

Chap 3. Stacks and Queues (1)

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3.1 Stacks



3.1 Stacks

- *Linear list.*
- One end is called *top*.
- The other end is called *bottom*.
- Additions to and removals from the *top* end only.

- A stack is a **LIFO** list.
 - *Last-In-First-Out*
-

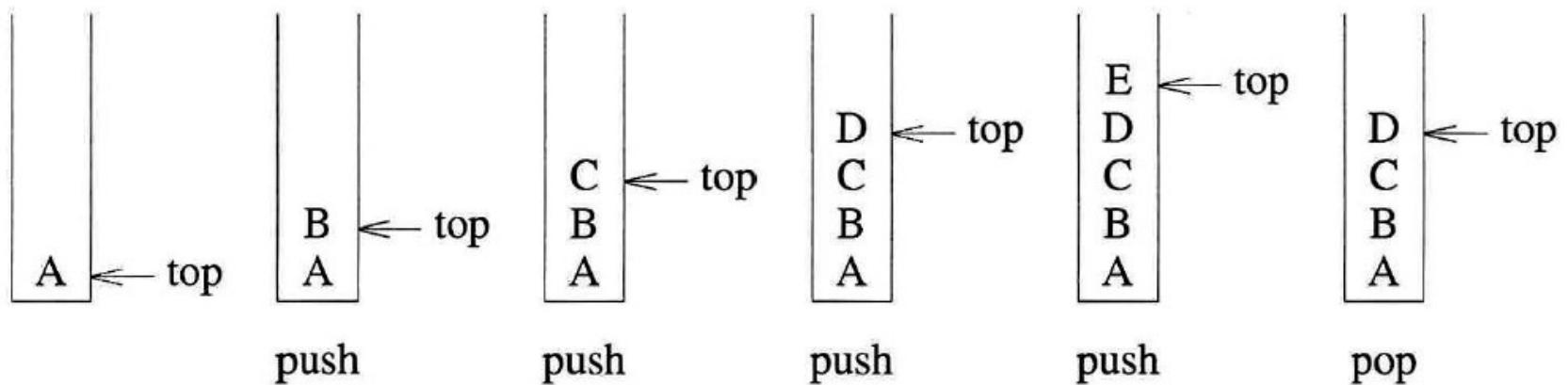


Figure 3.1: Inserting and deleting elements in a stack

ADT Stack is

objects: a finite ordered list with zero or more elements.

functions:

for all $stack \in Stack$, $item \in element$, $maxStackSize \in$ positive integer

$Stack \text{ CreateS}(maxStackSize) ::=$

create an empty stack whose maximum size is $maxStackSize$

$Boolean \text{ IsFull}(stack, maxStackSize) ::=$

if (number of elements in $stack == maxStackSize$)

return *TRUE*

else return *FALSE*

$Stack \text{ Push}(stack, item) ::=$

if ($\text{IsFull}(stack)$) *stackFull*

else insert *item* into top of *stack* and **return**

$Boolean \text{ IsEmpty}(stack) ::=$

if ($stack == \text{CreateS}(maxStackSize)$)

return *TRUE*

else return *FALSE*

$Element \text{ Pop}(stack) ::=$

if ($\text{IsEmpty}(stack)$) **return**

else remove and return the element at the top of the stack.

ADT 3.1: Abstract data type Stack

- Creation of Stack in C
 - Use a *1D array* to represent a stack.
 - Stack elements are stored in *stack[0]* through *stack[top]*.

Stack CreateS(maxStackSize) ::=

```
#define MAX_STACK_SIZE 100 /* maximum stack size */  
typedef struct {  
    int key;  
    /* other fields */  
    } element;  
element stack[MAX_STACK_SIZE];  
int top = -1;
```

Boolean IsEmpty(Stack) ::= [redacted]

Boolean IsFull(Stack) ::= [redacted]

• Implementation of Stack Operations

```
void push(element item)
{ /* add an item to the global stack */
    if (top >= MAX_STACK_SIZE-1)
        stackFull();
    stack[ ] = item;
}
```

Program 3.1

```
element pop()
{ /* delete and return the top element from the stack */
    if (top == -1)
        return stackEmpty(); /* returns an error key */
    return stack[ ];
}
```

