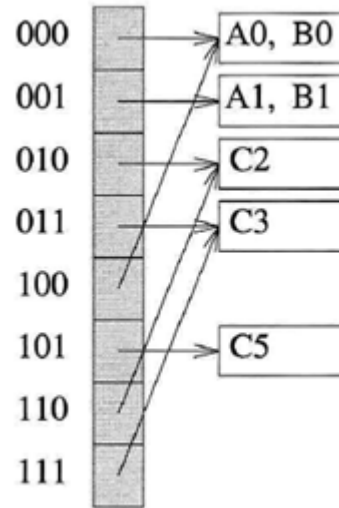
In the case least  $u$  is *less than or equal* to the directory depth

- ① The size of directory is not changed
- ② split the overflowed bucket using  $h(k, u)$   
000: A0, B0, 100:A4
- ③ update  $d[100]$  to point to the new bucket

# < Case 4 >

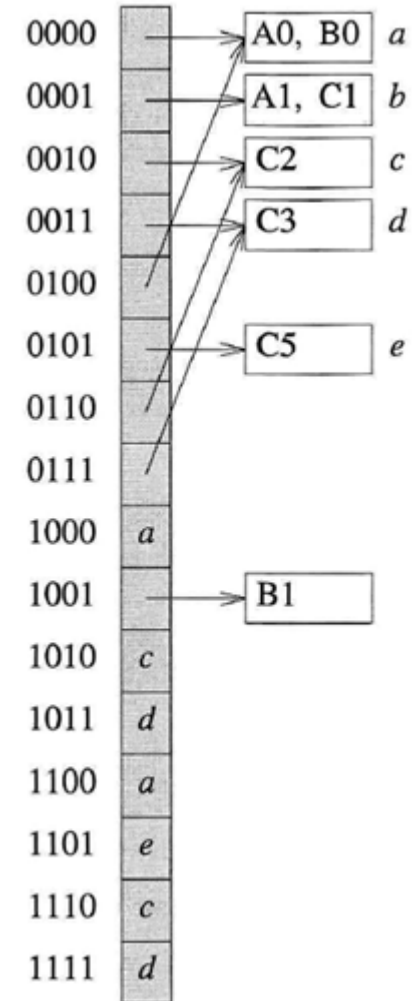
$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

(b) depth = 3



insert C1

(c)



$h(C1,3) = 001$   
*bucket overflow*

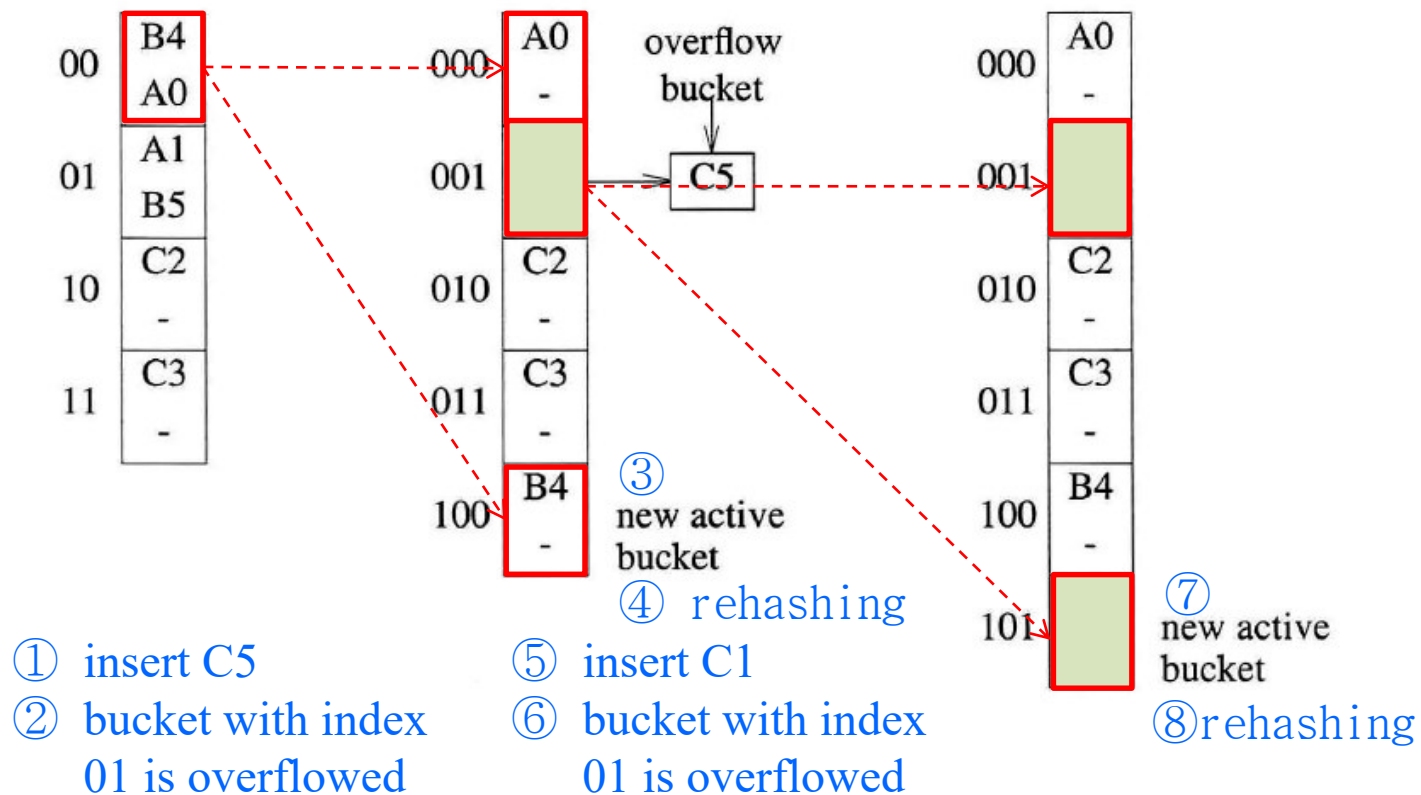
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## 8.3.3 Directoryless Dynamic Hashing

