Scale-invariant feature transform

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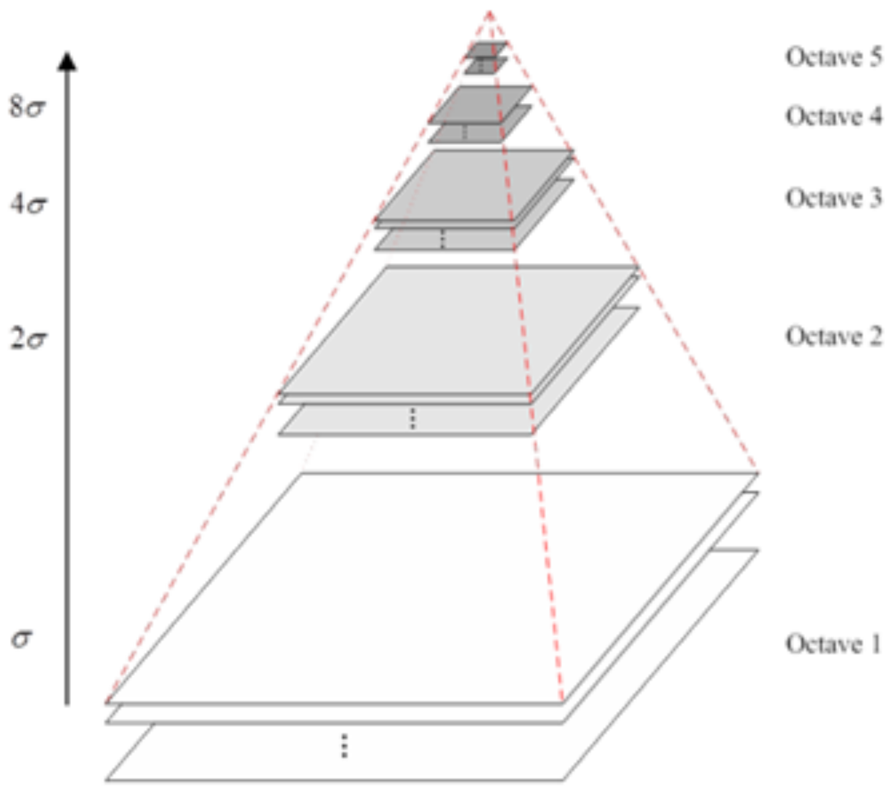