Program 3.2

```
void stackFull()
{
    fprintf(stderr, "Stack is full, cannot add element");
    exit(EXIT_FAILURE);
}
```

Program 3.3

3.2 Stacks Using Dynamic Arrays

```
Stack CreateS() ::= typedef struct {
    int key;
    /* other fields */
} element;
element *stack;
MALLOC(stack, sizeof(*stack));
int capacity = 1;
int top = -1;
```

※ **capacity** : maximum number of stack elements that may be stored in the array

stack → 

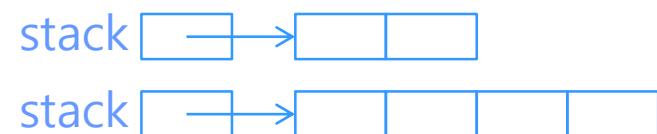
Boolean IsEmpty(Stack) ::= 

Boolean IsFull(Stack) ::= 

- *pop* : unchanged from Program 3.2
- *push, stackFull* : changed from Program 3.1&3.3
- ***Array Doubling***
 - When stack is full, double the capacity using REALLOC.

```
void stackFull()
{
    REALLOC(oldStack, stack, 2*capacity*sizeof(int));
    capacity *= 2;
}
```

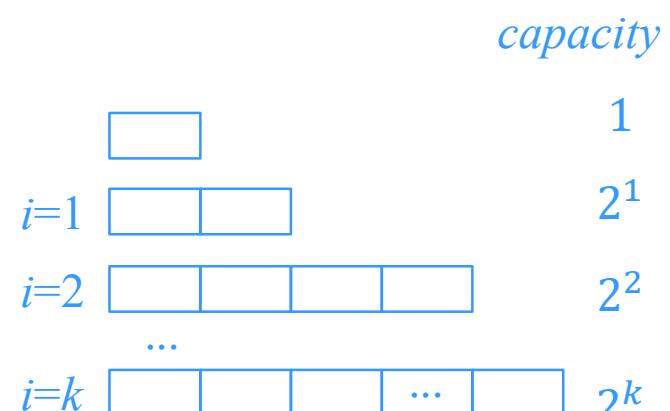
Program 3.4: Stack full with array doubling



- Time complexity of array doubling
 - One array doubling
 - Memory allocation : $O(1)$
 - Copy of an array element : $O(1)$
 - Copy of all array elements : $O(\text{capacity})$
 - All array doubling
 - N 번의 *push* 가 있었고 현재 *stack capacity*가 2^k 이라면 (k 번 *doubling*)

$$O\left(\sum_{i=1}^k 2^i\right) = O(2^{k+1}) = O(2^k)$$

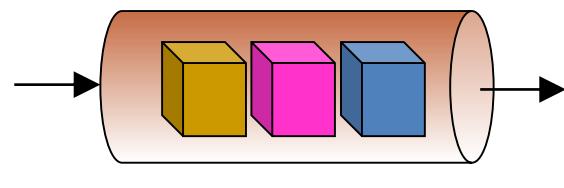
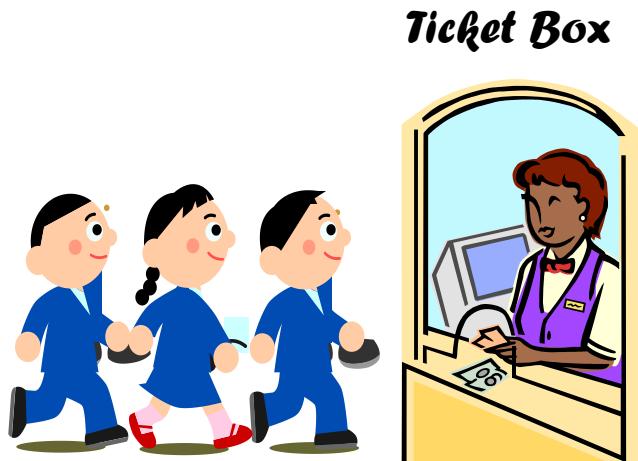
Doubling 시 copy수



What is difference between stack and Array?



3.3 Queues



(front)

(rear)

3.3 Queues

- *Linear list.*
- One end is called *front*.
- The other end is called *rear*.
- *Additions* are done at the *rear* only.
- *Removals* are made from the *front* only.



