

Chap 8. Hashing (2)

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8.3 Dynamic Hashing

8.3.1 Motivation for Dynamic Hashing

- It is necessary to increase *the size of a hash table* whenever its *loading density* exceeds a pre-specified threshold
 - Ex) b buckets with divisor $D = b$
 - 버킷수 증가
 - 2 $b+1$ buckets with divisor $D = 2b+1$
- But, rebuilding the hash table takes time
 Reduce the rebuild time using *dynamic hashing*

- Two forms of dynamic hashing
 - *Using directory*
 - *Directoryless*
- For both forms,
 - h maps keys into non-negative integers
 - The range of h is sufficiently large
 - $h(k, p)$: the integer formed by the p LSBs of $h(k)$
Ex) $h(12346, 5) \Rightarrow 12346$
 $h(12346, 3) \Rightarrow 346$

- Example

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

- keys composed of two characters
- Hash function $h(k)$
 - transforms keys into 6-bit nonnegative integers
 - letters A~C → 100, 101, 110
 - digits 0~7 → 000, 001, … ,111
 - $h(A0,1) = 100\ 000, 1 = 1$
 $h(A1,3) = 100\ 001, 1 = 1$
 $h(B1,4) = 011\ 001 = 9$
 $h(C1,6) = 110\ 001 = 49$

8.3.2 Dynamic Hashing Using Directories

- Use a directory d of pointers to buckets
- Directory depth
 - the number of bits of $h(k)$ used to index the directory
 - $h(k, t) : t$ is a directory depth
- The size of directory depends on directory depth
 - $h(k,2) \rightarrow$ directory size = $2^2 = 4$
 - $h(k,5) \rightarrow$ directory size = $2^5 = 32$
- The number of buckets is at most equal to the directory size.

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

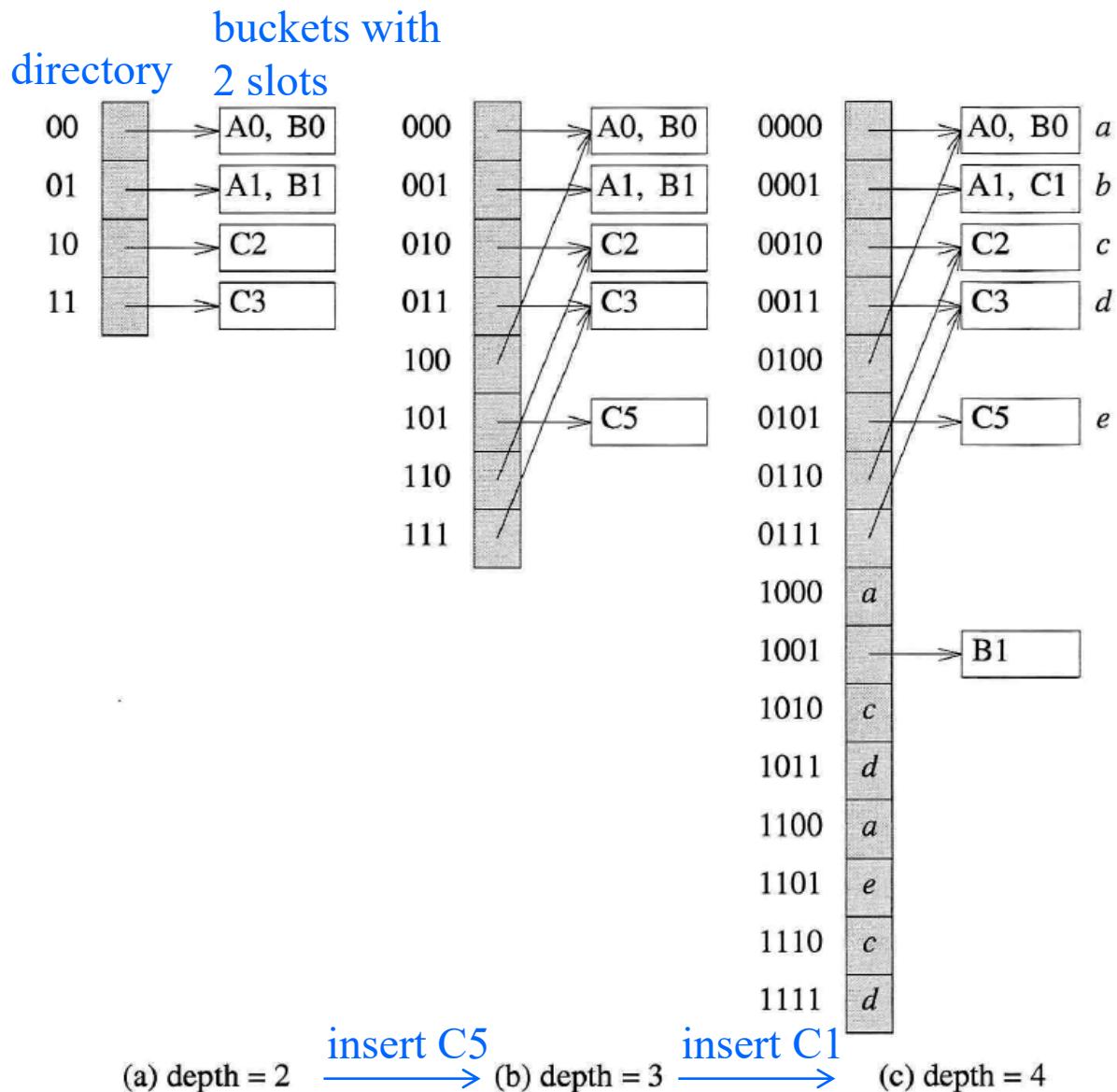
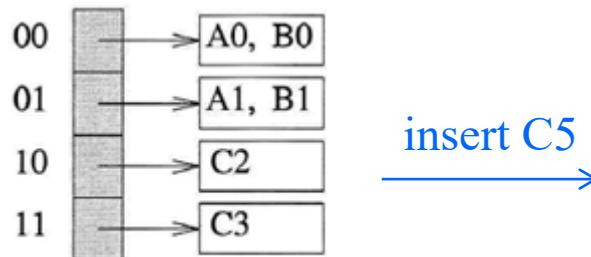


