

Chap 3. Stacks and Queues (2)

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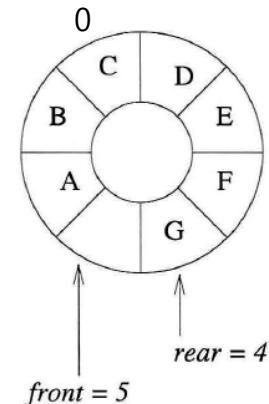
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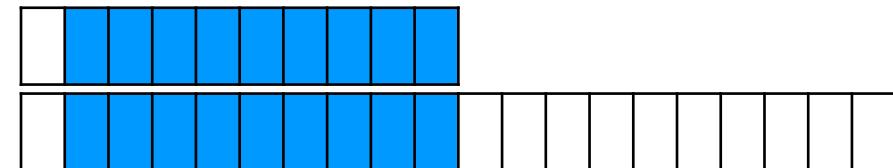
3.7 Multiple Stacks and Queues

3.4 Circular Queues Using Dynamically Allocated Arrays



queue	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
	C	D	E	F	G		A	B

(b) Flattened view of circular full queue
 $front = 5, rear = 4$



최악의 경우 이동

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
C	D	E	F	G		A	B								

$front = 5, rear = 4$
(c) After array doubling by *realloc*

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
C	D	E	F	G									A	B	

$front = 13, rear = 4$
(d) After shifting right segment

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
A	B	C	D	E	F	G									

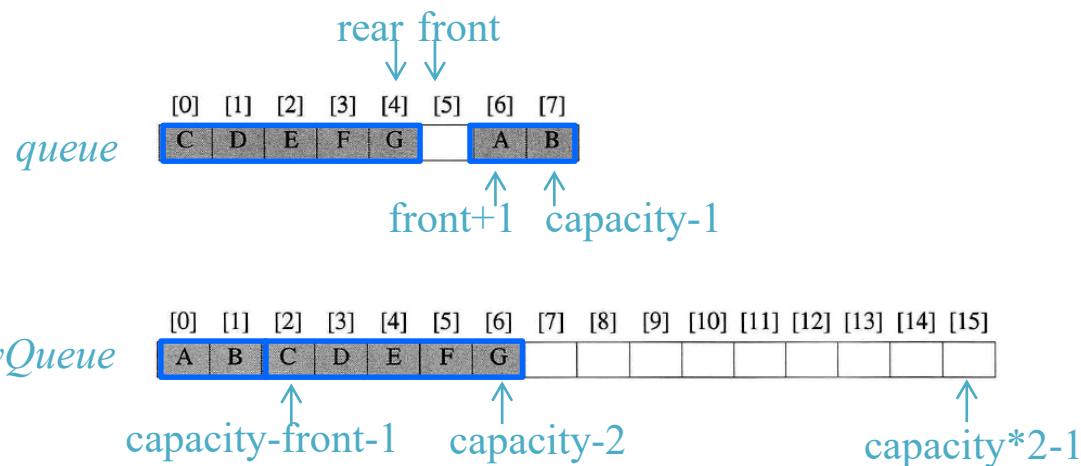
$front = 15, rear = 6$
(e) Alternative configuration by *malloc*

(case 1) (b) \rightarrow (c) \rightarrow (d)
(case 2) (b) \rightarrow (e), Program 3.10

Figure 3.7: Doubling queue capacity

- Figure 3.7 (b)→(e)

- (1) Create a new array *newQueue* of twice the capacity.
- (2) Copy the second segment (i.e., the elements *queue* [*front* + 1] through *queue* [*capacity* - 1]) to positions in *newQueue* beginning at 0.
- (3) Copy the first segment (i.e., the elements *queue* [0] through *queue* [*rear*]) to positions in *newQueue* beginning at *capacity* - *front* - 1.



```

void queueFull()
{
    int start;
    /* allocate an array with twice the capacity */
    element* newQueue;
    MALLOC(newQueue, 2 * capacity * sizeof(*queue));
}

/* copy from queue to newQueue */
start = (front+1) % capacity;

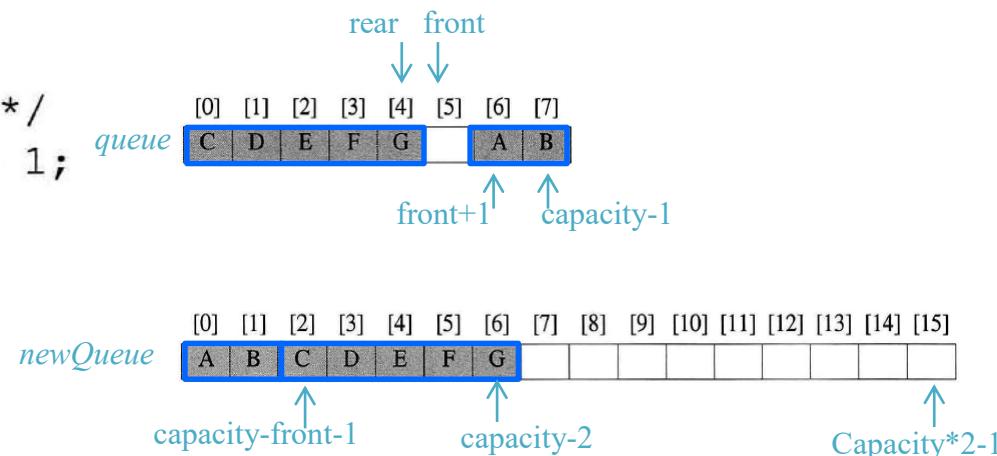
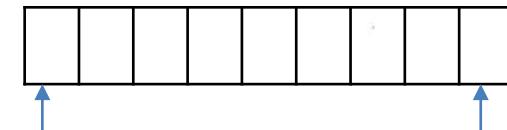
① if (start < 2)
    /* no wrap around */
    copy(queue+start, queue+start+capacity-1, newQueue);

② else
    /* queue wraps around */
    copy(queue+start, queue+capacity, newQueue);
    copy(queue, queue+rear+1, newQueue+capacity-start);

}

/* switch to newQueue */
front = 2 * capacity - 1; queue
rear = capacity - 2;
capacity *= 2;
free(queue);
queue = newQueue;
}

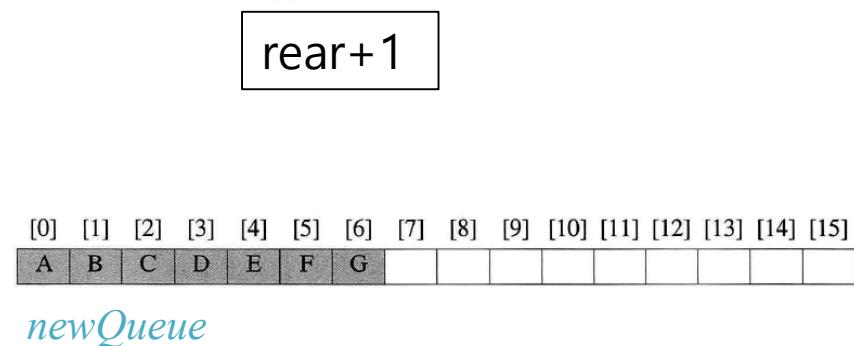
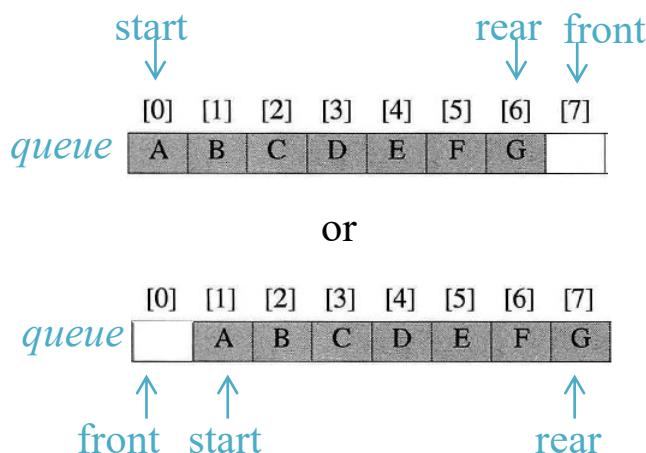
```