- A queue is a **FIFO** list.
 - *First-In-First-Out*
-

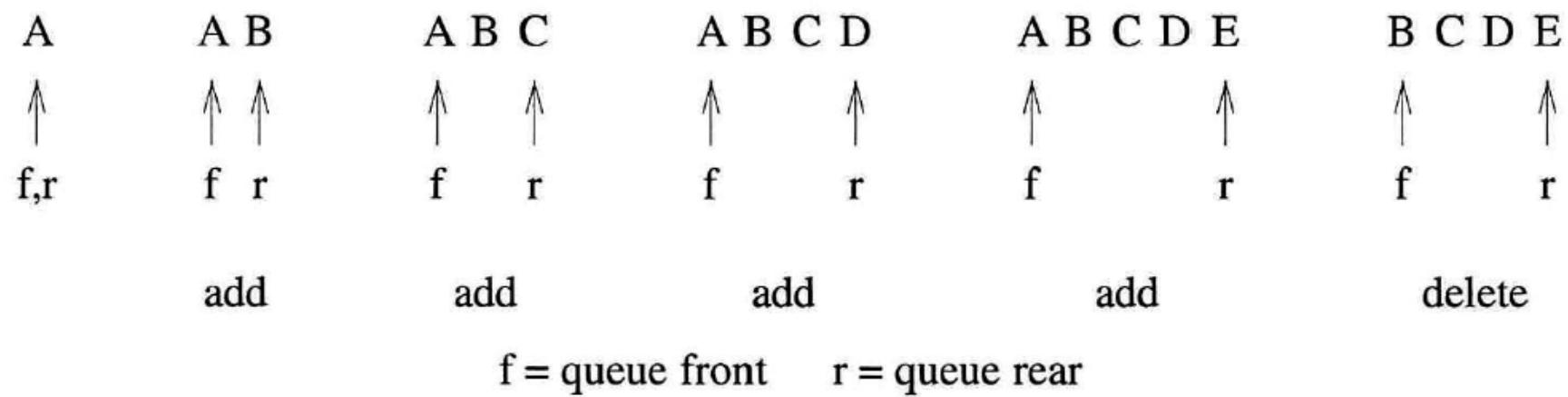


Figure 3.4: Inserting and deleting elements in a queue

ADT Queue is

objects: a finite ordered list with zero or more elements.

functions:

for all $queue \in Queue$, $item \in element$, $maxQueueSize \in$ positive integer

$Queue \text{ CreateQ}(maxQueueSize) ::=$

create an empty queue whose maximum size is $maxQueueSize$

$Boolean \text{ IsFullQ}(queue, maxQueueSize) ::=$

if (number of elements in $queue == maxQueueSize$)

return *TRUE*

else return *FALSE*

$Queue \text{ AddQ}(queue, item) ::=$

if (*IsFullQ(queue)*) *queueFull*

else insert *item* at rear of *queue* and return *queue*

$Boolean \text{ IsEmptyQ}(queue) ::=$

if ($queue == \text{CreateQ}(maxQueueSize)$)

return *TRUE*

else return *FALSE*

$Element \text{ DeleteQ}(queue) ::=$

if (*IsEmptyQ(queue)*) **return**

else remove and return the *item* at front of *queue*.

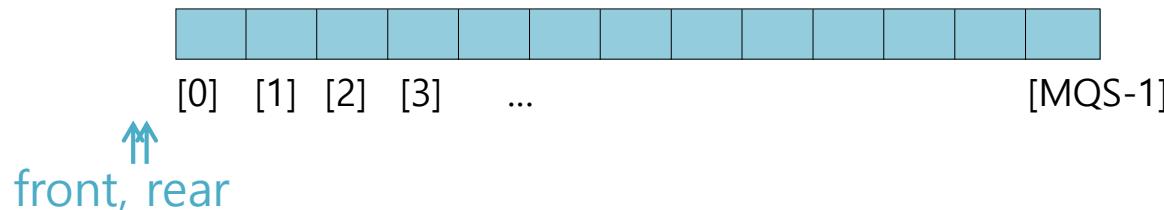
ADT 3.2: Abstract data type *Queue*

Representations of Queue

- Sequential representation
 - Uses an *1D array*
- Circular representation : *circular queue*
 - Uses an *1D array*
 - More efficient

