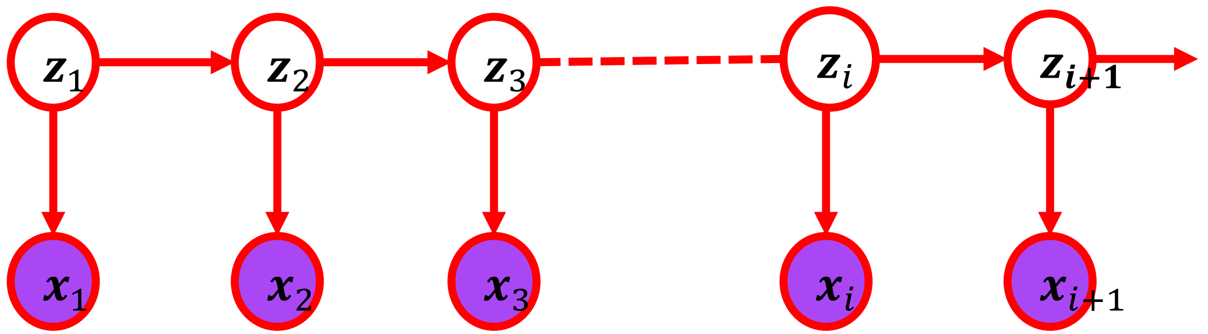


$$p(\boldsymbol{z}_1)p(\boldsymbol{z}_2|\boldsymbol{z}_1)p(\boldsymbol{z}_3|\boldsymbol{z}_2)\cdots p(\boldsymbol{z}_{i+1}|\boldsymbol{z}_i)\cdots p(\boldsymbol{z}_N|\boldsymbol{z}_{N-1})$$

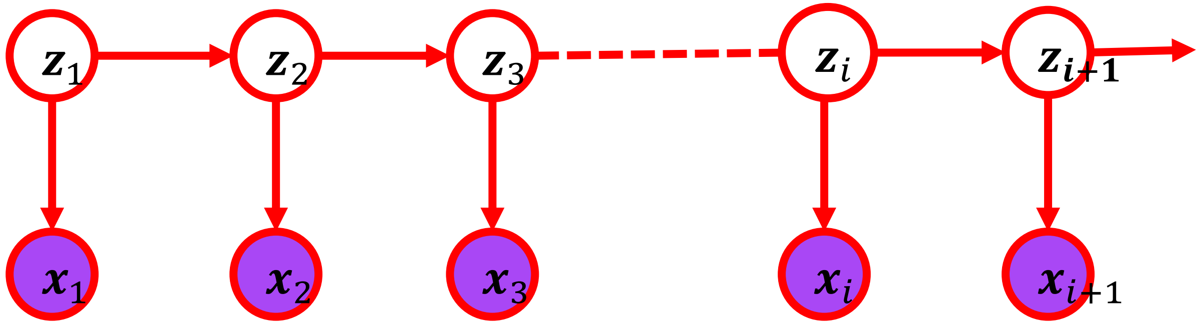
$$p(\boldsymbol{x}_1|\boldsymbol{z}_1)p(\boldsymbol{x}_2|\boldsymbol{z}_2)\cdots p(\boldsymbol{x}_i|\boldsymbol{z}_i)\cdots p(\boldsymbol{x}_N|\boldsymbol{z}_N)$$

$$p(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_N,\boldsymbol{z}_1,\cdots,\boldsymbol{z}_N) = p(\boldsymbol{z}_1)\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1})\right]\left[\prod_{i=1}^{N} p(\boldsymbol{x}_i|\boldsymbol{z}_i)\right]$$

$$p(\boldsymbol{X},\boldsymbol{Z}) = p(\boldsymbol{z}_1)\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1})\right]\left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j)\right]$$

$$p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta}) = p(\boldsymbol{z}_1|\boldsymbol{\pi})\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1},\boldsymbol{A})\right]\left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j,\boldsymbol{\phi})\right] \qquad \boldsymbol{\theta} = \{\boldsymbol{\pi},\boldsymbol{A},\boldsymbol{\phi}\}$$

HMM EM 알고리즘



$$\ln p(\boldsymbol{X}|\boldsymbol{\theta}) = \sum_{n=1}^{N} \ln p(\boldsymbol{x}_n|\boldsymbol{\theta}) = \sum_{n=1}^{N} \ln \sum_{\boldsymbol{z}_n} p(\boldsymbol{x}_n, \boldsymbol{z}_n|\boldsymbol{\theta})$$

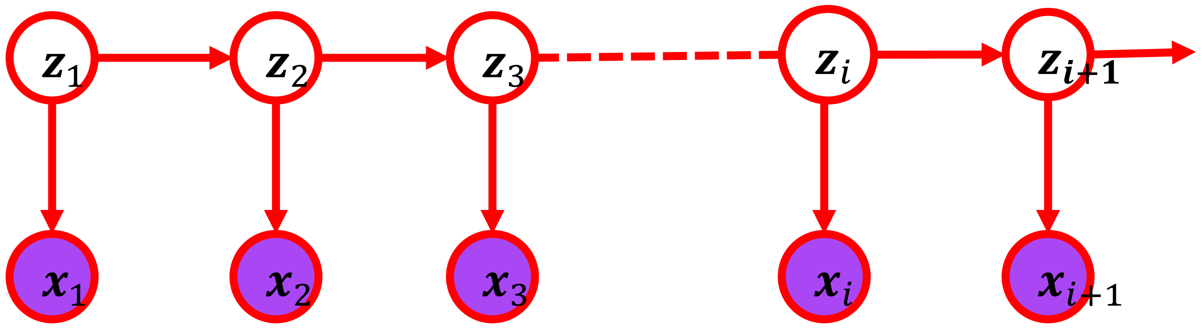
$$= \sum_{n=1}^{N} \sum_{\boldsymbol{z}_n} q(\boldsymbol{z}_n) \ln \left\{ \frac{p(\boldsymbol{x}_n, \boldsymbol{z}_n|\boldsymbol{\theta})}{q(\boldsymbol{z}_n)} \right\} - \sum_{\boldsymbol{z}_n} \sum_{n=1}^{N} q(\boldsymbol{z}_n) \ln \left\{ \frac{p(\boldsymbol{z}_n|\boldsymbol{x}_n, \boldsymbol{\theta})}{q(\boldsymbol{z}_n)} \right\}$$

E-step : $KL(q||p) = 0$ $\Rightarrow$ $q(\boldsymbol{z}_n) = p\left(\boldsymbol{z}_n \middle| \boldsymbol{x}_n, \boldsymbol{\theta}^{old}\right)$ $q(\boldsymbol{z}_n)$를 정의?