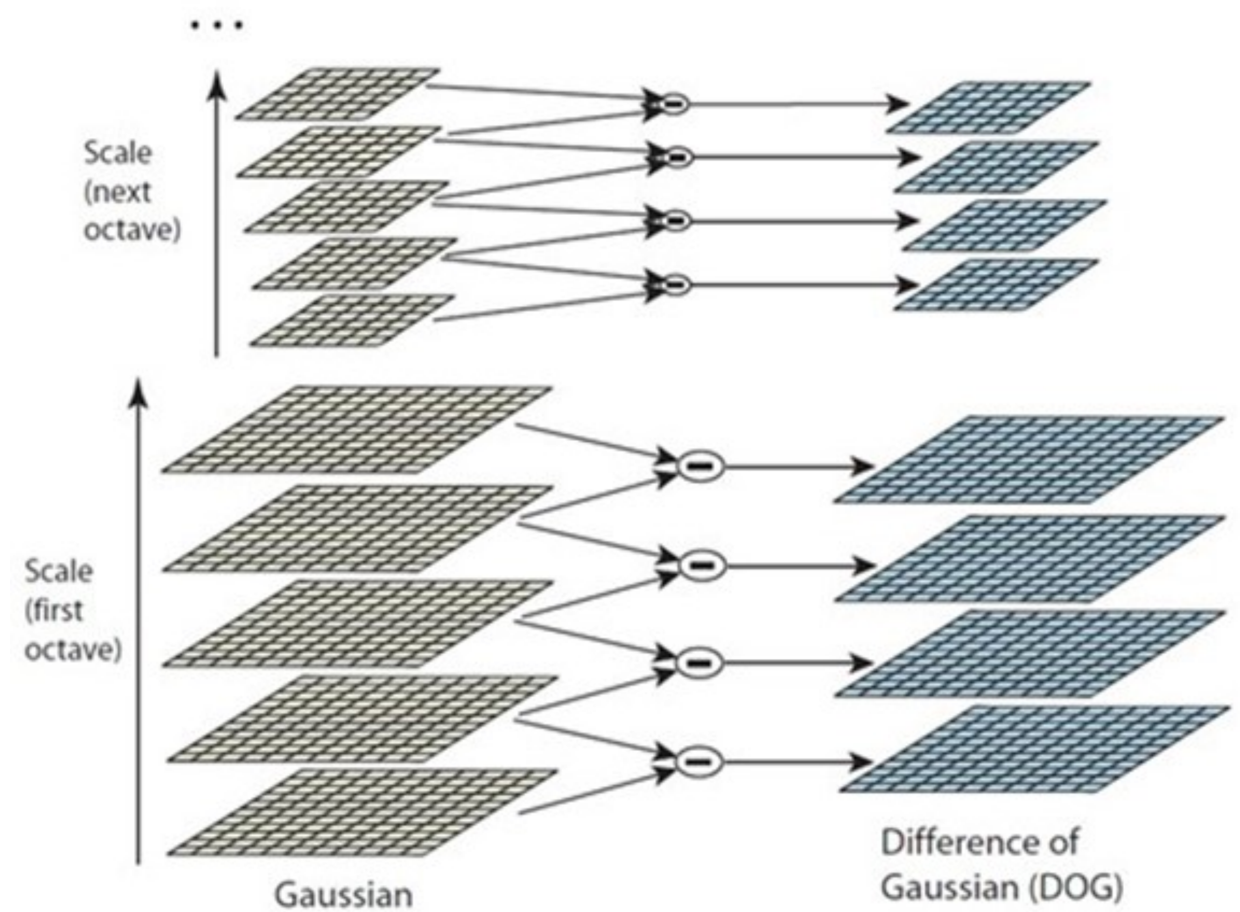
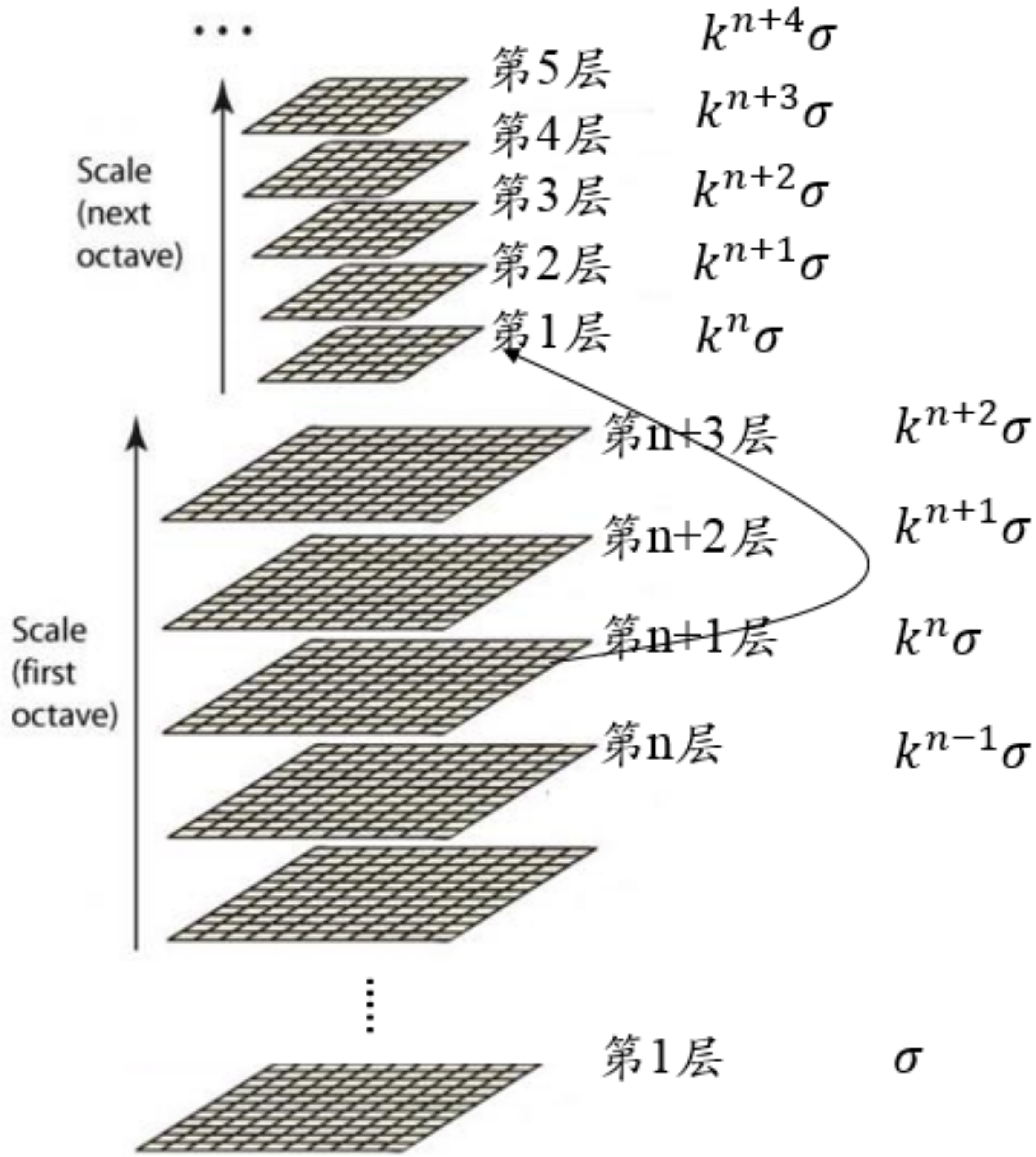
一、建立高斯差分金字塔