Sequential Representation

- Creation of Queue in C
 - Uses an 1D array, *queue*



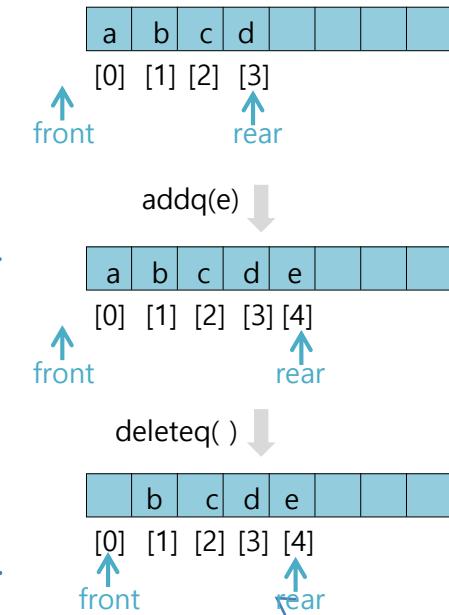
```
Queue CreateQ(maxQueueSize) ::=  
    #define MAX_QUEUE_SIZE 100 /* maximum queue size */  
    typedef struct {  
        int key;  
        /* other fields */  
        } element;  
    element queue[MAX_QUEUE_SIZE];  
    int rear = -1;  
    int front = -1;  
Boolean IsEmptyQ(queue) ::= [REDACTED]  
Boolean IsFullQ(queue) ::= [REDACTED]
```

Sequential Representation

• Implementation of Queue Operations

```
void addq(element item)
/* add an item to the queue */
if (rear == MAX_QUEUE_SIZE-1)
    queueFull();
queue[++rear] = item;
}
```

Program 3.5: Add to a queue



```
element deleteq()
/* remove element at the front of the queue */
if (front == rear)
    return queueEmpty(); /* return an error key */
return queue[ ];
```

Program 3.6: Delete from a queue

Sequential Representation

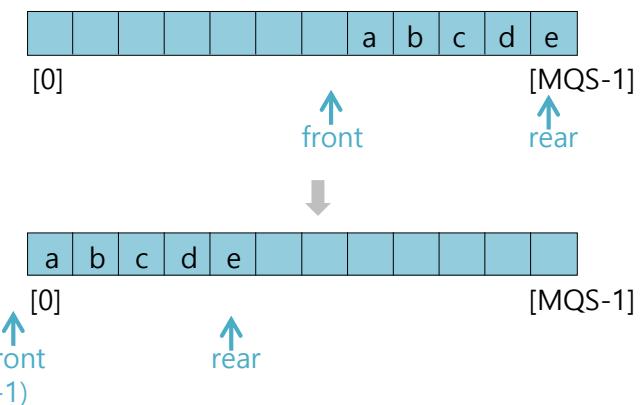
- Example: Job Scheduling by an OS

<i>front</i>	<i>rear</i>	$Q[0]$	$Q[1]$	$Q[2]$	$Q[3]$	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

Figure 3.5: Insertion and deletion from a sequential queue

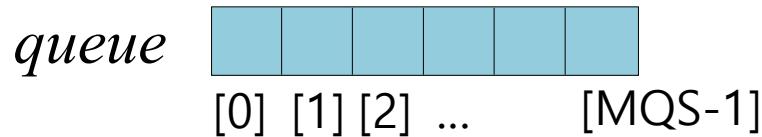
– queueFull

- array shifting : time-consuming
- Worst case time complexity,
 $O(MAX_QUEUE_SIZE)$

