Linear Programming
Cat One
Bsc Software Development
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Question (a)

A company produces AM and AM-FM radios. A plant of the company can be operated 24 hrs per week. Production of an AM radio will require 2hrs of production and AM-FM radio will require 3hrs each. An AM radio yields ksh 5,000 as profit and an AM-FM radio yields sh 10,000. The marketing department determined that a maximum of 15 AM and 10 AM-FM radios can be sold per week. Formulate the problem as a linear programming problem and solve it graphically

Solution

Maximize $z = 5000x_1 + 10{,}000x_2$

Subject to;

$$2x_1 + 3x_2 \leq 24$$

$$0 \leq x_1 \leq 15$$

$$0 \leq x_2 \leq 10$$

Plotting the constraints:-

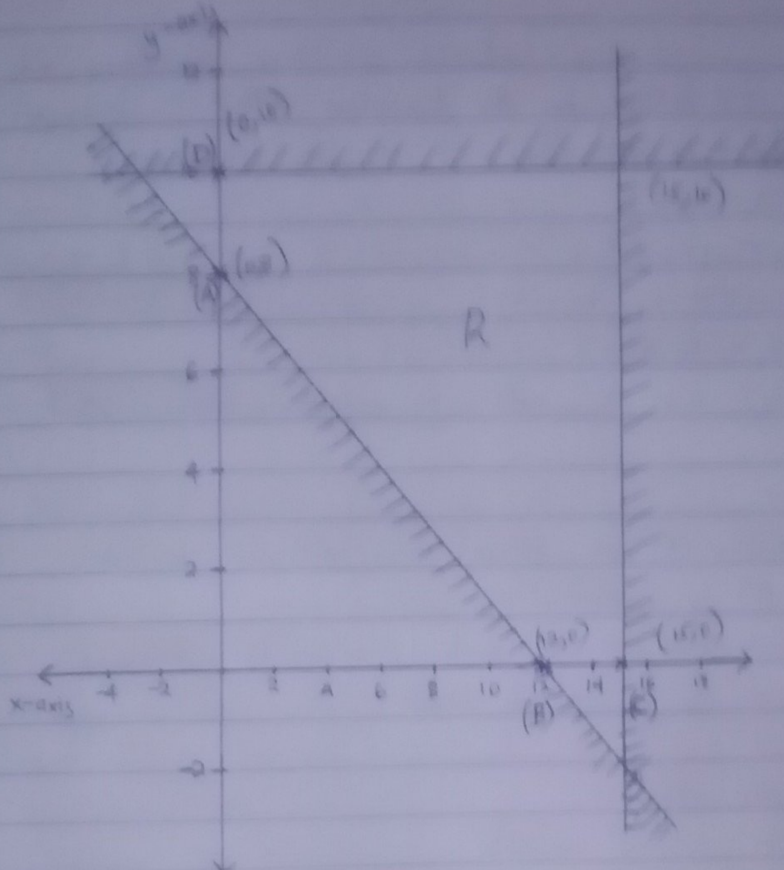
$2x_1 + 3x_2 \leq 24$

let $x_1 = 0$

$$\frac{3x_2}{3} = \frac{24}{3}$$

$$x_2 = 8$$

first point(A) - (0, 8)



Max co-ordinate / point = (15, 0)

$Z = 5000x_1 + 10{,}000x_2$

$Z_{max} = (5000 \times 15) + (10{,}000 \times 0)$

Maximum profit = ₦75,000

Question (k)

Solve the following LP problem by the two-phase method

Maximise $Z = 10x_1 + 7x_3 + x_4 + 5x_5$

Subject to $x_2 + x_3 + x_5 = 5$

$x_1 + x_2 + x_3 + x_5 = 5$

$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 10$

$x_1, x_2, x_3, x_4, x_5 \geq 0$.

Is it possible to solve this problem by a general simplex method

Solution

- If auxiliary problem (A) has optimal value < 0, we conclude that LP problem (P) is infeasible
- If (A) has optimal value = 0, we construct a feasible basis for (P) and solve it in second phase

Two phase simplex method

Phase 1 - eliminates artificial variables

Phase 2 - Optimum solution

Equality constraints don't have slack variables.

General form

Max $Z = 10x_1 + 0x_2 + 7x_3 + x_4 + 5x_5$

Subject to, $0x_1 + x_2 + x_3 + 0x_4 + x_5 = 5$

$x_1 + x_2 + x_3 + 0x_4 + x_5 = 5$

$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 10$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Converting the max function to a minimisation problem

Min $(-Z) = -10x_1 - 0x_2 - 7x_3 - x_4 - 5x_5$

Subject to, $0x_1 + x_2 + x_3 + 0x_4 + x_5 = 5$

$x_1 + x_2 + x_3 + 0x_4 + x_5 = 5$

$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 10$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

We convert the LPP into standard form by using artificial variables.