Program 3.10: Doubling queue capacity <Figure 3.7 (b)→(e)>

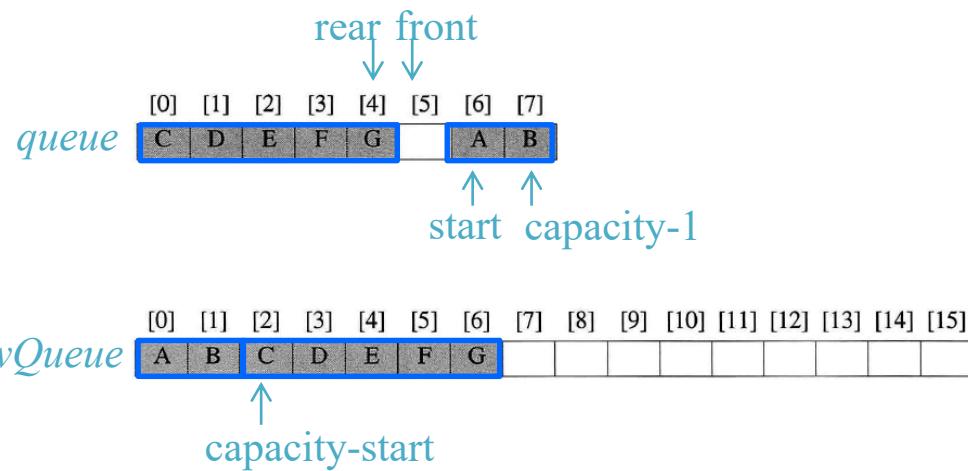
- $copy(a, b, c)$
 - copies elements from locations a through $b-1$ to locations beginning at c .
- ①

```
int start = (front+1) % capacity;
if (start < 2)
    /* no wrap around */
copy(queue+start, queue+start+capacity-1, newQueue);
```

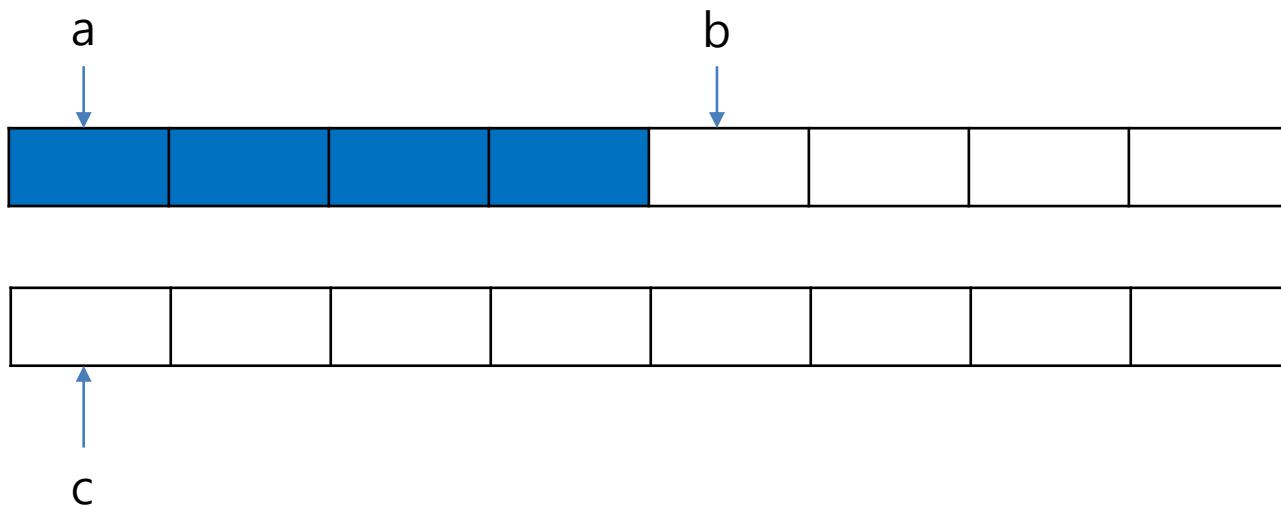


- ② else


```
/* queue wraps around */
copy(queue+start, queue+capacity, newQueue);
copy(queue, queue+rear+1, newQueue+capacity-start);
}
```



```
void copy( element *a, element *b, element *c)
{
while( a != b )
*c++ = *a++;
}
```



```
void addq(element item)
{/* add an item to the queue */
    rear = (rear + 1) % capacity;
    if (front == rear) {
        queueFull();                  /* double capacity */
        queue[+ + rear] = item;
    }
    else queue[rear] = item;
}
```

