Notes on Helix Fit

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1 Introduction to Helix Fit

Suppose we aim to determine a helical trajectory from a set of measured spatial points (or *hits*). A point on a helical trajectory, denoted as $\mathcal{H}(c_x, c_y, r, z_0, dz; \theta)$, can be described by a parameter θ and five coefficients: the radius r, the center coordinates of the circle in the transverse plane c_x and c_y, the longitudinal position at $\theta = 0$ denoted as z_0, ant the helical pitch dz.

To perform a χ^2 fit of the track to the measured points, we evaluate the distance from each point to the trajectory, normalize the distance by the associated spatial resolution σ, and then sum the squared normalized residuals to compute the total χ^2. Each hit provides three measurements, $\mathbf{X}_i \equiv (x_i, y_i, z_i)$. However, defining the distance between a hit and the helical trajectory requires identifying a corresponding point on the helix, $\mathbf{h}_i$, by choosing a specific parameter value θ_i for each hit. Details of this point-selection procedure will be discussed in the next session.

Selecting a point on the helix-i.e., *projecting* onto the θ parameter space-reduces one degree of freedom, resulting in only two effective degrees of freedom per hit. Consequently, the residual sum should contain two spatial components. Intuitively, the most natural choices are the radial component(in the transverse plane of the helix) and the vertical component (along the helix axis).

$$\chi^2 = \sum_i^n (\frac{\delta_{r,i}}{\sigma_{r,i}})^2 + (\frac{\delta_{V,i}}{\sigma_{V,i}})^2 - Cov(r, V)\frac{\delta_{r,i}}{\sigma_{r,i}}\frac{\delta_{V,i}}{\sigma_{V,i}} \tag{1}$$

Here, δ_r and δ_V are the radial and vertical components of the position difference $\mathbf{D}_i = \mathbf{h}_i - \mathbf{X}_i$, respectively, and $\text{Cov}(r, V)$ denotes their the covariance.

2 Point Selection in Helical Track

To evaluate (1) properly, we need a well-defined criterion for selecting each point $\mathbf{h}_i$ on the helix. The simplest approach is to determine θ_i by minimizing the squared spatial distance:

$$\begin{aligned}\delta_i^2 &= (\mathbf{h}_i - \mathbf{X}_i)^2 \\ &= \underbrace{(c_x + r\cos\theta - x_i)^2 + (c_y + r\sin\theta - y_i)^2}_{\delta_r^2} + \underbrace{(z_0 + dz\theta - z_i)^2}_{\delta_V^2}. \end{aligned} \quad (2)$$

This is referred to as the *Point of Closest approach*(PoC) method. However, minimizing the full distance introduces a non-zero (typically negative) correlation between the radial and vertical components, as seen from the minimization condition:

$$\frac{\partial\delta_i^2}{\partial\theta} = \frac{\partial\delta_r^2}{\partial\theta} + \frac{\partial\delta_V^2}{\partial\theta} = 0. \quad (3)$$

To avoid this undesired correlation from (3), alternative criteria for defining $\mathbf{h}_i$ can be considered, such as minimizing only one of the two components-either δ_r^2 or δ_V^2.

If we minimize δ_V^2, we obtain:

$$\frac{\partial\delta_V^2}{\partial\theta} = dz(z_0 + dz\theta - z_i) = 0 \to \theta = \frac{z_0 - z_i}{dz}. \quad (4)$$

However, this approach is extremely sensitive to vertical fluctuations when the pitch dz is small. In such cases, small variations in z lead to large changes in θ, which can destabilize the fit.

An alternative, referred to here as the *Minimal Radial Distance*(MRD) method[1] involves minimizing only δ_r^2. This method is generally more stable, as the radius tends to be large(on the order of hundreds to thousands of millimeters), while the spatial resolution σ_r is typically smaller than 10 mm:

$$\frac{\partial\delta_r^2}{\partial\theta} = -r(c_x + r\cos\theta - x_i)\sin\theta + r(c_y + r\sin\theta - y_i)\cos\theta = 0 \quad (5)$$

$$\tan\theta = \frac{\delta_y}{\delta_x}. \quad (6)$$

Figure 1 illustrates both (a) PoC method and (b) MRD method. Note that for tracks with small pitch ($dz \to 0$), (4) approaches zero and (3) effectively reduces to (5), implying that PoC and MRD converge to similar definitions.