Standard form

$\text{Min}(-z) = -10x_1 - 0x_2 - 7x_3 - x_4 - 5x_5$

Subject to, $0x_1 + x_2 + x_3 + 0x_4 + x_5 + A_1 = 5$

$x_1 + x_2 + x_3 + 0x_4 + x_5 + A_2 = 5$

$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 + A_3 = 10$

$x_1, x_2, x_3, x_4, x_5, A_1, A_2, A_3 \leq 0$

An initial basic ~~function~~ feasible solution is given by;

$x_1 = x_2 = x_4 = x_5 = 0, A_1 = 5, A_2 = 5, A_3 = 10$

Phase 1

Assign a cost -1 to the artificial variables A_1, A_2 and A_3 and cost 0 to other variables

Auxilliary LPP is,

$\text{Min } z = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 - A_1 - A_2 - A_3$

Subject to, $0x_1 + x_2 + x_3 + 0x_4 + x_5 + A_1 = 5$

$x_1 + x_2 + x_3 + 0x_4 + x_5 + A_2 = 5$

$2x_1 + 3x_2 + 4x_3 + x_4 + x_5 + A_3 = 10$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & A_1 & A_2 & A_3 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

			C_j	0	0	0	0	0	-1	-1	-1	Min
	C_B	B	X_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2	A_3	Ratio θ
R_1	-1	A_1	5	0	1	1	0	1	1	0	0	$5/1 = 5$
R_2	-1	A_2	5	1	1	1	0	1	0	1	0	$5/1 = 5$
R_3	-1	A_3	10	2	3	4	1	1	0	0	1	$10/4 = 2.5$
			$Z_j - C_j$	-3	-5	-6	-1	-3	0	0	0	

$Z_j - C_j = C_B x_j - C_j$

$Z_1 - C_1 = C_B x_1 - C_1$

$Z_1 - C_1 = (-1\times 0 + (-1)\times 1 + (-1)\times 2) - 0 = -3$

$Z_2 - C_2 = (-1\times 1 + (-1)\times 1 + (-1)\times 3) - 0 = -5$

$Z_3 - C_3 = (-1\times 1 + (-1)\times 1 + (-1)\times 4) - 0 = -6$

$Z_4 - C_4 = (-1\times 0 + (-1)\times 0 + (-1)\times 1) - 0 = -1$

$Z_5 - C_5 = (-1\times 1 + (-1)\times 1 + (-1)\times 1) - 0 = -3$

$Z_6 - C_6 = (-1\times 1) + (-1)\times 0 + (-1)\times 0) + 1 = 0$

$Z_7 - C_7 = (-1\times 0 + (-1)\times 1 + (-1)\times 0) + 1 = 0$

$Z_8 - C_8 = (-1\times 0 + (-1)\times 0 + (-1)\times 1) + 1 = 0$

Most negative value is -6 (x_3)

- Lowest minimum ratio shows the pivot element

Lowest minimum ratio = 2.5

pivot element = 4

New value = x_3

Basic variable = A_3 (eliminated)

Entering basic var = Row index = x_3
Departing basic var = A_3
Key element = 4

R_1 (new) = R_1 (old) ÷ 4
R_2 (new) = R_2 (old) - R_1 (new)

(i) For row that have key element:-

$$\text{NewRow value} = \frac{\text{each element of that Row } (A_3)}{\text{key element}}$$

(ii) For other rows:-

R' = old value - key column Value × NewRow value

R_1 key column value ⇒ 1
R_2 key column value ⇒ 1

	x_0	x_1	x_2	x_3	x_4	x_5	A_1	A_2	A_3
R_1' ⇒ R_1 (old value) →	5	0	1	1	0	1	1	0	0
Key column value R_3' = −	$\frac{10}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{4}$
	2.5	$-\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{3}{4}$	1	0	$-\frac{1}{4}$

	x_0	x_1	x_2	x_3	x_4	x_5	A_1	A_2	A_3
R_2' ⇒ R_2 →	5	1	1	1	0	1	0	1	0
− $1 * R_3$ →	$\frac{10}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{4}$
	2.5	$\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	1	$-\frac{1}{4}$

Pivot → $-\frac{1}{4}$
Row → x_5

Phase 2

C_j			10	0	7	1	5	0	0	0	Min
C_B	B	X_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2	A_3	Ratio θ
-1	A_1	2.5	$-\frac{1}{3}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{3}{4}$	1	0	$-\frac{1}{4}$	$\frac{5}{2} \div \frac{3}{4} = 3.3$
-10	A_2	2.5	$\frac{1}{3}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	1	$-\frac{1}{4}$	$\frac{5}{2} \div \frac{3}{4} = 3.3$
0	x_3	$\frac{10}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{4}$	$\frac{5}{2} \div \frac{1}{4} = 10$
$Z_j - C_j$		N/A	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{3}{2}$	-2	-2	$-\frac{1}{2}$	

- In phase 2, we are going to use the original variables from the objective function as (c_j)
- Since all $z_j - c_j \leq 0$, the optimality conditions are satisfied. Hence the optimal basic feasible solution is.

$$x_3 = \frac{5}{2},\ x_1 = 0,\ x_2 = 0,\ x_5 = 0,\ x_4 = 0$$

$$Z_{max} = -\left(-\frac{5}{2} \times 7\right)$$

$$Z_{max} = 17.5$$