Harris corner detection

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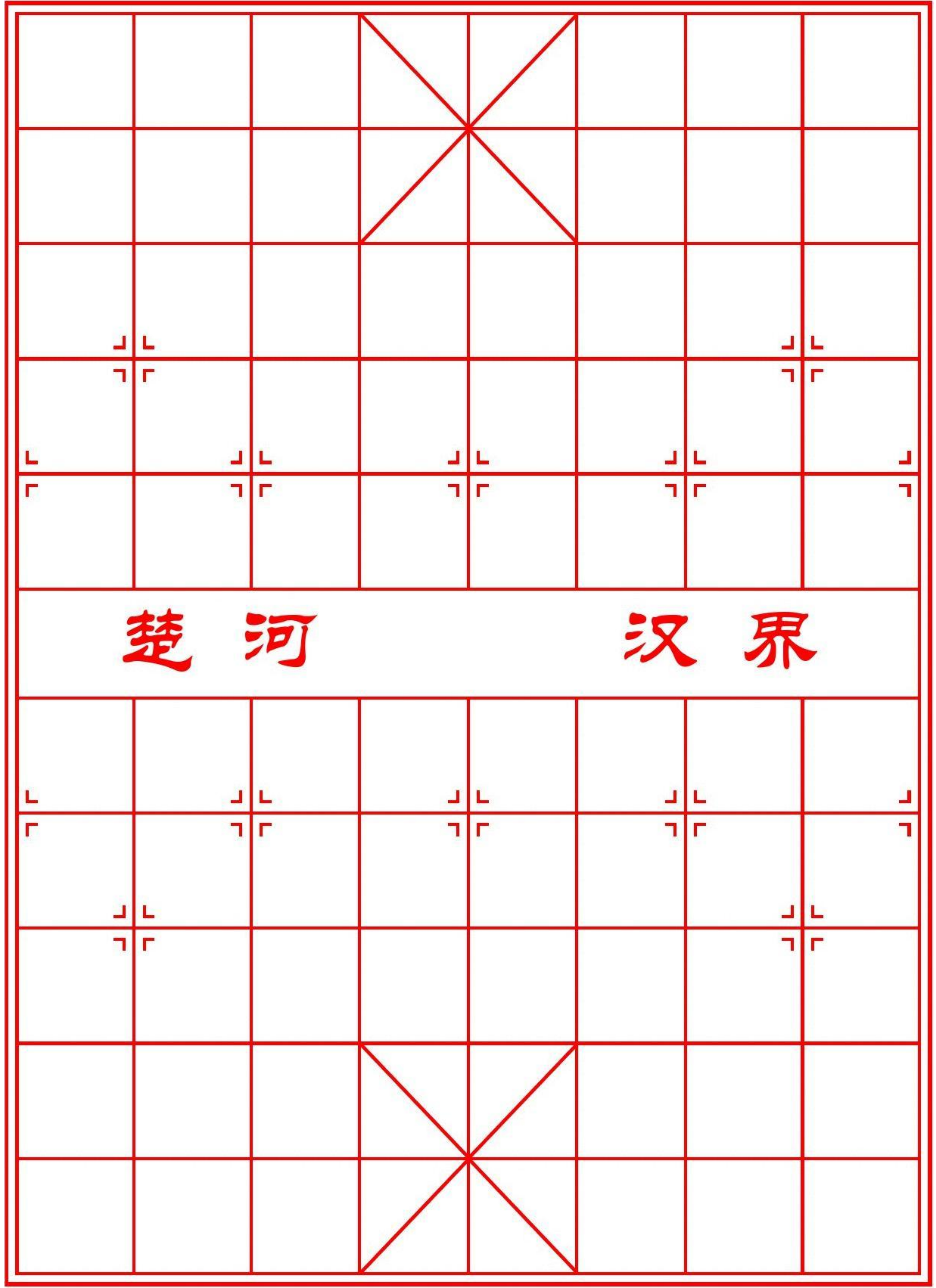
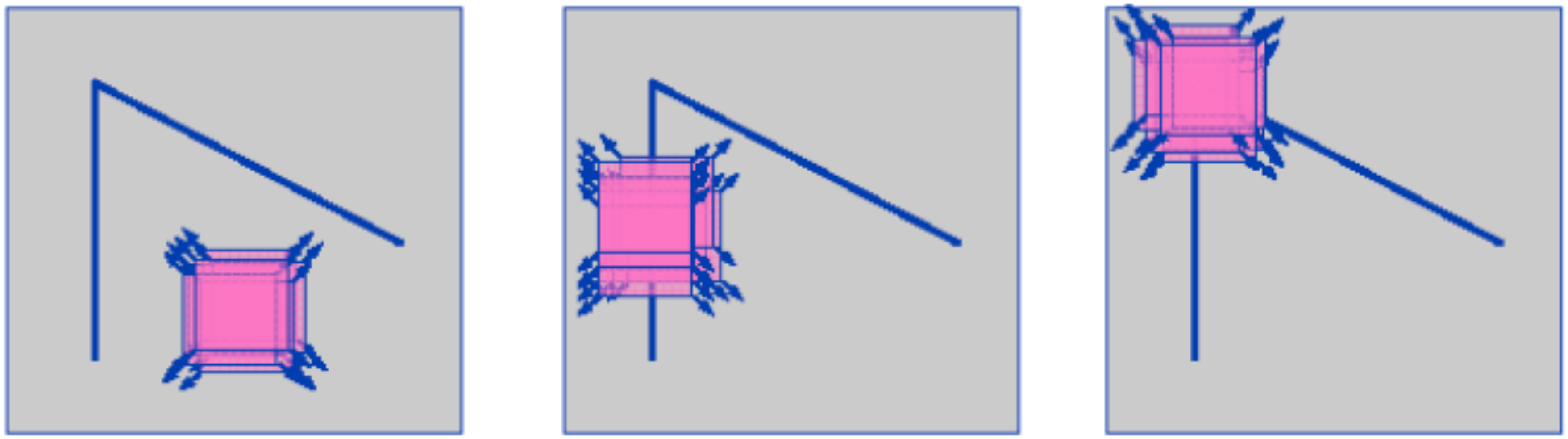