$$O = \left[\log_2\left(\min\left(M, N\right)\right)\right] - 3$$

S = n + 3



$$k = 2^{1/n}$$

$$\sigma_0 = \sqrt{1.6^2 - 0.5^2} = 1.52$$

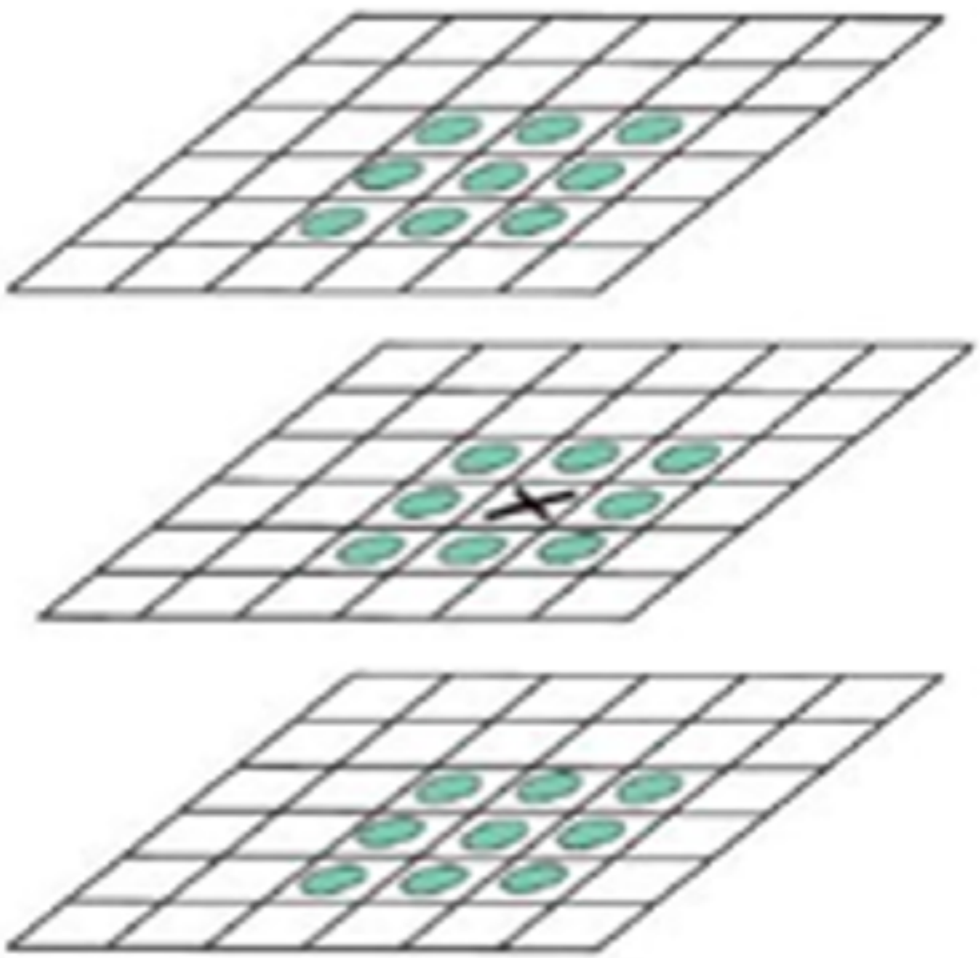
二、关键点(key points)位置确定

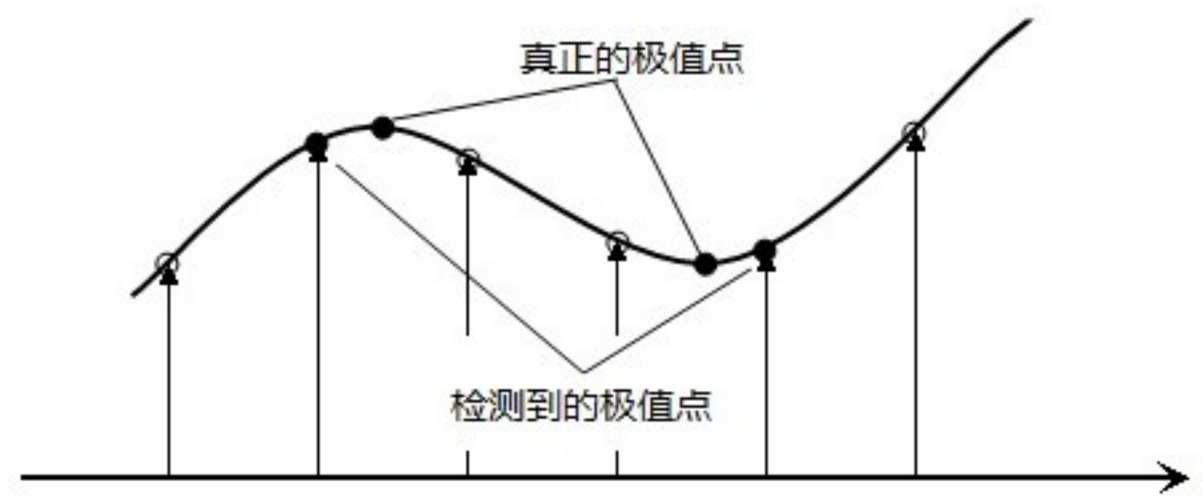
1.阈值化

abs(val) > 0.5*T/n

T = 0.04

2.在高斯差分金字塔中找极值





3.调整极值点位置

在检测到的极值点$X_0(x_0,y_0,\sigma_0)^T$ 处做三元二阶泰勒展开

$$f\left(\begin{bmatrix} x \\ y \\ \sigma \end{bmatrix}\right)=f\left(\begin{bmatrix} x_0 \\ y_0 \\ \sigma_0 \end{bmatrix}\right)+\left[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial \sigma}\right]\left(\begin{bmatrix} x \\ y \\ \sigma \end{bmatrix}-\begin{bmatrix} x_0 \\ y_0 \\ \sigma_0 \end{bmatrix}\right)$$

$$+\frac{1}{2}\left(\begin{bmatrix} x \\ y \\ \sigma \end{bmatrix}-\begin{bmatrix} x_0 \\ y_0 \\ \sigma_0 \end{bmatrix}\right)^T\begin{bmatrix} \frac{\partial^2 f}{\partial x\partial x},\frac{\partial^2 f}{\partial x\partial y},\frac{\partial^2 f}{\partial x\partial \sigma} \\ \frac{\partial^2 f}{\partial x\partial y},\frac{\partial^2 f}{\partial y\partial y},\frac{\partial^2 