

Chap 3. Stacks and Queues (1)

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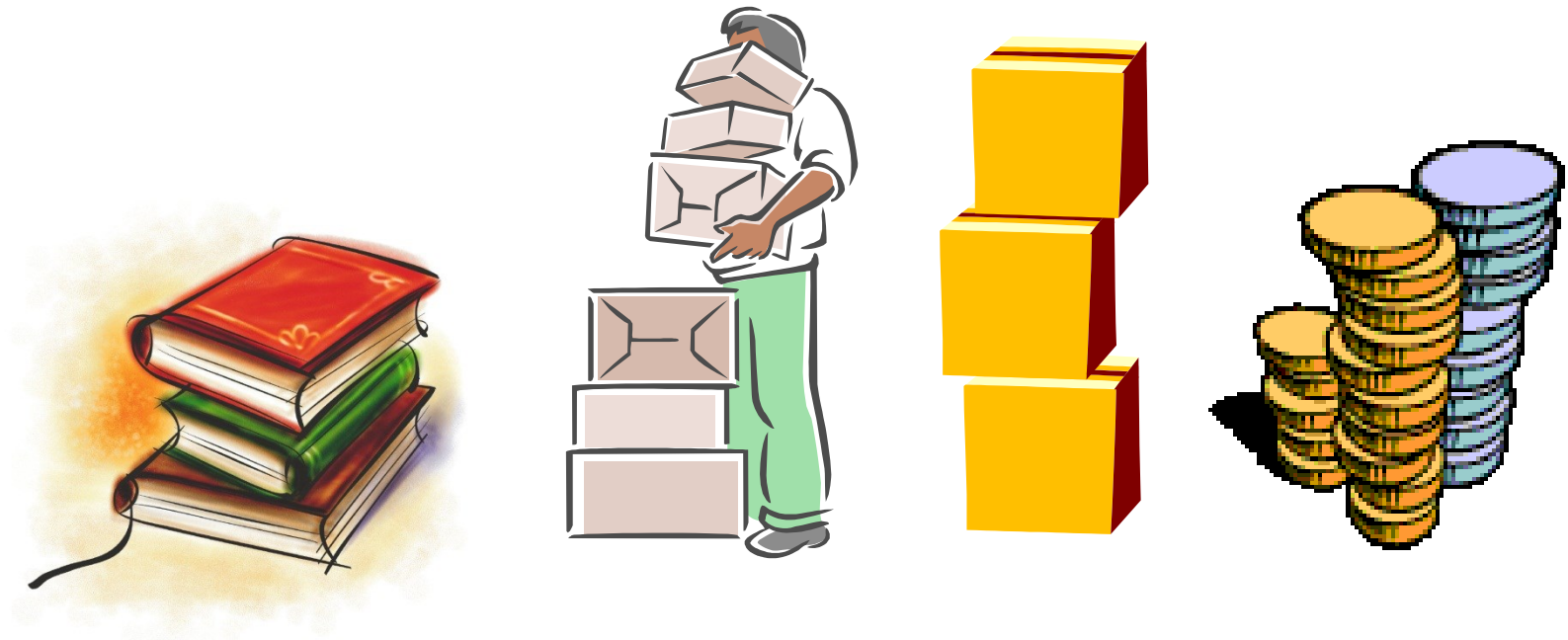
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3.1 Stacks



3.1 Stacks

- *Linear list.*
- One end is called *top*.
- The other end is called *bottom*.
- Additions to and removals from the *top* end only.