Figure 8.8: Dynamic hash tables with directories

< Case 1 >

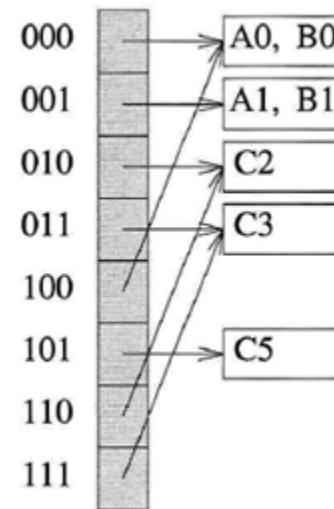
k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

(a) depth = 2



$h(C5, 2) = 01$
bucket overflow

(b)



the least u that $h(k, u)$ is not the same for all keys in the overflowed bucket ? **$u = 3$**

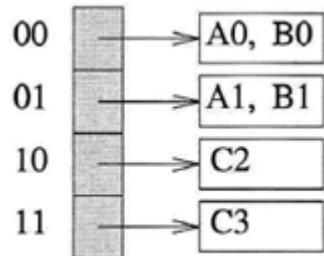
In the case least u is **greater than** the directory depth

- ① increase directory depth to $u = 3$
- ② **double** the directory size
- ③ the pointers in the original directory are duplicated
 $d[i+4]=d[i], 0 \leq i < 4$
- ④ split the overflowed bucket using $h(k, u)$
 001: A1, B1, 101:C5