Add to a circular queue

3.5 A Mazing Problem

- Rat in a maze
 - Experimental psychologists train rats to search mazes for food



- For us, a nice application of *stacks*
 - Searching the maze for an entrance to exit path.

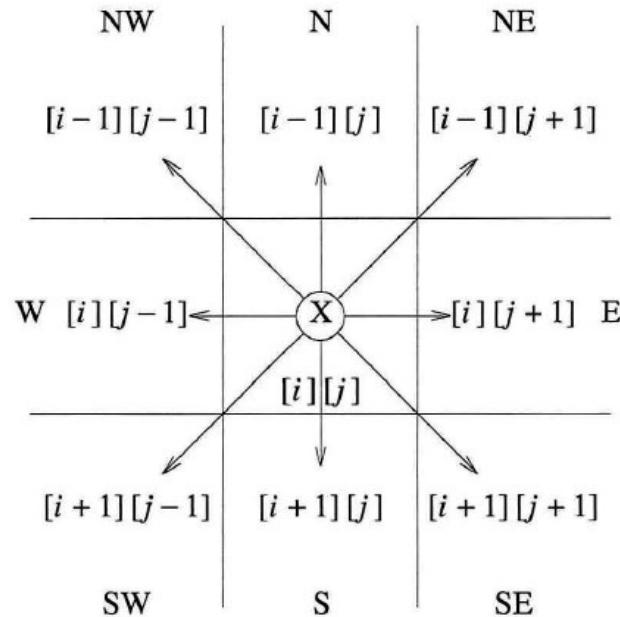
Implementation in C

- Representation of a maze
 - *A two-dimensional array, **maze***
 - 0 : the open paths, 1 : the barriers

maze[12,15]														
entrance														
0	1	0	0	0	1	1	0	0	0	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	0	1	1	1	1	0	1	1	0	1	1	0	0
1	1	0	1	0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	0	1
1	1	0	0	0	1	1	0	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0
0	1	0	0	1	1	1	1	0	1	1	1	1	1	0

exit

- The current location of the rat in the maze
 - *maze[row][col]*
- The possible *8 moves* from the current position



- Not every position has eight neighbors.
 - If $[row, col]$ is on a border, then less than eight.



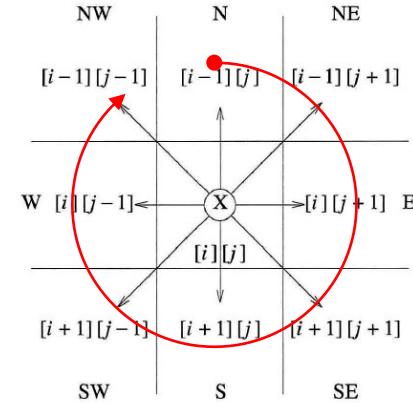
- To avoid checking for boarder conditions
 - We can surround the maze by a boarder of ones.

1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	0	1	0	1	1	1	1
1	1	1	1	1	0	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	0	1	0	1	1
1	0	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1

< $m \times p$ maze >
 $(m+2) \times (p+2)$ array, **maze**
 entrance : **maze[1][1]**
 exit : **maze[m][p]**

- Predefining the possible directions to move in an array ***move***

Name	Dir	<i>move[dir].vert</i>	<i>move[dir].horiz</i>
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1



```
typedef struct {
    short int vert;
    short int horiz;
} offsets;
offsets move[8]; /* array of moves for each direction */
```

- The position of the next move, ***maze[nextRow][nextCol]***

```
nextRow = row + move[dir].vert;
nextCol = col + move[dir].horiz;
```

- Since we do not know which choice is best,
 - we save our current position and
 - arbitrarily pick a possible move.
- By saving our current position,
 - we can *return to it* and *try another path* if we take a hopeless path.
- We examine the possible moves
 - starting from the north and moving clockwise.

- Maintaining a second 2D array, *mark*
 - to record the maze positions already checked
 - initialize the *mark*'s entries to *zero*
 - When we visit a position *maze[row][col]*,
we change *mark[row][col]* to *one*

1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	0	1	0	1	1	1	1
1	1	1	1	1	0	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	0	1	0	1	1
1	0	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1

maze

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

mark

현 위치 (r, c)에서 탐색방향<8이고 경로가 발견되지 않은 한 다음을 반복
현 위치 (r, c)에서 계산한 다음 위치 (nR, nC)에 대해

① if 출구인 경우

경로발견!