楚河
汉界

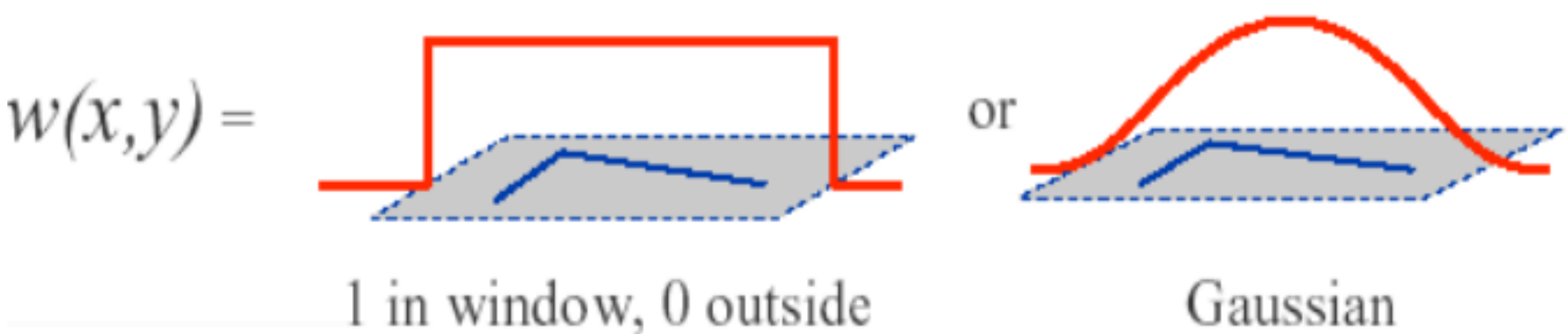
角的特征：



平坦区域
在所有方向没有
明显梯度变化

边缘区域
在某个方向有明显
梯度变化

角度边缘
在各个方向梯度值
有明显变化



$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uvf_{xy}x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ \ldots$ (Higher order terms)

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

$$\sum\left[I(x+u,y+v)-I(x,y)\right]^2$$

$$\approx\sum\left[I(x,y)+uI_x+vI_y-I(x,y)\right]^2$$ First order approx

$$=\sum u^2I_x^2+2uvI_xI_y+v^2I_y^2$$

$$=\sum\left[\begin{array}{cc} u & v \end{array}\right]\left[\begin{array}{cc} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{array}\right]\left[\begin{array}{c} u \\ v \end{array}\right]$$ Rewrite as matrix equation

$$=\left[\begin{array}{cc} u & v \end{array}\right]\left(\sum\left[\begin{array}{cc} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{array}\right]\right)\left[\begin{array}{c} u \\ v \end{array}\right]$$

$$E(u,v) \simeq \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

正交相似对角化

$$P \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} P^{-1}$$

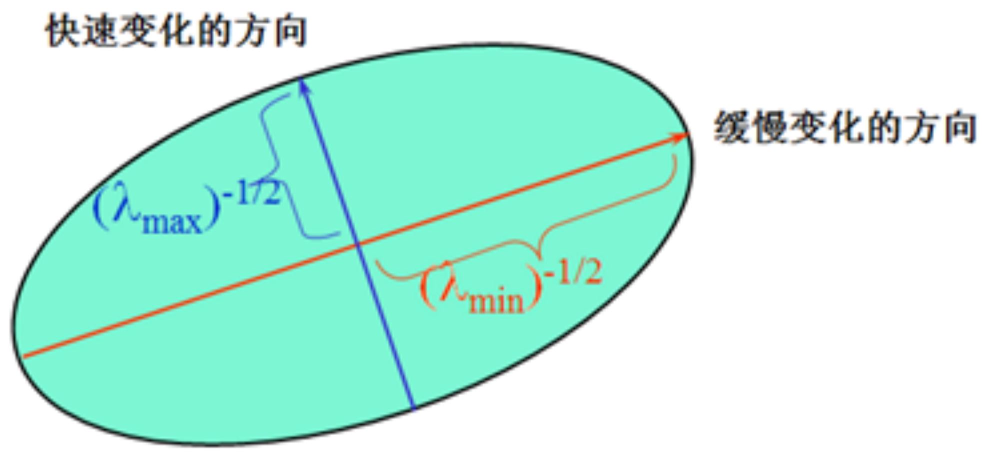
$$P^T = P^{-1}$$

$$E(u,v) = [u \ v] P \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} P^T [u,v]^T$$