- A stack is a **LIFO** list.
 - *Last-In-First-Out*

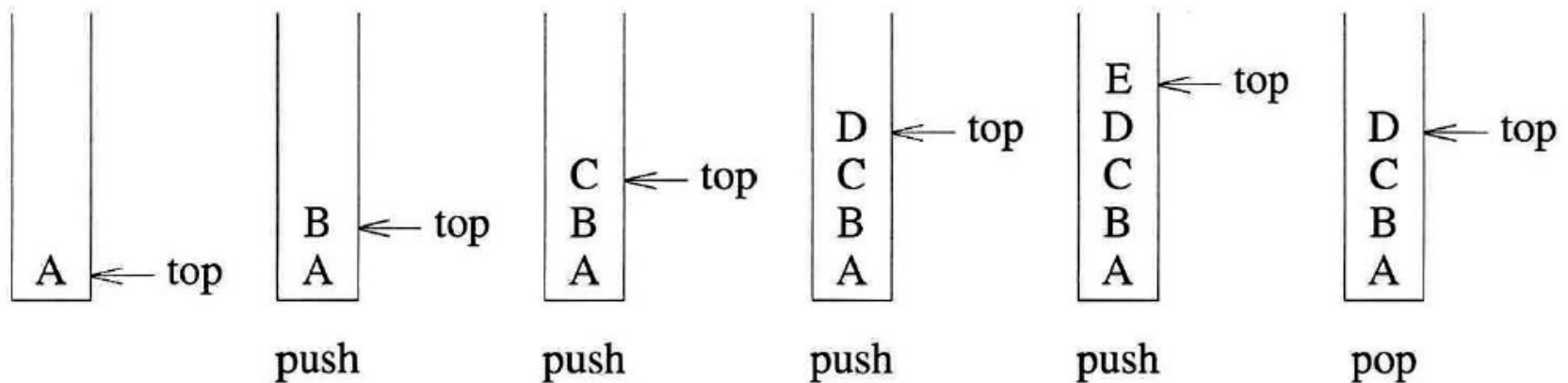


Figure 3.1: Inserting and deleting elements in a stack

ADT Stack is

objects: a finite ordered list with zero or more elements.

functions:

for all $stack \in Stack$, $item \in element$, $maxStackSize \in$ positive integer

$Stack\ CreateS(maxStackSize) ::=$

create an empty stack whose maximum size is $maxStackSize$

$Boolean\ IsFull(stack, maxStackSize) ::=$

if (number of elements in $stack == maxStackSize$)

return *TRUE*

else return *FALSE*

$Stack\ Push(stack, item) ::=$

if ($IsFull(stack)$) *stackFull*

else insert $item$ into top of $stack$ and return

$Boolean\ IsEmpty(stack) ::=$

if ($stack == CreateS(maxStackSize)$)

return *TRUE*

else return *FALSE*

$Element\ Pop(stack) ::=$

if ($IsEmpty(stack)$) return


else remove and return the element at the top of the stack.

ADT 3.1: Abstract data type *Stack*

- Creation of Stack in C
 - Use a *1D array* to represent a stack.
 - Stack elements are stored in *stack[0] through stack[top]*.

Stack CreateS(*maxStackSize*) ::=

```
#define MAX-STACK-SIZE 100 /* maximum stack size */
typedef struct {
    int key;
    /* other fields */
} element;
element stack[MAX-STACK-SIZE];
int top = -1;
```

Boolean IsEmpty(Stack) ::= 

Boolean IsFull(Stack) ::= 

• Implementation of Stack Operations

```
void push(element item)
{
    /* add an item to the global stack */
    if (top >= MAX_STACK_SIZE-1)
        stackFull();
    stack[ ] = item;
}
```

Program 3.1

```
element pop()
{
    /* delete and return the top element from the stack */
    if (top == -1)
        return stackEmpty(); /* returns an error key */
    return stack[ ];
}
```

Program 3.2

```
void stackFull()
{
    fprintf(stderr, "Stack is full, cannot add element");
    exit(EXIT_FAILURE);
}
```


Program 3.3

3.2 Stacks Using Dynamic Arrays

```
Stack CreateS() ::= typedef struct {  
    int key;  
    /* other fields */  
} element;  
element *stack;  
MALLOC(stack, sizeof(*stack));  
int capacity = 1;  
int top = -1;
```

