1. (a) $MA = \begin{bmatrix} 1 & 0 \\ -\frac{1}{\varepsilon} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \varepsilon & 1 \\ 0 & 1-\frac{1}{\varepsilon} \end{bmatrix} \approx \begin{bmatrix} \varepsilon & 1 \\ 0 & -\frac{1}{\varepsilon} \end{bmatrix} = U$

($\lim_{\varepsilon \to 0^+} \varepsilon$ 이므로 $\lim_{\varepsilon \to 0^+} \left(1 - \frac{1}{\varepsilon}\right) = \lim_{\varepsilon \to 0^+} \left(-\frac{1}{\varepsilon}\right)$)

$L = M^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{1}{\varepsilon} & 1 \end{bmatrix}$

$LU = \begin{bmatrix} 1 & 0 \\ \frac{1}{\varepsilon} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon & 1 \\ 0 & -\frac{1}{\varepsilon} \end{bmatrix} = \begin{bmatrix} \varepsilon & 1 \\ 1 & 0 \end{bmatrix} \neq A$

(b) $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \varepsilon & 1 \end{bmatrix}$ (pivoting)

$MB = \begin{bmatrix} 1 & 0 \\ -\varepsilon & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \varepsilon & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -\varepsilon+1 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = U$

$L = M^{-1} = \begin{bmatrix} 1 & 0 \\ \varepsilon & 1 \end{bmatrix}$

$LU = \begin{bmatrix} 1 & 0 \\ \varepsilon & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \varepsilon & \varepsilon+1 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 \\ \varepsilon & 1 \end{bmatrix} = B$

2. 양변에 $(A-uv^T)$를 곱하면

(i) 좌변: $(A-uv^T)(A-uv^T)^{-1} = E$

(ii) 우변: $(A-uv^T)\left(A^{-1}+A^{-1}u(1-v^TA^{-1}u)^{-1}v^TA^{-1}\right)$

$$= AA^{-1} + AA^{-1}u(1-v^TA^{-1}u)^{-1}v^TA^{-1} - uv^TA^{-1} - uv^TA^{-1}u(1-v^TA^{-1}u)^{-1}v^TA^{-1}$$

$$= E - uv^TA^{-1} + (1-v^TA^{-1}u)^{-1}(uv^TA^{-1} - uv^TA^{-1}uv^TA^{-1})$$ ($\because (1-v^TA^{-1}u)$는 스칼라)

$$= E - uv^TA^{-1} + (1-v^TA^{-1}u)^{-1}(u(1-v^TA^{-1}u)v^TA^{-1})$$

$$= E - uv^TA^{-1} + (1-v^TA^{-1}u)^{-1}(1-v^TA^{-1}u)uv^TA^{-1}$$

$$= E - uv^TA^{-1} + uv^TA^{-1}$$

$$= E$$

3. (i) $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix} = A^T$ 이므로 symmetric

(ii) $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \neq 0$ 에 대하여

$$x^TAx = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 - x_2 - x_3 & -x_1 + 3x_2 - x_3 & -x_1 - x_2 + 5x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_1^2 - x_1x_2 - x_1x_3 - x_1x_2 + 3x_2^2 - x_2x_3 - x_1x_3 - x_2x_3 + 5x_3^2$$