M-step : $\boldsymbol{\theta}^{new} = \underset{\boldsymbol{\theta}}{\arg\max}\, \mathcal{L}(q, \boldsymbol{\theta})$

$$\mathcal{L}(q, \boldsymbol{\theta}) = \sum_{n=1}^{N} \sum_{\boldsymbol{z}_n} p\left(\boldsymbol{z}_n \middle| \boldsymbol{x}_n, \boldsymbol{\theta}^{old}\right) \ln p(\boldsymbol{x}_n, \boldsymbol{z}_n|\boldsymbol{\theta}) + const$$

HMM EM 알고리즘



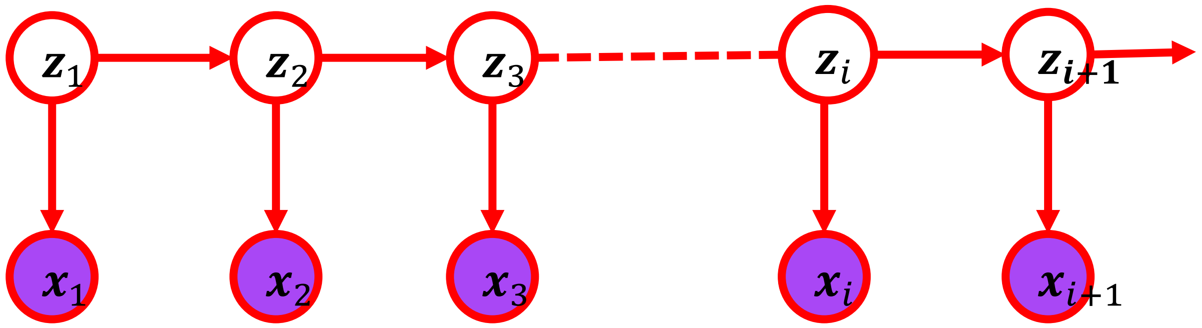
$$\ln p(\boldsymbol{X}|\boldsymbol{\theta}) = \sum_{n=1}^{N} \ln p(\boldsymbol{x_n}|\boldsymbol{\theta}) = \sum_{n=1}^{N} \ln \sum_{\boldsymbol{z_n}} p(\boldsymbol{x_n}, \boldsymbol{z_n}|\boldsymbol{\theta})$$

$$= \sum_{n=1}^{N} \sum_{\boldsymbol{z_n}} q(\boldsymbol{z_n}) \ln \left\{ \frac{p(\boldsymbol{x_n}, \boldsymbol{z_n}|\boldsymbol{\theta})}{q(\boldsymbol{z_n})} \right\} - \sum_{\boldsymbol{z_n}} \sum_{n=1}^{N} q(\boldsymbol{z_n}) \ln \left\{ \frac{p(\boldsymbol{z_n}|\boldsymbol{x_n}, \boldsymbol{\theta})}{q(\boldsymbol{z_n})} \right\}$$

$$p(\boldsymbol{X}, \boldsymbol{Z}) = p(\boldsymbol{z}_1) \left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}) \right] \left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j) \right]$$

$$\ln p(\boldsymbol{X}, \boldsymbol{Z}) = \ln p(\boldsymbol{z}_1) + \sum_{i=2}^{N} \ln p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}) + \sum_{j=1}^{N} \ln p(\boldsymbol{x}_j|\boldsymbol{z}_j)$$

HMM EM 알고리즘



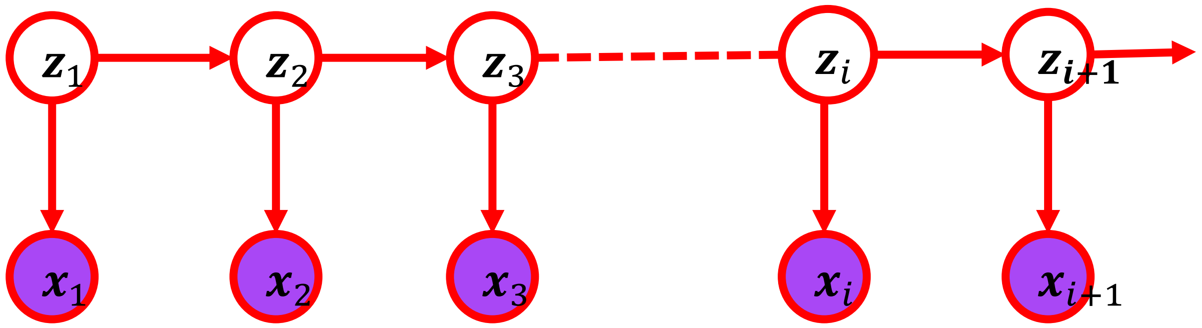
$$p(\boldsymbol{X},\boldsymbol{Z}) = p(\boldsymbol{z}_1)\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1})\right]\left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j)\right]$$

$$\ln\left\{\sum_{\boldsymbol{Z}} p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta})\right\} = \ln\left\{\sum_{\boldsymbol{Z}} p(\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_N,\boldsymbol{z}_1,\boldsymbol{z}_2,\cdots,\boldsymbol{z}_N|\boldsymbol{\theta})\right\} = \ln\left\{\sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p(\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_N,\boldsymbol{z}_1,\boldsymbol{z}_2,\cdots,\boldsymbol{z}_N|\boldsymbol{\theta})\right\}$$

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}^{old}) = \sum_{Z} p(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^{old}) \ln p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta})$$

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}^{old}) = \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^{old}) \ln\left\{p(\boldsymbol{z}_1|\boldsymbol{\theta})\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1},\boldsymbol{\theta})\right]\left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j,\boldsymbol{\theta})\right]\right\}$$

HMM EM 알고리즘



$$\gamma(\boldsymbol{z}_n) = p\left(\boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

$$\xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) = p\left(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{old}) = \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right) \ln\left\{p(\boldsymbol{z}_1|\boldsymbol{\theta})\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}, \boldsymbol{\theta})\right]\left[\prod_{j=1}^{N} p\left(\boldsymbol{x}_j\middle|\boldsymbol{z}_j, \boldsymbol{\theta}\right)\right]\right\}$$

$$= \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)\left\{\ln p(\boldsymbol{z}_1|\boldsymbol{\theta}) + \sum_{i=2}^{N}\ln p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}, \boldsymbol{\theta}) + \sum_{j=1}^{N}\ln p\left(\boldsymbol{x}_j\middle|\boldsymbol{z}_j, \boldsymbol{\theta}\right)\right\}$$

$$= \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_1|\boldsymbol{\theta}) + \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)\sum_{i=2}^{N}\ln p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}, \boldsymbol{\theta})$$

$$+ \sum_{\boldsymbol{z}_1}\sum_{\boldsymbol{z}_2}\cdots\sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)\sum_{j=1}^{N}\ln p\left(\boldsymbol{x}_j\middle|\boldsymbol{z}_j, \boldsymbol{\theta}\right)$$

$$\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta^{old}}\right)\ln p(\boldsymbol{z}_1|\boldsymbol{\theta})$$

$$=\sum_{\boldsymbol{z_1}}\ln p(\boldsymbol{z}_1|\boldsymbol{\theta})\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta^{old}}\right)$$

$$=\sum_{\boldsymbol{z_1}} p\left(\boldsymbol{z_1}\middle|\boldsymbol{X},\boldsymbol{\theta^{old}}\right)\ln p(\boldsymbol{z}_1|\boldsymbol{\theta})$$

$$\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\sum_{i=2}^{N}\ln p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1},\boldsymbol{\theta})$$

$$=\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\{\ln p(\boldsymbol{z}_2|\boldsymbol{z}_1,\boldsymbol{\theta})+\ln p(\boldsymbol{z}_3|\boldsymbol{z}_2,\boldsymbol{\theta})+\cdots+\ln p(\boldsymbol{z}_N|\boldsymbol{z}_{N-1},\boldsymbol{\theta})\}$$

$$=\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_2|\boldsymbol{z}_1,\boldsymbol{\theta})+\cdots+\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_N|\boldsymbol{z}_{N-1},\boldsymbol{\theta})$$

$$=\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}} p\left(\boldsymbol{z_1},\boldsymbol{z_2}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_2|\boldsymbol{z}_1,\boldsymbol{\theta})+\sum_{\boldsymbol{z_2}}\sum_{\boldsymbol{z_3}} p\left(\boldsymbol{z_2},\boldsymbol{z_3}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_3|\boldsymbol{z}_2,\boldsymbol{\theta})+$$

$$\cdots+\sum_{\boldsymbol{z_{N-1}}}\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_{N-1}},\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_N|\boldsymbol{z}_{N-1},\boldsymbol{\theta})$$

$$=\sum_{n=2}^{N}\sum_{\boldsymbol{z_{n-1}}}\sum_{\boldsymbol{z_n}} p\left(\boldsymbol{z_{n-1}},\boldsymbol{z_n}\middle|\boldsymbol{X},\boldsymbol{\theta}^{old}\right)\ln p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1},\boldsymbol{\theta})$$

$$\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\sum_{j=1}^{N}\ln p\left(\boldsymbol{x}_j\middle|\boldsymbol{z}_j,\boldsymbol{\theta}\right)$$

