

$$1. f(x) = x_1^3 + 2x_1^2 + 4x_1x_2 + x_2^2 + 2x_2 + 3$$

$$\nabla f(x) = \begin{bmatrix} 3x_1^2 + 4x_1 + 4x_2 \\ 2x_2 + 4x_1 + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1^2 + 4x_1 + 4x_2 = 0$$

$$2x_2 + 4x_1 + 2 = 0 \Rightarrow 2x_2 = -4x_1 - 2 \text{ 을 대입}$$

$$3x_1^2 + 4x_1 + 4x_2 = 0$$

$$\Rightarrow 3x_1^2 + 4x_1 - 8x_1 - 4 = 0$$

$$\Rightarrow 3x_1^2 - 4x_1 - 4 = 0$$

$$\Rightarrow (x_1 - 2)(3x_1 + 2) = 0$$

$$\Rightarrow x_1 = 2, -\frac{2}{3}$$

$$x_2 = -5, -\frac{1}{3}$$

$$\text{critical points : } (2, -5), (-\frac{2}{3}, -\frac{1}{3})$$

$$H_f(x) = \begin{bmatrix} 6x_1 + 4 & 4 \\ 4 & 2 \end{bmatrix}$$

$$\text{i) } H_f(2, -5) = \begin{bmatrix} 16 & 4 \\ 4 & 2 \end{bmatrix}$$

$$(\lambda - 16)(\lambda - 2) - 16 = 0$$

$$\Rightarrow \lambda - 18\lambda + 16 = 0$$

$$\lambda = 9 \pm \sqrt{65} \text{ 이므로 } (2, -5) \text{ 는 local minimum 이다.}$$

$$\text{ii) } H_f(-\frac{2}{3}, -\frac{1}{3}) = \begin{bmatrix} 0 & 4 \\ 4 & 2 \end{bmatrix}$$

$$\lambda(\lambda - 2) - 16 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda - 16 = 0$$

$$\lambda = 1 \pm \sqrt{17} \text{ 이므로 } (-\frac{2}{3}, -\frac{1}{3}) \text{ 는 local minimum 이 아니다.}$$

따라서, $(2, -5)$ 가 local minimum of f 이다.

$$2. f(x) = 2x_1^2 + 5x_2^2 \text{ subject to } h(x) = x_1 - x_2 + 7 \leq 0$$

$$\nabla f(x) = \begin{bmatrix} 4x_1 \\ 10x_2 \end{bmatrix} \quad J_h(x) = [1 \quad -1]$$

$$\nabla_x \iota(x, \lambda) = f(x) + J_h^T(x) \lambda = \begin{bmatrix} 4x_1 \\ 10x_2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} \nabla_x \iota(x, \lambda) \\ h(x) \end{bmatrix} = \begin{bmatrix} 4x_1 + \lambda \\ 10x_2 - \lambda \\ x_1 - x_2 + 7 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 1 \\ 0 & 10 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -7 \end{bmatrix}$$

$$x_1 = x_2 - 7 \quad x_1 = x_2 - 7 \quad x_1 = -5$$

$$4x_1 + \lambda = 0 \Rightarrow 4x_2 - 28 + \lambda = 0 \Rightarrow x_2 = 2$$

$$10x_2 - \lambda = 0 \quad 10x_2 - \lambda = 0 \quad \lambda = 20$$

3.

(a)

명령 창

```

-0.6361
-1.9198

fx >>

```

편집기 - C:\Users\Wk\Desktop\자료W4학년W2학기W수치해석과 최적화\WMATLABW11주차\HW11_prob3_a.m

```

1 clc; clear;
2
3 a=-2;
4 b=0;
5 r=(5*0.5-1)/2;
6
7 x1=a+(1-r)*(b-a);
8 x2=a+r*(b-a);
9
10 f1=fnc_f(x1);
11 f2=fnc_f(x2);
12
13 tol=1e-6;
14 while (b-a)>tol
15     if f1>f2
16         a=x1;
17         x1=x2;
18         f1=f2;
19         x2=a+r*(b-a);
20         f2=fnc_f(x2);
21     else
22         b=x1;
23         x2=x1;
24         f2=f1;
25         x1=a+(1-r)*(b-a);
26         f1=fnc_f(x1);
27     end
28 end
29 disp(x1)
30 disp(f1)
31
32 function f=fnc_f(x)
33     f=5*x*exp(-x^2)+0.5*x^2;
34 end
35

```

작업 공간

이름	값
a	-0.6361
b	-0.6361
f1	-1.9198
f2	-1.9198
r	0.6180
tol	1.0000e-06
x1	-0.6361
x2	-0.6361

(b)

