Handwriting problem
Give an answer and prove it or show the calculation process in detail.

1. (Handwriting Problem) Let A be an m × n matrix with m > n.
(a) What is the maximum number of nonzero singular values that A can have?

The number of nonzero singular values of A equals the rank of A^TA.

$A^TA = n \times n$ matrix이므로 $rank(A^TA) \leq n$이다.
따라서 $rank(A) = n$일 때, the maximum number of nonzero singular values이다.

(b) If rank(A) = k, how many nonzero singular values does A have?
rank(A)=k일 때, A는 k개의 nonzero singular values를 갖는다.

2. (Handwriting Problem) Determine the Householder transformation, i.e. nd H or v, that
(a) Annihilates all but the first entry of the vector [1 1 1 1]T

$$v_1 = a_1 - \alpha_1 e_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} - \begin{bmatrix}\alpha_1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} - \begin{bmatrix}-2\\0\\0\\0\end{bmatrix} = \begin{bmatrix}3\\1\\1\\1\end{bmatrix}$$

$$H_1 = I - 2\frac{v_1 v_1^T}{v_1^T v_1} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} - \frac{1}{6}\begin{bmatrix}9&3&3&3\\3&1&1&1\\3&1&1&1\\3&1&1&1\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}\\-\frac{1}{2}&\frac{5}{6}&-\frac{1}{6}&-\frac{1}{6}\\-\frac{1}{2}&-\frac{1}{6}&\frac{5}{6}&-\frac{1}{6}\\-\frac{1}{2}&-\frac{1}{6}&-\frac{1}{6}&\frac{5}{6}\end{bmatrix}$$

(b) Annihilates all but the first two entry of the vector [1 1 1 1]T

$$v_2 = \hat{a_2} - \alpha_2 e_2 = \begin{bmatrix}0\\1\\1\\1\end{bmatrix} - \begin{bmatrix}0\\\alpha_2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\1\\1\end{bmatrix} - \begin{bmatrix}0\\-\sqrt{3}\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1+\sqrt{3}\\1\\1\end{bmatrix} \approx \begin{bmatrix}0\\2.7\\1\\1\end{bmatrix}$$

$$H_2 = I - 2\frac{v_2 v_2^T}{v_2^T v_2} = \begin{bmatrix}1&0&0&0\\0&-0.57&-0.57&-0.57\\0&-0.57&0.78&-0.21\\0&-0.57&-0.21&0.78\end{bmatrix}$$

Matlab problem

Give an answer and provide the Matlab code.

1. (Matlab Problem) We are going to solve the linear system Ax = b where

명령 창

```
x1 =

    0.4562
   -0.2294
   -0.0717
   -0.3041


x2 =

    0.4562
   -0.2294
   -0.0717
   -0.3041


x3 =

   -0.0241
   -0.0089
    0.0544
   -0.0707

fx >>
```

편집기 - C:₩Users₩k₩Desktop₩자료₩4학년₩2학기₩수치해석과 최적화₩MATLAB₩13₩HW13_prob1.m

HW13_prob1.m

```
clearvars; close all; clc

A=[2 1 -2 5; -3 -1 2 -4;-1 1 -1 1;3 -1 2 -5;1 2 1 3; -1 -2 -5 1;4 3 -3 2;2 -3 -3 2];
b=[1;4;-5;1;-2;-1;3;2];
x1=gs_fnc(A,b)
x2=h_func(A,b)
x3=svd_func(A,b)

% Gram-Suhmidt
function x=gs_fnc(A,b)
    [m,n]=size(A);
    R=zeros(n);
    for k=1:n
        Q(:,k)=A(:,k);
        for j=1:k-1
            R(j,k)=transpose(Q(:,j))*A(:,k);
            Q(:,k)=Q(:,k)-R(j,k)*Q(:,j);
        end
        R(k,k)=norm(Q(:,k),2);
        Q(:,k)=Q(:,k)/R(k,k);
    end
    b=Q'*b;
    x=bs(R,b);
end

% Householder QR Factorization
function x=h_func(A, b)
    [m, n]=size(A);
    I=eye(m);
    for k=1:n
        alpha(k)=-sign(A(k,k))*norm(A(k:m,k),2);
        ek=I(:,k);
        v(:,k)=[zeros(k-1,1); A(k:m,k)]-alpha(k)*ek;
        beta(k)=norm(v(:,k),2)^2;
        if beta(k) ~= 0
            for j=k:n
                gamma(j)=dot(v(:,k),A(:,j));
                A(:,j)=A(:,j)-(2*gamma(j)/beta(k))*v(:,k);
            end
            b=b-(2*dot(v(:,k),b)/beta(k))*v(:,k);
        end
    end
    x=bs(A,b);
```

(a) Find x using Gram-Schmidt QR Factorization with backward substitution. [10]

```
x1 =

    0.4562
   -0.2294
   -0.0717
   -0.3041
```

(b) Find x using Householder QR Factorization with backward substitution. [10]

```
x2 =

    0.4562
   -0.2294
   -0.0717
   -0.3041
```

(c) Find x using singular value decomposition (SVD). [10]

```
x3 =

   -0.0241
   -0.0089
    0.0544
   -0.0707
```

Don't use Matlab built-in function for matrix inversion, e.g. pinv and back-slash.

Don't use Matlab built-in function for QR factorization, e.g. qr.

You can use Matlab built-in function for SVD, e.g. svd.