Handwriting problem

1. A의 열이 선형독립이면 해가 유일하다.

2. $x=[x_1\ \ x_2\ \ \dots\ \ x_n]^T$일 때,

(a) $f(x)=b^Tb=\begin{bmatrix} b_1\, b_2\, ...\, b_n \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_n \end{bmatrix}=b_1^2+b_2^2+\dots+b_n^2$에 대해 x에 대한 미분하면 $f'(x)=0$이다.

(b) $f(x)=x^Tc=[x_1\ \ x_2\ \ \dots\ \ x_n]\begin{bmatrix} c_1 \\ c_2 \\ . \\ . \\ . \\ c_n \end{bmatrix}=c_1x_1+c_2x_2+\dots+c_nx_n$에 대해 x에 대한 미분하면 $f'(x)=c_1+c_2+\dots+c_n=c$

(c) $f(x)=x^TMx=\begin{bmatrix} x_1\, x_2\, ...\, x_n \end{bmatrix}\begin{bmatrix} M_{11}\ M_{12}\ ...\ M_{1n} \\ M_{21}\ M_{22}\ ...\ M_{2n} \\ .\quad\ .\ \ \cdots\ \ . \\ .\quad\ .\ \ \cdots\ \ . \\ .\quad\ .\ \ \cdots\ \ . \\ M_{n1}\ M_{n2}\ ...\ M_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix}$

$=\begin{bmatrix} x_1\, x_2\, ...\, x_n \end{bmatrix}\begin{bmatrix} M_{11}x_1+M_{12}x_2+\dots+M_{1n}x_n \\ M_{21}x_1+M_{22}x_2+\dots+M_{2n}x_n \\ . \\ . \\ . \\ M_{n1}x_1+M_{n2}x_2+\dots+M_{nn}x_n \end{bmatrix}$

$=\begin{bmatrix} x_1\, x_2\, ...\, x_n \end{bmatrix}\begin{bmatrix} t_1 \\ \ \end{bmatrix}$

에 대해 x에 대한 미분하면 $f'(x)=t_1+t_2+\dots+t_n=Mx$

3. Q is orthogonal if $Q^TQ=I$

(a) $\begin{bmatrix} 1\,1 \\ 1\,0 \end{bmatrix}\begin{bmatrix} 1\,1 \\ 1\,0 \end{bmatrix}=\begin{bmatrix} 2\,1 \\ 1\,1 \end{bmatrix}$이므로 orthogonal이 아니다.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}=\begin{bmatrix} 1\,0 \\ 0\,1 \end{bmatrix}$이므로 orthogonal이다.

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}=\begin{bmatrix} 4 & 0 \\ 0 & 1/4 \end{bmatrix}$이므로 orthogonal이 아니다.

(d) $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \sqrt{2} & \frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{2}}{2} & \sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}=\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{2} \end{bmatrix}$이므로 orthogonal이 아니다.

4.

$$a_1=\begin{bmatrix}1\\0\\0\\-1\\-1\\0\end{bmatrix},\ a_2=\begin{bmatrix}0\\1\\0\\1\\0\\-1\end{bmatrix},\ a_3=\begin{bmatrix}0\\0\\1\\0\\1\\1\end{bmatrix}$$

(i) $r_{1,1}=\|a_1\|_2=\sqrt{1+0+0+1+1}=\sqrt{3}\,,$ $\qquad q_1=a_1/r_{1,1}=\begin{bmatrix}\frac{1}{\sqrt{3}}\\0\\0\\-\frac{1}{\sqrt{3}}\\-\frac{1}{\sqrt{3}}\\0\end{bmatrix}$

$$r_{1,2}=q_1^Ta_2=\left[\frac{1}{\sqrt{3}}\ 0\ 0-\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}}\ 0\right]\begin{bmatrix}0\\1\\0\\1\\0\\-1\end{bmatrix}=-\frac{1}{\sqrt{3}}\,,$$