명령 창

```

-0.6361
-1.9198

fx >>

```

편집기 - C:\Users\Wk\Desktop\자료W4학년W2학기W수치해석과 최적화\WMATLABW11주차\HW11_prob3_b.m

```

1 clc; clear;
2
3 x=-2;
4 for k=1:10
5     x=fnc_g(x)
6 end
7
8
9 function xk=fnc_g(x)
10     xk = x - (5*exp(-x^2) - 10*(x^2)*exp(-x^2) + x) / (-10*x*exp(-x^2) - 20*x*exp(-x^2) + 20*(x^3)*exp(-x^2) + 1);
11 end

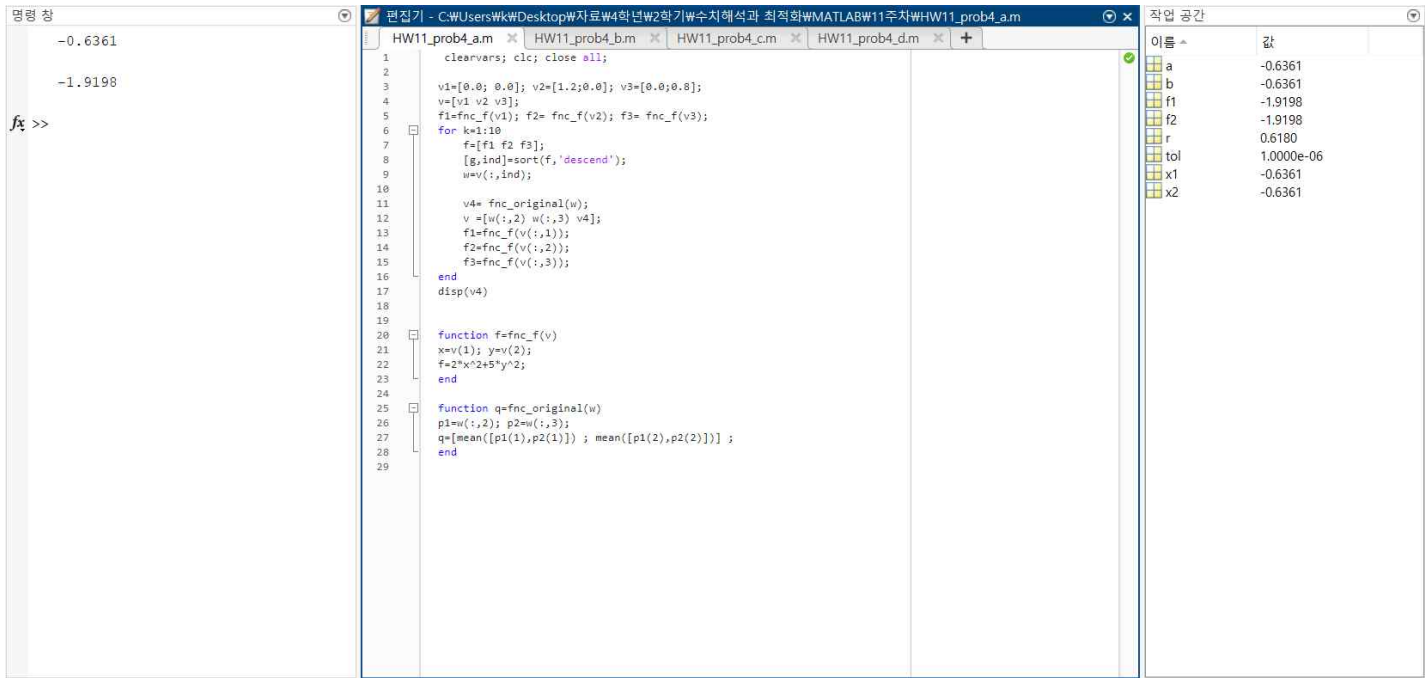
```

