1.

We are going to approximate cos(π/4) without a calculator or computer

Instead of true input x = π/4, we use $\hat{x}$ = 3/4

Instead of true function f(x) = cos(x), we use truncated Taylor series such as $\hat{f}$(x) =1 − x^2/2

We obtain approximate result $\hat{f}(\hat{x})$ = 23/32

To four digits, true result is y = cos(π/4) ≈ 0.7071

Computational error:

$\hat{f}(\hat{x})$ − f($\hat{x}$) = 23/32− cos(3/4) ≈ -0.0129

Propagated data error:

f($\hat{x}$) − f(x) = cos(3/4) − cos(π/4) ≈ -0.0246

Total error:

$\hat{f}(\hat{x})$ − f(x) = -0.0129 − 0.0246 = -0.0375

2.

$f(x) = x^3 \quad \Rightarrow \quad f'(x) = 3x^2$

$$cond \approx \left| \frac{xf'(x)}{f(x)} \right| = \left| \frac{x(3x^2)}{x^3} \right| = 3$$

3.

IEEE DP (β = 2, p = 53, L = −1022, U = 1023)

$2(\beta-1)\beta^{p-1}(U-L+1)+1 = 2\times 1\times 2^{52}\times(1023+1022+1)+1 = 2^{53}\times 2046+1$

4. (a) $\beta^{U+1}(1-\beta^{-p}) = 2^2(1-2^{-3}) = \frac{7}{2}$

(b) $\beta^L = \frac{1}{2}$

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