- Known as *linear dynamic hashing*
- Assumption
  - this array is as large as possible and so there is no possibility of increasing its size dynamically.

- Two variables  $q$  and  $r$ ,  $0 \leq q < 2^r$ 
  - $r$  is the number of bits of  $h(k)$  used to index into the hash table
  - $q$  is the bucket that will split next
- *At any time, only buckets  $0$  through  $2^r + q - 1$  are active*

$k$	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

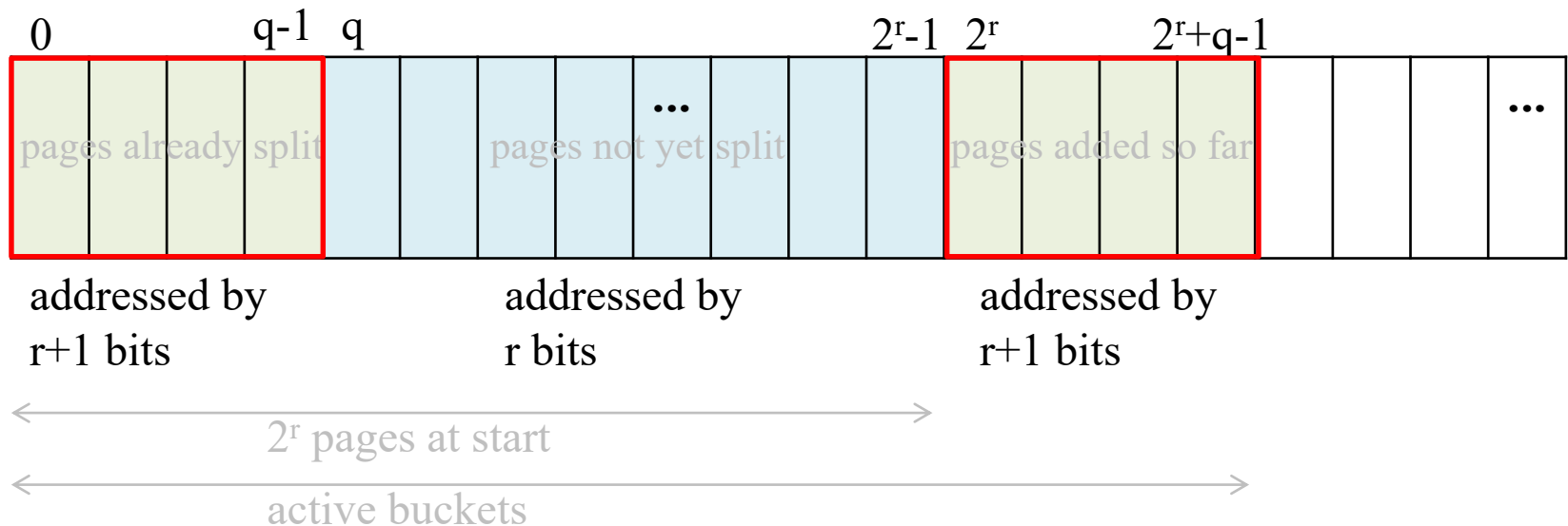


(a)  $r = 2, q = 0$

(b) Insert C5,  $r = 2, q = 1$

(c) Insert C1,  $r = 2, q = 2$

**Figure 8.9:** Inserting into a directoryless dynamic hash table



In case  $q$  becomes  $2^r$ , we increment  $r$  by 1 and reset  $q$  to 0 .

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**if**  $(h(k, r) < q)$  search the chain that begins at bucket  $h(k, r+1)$ ;  
**else** search the chain that begins at bucket  $h(k, r)$ ;

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**Program 8.5:** Searching a directoryless hash table