f}{\partial y\partial \sigma} \\ \frac{\partial^2 f}{\partial x\partial \sigma},\frac{\partial^2 f}{\partial y\partial \sigma},\frac{\partial^2 f}{\partial \sigma\partial \sigma} \end{bmatrix}\left(\begin{bmatrix} x \\ y \\ \sigma \end{bmatrix}-\begin{bmatrix} x_0 \\ y_0 \\ \sigma_0 \end{bmatrix}\right)$$

矢量形式： $f(X)=f(\boldsymbol{X_0})+\frac{\partial f^T}{\partial \boldsymbol{X}}\hat{\boldsymbol{X}}+\frac{1}{2}\hat{\boldsymbol{X}}^T\frac{\partial^2 f}{\partial \boldsymbol{X}^2}\hat{\boldsymbol{X}}$

$$f(X) = f(\boldsymbol{X_0}) + \frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}} + \frac{1}{2}\widehat{\boldsymbol{X}}^T\frac{\partial^2 f}{\partial \boldsymbol{X}^2}\widehat{\boldsymbol{X}}$$

f(x)求导： $\frac{\partial f(X)}{\partial \boldsymbol{X}} = \frac{\partial f^T}{\partial \boldsymbol{X}} + \frac{1}{2}\left(\frac{\partial^2 f}{\partial \boldsymbol{X}^2} + \frac{\partial^2 f^T}{\partial \boldsymbol{X}^2}\right)\hat{X} = \frac{\partial f^T}{\partial \boldsymbol{X}} + \frac{\partial^2 f}{\partial \boldsymbol{X}^2}\widehat{\boldsymbol{X}}$