$$E(u,v) = [u' \ v'] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} u' \\ v' \end{bmatrix}$$

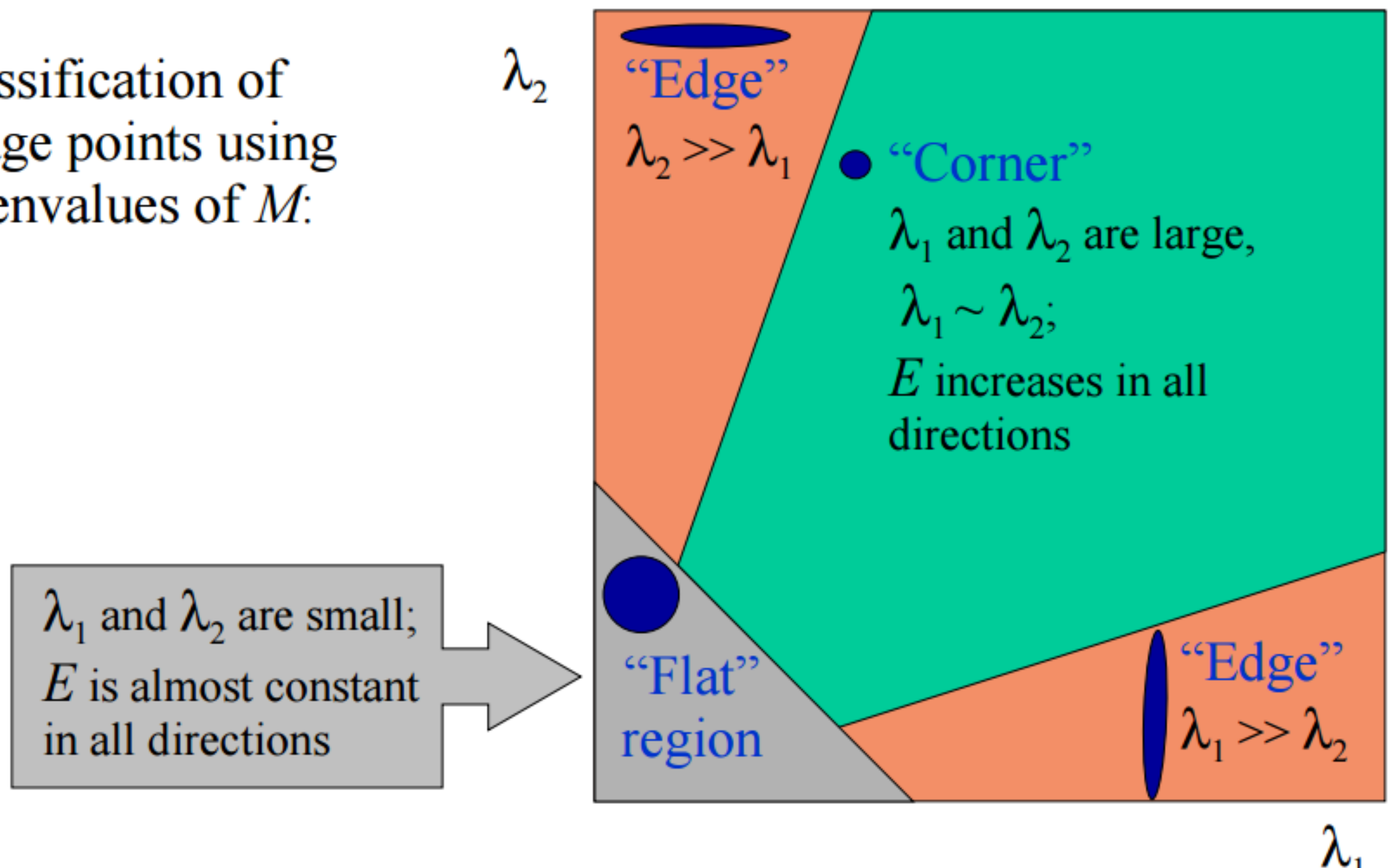
$$= \lambda_1 (u')^2 + \lambda_2 (v')^2$$

$$= \frac{(u')^2}{\frac{1}{\lambda_1}} + \frac{(v')^2}{\frac{1}{\lambda_2}}$$



$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

Classification of image points using eigenvalues of *M*:



$$R = \det M - k\,(\operatorname{trace} M)^2$$