※ **capacity** : maximum number of stack elements that may be stored in the array

stack → 

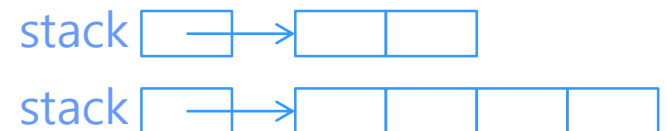
Boolean IsEmpty(Stack) ::= 

Boolean IsFull(Stack) ::= 

- *pop* : unchanged from Program 3.2
- *push, stackFull* : changed from Program 3.1&3.3
- ***Array Doubling***
 - When stack is full, double the capacity using REALLOC.

```
void stackFull()
{
    REALLOC(oldStack, stack, 2*capacity*sizeof(int));
    capacity *= 2;
}
```

Program 3.4: Stack full with array doubling



- Time complexity of array doubling

- One array doubling

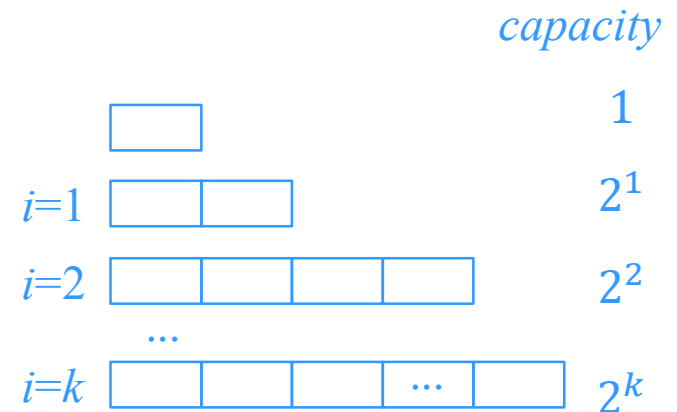
- Memory allocation : $O(1)$
- Copy of an array element : $O(1)$
- Copy of all array elements : $O(\text{capacity})$

- All array doubling

- N 번의 *push* 가 있었을 때 현재 *stack capacity* 가 2^k 이라면 (k 번 *doubling*)

$$O\left(\sum_{i=1}^k 2^i\right) = O(2^{k+1}) = O(2^k)$$

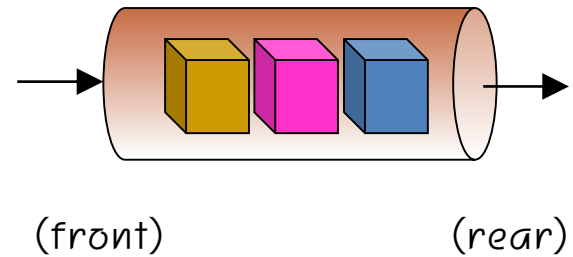
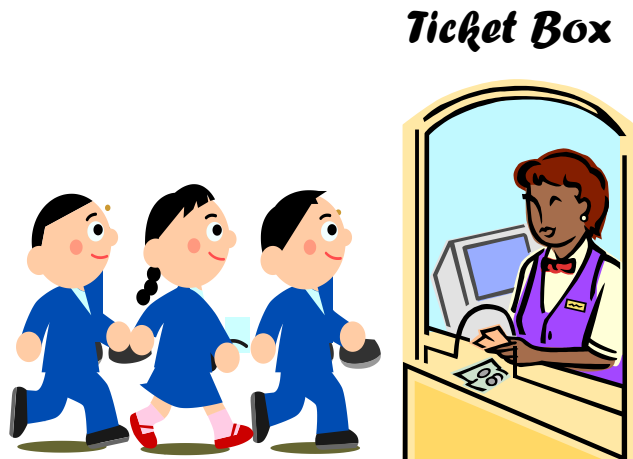
Doubling 시 copy 수



What is difference between stack and Array?



3.3 Queues



3.3 Queues

- *Linear list.*
- One end is called *front*.
- The other end is called *rear*.
- *Additions* are done at the *rear* only.
- *Removals* are made from the *front* only.