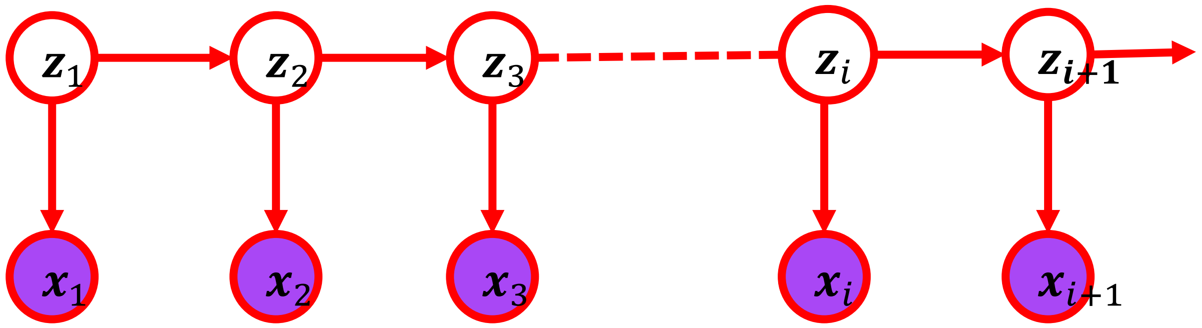
$$=\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\{\ln p(\boldsymbol{x}_1|\boldsymbol{z}_1,\boldsymbol{\theta})+\ln p(\boldsymbol{x}_2|\boldsymbol{z}_2,\boldsymbol{\theta})+\cdots+\ln p(\boldsymbol{x}_N|\boldsymbol{z}_N,\boldsymbol{\theta})\}$$

$$=\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_1|\boldsymbol{z}_1,\boldsymbol{\theta})+\cdots+\sum_{\boldsymbol{z_1}}\sum_{\boldsymbol{z_2}}\cdots\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_1},\boldsymbol{z_2},\cdots,\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_N|\boldsymbol{z}_N,\boldsymbol{\theta})$$

$$=\sum_{\boldsymbol{z_1}} p\left(\boldsymbol{z_1}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_1|\boldsymbol{z}_1,\boldsymbol{\theta})+\sum_{\boldsymbol{z_2}} p\left(\boldsymbol{z_2}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_2|\boldsymbol{z}_2,\boldsymbol{\theta})+\cdots+\sum_{\boldsymbol{z_N}} p\left(\boldsymbol{z_N}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_N|\boldsymbol{z}_N,\boldsymbol{\theta})$$

$$=\sum_{n=1}^{N}\sum_{\boldsymbol{z_n}} p\left(\boldsymbol{z_n}\middle|\boldsymbol{X},\boldsymbol{\theta}^{\boldsymbol{old}}\right)\ln p(\boldsymbol{x}_n|\boldsymbol{z}_n,\boldsymbol{\theta})$$

HMM EM 알고리즘



$$\gamma(\boldsymbol{z}_n) = p\left(\boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

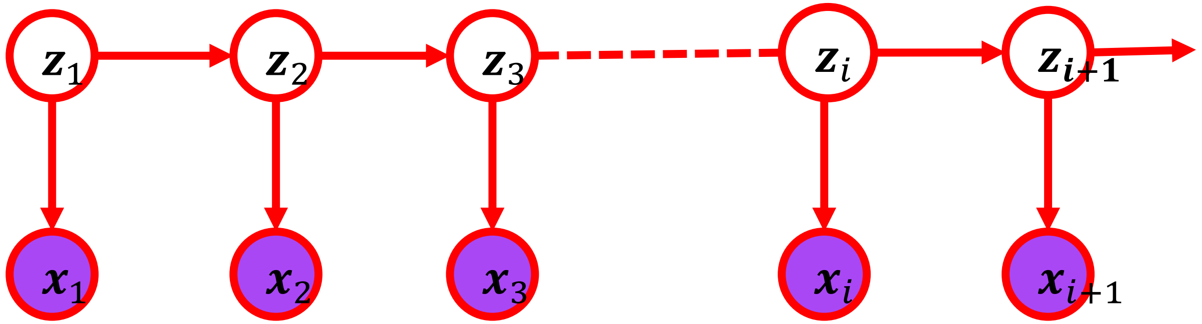
$$\xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) = p\left(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

$$Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{old}\right) = \sum_{\boldsymbol{z}_1} \sum_{\boldsymbol{z}_2} \cdots \sum_{\boldsymbol{z}_N} p\left(\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right) \ln \left\{ p(\boldsymbol{z}_1|\boldsymbol{\theta}) \left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1}, \boldsymbol{\theta})\right] \left[\prod_{j=1}^{N} p\left(\boldsymbol{x}_j|\boldsymbol{z}_j, \boldsymbol{\theta}\right)\right] \right\}$$

$$= \sum_{\boldsymbol{z}_1} p\left(\boldsymbol{z}_1 \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right) \ln p(\boldsymbol{z}_1|\boldsymbol{\theta}) + \sum_{n=2}^{N} \sum_{\boldsymbol{z}_{n-1}} \sum_{\boldsymbol{z}_n} p\left(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right) \ln p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}, \boldsymbol{\theta}) + \sum_{n=1}^{N} \sum_{\boldsymbol{z}_n} p\left(\boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right) \ln p(\boldsymbol{x}_n|\boldsymbol{z}_n, \boldsymbol{\theta})$$

$$= \sum_{\boldsymbol{z}_1} \gamma(\boldsymbol{z}_1) \ln p(\boldsymbol{z}_1|\boldsymbol{\theta}) + \sum_{n=2}^{N} \sum_{\boldsymbol{z}_{n-1}} \sum_{\boldsymbol{z}_n} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) \ln p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}, \boldsymbol{\theta}) + \sum_{n=1}^{N} \sum_{\boldsymbol{z}_n} \gamma(\boldsymbol{z}_n) \ln p(\boldsymbol{x}_n|\boldsymbol{z}_n, \boldsymbol{\theta})$$

HMM EM 알고리즘



$$\gamma(\boldsymbol{z}_n) = p\left(\boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

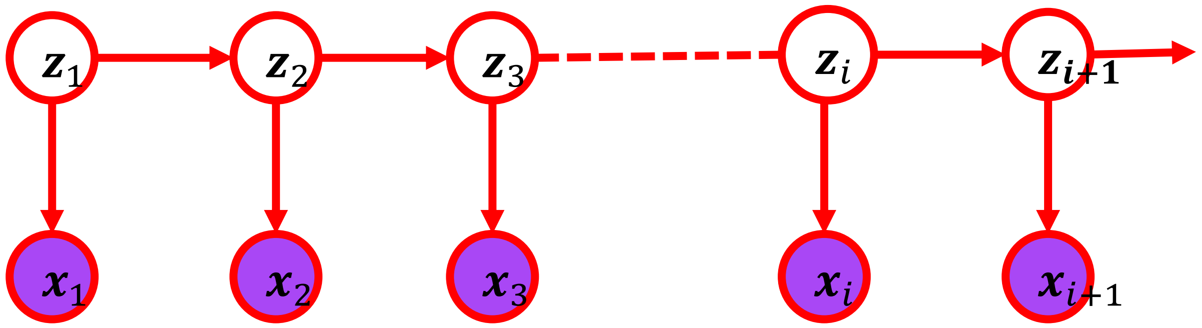
$$\xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) = p\left(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n \middle| \boldsymbol{X}, \boldsymbol{\theta}^{old}\right)$$