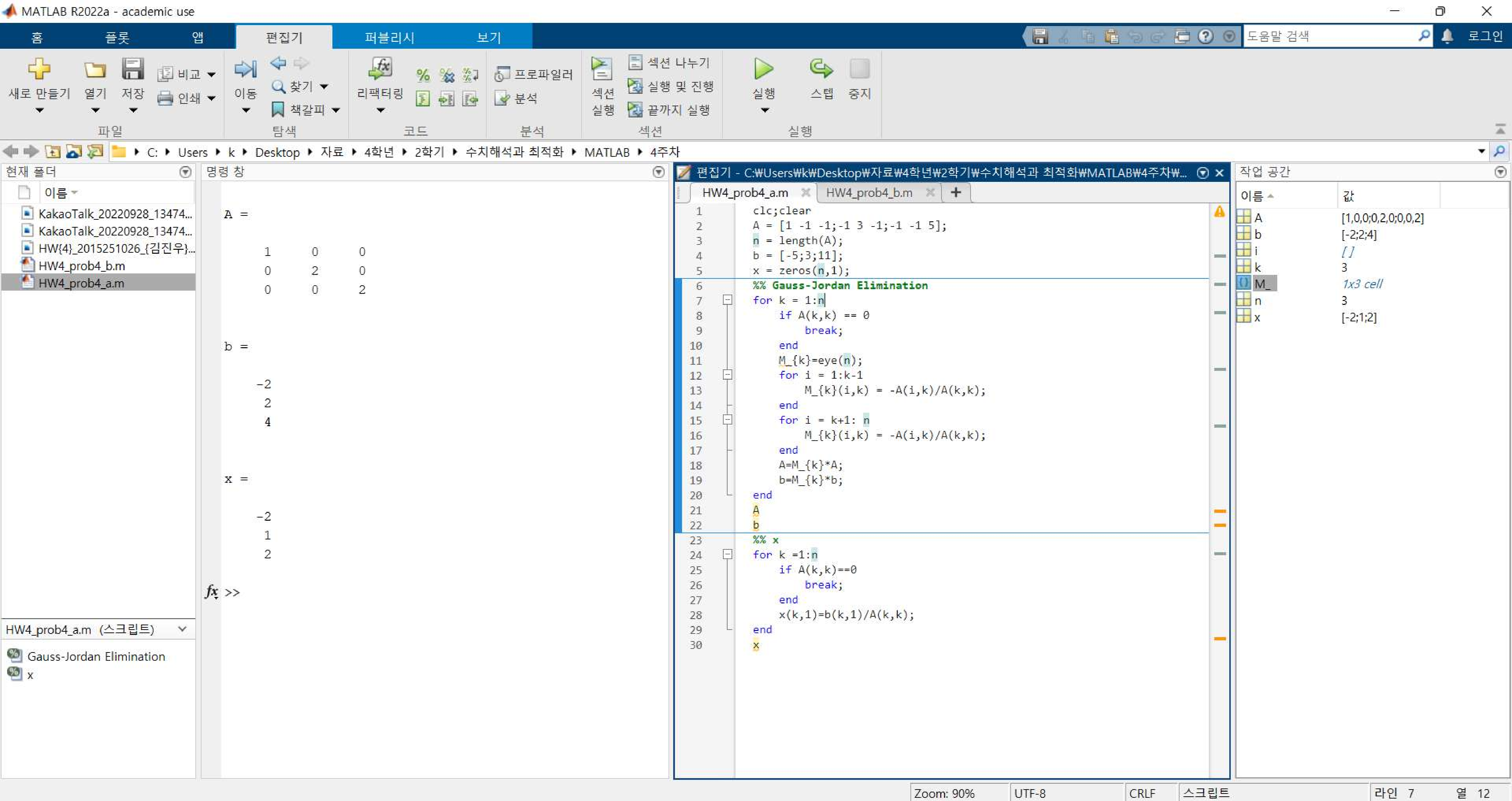
$$= x_1^2 + 3x_2^2 + 5x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_1x_3$$

$$= \left(\frac{\sqrt{2}}{2}x_1 - \sqrt{2}x_2\right)^2 + \left(\frac{\sqrt{2}}{2}x_1 - \sqrt{2}x_3\right)^2 + \left(\frac{\sqrt{2}}{2}x_2 - \sqrt{2}x_3\right)^2 + \frac{1}{2}x_2^2 + x_3^2$$

$$> 0$$

$\therefore$ positive definite

(i) (ii)에 의하여 A는 symmetric positive definite 이다.