② else if 이동가능하고 이전에 방문하지 않은 경우

push(백트래킹 후 탐색할 위치와 방향) // push($r, c, d++$)

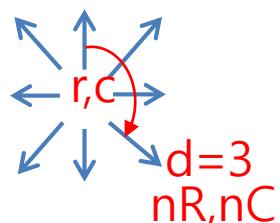
다음위치 방문했음을 표시

다음위치로 이동

③ else

탐색방향증가 // $d++$

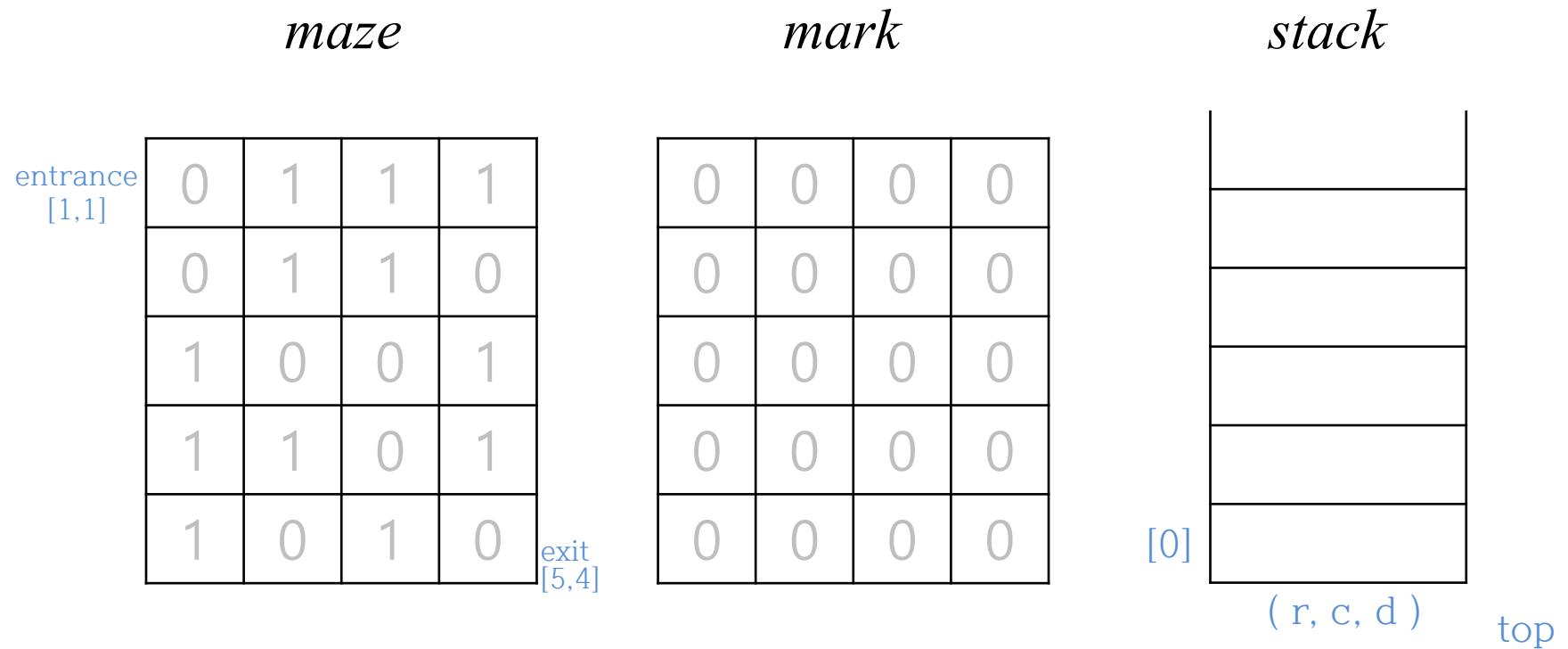
현 위치에서 탐색방향==8이면 스택에서 돌아갈 위치를 가져와서
위의 과정을 반복, 스택이 empty이면 경로 발견 실패



Q1. 언제 push를 수행하는가?

Q2. 언제 pop을 수행하는가?

Maze Search : Example



Program initialization

Maze Search : Example

maze

	nR, nC	
entrance	[1,1]	r, c
0	1	1
1	1	1
1	0	0
1	1	0
1	0	1
exit	[5,4]	

mark

1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

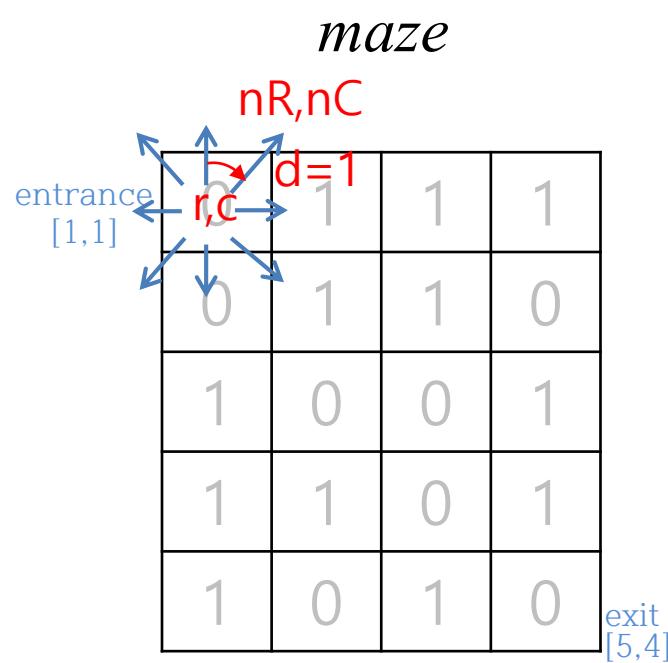
[0]
1, 1, 1
top

Initialization of function path()

(1, 1) 방문

push(1, 1, 1)

Maze Search : Example



mark

1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]

top

pop()
 $(r, c, d) = (1, 1, 1)$

Maze Search : Example

maze

entrance	[1,1]
	r, c
	$d=2$
	nR, nC
	exit [5,4]
0	1
1	1
1	0
1	0

mark

1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]

top

Maze Search : Example

maze

entrance	[1,1]
rc	0
d=3	nR,nC
1	1
1	1
1	0
1	0
1	1
1	0
exit	[5,4]

mark

1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]
top

Maze Search : Example

maze

entrance [1,1]	1	1	1
<i>r,c</i>	1	1	0
<i>d=4</i> <i>nR,nC</i>	1	0	1
1	1	0	1
1	0	1	0

exit [5,4]

mark

1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]

top

Maze Search : Example

maze

entrance [1,1]	0	1	1	1
	d=0 nR, nC r, c	1	1	0
	1	0	0	1
	1	1	0	1
	1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]	1, 1, 5	top

push(1, 1, 5)
 $(1, 2)$ 방문, $(r, c, d) = (1, 2, 0)$

Maze Search : Example

maze

entrance	[1,1]	0	nR,nC	1	1
		2	r,c	d=1	
		1	0	0	1
		1	1	0	1
		1	0	1	0

exit [5,4]

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]	1, 1, 5	top	

Maze Search : Example

maze

0	1	1	1
1	1	1	0
0	0	0	1
1	1	0	1
1	0	1	0

entrance [1,1] exit [5,4]

Diagram showing a 5x4 maze grid. The entrance is at [1,1] and the exit is at [5,4]. The current position is at [1,2] (labeled r,c). A red arrow labeled d=2 points to the next position [2,2] (labeled nR,nC).

