Notes on Helix Fit

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1 Introduction to Helix Fit

Let us say, we want to determine a helical trajectory from a set of measured spatial points(or, hits). A point in a helical trajectory, $\mathcal{H}(c_x, c_y, r, z0, dz; \theta)$, can be represented by a parameter θ and five coefficients, which corresponds to the radius r, x and y of the center of the circle c_x, c_y, axis position at $\theta = 0$ z_0, and helical pitch dz.

To conduct a χ^2 fit of the track from a set of measured spatial points(or, **hits**), we need to evaluate the distance of each point from the trajectory, estimate the pull by dividing it into spatial residual σ, and then sum up the square of the pulls to determine the χ^2. Each hit comprise of three measurements, $\mathbf{X}_i \equiv (x_i, y_i, z_i)$. However, to define the distance between helical track and spatial point, *we need to define a specific point*($\mathbf{h}_i$) by selecting specific parameter, θ_i, in the helix($H(c_x, c_y, r, z0, dz; \theta)$) for each measurement $\mathbf{X}_i$. This point selection will be discussed in the next session. Selection of the point results in a *loss of one degree of freedom, giving only two degrees of freedom for each hit.* Then, the summation for pull squared should have two spatial components. Our intuition says that, **radial**(along the plane where the circle lies) component and **vertical**(along the axis) component are the best choices.

$$\chi^2 = \sum_i^n (\frac{\delta_{r,i}}{\sigma_{r,i}})^2 + (\frac{\delta_{V,i}}{\sigma_{V,i}})^2 - Cov(r, V)\frac{\delta_{r,i}}{\sigma_{r,i}}\frac{\delta_{V,i}}{\sigma_{V,i}} \tag{1}$$

where δ_r and δ_V are the radial and vertical component squared, respectively, for the position difference $\mathbf{D}_i = \mathbf{h}_i - \mathbf{X}_i$, and $Cov(r, V)$ is the covariance between δ_r and δ_V.

2 Point Selection in Helical Track

To evaluate (1) properly, we need a good criteria to define $\mathbf{h}_i$. The simplest way is to get θ_i by minimizing

$$\begin{aligned}\delta_i^2 &= (\mathbf{h}_i - \mathbf{X}_i)^2 \\ &= \underbrace{(c_x + r\cos\theta - x_i)^2 + (c_y + r\sin\theta - y_i)^2}_{\delta_r^2} + \underbrace{(z_0 + dz\theta - z_i)^2}_{\delta_V^2}.\end{aligned} \tag{2}$$

This is known as *Point of Closest approach*(PoC) method. However, with this definition, we have non-zero (anti) correlation between r and V, arising from the minimizing condition:

$$\frac{\partial \delta_i^2}{\partial \theta} = \frac{\partial \delta_r^2}{\partial \theta} + \frac{\partial \delta_V^2}{\partial \theta} = 0. \tag{3}$$

In order to avoid this problem, we could define $\mathbf{h}_i$ by implying criteria other than (2) to avoid (anti) correlation between two variables from (3). We can either minimize δ_r^2 or δ_V^2 only.

If we minimize δ_V^2, resulting θ from

$$\frac{\partial \delta_V^2}{\partial \theta} = dz(z_0 + dz\theta - z_i) = 0 \rightarrow \theta = \frac{z_0 - z_i}{dz} \tag{4}$$

will be is extremely sensitive to small variance in vertical direction for tracks with low pitch angle($dz \rightarrow 0$), leading to high instability for θ_i selection, hence degrading the fitting result.