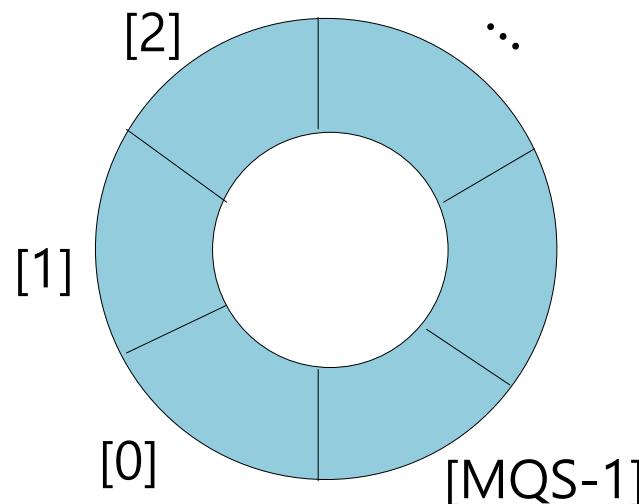


Circular Queue

- Uses an 1D array, *queue*



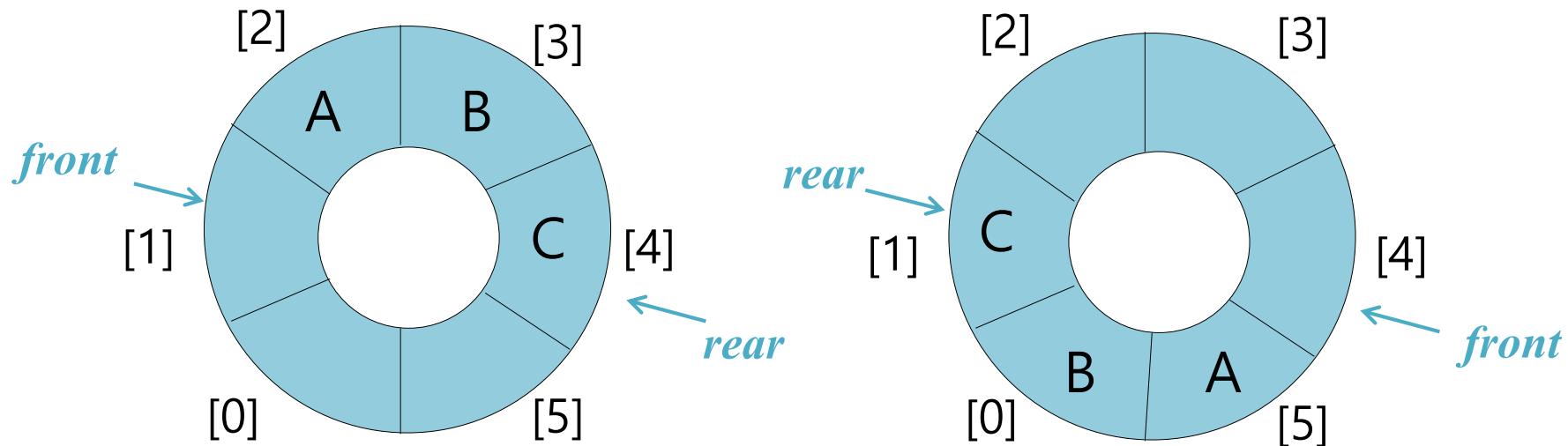
- Circular view of an 1D array



Initial values
 $front = rear = 0$

Circular Queue

- integer variables *front* and *rear*.
 - *front* is *one position counterclockwise from first element*
 - *rear* gives position of last element