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]	1, 1, 5	top	

Maze Search : Example

maze

0	1	1	1
1	1	1	0
0	0	0	1
1	1	0	1
1	0	1	0

entrance [1,1] exit [5,4]

r, c

$d=3$

nR, nC

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]	1, 1, 5		top

Maze Search : Example

maze

0	1	1	1
0	1	1	0
1	1	0	1
1	1	0	1
1	0	1	0

entrance [1,1] exit [5,4]

r, c *d=0* *nR, nC*

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4			top
[0]			
1, 1, 5			

push(2, 1, 4)
 $(3, 2)$ 방문, $(r, c, d) = (3, 2, 0)$

Maze Search : Example

maze

entrance	[1,1]		
0	1	1	1
0	1	nR,nC 1	0
1	1	d=1 r,c	0
1	1	0	1
1	0	1	0

exit [5,4]

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4	top
[0]	
1, 1, 5	

Maze Search : Example

maze

entrance	[1,1]
0	1
0	1
1	1
1	0

exit [5,4]

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4	top
[0]	
1, 1, 5	

Maze Search : Example

maze

0	1	1	1
0	1	1	0
1	0	r, c	1
1	1	0	1
1	0	1	0

entrance [1,1] exit [5,4]

d=0
nR, nC

mark

1	0	0	0
1	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3			top
2, 1, 4			
1, 1, 5			
[0]			

push(3, 2, 3)
 $(3, 3)$ 방문, $(r, c, d) = (3, 3, 0)$

Maze Search : Example

maze

entrance	0	1	1	1
[1,1]	0	1	1	1
	0	1	1	nR,nC
	1	0	r,c	d=1
	1	1	0	1
	1	0	1	0
exit	5,4			

mark

1	0	0	0
1	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

stack

	3, 2, 3	
	2, 1, 4	
[0]	1, 1, 5	

Maze Search : Example

maze

0	1	1	1	d=0 nR, nC
0	1	1	0	r, c
1	0	0	1	
1	1	0	1	
1	0	1	0	exit [5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