- A queue is a **FIFO** list.
 - *First-In-First-Out*

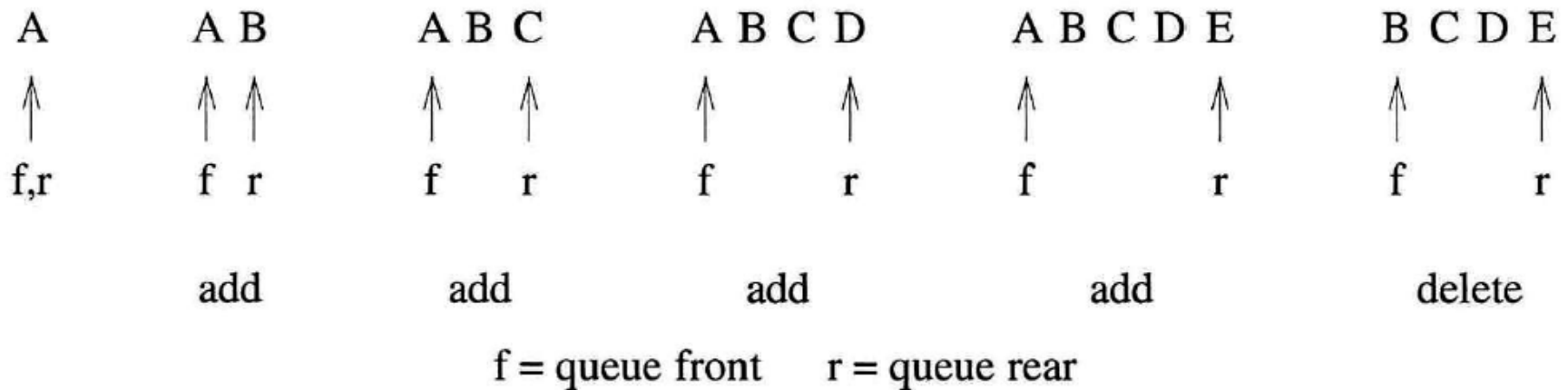


Figure 3.4: Inserting and deleting elements in a queue

ADT *Queue* is

objects: a finite ordered list with zero or more elements.

functions:

for all $queue \in Queue$, $item \in element$, $maxQueueSize \in$ positive integer

Queue CreateQ($maxQueueSize$) ::=

create an empty queue whose maximum size is $maxQueueSize$

Boolean IsFullQ($queue$, $maxQueueSize$) ::=

if (number of elements in $queue == maxQueueSize$)

return *TRUE*

else return *FALSE*

Queue AddQ($queue$, $item$) ::=

if (IsFullQ($queue$)) *queueFull*

else insert $item$ at rear of $queue$ and return $queue$

Boolean IsEmptyQ($queue$) ::=

if ($queue ==$ CreateQ($maxQueueSize$))

return *TRUE*

else return *FALSE*

Element DeleteQ($queue$) ::=

if (IsEmptyQ($queue$)) return

else remove and return the $item$ at front of $queue$.

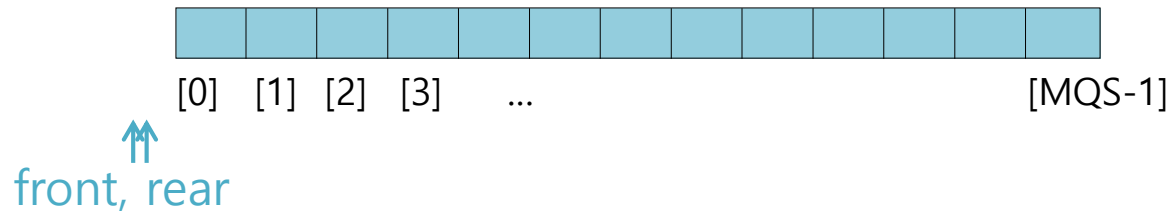
ADT 3.2: Abstract data type *Queue*

Representations of Queue

- Sequential representation
 - Uses an *1D array*
- Circular representation : *circular queue*
 - Uses an *1D array*
 - More efficient

Sequential Representation

- Creation of Queue in C
 - Uses an 1D array, *queue*



Queue CreateQ(maxQueueSize) ::=

```
#define MAX_QUEUE_SIZE 100 /* maximum queue size */
typedef struct {
    int key;
    /* other fields */
} element;
element queue[MAX_QUEUE_SIZE];
int rear = -1;
int front = -1;
```