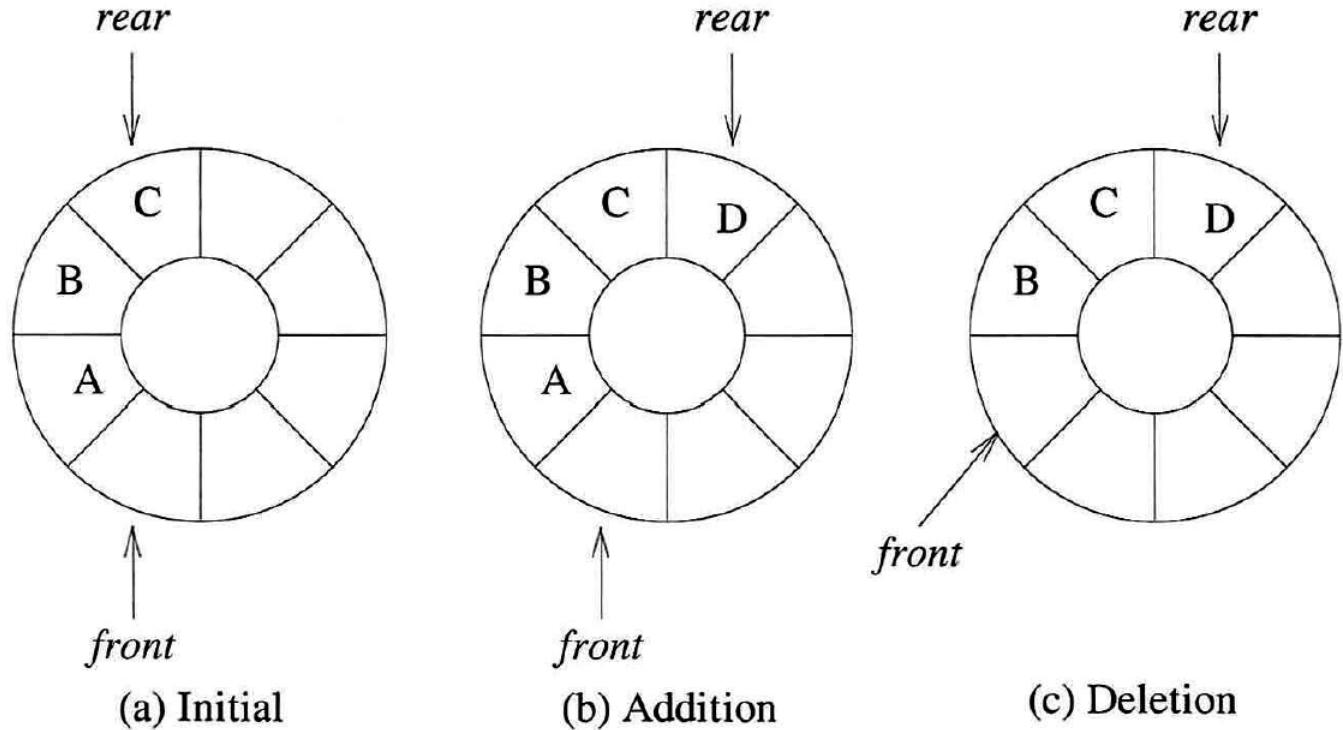
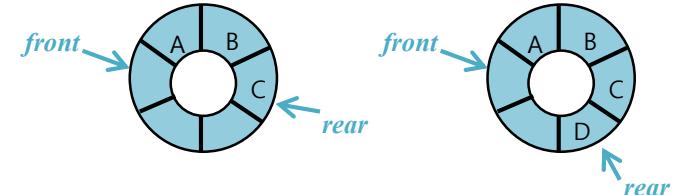


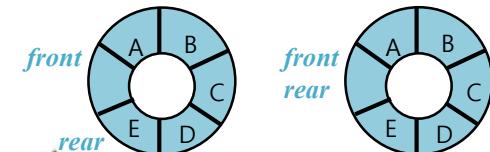
Figure 3.6: Circular queue

Circular Queue

- Add an element in the circular queue.
 - Move *rear* one clockwise.
 - Queue Full Check
 - Then put into *queue[rear]*.



```
void addq(element item)
{ /* add an item to the queue */
    if (/* condition */)
        queueFull(); /* print error and exit */
    queue[rear] = item;
}
```

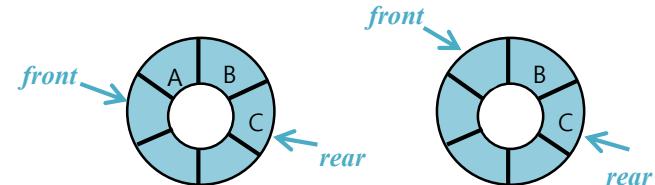


Program 3.7: Add to a circular queue

a maximum of MAX_QUEUE_SIZE-1 elements in the queue at any time!

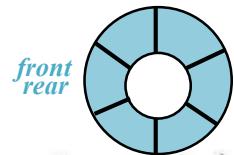
Circular Queue

- Delete an element from the circular queue.
 - Queue Empty check
 - Move *front* one clockwise.
 - Then extract from *queue[front]*.



```
element deleteq()
{ /* remove front element from the queue */

    if (front == rear)
        return queueEmpty(); /* return an error key */
    front = (front+1) % MAX_QUEUE_SIZE;
    return queue[front];
}
```



Program 3.8: Delete from a circular queue