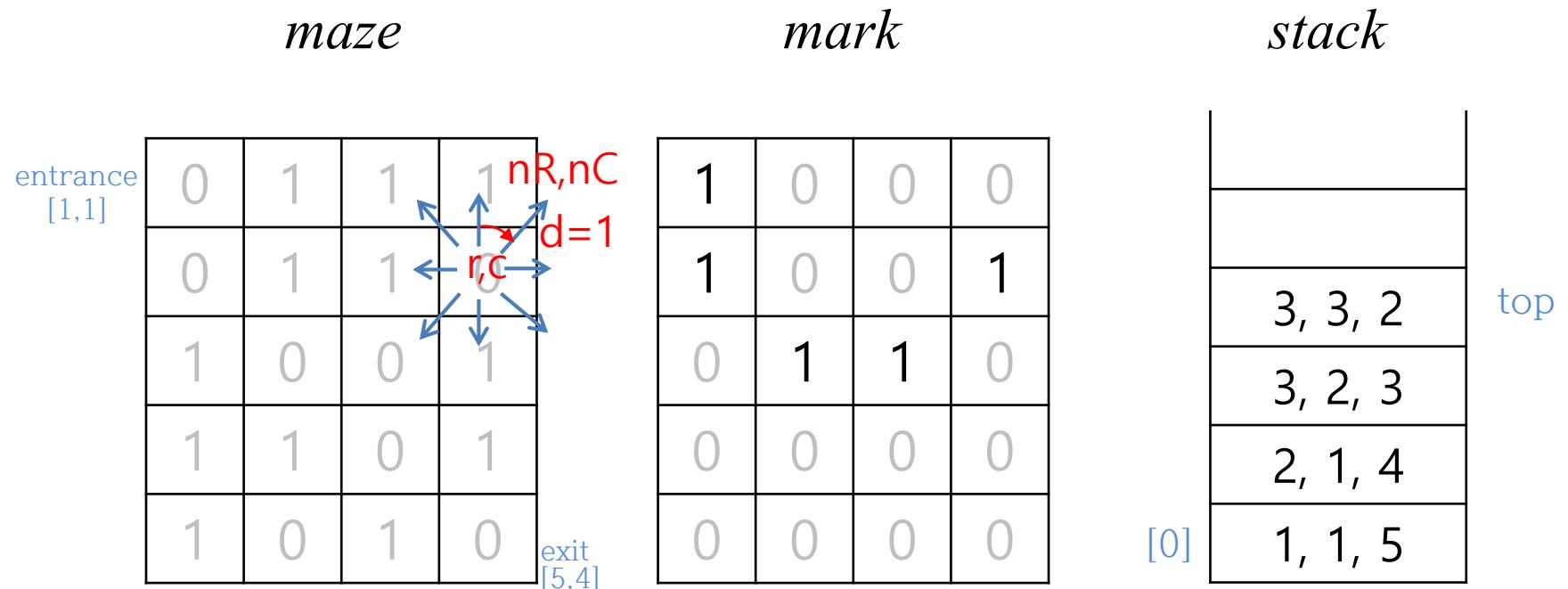
stack

top

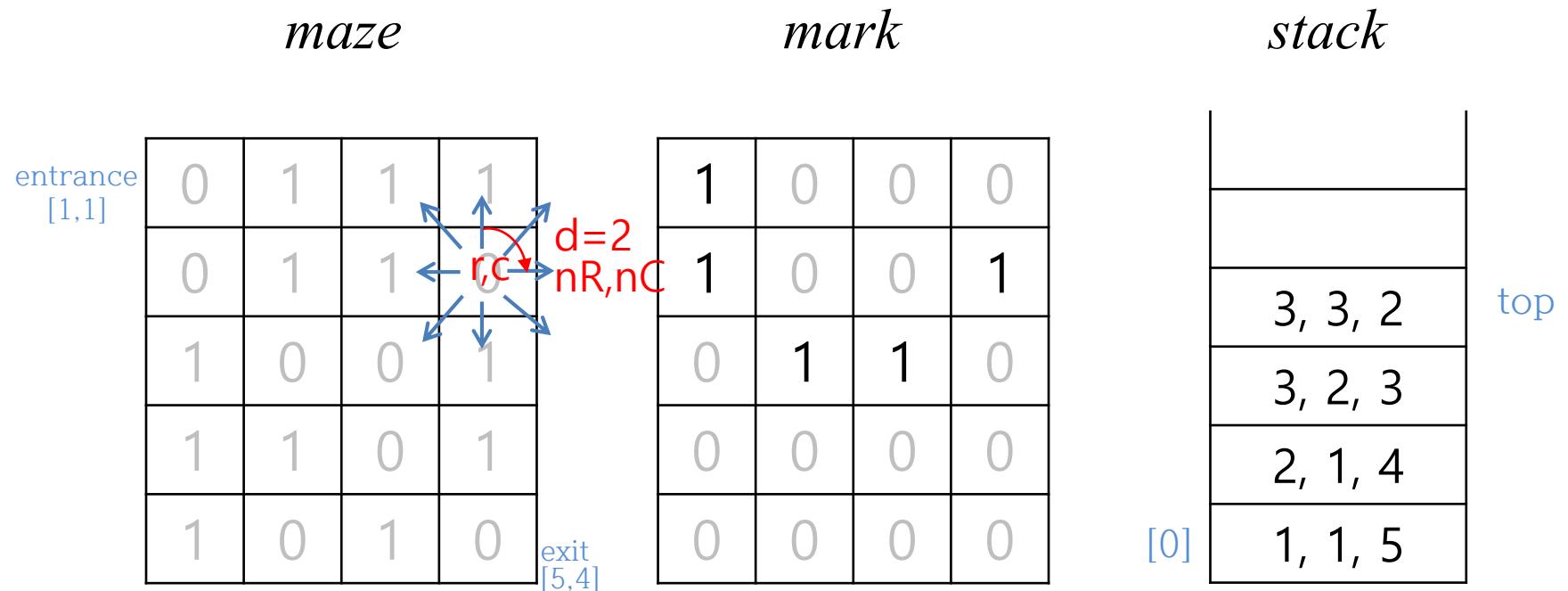
[0]
3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

push(3, 3, 2)
 $(2, 4)$ 방문, $(r, c, d) = (2, 4, 0)$

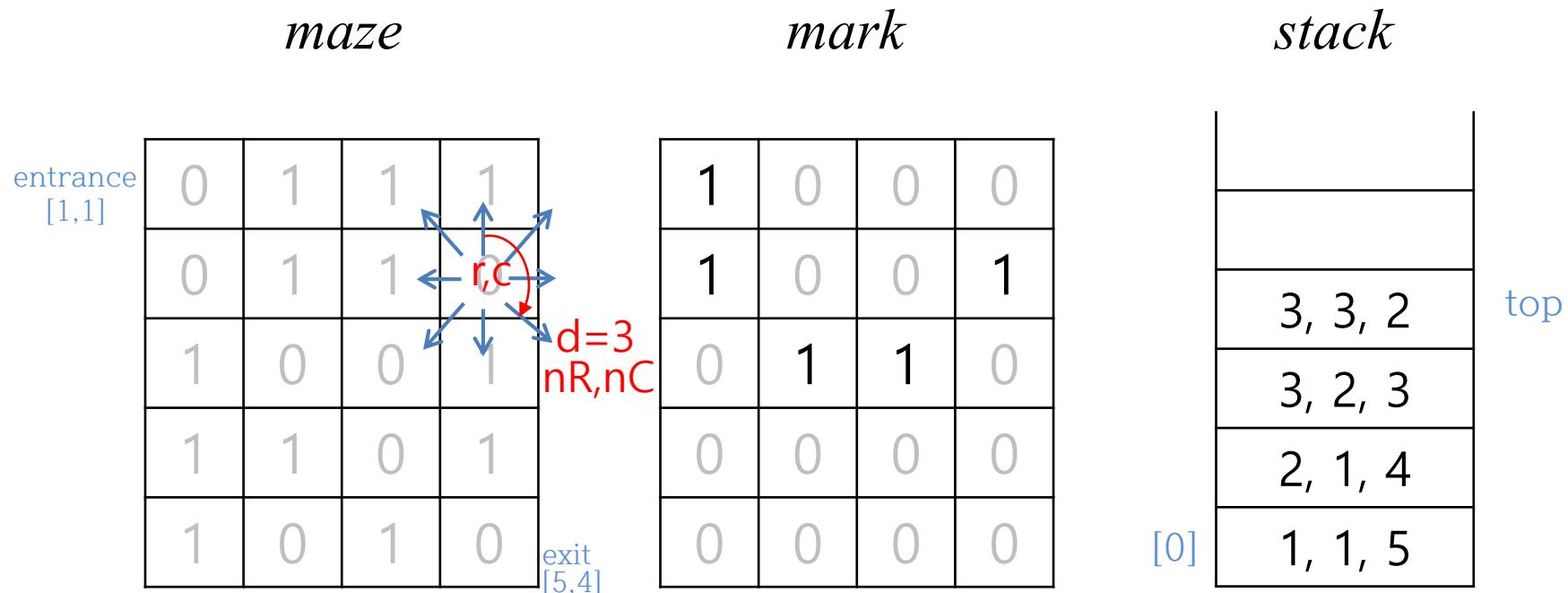
Maze Search : Example



Maze Search : Example



Maze Search : Example



Maze Search : Example

maze

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

entrance [1,1] exit [5,4]