< Case 2 >

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

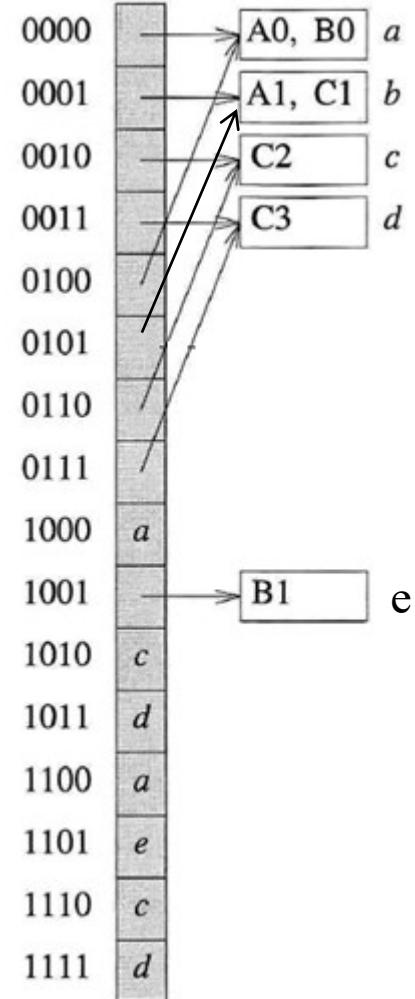
(a) depth = 2



insert C1
→

$h(C1, 2) = 01$
bucket overflow

the least u that $h(k, u)$ is not the same for all keys in the overflowed bucket ? **$u = 4$**



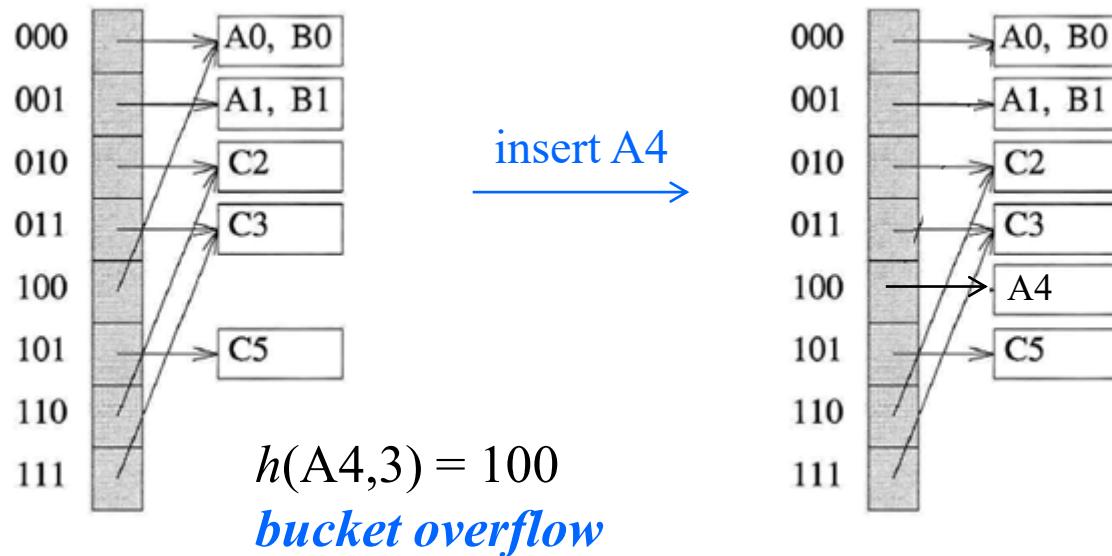
In the case least u is **greater than** the directory depth

- ① increase directory depth to $u = 4$
- ② **quadruple** the directory size
- ③ the pointers in the original directory are replicated 3 times,
- ④ split the overflowed bucket using $h(k, u)$
0001: A1, C1, 1001:B1

< Case 3 >

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

(b) depth = 3



the least u that $h(k,u)$ is not the same for all keys in the overflowed bucket ? **$u = 3$**

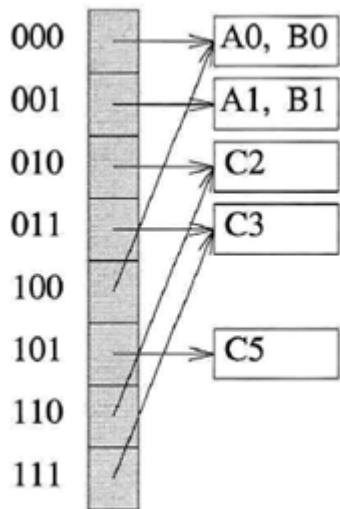
In the case least u is *less than or equal* to the directory depth

- ① The size of directory is not changed
- ② split the overflowed bucket using $h(k, u)$
000: A0, B0, 100:A4
- ③ update d[100] to point to the new bucket