令导数为零解得： $\widehat{\boldsymbol{X}} = -\frac{\partial^2 f^{-1}}{\partial \boldsymbol{X}^2}\frac{\partial f}{\partial \boldsymbol{X}}$

代入f(x): $f(X) = f(\boldsymbol{X_0}) + \frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}} + \frac{1}{2}\left(-\frac{\partial^2 f^{-1}}{\partial \boldsymbol{X}^2}\frac{\partial f}{\partial \boldsymbol{X}}\right)^T\frac{\partial^2 f}{\partial \boldsymbol{X}^2}\left(-\frac{\partial^2 f^{-1}}{\partial \boldsymbol{X}^2}\frac{\partial f}{\partial \boldsymbol{X}}\right)$

$$= f(\boldsymbol{X_0}) + \frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}} + \frac{1}{2}\frac{\partial f^T}{\partial \boldsymbol{X}}\frac{\partial^2 f^{-T}}{\partial \boldsymbol{X}^2}\frac{\partial^2 f}{\partial \boldsymbol{X}^2}\frac{\partial^2 f^{-1}}{\partial \boldsymbol{X}^2}\frac{\partial f}{\partial \boldsymbol{X}}$$

$$= f(\boldsymbol{X_0}) + \frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}} + \frac{1}{2}\frac{\partial f^T}{\partial \boldsymbol{X}}\frac{\partial^2 f^{-1}}{\partial \boldsymbol{X}^2}\frac{\partial f}{\partial \boldsymbol{X}}$$

$$= f(\boldsymbol{X_0}) + \frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}} + \frac{1}{2}\frac{\partial f^T}{\partial \boldsymbol{X}}(-\widehat{\boldsymbol{X}})$$

$$= f(\boldsymbol{X_0}) + \frac{1}{2}\frac{\partial f^T}{\partial \boldsymbol{X}}\widehat{\boldsymbol{X}}$$

还有一些细节问题：迭代次数限制、解超出一定范围舍去

4.舍去低对比度的点

若 $\left|f(X)\right| < \frac{T}{n}$,则舍去点X

5.边缘效应的去除

$$H(x,y) = \begin{bmatrix} D_{xx}(x,y) & D_{xy}(x,y) \\ D_{xy}(x,y) & D_{yy}(x,y) \end{bmatrix}$$

$$\mathrm{Tr}(H) = D_{xx} + D_{yy} = \alpha + \beta$$

其中: $\alpha > \beta$; 且 $\alpha = \gamma\beta$

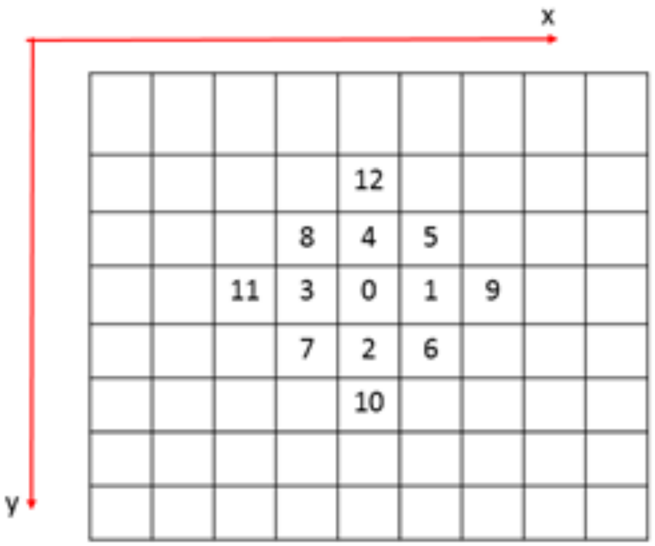
$$\mathrm{Det}(H) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta$$

若 $\mathrm{Det}(H) < 0$ 舍去点X

$$\frac{\mathrm{Tr}(H)^2}{\mathrm{Det}(H)} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(\gamma\beta+\beta)^2}{\gamma\beta^2} = \frac{(\gamma+1)^2}{\gamma}$$

若不满足 $\frac{\mathrm{Tr}(H)}{\mathrm{Det}(H)} < \frac{(\gamma+1)^2}{\gamma}$ 舍去点X (建议 γ 取 10.0)