$$p(\boldsymbol{z}_1|\boldsymbol{\pi}) = \prod_{i=1}^{K} \pi_i^{z_{ni}} \qquad p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}, \boldsymbol{A}) = \prod_{i=1}^{K}\prod_{j=1}^{K} A_{j,i}^{z_{n-1,j} z_{n,i}} \qquad p(\boldsymbol{x}_n|\boldsymbol{z}_n, \boldsymbol{\phi}) = \prod_{i=1}^{K} p(\boldsymbol{x}_n|\phi_i)^{z_{n,i}}$$

$$Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{old}\right) = \sum_{\boldsymbol{z_1}} \gamma(\boldsymbol{z}_1) \ln p(\boldsymbol{z}_1|\boldsymbol{\theta}) + \sum_{n=2}^{N} \sum_{\boldsymbol{z_{n-1}}} \sum_{\boldsymbol{z_n}} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) \ln p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}, \boldsymbol{\theta}) + \sum_{n=1}^{N} \sum_{\boldsymbol{z_n}} \gamma(\boldsymbol{z}_n) \ln p(\boldsymbol{x}_n|\boldsymbol{z}_n, \boldsymbol{\theta})$$

$$= \sum_{i=1}^{K} \left\{ \sum_{\boldsymbol{z_1}} \gamma(\boldsymbol{z}_1) z_{1i} \right\} \ln \pi_i + \sum_{n=2}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \left\{ \sum_{\boldsymbol{z_{n-1}}} \sum_{\boldsymbol{z_n}} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) z_{n-1,j} z_{n,i} \right\} \ln A_{j,i} + \sum_{n=1}^{N} \sum_{i=1}^{K} \left\{ \sum_{\boldsymbol{z_n}} \gamma(\boldsymbol{z}_n) z_{n,i} \right\} \ln p(\boldsymbol{x}_n|\phi_i)$$

HMM EM 알고리즘



$$\gamma\left(z_{n,i}\right) = \mathbb{E}\left[z_{n,i}\right] = \sum_{\boldsymbol{z_n}} \gamma(\boldsymbol{z}_n) z_{n,i}$$

$$\xi\left(z_{n-1,j} z_{n,i}\right) = \mathbb{E}\left[z_{n-1,j} z_{n,i}\right] = \sum_{\boldsymbol{z_{n-1}}} \sum_{\boldsymbol{z_n}} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) z_{n-1,j} z_{n,i}$$

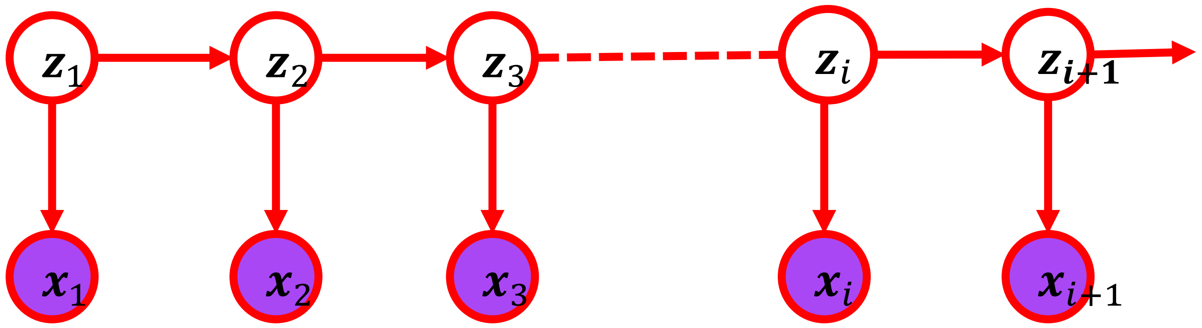
$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{\boldsymbol{old}}) = \sum_{i=1}^{K} \left\{ \sum_{\boldsymbol{z_1}} \gamma(\boldsymbol{z}_1) z_{1i} \right\} \ln \pi_i + \sum_{n=2}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \left\{ \sum_{\boldsymbol{z_{n-1}}} \sum_{\boldsymbol{z_n}} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) z_{n-1,j} z_{n,i} \right\} \ln A_{j,i} + \sum_{n=1}^{N} \sum_{i=1}^{K} \left\{ \sum_{\boldsymbol{z_n}} \gamma(\boldsymbol{z}_n) z_{n,i} \right\} \ln p(\boldsymbol{x}_n | \phi_i)$$

$$= \sum_{i=1}^{K} \gamma\left(z_{1,i}\right) \ln \pi_i + \sum_{n=2}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \xi\left(z_{n-1,j} z_{n,i}\right) \ln A_{j,i} + \sum_{n=1}^{N} \sum_{i=1}^{K} \gamma\left(z_{n,i}\right) \ln p(\boldsymbol{x}_n | \phi_i)$$

HMM EM 알고리즘

$$p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta}) = p(\boldsymbol{z}_1|\boldsymbol{\pi})\left[\prod_{i=2}^{N} p(\boldsymbol{z}_i|\boldsymbol{z}_{i-1},\boldsymbol{A})\right]\left[\prod_{j=1}^{N} p(\boldsymbol{x}_j|\boldsymbol{z}_j,\boldsymbol{\phi})\right]$$

$$\boldsymbol{\theta} = \{\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{\phi}\}$$



E-step :

$$\gamma\left(z_{n,i}\right) = \mathbb{E}\left[z_{n,i}\right] = \sum_{\boldsymbol{z_n}} \gamma(\boldsymbol{z}_n) z_{n,i} = p\left(z_{n,i}\middle|\boldsymbol{X}, \boldsymbol{\theta}^{\boldsymbol{old}}\right)$$

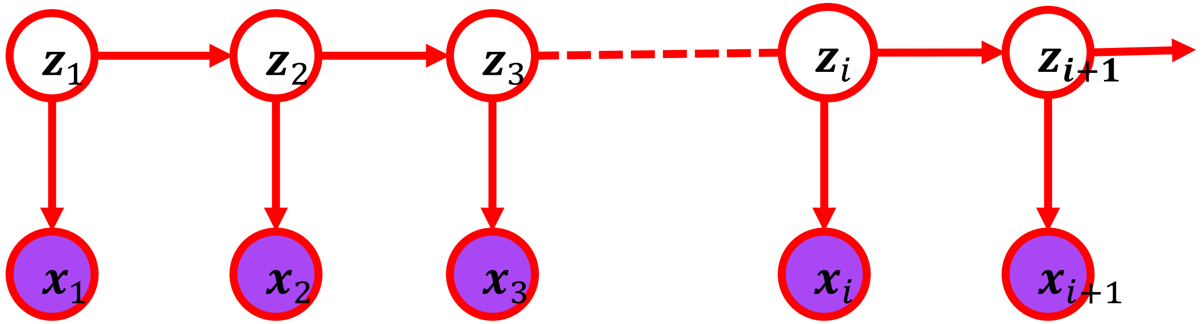
$$\xi\left(z_{n-1,j}, z_{n,i}\right) = \mathbb{E}\left[z_{n-1,j}, z_{n,i}\right] = \sum_{\boldsymbol{z_{n-1}}} \sum_{\boldsymbol{z_n}} \xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) z_{n-1,j} z_{n,i} = p\left(z_{n-1,j}, z_{n,i}\middle|\boldsymbol{X}, \boldsymbol{\theta}^{\boldsymbol{old}}\right)$$

M-step :

$$Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{\boldsymbol{old}}\right) = \sum_{i=1}^{K} \gamma\left(z_{1,i}\right) \ln \pi_i + \sum_{n=2}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \xi\left(z_{n-1,j} z_{n,i}\right) \ln A_{j,i} + \sum_{n=1}^{N} \sum_{i=1}^{K} \gamma\left(z_{n,i}\right) \ln p(\boldsymbol{x}_n|\phi_i)$$

