

Handwriting problem

1. A의 열이 선형독립이면 해가 유일하다.

2. $x = [x_1 \ x_2 \ \dots \ x_n]^T$ 일 때,

$$(a) \ f(x) = b^T b = [b_1 \ b_2 \ \dots \ b_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = b_1^2 + b_2^2 + \dots + b_n^2 \text{에 대해 } x \text{에 대한 미분하면 } f'(x) = 0 \text{이다.}$$

$$(b) \ f(x) = x^T c = [x_1 \ x_2 \ \dots \ x_n] \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \text{에 대해 } x \text{에 대한 미분하면 } f'(x) = c_1 + c_2 + \dots + c_n = c$$

$$(c) \ f(x) = x^T M x = [x_1 \ x_2 \ \dots \ x_n] \begin{bmatrix} M_{11} & M_{12} & \dots & M_{1n} \\ M_{21} & M_{22} & \dots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \dots & M_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= [x_1 \ x_2 \ \dots \ x_n] \begin{bmatrix} M_{11}x_1 + M_{12}x_2 + \dots + M_{1n}x_n \\ M_{21}x_1 + M_{22}x_2 + \dots + M_{2n}x_n \\ \vdots \\ M_{n1}x_1 + M_{n2}x_2 + \dots + M_{nn}x_n \end{bmatrix}$$

$$= [x_1 \ x_2 \ \dots \ x_n] \begin{bmatrix} t_1 \\ \vdots \\ t_n \end{bmatrix}$$

에 대해 x 에 대한 미분하면 $f'(x) = t_1 + t_2 + \dots + t_n = Mx$

3. Q is orthogonal if $Q^T Q = I$

$$(a) \ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{이므로 orthogonal이 아니다.}$$

$$(b) \ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{이므로 orthogonal이다.}$$

$$(c) \ \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1/4 \end{bmatrix} \text{이므로 orthogonal이 아니다.}$$

$$(d) \ \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \sqrt{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 5 \end{bmatrix} \text{이므로 orthogonal이 아니다.}$$

4.

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$(i) \quad r_{1,1} = \|a_1\|_2 = \sqrt{1+0+0+1+1} = \sqrt{3},$$

$$q_1 = a_1/r_{1,1} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ 0 \end{bmatrix}$$

$$r_{1,2} = q_1^T a_2 = \left[\frac{1}{\sqrt{3}} \ 0 \ 0 - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \ 0 \right] \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = -\frac{1}{\sqrt{3}},$$

$$r_{1,3} = q_1^T a_3 = \left[\frac{1}{\sqrt{3}} \ 0 \ 0 - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \ 0 \right] \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = -\frac{1}{\sqrt{3}}$$

$$a_2 \Leftarrow a_2 - r_{1,2} q_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 1 \\ 0 \\ \frac{2}{3} \\ -\frac{1}{3} \\ -1 \end{bmatrix} \quad a_3 \Leftarrow a_3 - r_{1,3} q_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \\ -\frac{1}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix}$$

$$(ii) \quad r_{2,2} = \|a_2\|_2 = \sqrt{\frac{1}{9} + 1 + 0 + \frac{4}{9} + \frac{1}{9} + 1} = \sqrt{\frac{8}{3}},$$

$$q_2 = a_2/r_{2,2} = \begin{bmatrix} \frac{\sqrt{6}}{12} \\ \frac{\sqrt{6}}{4} \\ 0 \\ \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{12} \\ -\frac{\sqrt{6}}{4} \end{bmatrix}$$

$$r_{2,3} = q_2^T a_3 = \left[\frac{\sqrt{6}}{12} \ \frac{\sqrt{6}}{4} \ 0 \ \frac{\sqrt{6}}{6} - \frac{\sqrt{6}}{12} - \frac{\sqrt{6}}{4} \right] \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \\ -\frac{1}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} = \frac{\sqrt{6}}{36} + 0 + 0 - \frac{\sqrt{6}}{18} - \frac{\sqrt{6}}{18} - \frac{\sqrt{6}}{4} = -\frac{\sqrt{6}}{3}$$

$$a_3 \Leftarrow a_3 - r_{2,3}q_2 = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \\ -\frac{1}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} + \frac{\sqrt{6}}{3} \begin{bmatrix} \frac{\sqrt{6}}{12} \\ \frac{\sqrt{6}}{4} \\ 0 \\ \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{12} \\ -\frac{\sqrt{6}}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$(iii) \quad r_{3,3} = \|a_3\|_2 = \sqrt{\frac{1}{4} + \frac{1}{4} + 1 + 0 + \frac{1}{4} + \frac{1}{4}} = \sqrt{2} \qquad q_3 = a_3/r_{3,3} = \begin{bmatrix} \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} \end{bmatrix}$$

$$\therefore Q = [q_1 \, q_2 \, q_3] = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ -\frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix}, \quad R = \begin{bmatrix} \sqrt{3} - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{8}{3}} - \frac{\sqrt{6}}{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ -\frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix} \begin{bmatrix} \sqrt{3} - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{8}{3}} - \frac{\sqrt{6}}{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

Matlab problem

1.

명령 창

```
>> Q

Q =

    0.5774    0.2041    0.3536
         0    0.6124    0.3536
         0         0    0.7071
   -0.5774    0.4082   -0.0000
   -0.5774   -0.2041    0.3536
         0   -0.6124    0.3536

>> R

R =

    1.7321   -0.5774   -0.5774
         0    1.6330   -0.8165
         0         0    1.4142

>> Q*R

ans =

    1.0000   -0.0000    0.0000
         0    1.0000    0.0000
         0         0    1.0000
   -1.0000    1.0000    0.0000
   -1.0000    0.0000    1.0000
         0   -1.0000    1.0000
```

편집기 - C:\Users\kk\Desktop\자료\4학년\2학기\수치해석과 최적화\MATLAB\...

HW12_prob1.m

```
1 clearvars; close all; clc
2 A=[1 0 0;0 1 0;0 0 1;-1 1 0;-1 0 1;0 -1 1];
3 [m,n]=size(A);
4 R=zeros(3);
5
6 for k=1:n
7     R(k,k)=norm(A(:,k),2);
8     Q(:,k)=A(:,k)/R(k,k);
9     for j=k+1:n
10        R(k,j)=transpose(Q(:,k))*A(:,j);
11        A(:,j)=A(:,j)-R(k,j)*Q(:,k);
12    end
13 end
14 A=[1 0 0;0 1 0;0 0 1;-1 1 0;-1 0 1;0 -1 1];
```

작업 공간

이름	값
A	6x3 double
ans	6x3 double
j	[]
k	3
m	6
n	3
Q	6x3 double
R	[1.7321,-0.5774

fx >> |