Boolean IsEmptyQ(queue) ::=

Boolean IsFullQ(queue) ::=

Sequential Representation

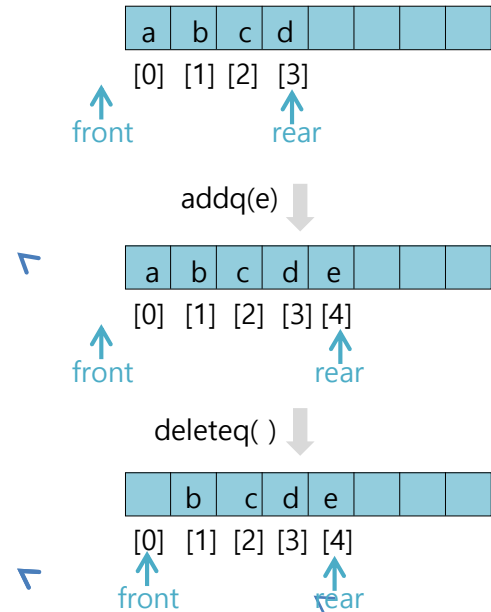
- Implementation of Queue Operations

```
void addq(element item)
{/* add an item to the queue */
    if (rear == MAX_QUEUE_SIZE-1)
        queueFull();
    queue[++rear] = item;
}
```

Program 3.5: Add to a queue

```
element deleteq()
{/* remove element at the front of the queue */
    if (front == rear)
        return queueEmpty(); /* return an error key */
    return queue[front];
}
```

Program 3.6: Delete from a queue



Sequential Representation

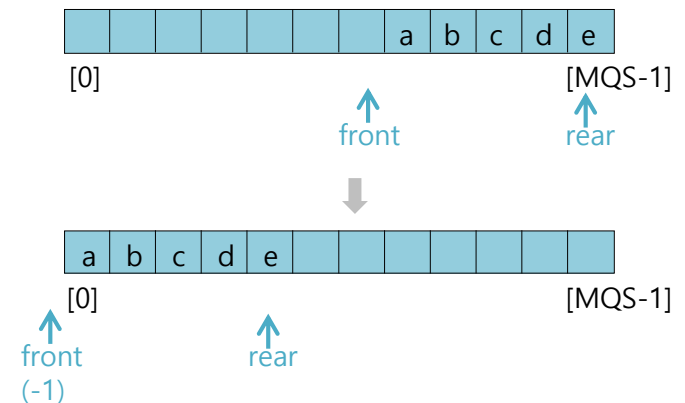
- Example: Job Scheduling by an OS

<i>front</i>	<i>rear</i>	<i>Q</i> [0]	<i>Q</i> [1]	<i>Q</i> [2]	<i>Q</i> [3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

Figure 3.5: Insertion and deletion from a sequential queue

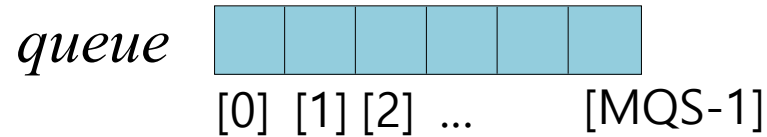
- queueFull

- array shifting : time-consuming
- Worst case time complexity,
 $O(MAX_QUEUE_SIZE)$