$$r_{1,3}=q_1^Ta_3=\left[\frac{1}{\sqrt{3}}\ 0\ 0-\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}}\ 0\right]\begin{bmatrix}0\\0\\1\\0\\1\\1\end{bmatrix}=-\frac{1}{\sqrt{3}}$$

$$a_2\Leftarrow a_2-r_{1,2}q_1=\begin{bmatrix}0\\1\\0\\1\\0\\-1\end{bmatrix}+\frac{1}{\sqrt{3}}\begin{bmatrix}\frac{1}{\sqrt{3}}\\0\\0\\-\frac{1}{\sqrt{3}}\\-\frac{1}{\sqrt{3}}\\0\end{bmatrix}=\begin{bmatrix}\frac{1}{3}\\1\\0\\\frac{2}{3}\\-\frac{1}{3}\\-1\end{bmatrix}\qquad a_3\Leftarrow a_3-r_{1,3}q_1=\begin{bmatrix}0\\0\\1\\0\\1\\1\end{bmatrix}+\frac{1}{\sqrt{3}}\begin{bmatrix}\frac{1}{\sqrt{3}}\\0\\0\\-\frac{1}{\sqrt{3}}\\-\frac{1}{\sqrt{3}}\\0\end{bmatrix}=\begin{bmatrix}\frac{1}{3}\\0\\1\\-\frac{1}{3}\\\frac{2}{3}\\1\end{bmatrix}$$

(ii) $r_{2,2}=\|a_2\|_2=\sqrt{\frac{1}{9}+1+0+\frac{4}{9}+\frac{1}{9}+1}=\sqrt{\frac{8}{3}}\,,$ $\qquad q_2=a_2/r_{2,2}=\begin{bmatrix}\frac{\sqrt{6}}{12}\\\frac{\sqrt{6}}{4}\\0\\\frac{\sqrt{6}}{6}\\-\frac{\sqrt{6}}{12}\\-\frac{\sqrt{6}}{4}\end{bmatrix}$

$$r_{2,3}=q_2^Ta_3=\left[\frac{\sqrt{6}}{12}\ \frac{\sqrt{6}}{4}\ 0\ \frac{\sqrt{6}}{6}-\frac{\sqrt{6}}{12}-\frac{\sqrt{6}}{4}\right]\begin{bmatrix}\frac{1}{3}\\0\\1\\-\frac{1}{3}\\\frac{2}{3}\\1\end{bmatrix}=\frac{\sqrt{6}}{36}+0+0-\frac{\sqrt{6}}{18}-\frac{\sqrt{6}}{18}-\frac{\sqrt{6}}{4}=-\frac{\sqrt{6}}{3}$$

$$a_3 \Leftarrow a_3 - r_{2,3}q_2 = \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \\ -\frac{1}{3} \\ \frac{2}{3} \\ 1 \end{bmatrix} + \frac{\sqrt{6}}{3} \begin{bmatrix} \frac{\sqrt{6}}{12} \\ \frac{\sqrt{6}}{4} \\ 0 \\ \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{12} \\ -\frac{\sqrt{6}}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

(iii) $r_{3,3} = \| a_3 \|_2 = \sqrt{\frac{1}{4} + \frac{1}{4} + 1 + 0 + \frac{1}{4} + \frac{1}{4}} = \sqrt{2}$ $\quad q_3 = a_3 / r_{3,3} = \begin{bmatrix} \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} \end{bmatrix}$

$$\therefore Q = [q_1\, q_2\, q_3] = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ -\frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix}, \quad R = \begin{bmatrix} \sqrt{3} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{8}{3}} & -\frac{\sqrt{6}}{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ -\frac{1}{\sqrt{3}} & \frac{\sqrt{6}}{6} & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{\sqrt{6}}{12} & \frac{\sqrt{2}}{4} \\ 0 & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix} \begin{bmatrix} \sqrt{3} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{8}{3}} & -\frac{\sqrt{6}}{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

Matlab problem

1.