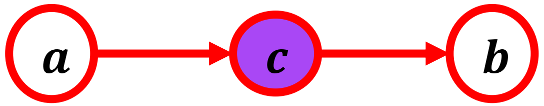
알파 베타 알고리즘

$\boldsymbol{c}$는 머리 대 꼬리($\boldsymbol{head-to-tail}$) 노드



$$p(\boldsymbol{a},\boldsymbol{b}|\boldsymbol{c})=\frac{p(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c})}{p(\boldsymbol{c})}=\frac{p(\boldsymbol{b}|\boldsymbol{c})p(\boldsymbol{c}|\boldsymbol{a})p(\boldsymbol{a})}{p(\boldsymbol{c})}$$

$$=p(\boldsymbol{a}|\boldsymbol{c})p(\boldsymbol{b}|\boldsymbol{c})$$



$$p(\boldsymbol{x_1},\boldsymbol{x_2},\cdots,\boldsymbol{x_n},\boldsymbol{x_{n+1}}\cdots,\boldsymbol{x_N}|\boldsymbol{z_n})=p(\boldsymbol{x_1},\boldsymbol{x_2},\cdots,\boldsymbol{x_n}|\boldsymbol{z_n})p(\boldsymbol{x_{n+1}}\cdots,\boldsymbol{x_N}|\boldsymbol{z_n})$$

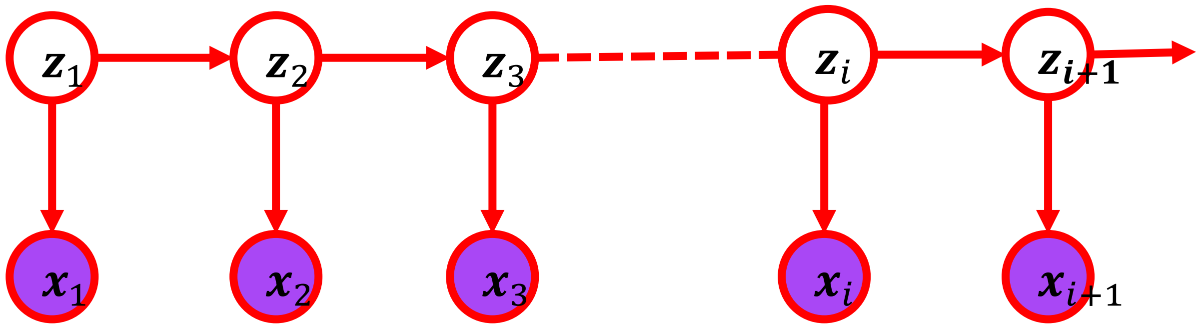
$$\gamma(\boldsymbol{z}_n)=p(\boldsymbol{z}_n|\boldsymbol{X})=\frac{p(\boldsymbol{X}|\boldsymbol{z}_n)p(\boldsymbol{z}_n)}{p(\boldsymbol{X})}=\frac{p(\boldsymbol{x_1},\boldsymbol{x_2},\cdots,\boldsymbol{x_n}|\boldsymbol{z_n})p(\boldsymbol{x_{n+1}}\cdots,\boldsymbol{x_N}|\boldsymbol{z_n})p(\boldsymbol{z}_n)}{p(\boldsymbol{X})}$$

$$=\frac{p(\boldsymbol{x_1},\boldsymbol{x_2},\cdots,\boldsymbol{x_n},\boldsymbol{z_n})p(\boldsymbol{x_{n+1}}\cdots,\boldsymbol{x_N}|\boldsymbol{z_n})}{p(\boldsymbol{X})}=\frac{\alpha(\boldsymbol{z_n})\beta(\boldsymbol{z_n})}{p(\boldsymbol{X})}$$

$$\alpha(\boldsymbol{z_n})\equiv p(\boldsymbol{x_1},\boldsymbol{x_2},\cdots,\boldsymbol{x_n},\boldsymbol{z_n})$$

$$\beta(\boldsymbol{z_n})\equiv p(\boldsymbol{x_{n+1}}\cdots,\boldsymbol{x_N}|\boldsymbol{z_n})$$

알파 베타 알고리즘



$$\alpha(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \boldsymbol{z_n})$$

$$\beta(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_{n+1}} \cdots, \boldsymbol{x_N} | \boldsymbol{z_n})$$

$$\gamma(\boldsymbol{z}_n) = p(\boldsymbol{z}_n | \boldsymbol{X}\,) = \frac{p(\boldsymbol{X}|\boldsymbol{z_n})p(\boldsymbol{z_n})}{p(\boldsymbol{X})} == \frac{\alpha(\boldsymbol{z_n})\beta(\boldsymbol{z_n})}{p(\boldsymbol{X})}$$

$$\begin{aligned}
\alpha(\boldsymbol{z_n}) &= p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \boldsymbol{z_n}) \\
&= p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n} | \boldsymbol{z_n}) p(\boldsymbol{z}_n) \\
&= p(\boldsymbol{x_n}|\boldsymbol{z_n}) p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}} | \boldsymbol{z_n}) p(\boldsymbol{z}_n) \rightarrow d\text{분리이용} \\
&= p(\boldsymbol{x_n}|\boldsymbol{z_n}) p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}, \boldsymbol{z_n}) \\
&= p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}, \boldsymbol{z_{n-1}}, \boldsymbol{z_n}) \\
&= p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}, \boldsymbol{z_n} | \boldsymbol{z_{n-1}}) p(\boldsymbol{z_{n-1}}) \\
&= p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}} | \boldsymbol{z_{n-1}}) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}}) p(\boldsymbol{z_{n-1}}) \rightarrow d\text{분리이용}
\end{aligned}$$

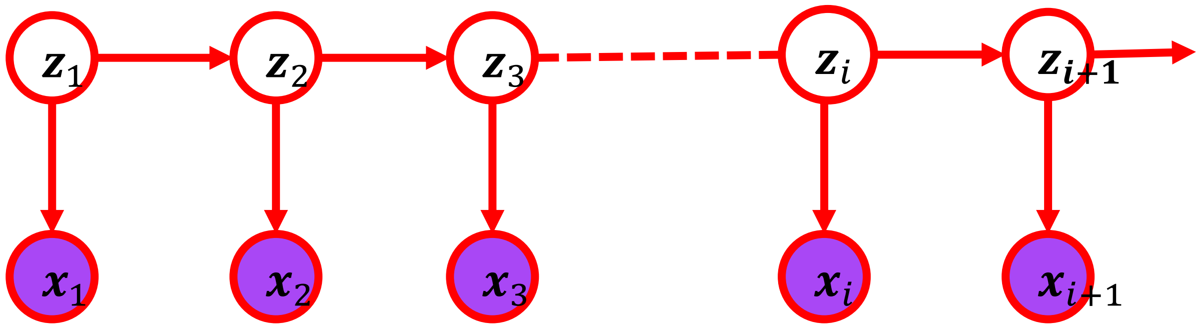
$$= p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}, \boldsymbol{z_{n-1}}\,) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

$$\alpha(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \alpha(\boldsymbol{z_{n-1}}) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

알파 베타 알고리즘

$$\alpha(\mathbf{z_n}) \equiv p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \mathbf{z_n})$$

$$\beta(\mathbf{z_n}) \equiv p(\boldsymbol{x_{n+1}} \cdots, \boldsymbol{x_N} | \mathbf{z_n})$$