附：有限差分求导法



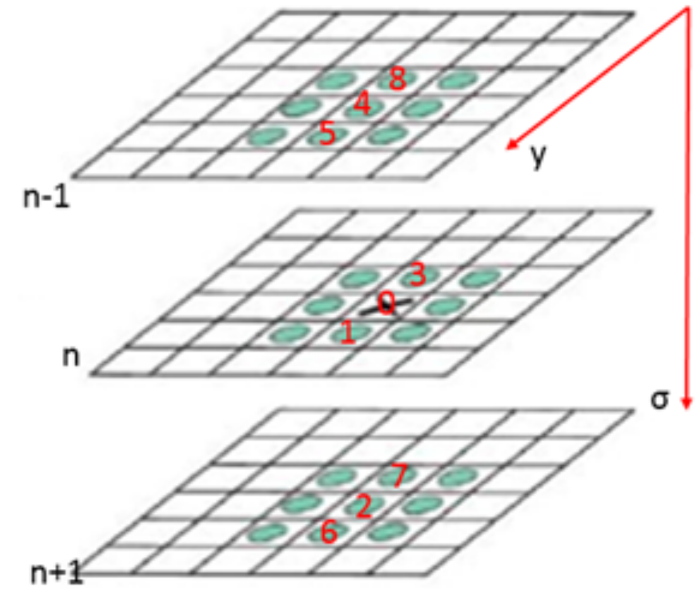
$$\left(\frac{\partial f}{\partial x}\right)=\frac{f_1-f_3}{2h} \qquad (1)$$

$$\left(\frac{\partial f}{\partial y}\right)=\frac{f_2-f_4}{2h} \qquad (2)$$

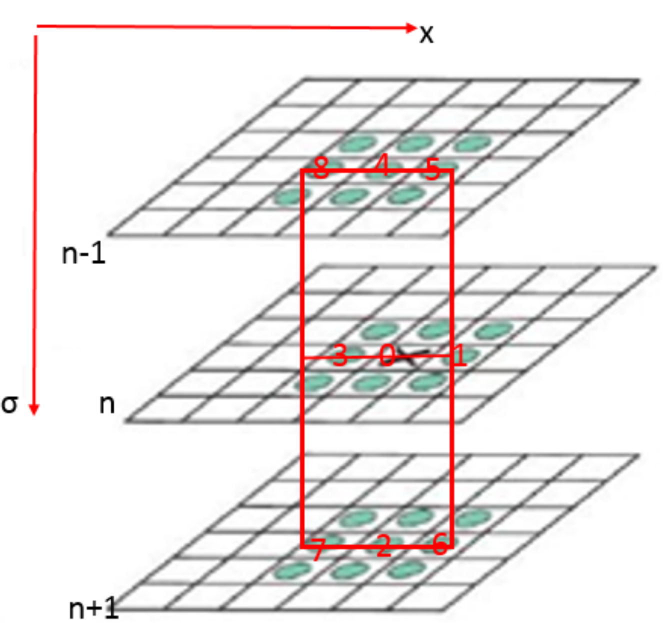
$$\left(\frac{\partial^2 f}{\partial x^2}\right)=\frac{f_1+f_3-2f_0}{h^2} \qquad (3)$$

$$\left(\frac{\partial^2 f}{\partial y^2}\right)=\frac{f_2+f_4-2f_0}{h^2} \qquad (4)$$

$$\left(\frac{\partial^2 f}{\partial x\partial y}\right)=\frac{(f_8+f_6)-(f_5+f_7)}{4h^2} \qquad (5)$$



$$\left(\frac{\partial^2 f}{\partial y\partial \sigma}\right)=\frac{(f_8+f_6)-(f_5+f_7)}{4h^2} \qquad (9)$$