3 Position Residual Comparison with HypTPC Data

Figure 2 compares the radial residuals obtained using (a) the PoC Method and (b) the MRD method for scattered π tracks from a ^{12}C target in a 1T B-field. As discussed in the previous section, the PoC method minimizes the total squared distance, $\delta_r^2 + \delta_T^2$, so the radial component δ_r alone is not necessarily minimized. This results in the dip structure observed in (a).

[1]A standard name for this method is not commonly found in literature.

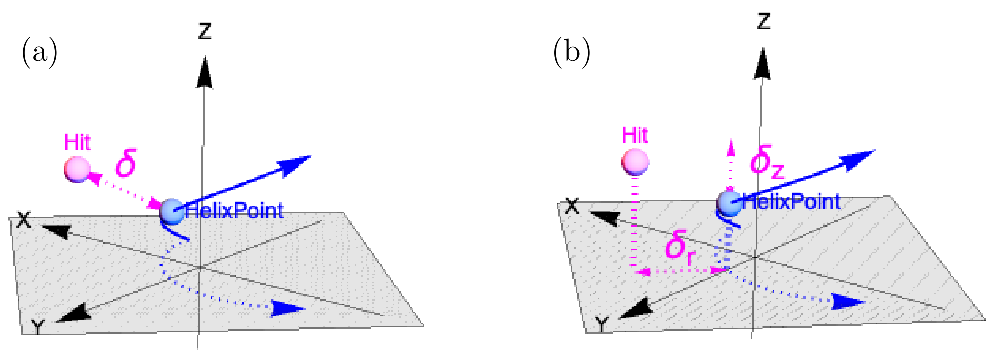

Figure 1

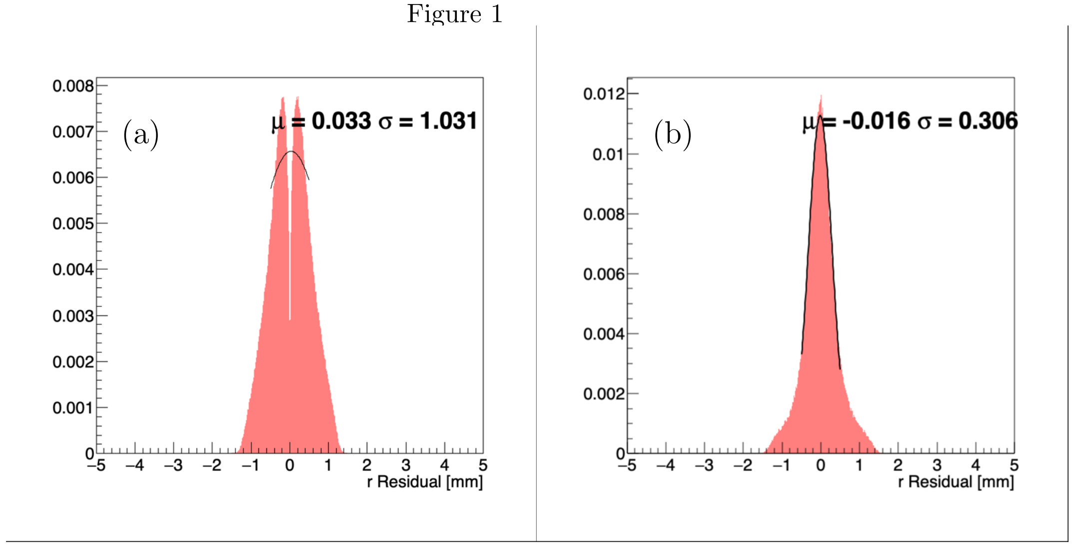

Figure 2

4 Position Resolution

4.1 Radial Resolution

To be written... Check [1]

4.2 Vertical Resolution

To be written...

5 Momentum Resolution in HypTPC Beamthrough Analysis

To be written...

References

[1] TPC Spatial Resolution Study from W.S. Jung
https://lambda.phys.tohoku.ac.jp/hyptpc-wiki/lib/exe/fetch.php?media=document:analysis:tpcspatialresol