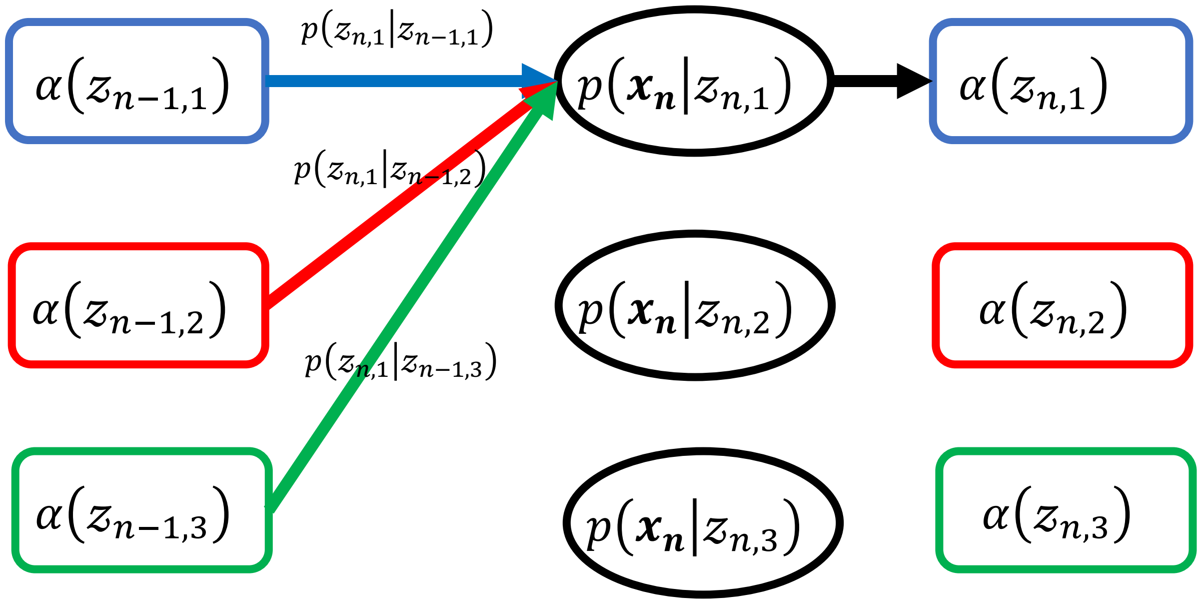


$$\gamma(\mathbf{z}_n) = p(\mathbf{z}_n | \boldsymbol{X}) = \frac{p(\boldsymbol{X}|\mathbf{z}_n)p(\mathbf{z}_n)}{p(\boldsymbol{X})} = \frac{\alpha(\mathbf{z_n})\beta(\mathbf{z_n})}{p(\boldsymbol{X})}$$

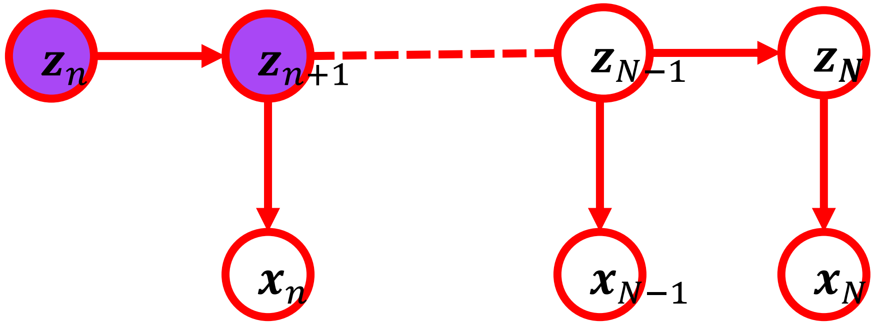
$$\alpha(\mathbf{z_n}) = p(\boldsymbol{x_n}|\mathbf{z_n}) \sum_{\mathbf{z_{n-1}}} \alpha(\mathbf{z_{n-1}}) p(\mathbf{z_n}|\mathbf{z_{n-1}})$$

$$\alpha(\mathbf{z_1}) = p(\boldsymbol{x_1}, \mathbf{z_1})$$

$$\alpha(z_{n,i}) = p(\boldsymbol{x_n}|z_{n,i}) \sum_{j=1}^{K} \alpha(z_{n-1,j}) p(z_{n,i}|z_{n-1,j})$$



알파 베타 알고리즘



$$\alpha(\boldsymbol{z}_n) \equiv p(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n, \boldsymbol{z}_n)$$

$$\beta(\boldsymbol{z}_n) \equiv p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N | \boldsymbol{z}_n)$$

$$\gamma(\boldsymbol{z}_n) = p(\boldsymbol{z}_n|\boldsymbol{X}) = \frac{p(\boldsymbol{X}|\boldsymbol{z}_n)p(\boldsymbol{z}_n)}{p(\boldsymbol{X})} = \frac{\alpha(\boldsymbol{z}_n)\beta(\boldsymbol{z}_n)}{p(\boldsymbol{X})}$$

$$\beta(\boldsymbol{z}_n) = p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N|\boldsymbol{z}_n) = \sum_{\boldsymbol{z}_{n+1}} p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N, \boldsymbol{z}_{n+1}|\boldsymbol{z}_n)$$

$$= \sum_{\boldsymbol{z}_{n+1}} p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N | \boldsymbol{z}_n, \boldsymbol{z}_{n+1}) p(\boldsymbol{z}_{n+1}|\boldsymbol{z}_n) \rightarrow \boldsymbol{d}\text{분리이용}$$

$$= \sum_{\boldsymbol{z}_{n+1}} p(\boldsymbol{x}_{n+2} \cdots, \boldsymbol{x}_N | \boldsymbol{z}_{n+1}) p(\boldsymbol{x}_{n+1}|\boldsymbol{z}_{n+1}) p(\boldsymbol{z}_{n+1}|\boldsymbol{z}_n)$$

$$\beta(\boldsymbol{z}_n) = \sum_{\boldsymbol{z}_{n+1}} \beta(\boldsymbol{z}_{n+1}) p(\boldsymbol{x}_{n+1}|\boldsymbol{z}_{n+1}) p(\boldsymbol{z}_{n+1}|\boldsymbol{z}_n)$$

$$p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N | \boldsymbol{z}_n, \boldsymbol{z}_{n+1}) = \frac{p(\boldsymbol{x}_{n+1}, \cdots, \boldsymbol{x}_N, \boldsymbol{z}_n, \boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_n, \boldsymbol{z}_{n+1})}$$

$$= \frac{\dfrac{p(\boldsymbol{x}_{n+1}, \cdots, \boldsymbol{x}_N, \boldsymbol{z}_n, \boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_{n+1})}}{\dfrac{p(\boldsymbol{z}_n, \boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_{n+1})}}$$

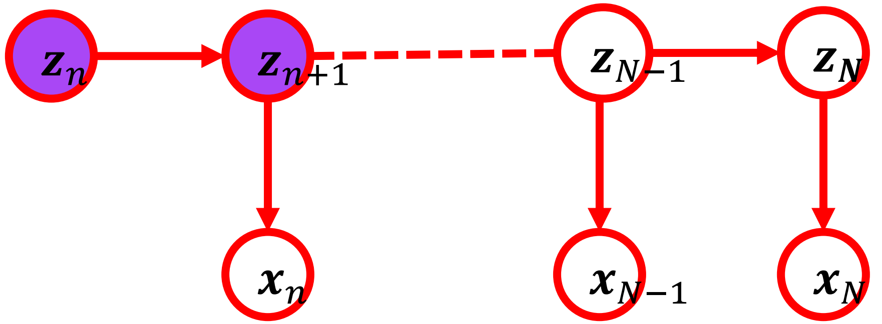
$$= \frac{p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N, \boldsymbol{z}_n | \boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_n|\boldsymbol{z}_{n+1})}$$

$$= \frac{p(\boldsymbol{x}_{n+1} \cdots, \boldsymbol{x}_N | \boldsymbol{z}_{n+1}) p(\boldsymbol{z}_n|\boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_n|\boldsymbol{z}_{n+1})}$$

알파 베타 알고리즘

$$\alpha(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \boldsymbol{z_n})$$

$$\beta(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_{n+1}} \cdots, \boldsymbol{x_N} | \boldsymbol{z_n})$$