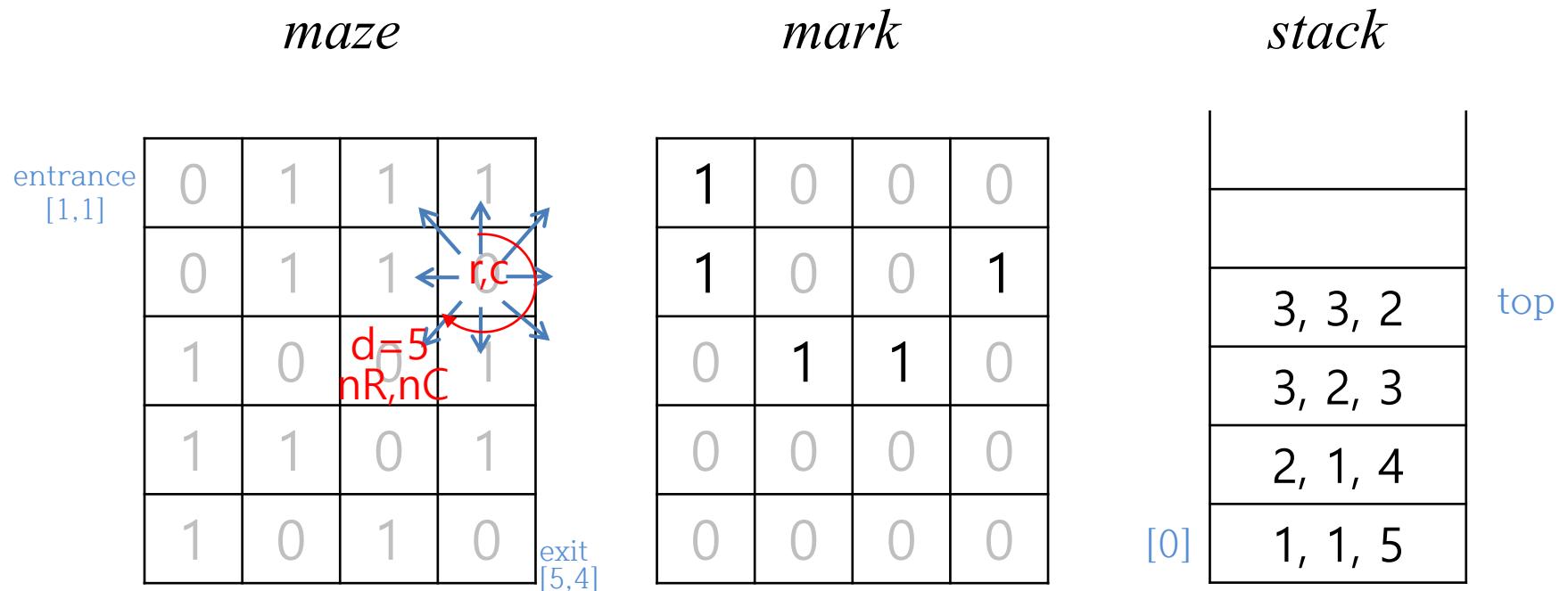
mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2		top
3, 2, 3		
2, 1, 4		
1, 1, 5		
[0]		

Maze Search : Example



Maze Search : Example

maze

0	1	1	1	
0	1	1	1	
1	0	0	1	
1	1	0	1	
1	0	1	0	

entrance [1,1] exit [5,4]

$d=6$
nR, nC
r, c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2			top
3, 2, 3			
2, 1, 4			
1, 1, 5			
[0]			

Maze Search : Example

maze

0	1	1	d=7 nR, nC	1
0	1	1		
1	0	0		1
1	1	0		1
1	0	1		0

entrance [1,1] exit [5,4]

r,c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2		top
3, 2, 3		
2, 1, 4		
1, 1, 5		
[0]		

Maze Search : Example

maze

0	1	1	1	d=8
0	1	1	0	r,c
1	0	0	1	
1	1	0	1	
1	0	1	0	exit [5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2		top
3, 2, 3		
2, 1, 4		
1, 1, 5		
[0]		

(d < 8 && !found) ? No!

(top > -1 && !found) ? Yes!

Maze Search : Example

maze

entrance [1,1]	0	1	1	1
	0	1	1	0
	1	0	1	1
	1	1	0	1
	1	0	1	0

exit [5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3			top
2, 1, 4			
1, 1, 5			
[0]			

pop()

(r, c, d) = (3, 3, 2)

Maze Search : Example

maze

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
[0]
1, 1, 5

top

[0]

Maze Search : Example

maze

0	1	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	1	
1	0	1	0	

entrance [1,1]

exit [5,4]

r, c

$d = 4$

nR, nC

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3			
2, 1, 4			
1, 1, 5			
[0]			

top

Maze Search : Example

maze

entrance [1,1]	0	1	1	1
	0	1	1	0
	1	0	0	1
	1	1	0	1
	1	0	1	0

d=0
nR, nC
r, c

exit
[5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5			top
3, 2, 3			
2, 1, 4			
1, 1, 5			
[0]			

push(3, 3, 5)
(4, 3) 방문, (r, c, d) = (4, 3, 0)

Maze Search : Example

maze

0	1	1	1
0	1	1	0
1	0	0	1
1	1	r,c	d=1
1	0	1	0

entrance [1,1]

exit [5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5		top
3, 2, 3		
2, 1, 4		
1, 1, 5		
[0]		

Maze Search : Example

maze

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

entrance
[1,1]

exit
[5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5		top
3, 2, 3		
2, 1, 4		
1, 1, 5		
[0]		