Another choice, say *Minimal Radial Distance*(MRD)[1] method is to minimize δ_r^2, which would obviously be unstable for small radius ($r \simeq \sigma_r$). However, this is not the usual case because r typically ranges from few hundreds to thousands of millimeters, while $\sigma_r < 10$ mm. Then, we have

$$\frac{\partial \delta_r^2}{\partial \theta} = -r(c_x + r\cos\theta - x_i)\sin\theta + r(c_y + r\sin\theta - y_i)\cos\theta = 0 \tag{5}$$

$$\tan\theta = \frac{\delta_y}{\delta_x}. \tag{6}$$

Figure 1 illustrates the (a) PoC method and (b) MRD method. Note that, for small dz tracks, (4)$\simeq$ 0 hence (3)$\simeq$ (5) so that PoC and MRD becomes essentially similar.

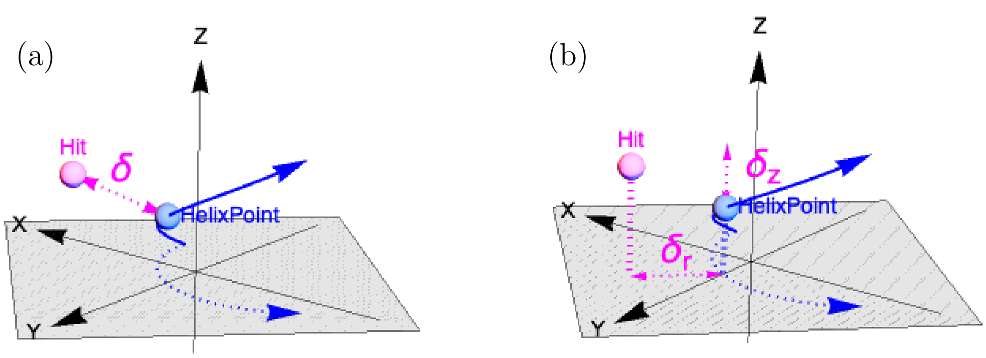

Figure 1

[1]I can't find conventional name for this method

3 Result Comparison in HypTPC

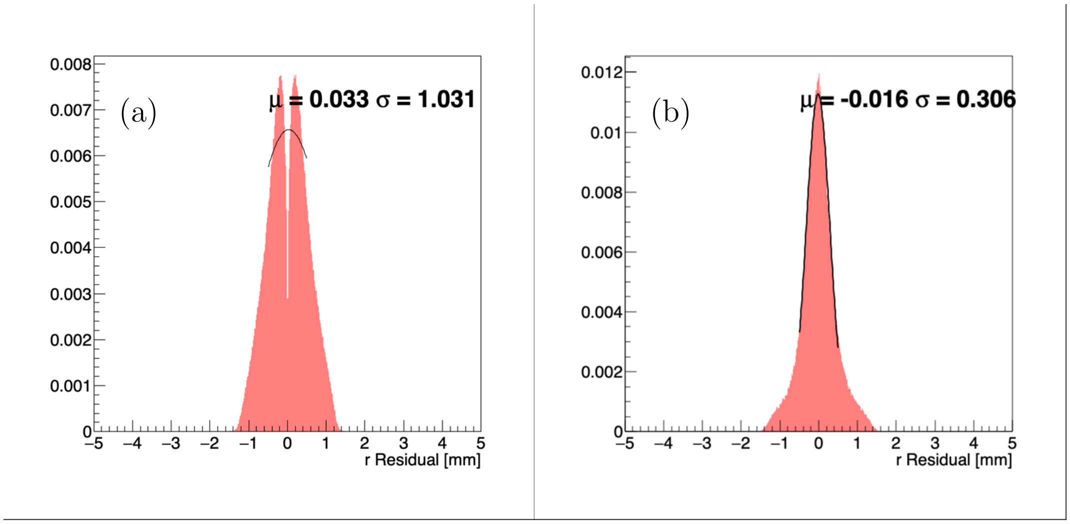

Figure 2

Figure 2 shows the comparison of radial residuals between (a) PoC Method and (b) MRD method for scattered πs from ^{12}C target, in 1T B-field. As discussed in the above section, PoC method minimizes $\delta_r^2 + \delta_T^2$, hence δ_r may not be in minimum. As so we observe a dip structure in (a).