작업 공간

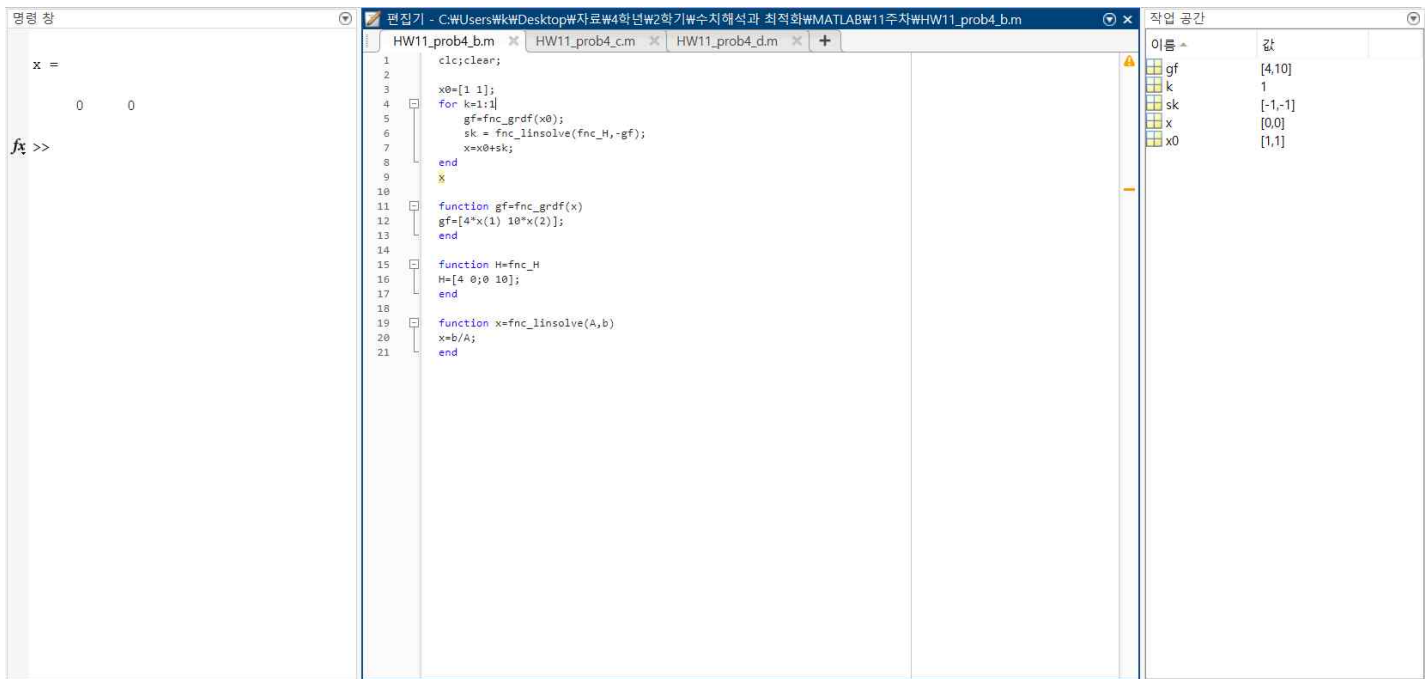
이름	값
a	-0.6361
b	-0.6361
f1	-1.9198
f2	-1.9198
r	0.6180
tol	1.0000e-06
x1	-0.6361
x2	-0.6361

4.

(a)



(b)



(c)

The MATLAB interface displays the code for `HW11_prob4_c.m` in the editor. The code defines a function `fnc_gf` and a function `fnc_falpha`. The main script calculates `xnew` based on the function `fnc_gf` and the function `fnc_falpha`.

```
1 clc;clear;
2
3 x0=[5 1];
4 xold=x0;
5
6 for k=1:10
7     sk=fnc_gf(xold);
8     ak= fnc_falpha(xold, 1);
9     xnew=xold+ak*sk;
10    xold=xnew;
11 end
12 xnew
13
14 function gf=fnc_gf(x)
15     gf=[4*x(1) 10*x(2)];
16 end
17
18 function ak=fnc_falpha(x,a)
19     x1=x(1); x2=x(2);
20     for k=1:10
21         bj=16*x1^2*(4*a-1)+100*x2^2*(10*a-1);
22         bm=64*x1^2+1000*x2^2;
23         a=bj/bm;
24     end
25     ak=a;
26 end
```

The Command Window shows the output of the script:

```
xnew =
    1.0e-03 *
    0.1580    0.0316
```

The Workspace window shows the variables:

이름	값
ak	0.1136
k	10
sk	[-0.0012,0.0023]
x0	[5,1]
xnew	[1.5800e-04,3.16...
xold	[1.5800e-04,3.16...

(d)

The MATLAB interface displays the code for `HW11_prob4_d.m` in the editor. The code defines a function `fnc_gf` and a function `fnc_falpha`. The main script calculates `xk` based on the function `fnc_gf` and the function `fnc_falpha`.

```
1 clc;clear;
2
3 x0=[1 1];
4 xk=x0;
5 gk= fnc_gf(xk);
6 sk=gk;
7
8 for k=1:10
9     ak= fnc_falpha(xk, 1);
10    xk=xk+ak*sk;
11    bk = gk'*transpose(gk);
12    gk=fnc_gf(xk);
13    bk = (gk'*transpose(gk))/bk;
14    sk = -gk/bk*sk;
15 end
16 xk
17
18 function gf=fnc_gf(x)
19     gf=[4*x(1) 10*x(2)];
20 end
21
22 function ak=fnc_falpha(x,a)
23     x1=x(1); x2=x(2);
24     for k=1:10
25         bj=16*x1^2*(4*a-1)+100*x2^2*(10*a-1);
26         bm=64*x1^2+1000*x2^2;
27         a=bj/bm;
28     end
29     ak=a;
30 end
```

The Command Window shows the output of the script:

```
xk =
    1.0e-05 *
   -0.0878   -0.1290
```

The Workspace window shows the variables:

이름	값
ak	0.1015
bk	0.0535
gk	[-3.5103e-06,-1.2...
k	10
sk	[4.2409e-06,1.52...
x0	[1,1]
xk	[-8.7758e-07,-1.2...