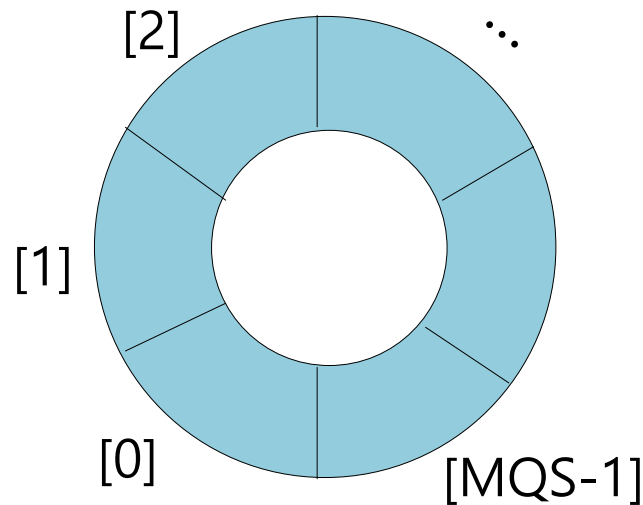


Circular Queue

- Uses an 1D array, *queue*



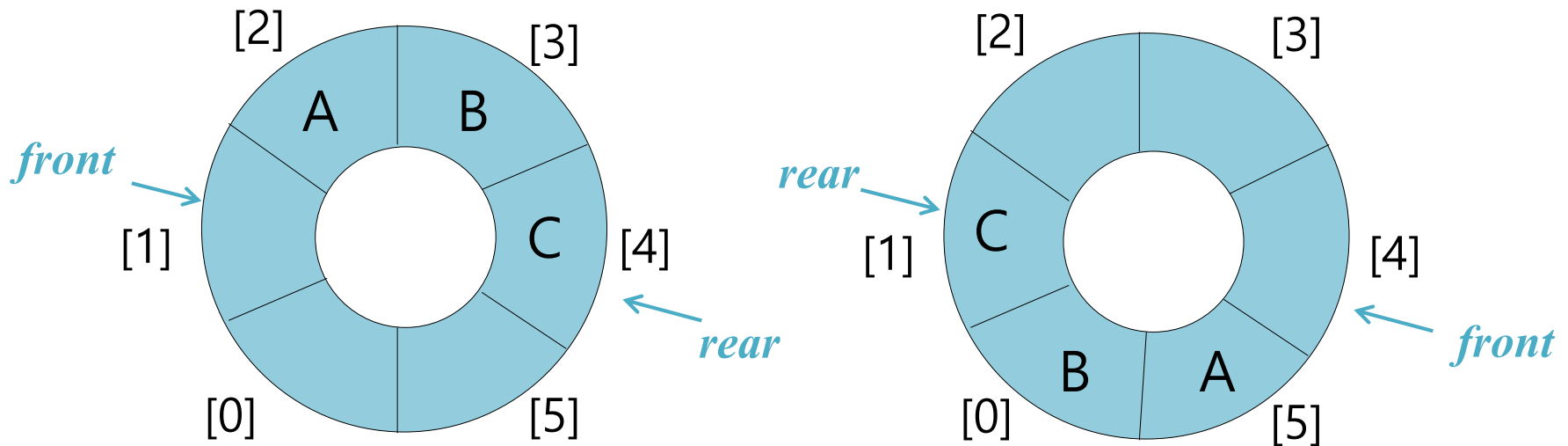
- Circular view of an 1D array



Initial values
front = rear = 0

Circular Queue

- integer variables *front* and *rear*.
 - front* is one position counterclockwise from first element
 - rear* gives position of last element