< Case 4 >

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

(b) depth = 3

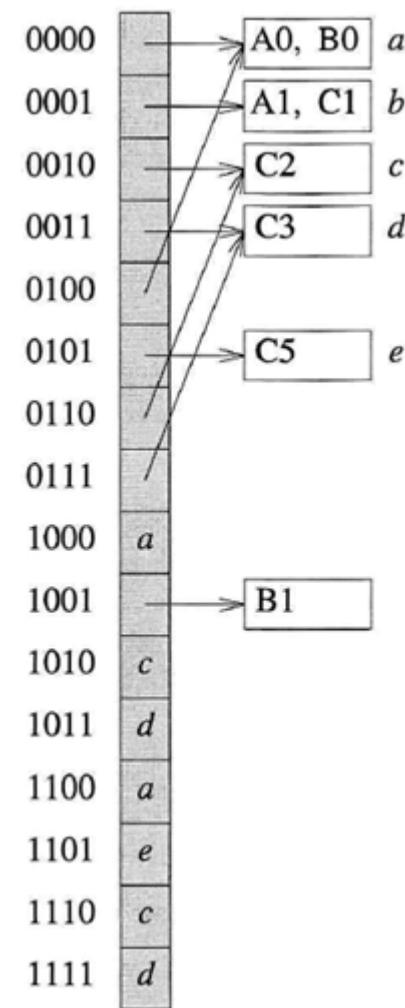


insert C1
→

$h(C1,3) = 001$
bucket overflow

...

(c)