Maze Search : Example

maze

entrance [1,1]	0	1	1	1
	0	1	1	0
	1	0	0	1
	1	1	r, c	1
	1	0	d=3	nR, nC exit [5,4]

mark

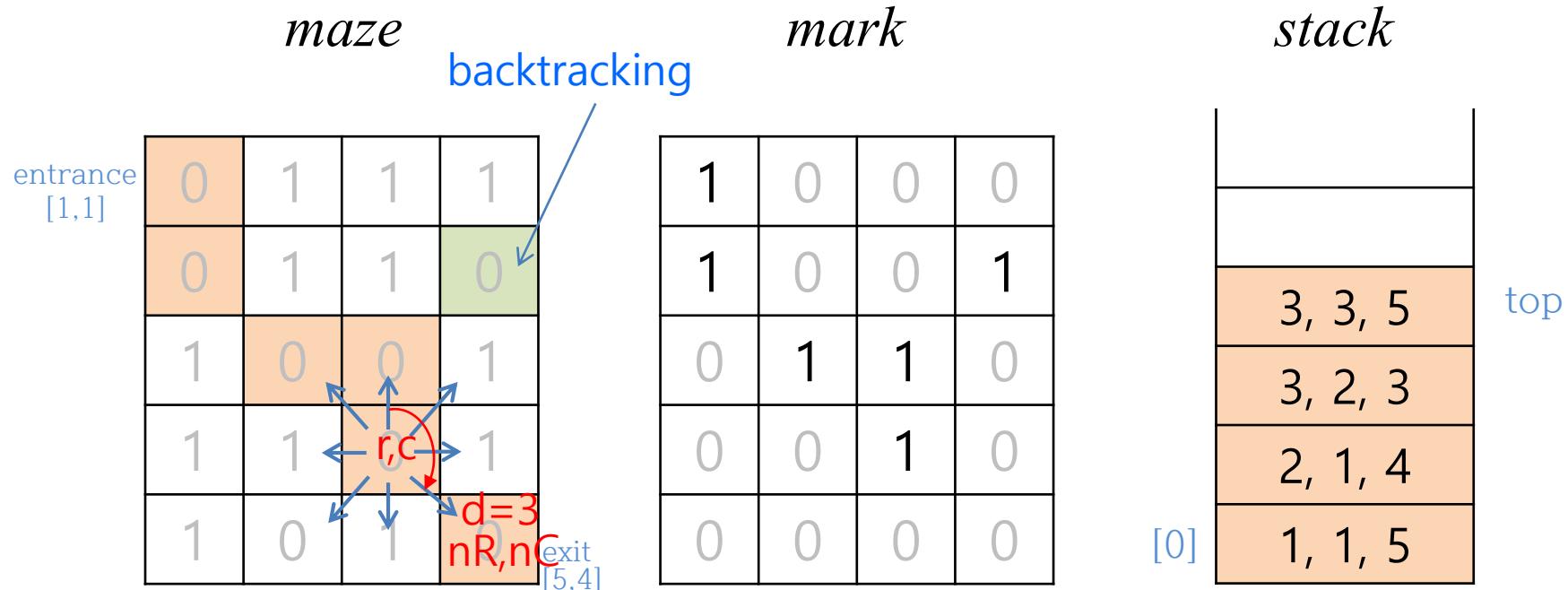
1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5			top
3, 2, 3			
2, 1, 4			
1, 1, 5			
[0]			

다음 위치 (nR, nC)가 출구(EXIT_ROW, EXIT_COL)임
 경로발견!

Maze Search : Example



< 경로 출력 순서 >

- ① stack[0] → stack[top]
- ② 현재 위치 (r, c)
- ③ 출구 위치 (EXIT_ROW, EXIT_COL)

path: (1, 1), (2, 1), (3, 2), (3, 3), (4, 3), (5, 4)

- Stack
 - Use the implementation of section 3.1 or 3.2

```
typedef struct {
    short int row;
    short int col;
    short int dir;
} element;
```

- Capacity
 - Each position in the maze is visited no more than once.
 - An $m \times p$ maze has at most mp zeroes.
 - ***mp*** is sufficient for the stack capacity.

```
initialize a stack to the maze's entrance coordinates and
direction to north;
while (stack is not empty) {
    /* move to position at top of stack */
    <row,col,dir> = delete from top of stack;
    while (there are more moves from current position) {
        <nextRow, nextCol> = coordinates of next move;
        dir = direction of move;
        if ((nextRow == EXIT-ROW) && (nextCol == EXIT-COL))
            success;
        if (maze[nextRow] [nextCol] == 0 &&
            mark[nextRow] [nextCol] == 0) {
            /* legal move and haven't been there */
            mark[nextRow] [nextCol] = 1;
            /* save current position and direction */
            add <row,col,dir> to the top of the stack;
            row = nextRow;
            col = nextCol;
            dir = north;
        }
    }
    printf("No path found\n");
}
```

Program 3.11: Initial maze algorithm

```

void path(void)
{ /* output a path through the maze if such a path exists */
    int i, row, col, nextRow, nextCol, dir, found = FALSE;
    element position;
    mark[1][1] = 1; top = 0;
    stack[0].row = 1; stack[0].col = 1; stack[0].dir = 1;
    while (top > -1 && !found) {
        position = pop();
        row = position.row; col = position.col;
        dir = position.dir;
        while (dir < 8 && !found) {
            /* move in direction dir */
            nextRow = row + move[dir].vert;
            nextCol = col + move[dir].horiz;
            if (nextRow == EXIT_ROW && nextCol == EXIT_COL)
                found = TRUE;
            else if ( !maze[nextRow][nextCol] &&
                      !mark[nextRow][nextCol]) {
                mark[nextRow][nextCol] = 1;
                position.row = row; position.col = col;
                position.dir = ++dir;
                push(position);
                row = nextRow; col = nextCol; dir = 0;
            }
            else ++dir;
        }
    }
    if (found) {
        printf("The path is:\n");
        printf("row col\n");
        for (i = 0; i <= top; i++)
            printf("%2d%5d", stack[i].row, stack[i].col);
        printf("%2d%5d\n", row, col);
        printf("%2d%5d\n", EXIT_ROW, EXIT_COL);
    }
    else printf("The maze does not have a path\n");
}

```

Analysis of *path*:

- each position within the maze is visited no more than once,
- worst case complexity : **O(*mp*)**, for $m \times p$ maze