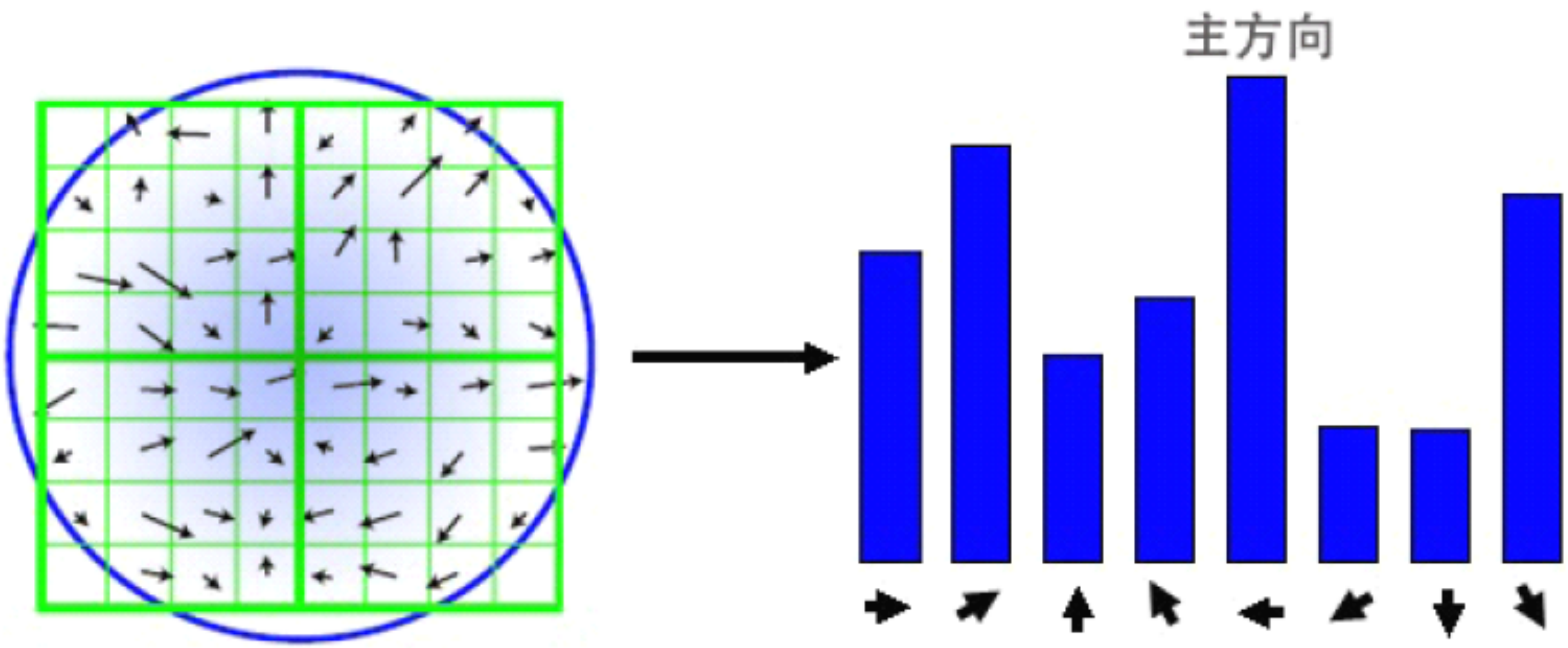
$$\left(\frac{\partial f}{\partial \sigma}\right)=\frac{f_2-f_4}{2h} \qquad (6)$$

$$\left(\frac{\partial^2 f}{\partial \sigma^2}\right)=\frac{f_2+f_4-2f_0}{h^2} \qquad (7)$$

$$\left(\frac{\partial^2 f}{\partial x\partial \sigma}\right)=\frac{(f_8+f_6)-(f_5+f_7)}{4h^2} \qquad (8)$$

三、为关键点赋予方向

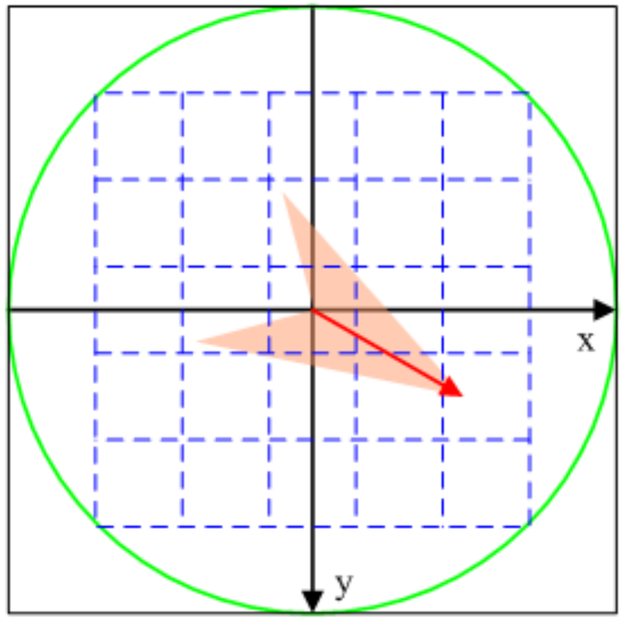
统计 以特征点为圆心，以该特征点所在的高斯图像的尺度的1.5倍为半径的圆内的所有的像素的梯度方向及其梯度幅值，并做 1.5σ 的高斯滤波