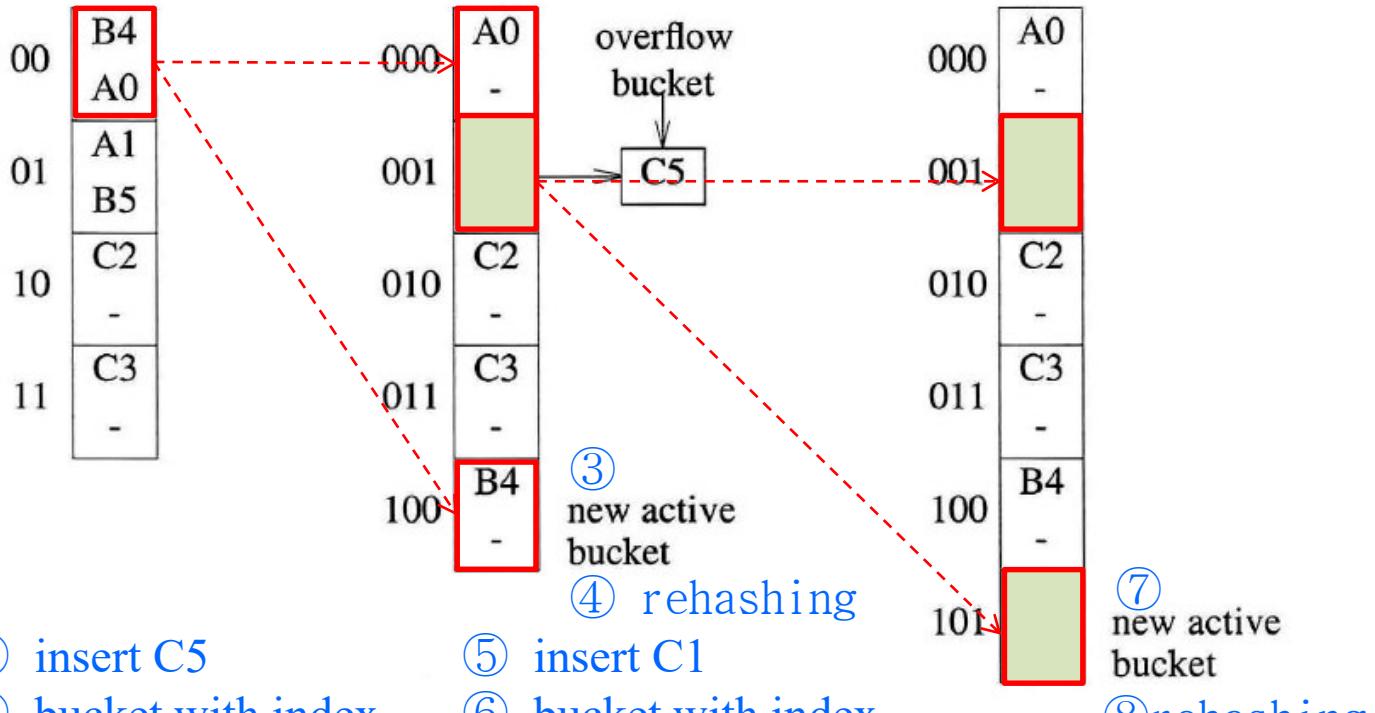


8.3.3 Directoryless Dynamic Hashing

- Known as *linear dynamic hashing*
- Assumption
 - this array is as large as possible and so there is no possibility of increasing its size dynamically.

- Two variables q and r , $0 \leq q < 2^r$
 - r is the number of bits of $h(k)$ used to index into the hash table
 - q is the bucket that will split next
- At any time, *only buckets 0 through 2^r+q-1 are active*

k	$h(k)$
A0	100 000
A1	100 001
B0	101 000
B1	101 001
C1	110 001
C2	110 010
C3	110 011
C5	110 101

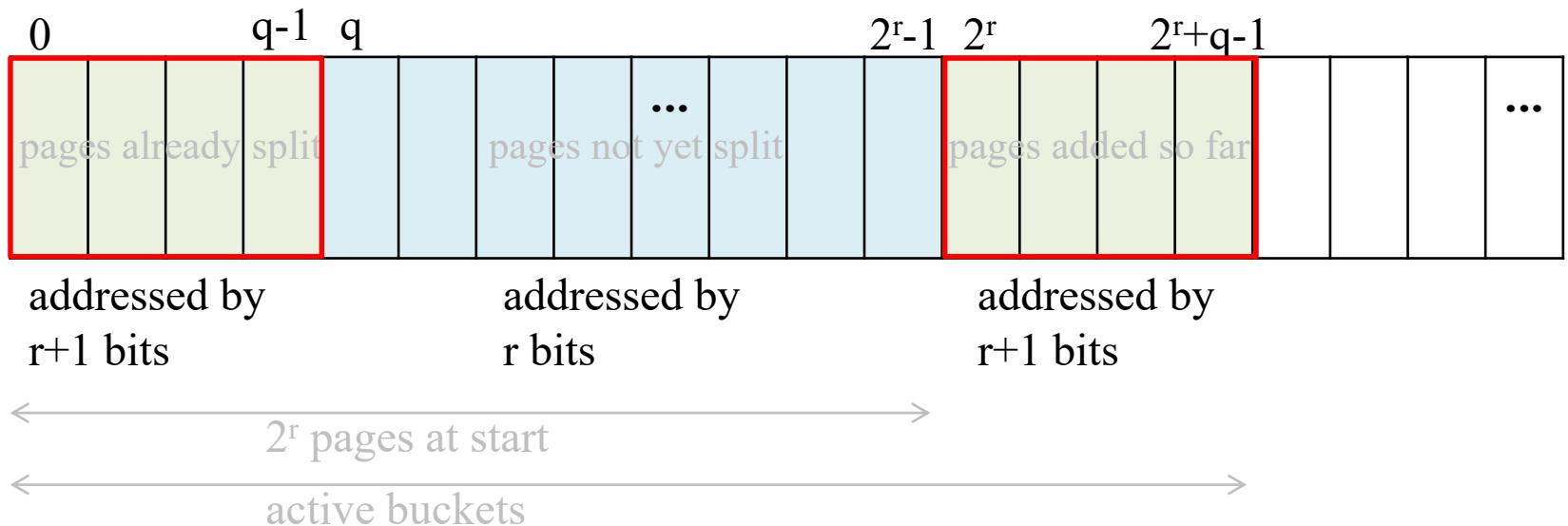


(a) $r = 2, q = 0$

(b) Insert $C5$, $r = 2, q = 1$

(c) Insert $C1$, $r = 2, q = 2$

Figure 8.9: Inserting into a directoryless dynamic hash table



In case q becomes 2^r , we increment r by 1 and reset q to 0 .

if ($h(k, r) < q$) search the chain that begins at bucket $h(k, r+1)$;
else search the chain that begins at bucket $h(k, r)$;

Program 8.5: Searching a directoryless hash table