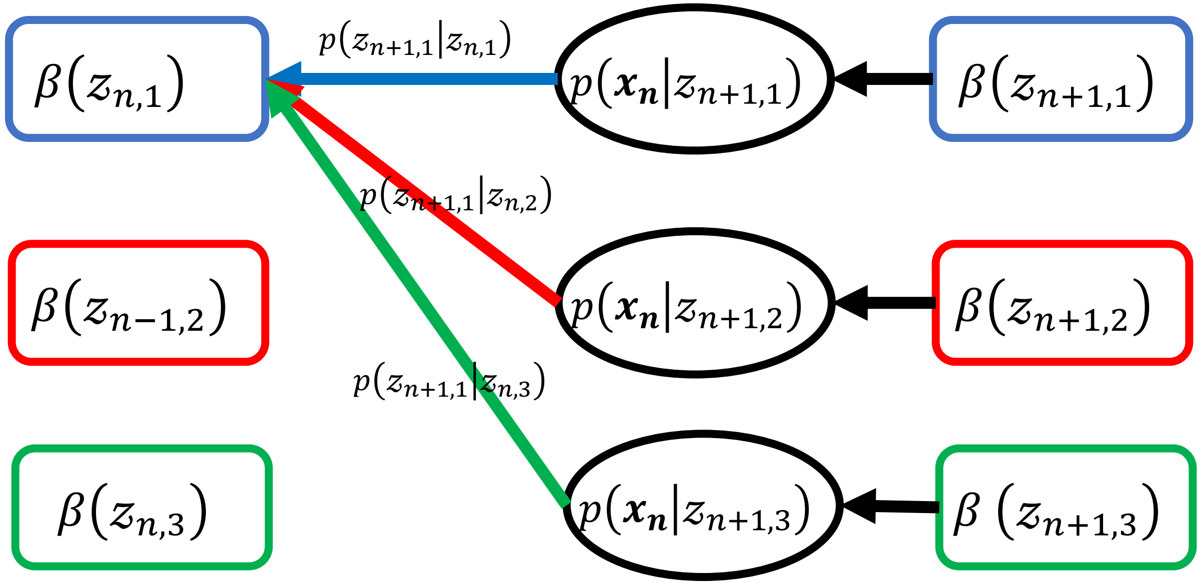


$$\gamma(\boldsymbol{z}_n) = p(\boldsymbol{z}_n | \boldsymbol{X}) = \frac{p(\boldsymbol{X}|\boldsymbol{z}_n)p(\boldsymbol{z}_n)}{p(\boldsymbol{X})} = \frac{\alpha(\boldsymbol{z_n})\beta(\boldsymbol{z_n})}{p(\boldsymbol{X})}$$

$$\beta(\boldsymbol{z_n}) = \sum_{\boldsymbol{z_{n+1}}} \beta(\boldsymbol{z_{n+1}}) p(\boldsymbol{x_{n+1}} | \boldsymbol{z_{n+1}}) p(\boldsymbol{z_{n+1}} | \boldsymbol{z_n})$$

$$\beta(\boldsymbol{z_N}) = \boldsymbol{1}$$

$$\beta(z_{n,i}) = \sum_{j=1}^{K} \beta(z_{n+1,j}) p(\boldsymbol{x_{n+1}} | z_{n+1,j}) p(z_{n+1,j} | z_{n,i})$$



알파 베타 알고리즘

$$\alpha(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \boldsymbol{z_n})$$

$$\beta(\boldsymbol{z_n}) \equiv p(\boldsymbol{x_{n+1}} \cdots, \boldsymbol{x_N} | \boldsymbol{z_n})$$

$$\xi(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n) = p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n | \boldsymbol{X}) = \frac{p(\boldsymbol{X}, \boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}{p(\boldsymbol{X})} = \frac{p(\boldsymbol{X}|\boldsymbol{z}_{n-1}, \boldsymbol{z}_n)p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}{p(\boldsymbol{X})}$$

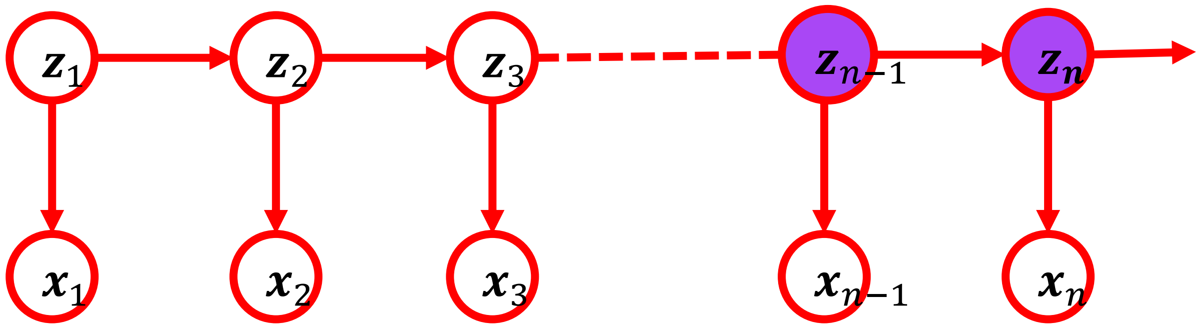
$$= \frac{p(\boldsymbol{X}|\boldsymbol{z}_{n-1}, \boldsymbol{z}_n)p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})p(\boldsymbol{z}_{n-1})}{p(\boldsymbol{X})}$$

$$= \frac{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}} | \boldsymbol{z}_{n-1})p(\boldsymbol{x_n} | \boldsymbol{z}_n)p(\boldsymbol{x_{n+1}}, \boldsymbol{x_{n+2}}, \cdots, \boldsymbol{x_N} | \boldsymbol{z}_n)p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})p(\boldsymbol{z}_{n-1})}{p(\boldsymbol{X})}$$

$$= \frac{\alpha(\boldsymbol{z_{n-1}})p(\boldsymbol{x_n} | \boldsymbol{z}_n)p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})\beta(\boldsymbol{z_n})}{p(\boldsymbol{X})}$$

참고0

참고1



$\rightarrow p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_n} \,|\mathbf{z}_{n-1}, \mathbf{z}_n)$

$$= p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}} \,|\mathbf{z}_{n-1}, \mathbf{z}_n) p(\,\mathbf{x_n} \,|\mathbf{z}_{n-1}, \mathbf{z}_n)$$ $\rightarrow d$분리이용

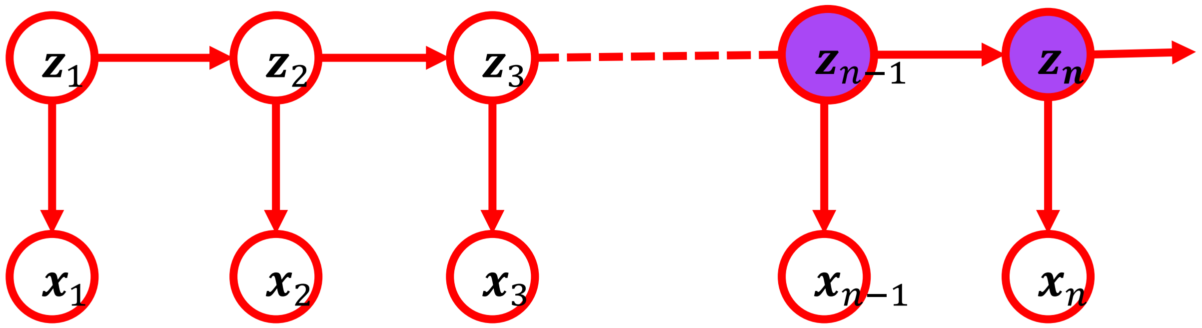
$$= \frac{p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}}, \mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_{n-1}, \mathbf{z}_n)} \frac{p(\mathbf{x_n}, \mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_{n-1}, \mathbf{z}_n)}$$