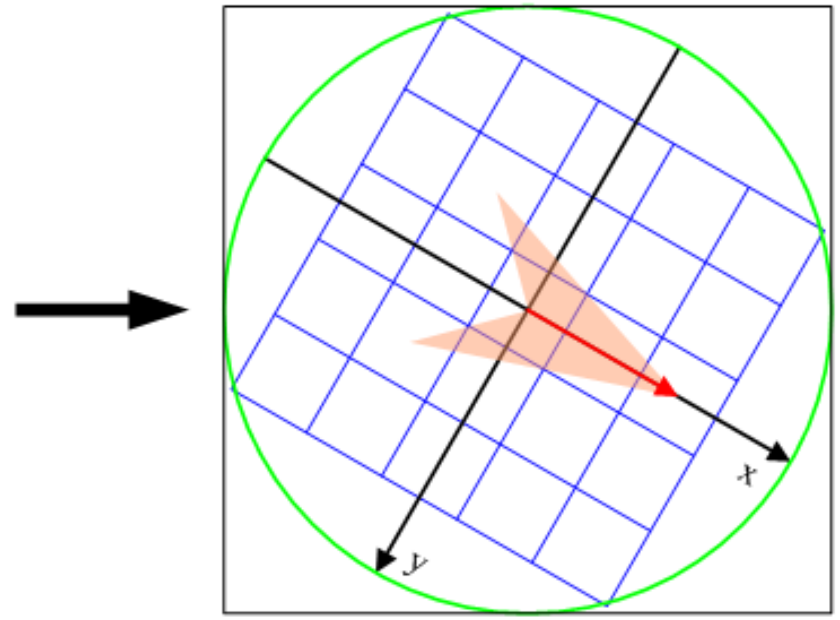
（在最接近关键点尺度值σ的高斯图像上进行统计）


Original Image
SIFT Features

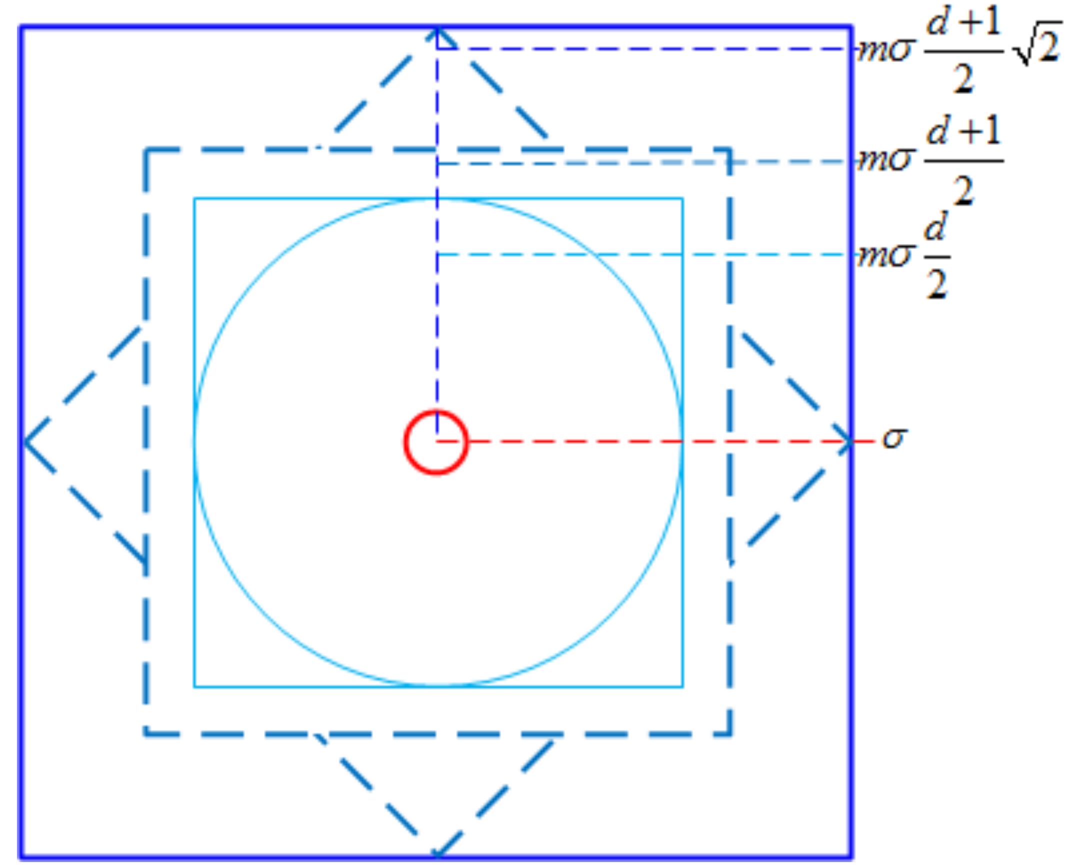
四、构建关键点描述符



x
y



x
y



$m\sigma\frac{d+1}{2}\sqrt{2}$
$m\sigma\frac{d+1}{2}$
$m\sigma\frac{d}{2}$
σ