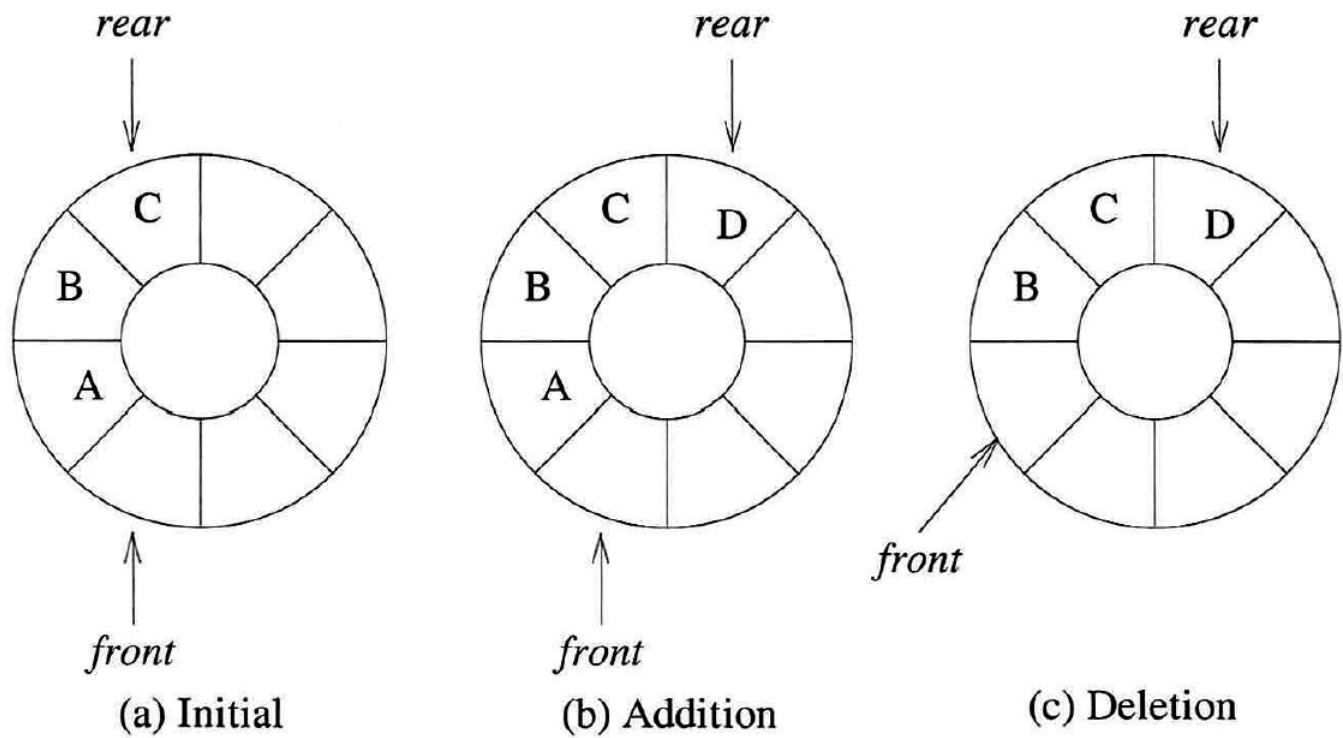
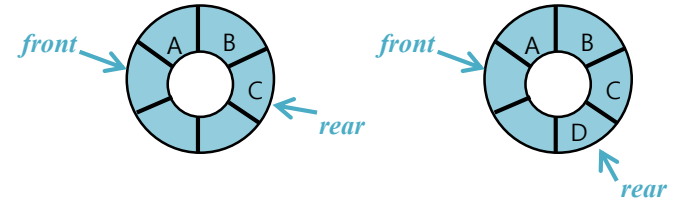


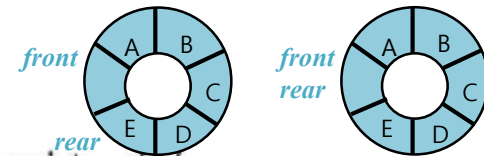
Figure 3.6: Circular queue

Circular Queue

- Add an element in the circular queue.
 - Move *rear* one clockwise.
 - Queue Full Check
 - Then put into *queue[rear]*.



```
void addq(element item)
{ /* add an item to the queue */
    if ( )
        queueFull(); /* print error and exit */
    queue[rear] = item;
}
```

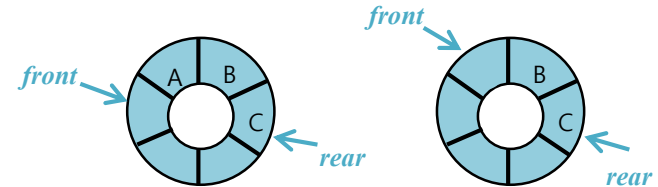


Program 3.7: Add to a circular queue

a maximum of `MAX_QUEUE_SIZE-1`
elements in the queue at any time!

Circular Queue

- Delete an element from the circular queue.
 - Queue Empty check
 - Move *front* one clockwise.
 - Then extract from *queue[front]*.



```
element deleteq()
{/* remove front element from the queue */

    if (front == rear)
        return queueEmpty(); /* return an error key */
    front = (front+1) % MAX_QUEUE_SIZE;
    return queue[front];
}
```

The diagram shows a circular queue with three segments. Two blue arrows, one labeled 'front' and one labeled 'rear', both point to the same segment, indicating that the queue is empty.

Program 3.8: Delete from a circular queue