MATLAB R2022a - academic use
홈
플롯
앱
편집기
퍼블리시
보기
도움말 검색
로그인
새로 만들기
열기
저장
비교
인쇄
파일
이동
찾기
책갈피
탐색
리팩터링
코드
프로파일러
분석
분석
섹션 실행
섹션 나누기
실행 및 진행
끝까지 실행
섹션
실행
스텝
중지
실행
C: ▸ Users ▸ k ▸ Desktop ▸ 자료 ▸ 4학년 ▸ 2학기 ▸ 수치해석과 최적화 ▸ MATLAB ▸ 4주차
현재 폴더
이름
KakaoTalk_20220928_13474...
KakaoTalk_20220928_13474...
HW{4}_2015251026_{김진우}...
HW4_prob4_b.m
HW4_prob4_a.m
HW4_prob4_a.m (스크립트)
Gauss-Jordan Elimination
x
명령 창
A =
1 0 0
0 2 0
0 0 2
b =
-2
2
4
x =
-2
1
2
>>
편집기 - C:₩Users₩k₩Desktop₩자료₩4학년₩2학기₩수치해석과 최적화₩MATLAB₩4주차₩...
HW4_prob4_a.m
HW4_prob4_b.m
clc;clear
A = [1 -1 -1;-1 3 -1;-1 -1 5];
n = length(A);
b = [-5;3;11];
x = zeros(n,1);
%% Gauss-Jordan Elimination
for k = 1:n
if A(k,k) == 0
break;
end
M_{k}=eye(n);
for i = 1:k-1
M_{k}(i,k) = -A(i,k)/A(k,k);
end
for i = k+1: n
M_{k}(i,k) = -A(i,k)/A(k,k);
end
A=M_{k}*A;
b=M_{k}*b;
end
A
b
%% x
for k =1:n
if A(k,k)==0
break;
end
x(k,1)=b(k,1)/A(k,k);
end
x
작업 공간
이름
값
A
[1,0,0;0,2,0;0,0,2]
b
[-2;2;4]
i
[]
k
3
M_
1x3 cell
n
3
x
[-2;1;2]
Zoom: 90%
UTF-8
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스크립트
라인 7
열 12

MATLAB R2022a - academic use

편집기 - C:\Users\k\Desktop\자료\4학년\2학기\수치해석과 최적화\MATLAB\4주차\...

HW4_prob4_a.m | HW4_prob4_b.m

```
clc;clear
A = [1 -1 -1;-1 3 -1;-1 -1 5];
n = length(A);
b = [-5;3;11];
x = zeros(n,1);
%% Cholesky factorization
for k=1:n
    A(k,k)=sqrt(A(k,k));
    for i=k+1:n
            A(i,k)=A(i,k)/A(k,k);
    end
    for j=k+1:n
        for i=j:n
            A(i,j)=A(i,j)-A(i,k)*A(j,k);
        end
    end
end
L=tril(A); %하부만 가져온다.
L
%% x
U=L.';
M=inv(L);
b=M*b;

%Ux=Mb
for j = n:-1:1
    if U(j,j) == 0
        break;
    end
    x(j,1) = b(j,1)/U(j,j);
    for i=1:(j-1)
M1 = [1 0 0;
    -U(2,1)/U(1,1) 1 0;
    -U(3,1)/U(1,1) 0 1];
        b(i,1)=b(i,1)-U(i,j)*x(j,1);
    end
end
x
```

명령 창

```
L =

    1.0000         0         0
   -1.0000    1.4142         0
   -1.0000   -1.4142    1.4142


x =

   -2.0000
    1.0000
    2.0000

>>
```

작업 공간

이름	값
A	[1,-1,-1;-1,1.4142,...
b	[-2.0000;1.4142;2...
i	[]
j	1
k	3
L	[1,0,0;-1,1.4142,0;...
M	[1,0,0;0.7071,0.70...
M1	[1,0,0;0,1,0;0,0,1]
n	3
U	[1,-1,-1;0,1.4142,-...
x	[-2.0000;1.0000;2...

현재 폴더: KakaoTalk_20220928_13474..., KakaoTalk_20220928_13474..., HW{4}_2015251026_{김진우}..., HW4_prob4_b.m, HW4_prob4_a.m, 20220928_135956.png

Zoom: 90% | UTF-8 | CRLF | 스크립트 | 라인 17 | 열 4