$$= \frac{\dfrac{p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}}, \mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_{n-1})}}{\dfrac{p(\mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_{n-1})}} \frac{\dfrac{p(\mathbf{x_n}, \mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_n)}}{\dfrac{p(\mathbf{z}_{n-1}, \mathbf{z}_n)}{p(\mathbf{z}_n)}} = \frac{p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}}\,, \mathbf{z}_n|\mathbf{z}_{n-1}\,)}{p(\mathbf{z}_n|\mathbf{z}_{n-1}\,)} \frac{p(\mathbf{x_n}\,, \mathbf{z}_{n-1}|\mathbf{z}_n\,)}{p(\mathbf{z}_{n-1}|\mathbf{z}_n\,)}$$

$$= \frac{p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}} \;|\mathbf{z}_{n-1}\,) p(\mathbf{z}_n|\mathbf{z}_{n-1}\,)}{p(\mathbf{z}_n|\mathbf{z}_{n-1}\,)} \frac{p(\mathbf{x_n} \;|\mathbf{z}_n\,)(\mathbf{z}_{n-1}|\mathbf{z}_n\,)}{p(\mathbf{z}_{n-1}|\mathbf{z}_n\,)}$$

$$= p(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_{n-1}} \;|\mathbf{z}_{n-1}\,) p(\mathbf{x_n} \;|\mathbf{z}_n\,)$$

참고2



$\rightarrow p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N \, | \boldsymbol{z}_{n-1}, \boldsymbol{z}_n)$

$$= \frac{p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N, \boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}{p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}$$

$\rightarrow d$분리이용

$$= \frac{\dfrac{p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N, \boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}{p(\boldsymbol{z}_n)}}{\dfrac{p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n)}{p(\boldsymbol{z}_n)}}$$

$$= \frac{p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N, \boldsymbol{z}_{n-1} | \boldsymbol{z}_n)}{p(\boldsymbol{z}_{n+1} | \boldsymbol{z}_n)} = \frac{p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N \, | \boldsymbol{z}_n) p(\boldsymbol{z}_{n-1} | \boldsymbol{z}_n)}{p(\boldsymbol{z}_{n+1} | \boldsymbol{z}_n)}$$

$$= p(\boldsymbol{x}_{\boldsymbol{n}+\mathbf{1}}, \boldsymbol{x}_{\boldsymbol{n}+\mathbf{2}}, \cdots, \boldsymbol{x}_N \, | \boldsymbol{z}_n)$$

$$\alpha(\boldsymbol{z_N}) = p(\boldsymbol{X_N}, \boldsymbol{z_N})$$

$$p(\boldsymbol{x}_{N+1}|\boldsymbol{X}) = \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}, \boldsymbol{z}_{N+1}|\boldsymbol{X}) = \sum_{\boldsymbol{z}_{N+1}} \frac{p(\boldsymbol{X}, \boldsymbol{x}_{N+1}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{X})} \frac{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})} = \sum_{\boldsymbol{z}_{N+1}} \frac{p(\boldsymbol{X}, \boldsymbol{x}_{N+1}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})} \frac{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{X})}$$

$$= \sum_{\boldsymbol{z}_{N+1}} \frac{\frac{p(\boldsymbol{X}, \boldsymbol{x}_{N+1}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{z}_{N+1})}}{\frac{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{z}_{N+1})}} \frac{p(\boldsymbol{X}, \boldsymbol{z}_{N+1})}{p(\boldsymbol{X})} = \sum_{\boldsymbol{z}_{N+1}} \frac{p(\boldsymbol{X}, \boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1})}{p(\boldsymbol{X}|\boldsymbol{z}_{N+1})} p(\boldsymbol{z}_{N+1}|\boldsymbol{X}) = \sum_{\boldsymbol{z}_{N+1}} \frac{p(\boldsymbol{X}\,|\boldsymbol{z}_{N+1}) p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1})}{p(\boldsymbol{X}|\boldsymbol{z}_{N+1})} p(\boldsymbol{z}_{N+1}|\boldsymbol{X})$$

$$= \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) p(\boldsymbol{z}_{N+1}|\boldsymbol{X}) = \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) \sum_{\boldsymbol{z}_N} p(\boldsymbol{z}_N, \boldsymbol{z}_{N+1}|\boldsymbol{X}) = \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) \sum_{\boldsymbol{z}_N} p(\boldsymbol{z}_{N+1}|\boldsymbol{z}_N) p(\boldsymbol{z}_N\,|\boldsymbol{X})$$

$$= \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) \sum_{\boldsymbol{z}_N} p(\boldsymbol{z}_{N+1}|\boldsymbol{z}_N) \frac{p(\boldsymbol{X}, \boldsymbol{z_N})}{p(\boldsymbol{X})} = \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) \sum_{\boldsymbol{z}_N} p(\boldsymbol{z}_{N+1}|\boldsymbol{z}_N) \frac{\alpha(\boldsymbol{z_N})}{p(\boldsymbol{X})}$$

$$c_{N+1} \equiv p(\boldsymbol{x}_{N+1}|\boldsymbol{X}) = \frac{1}{p(\boldsymbol{X})} \sum_{\boldsymbol{z}_{N+1}} p(\boldsymbol{x}_{N+1}|\boldsymbol{z}_{N+1}) \sum_{\boldsymbol{z}_N} p(\boldsymbol{z}_{N+1}|\boldsymbol{z}_N) \alpha(\boldsymbol{z_N})$$

$$\hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x}_2, \cdots, \boldsymbol{x_n}) = \frac{\alpha(\boldsymbol{z_n})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n})}$$

$$\alpha(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \alpha(\boldsymbol{z_{n-1}}) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

$$= p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}, \boldsymbol{z_n})$$

$$c_1 = p(\boldsymbol{x_1})$$

$$c_2 = p(\boldsymbol{x_2}|\boldsymbol{x_1})$$

$$c_3 = p(\boldsymbol{x_3}|\boldsymbol{x_1}, \boldsymbol{x_2})$$

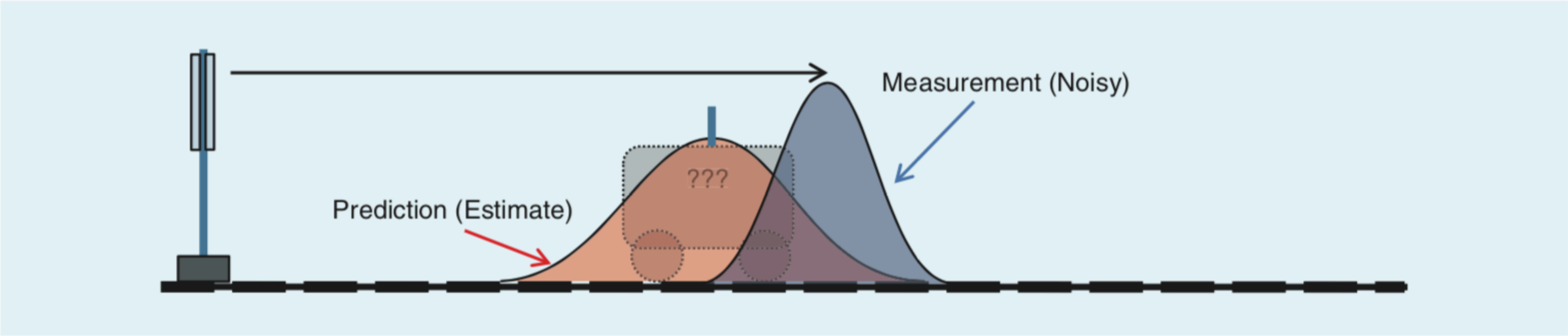
$$c_4 = p(\boldsymbol{x_4}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3})$$

$$\frac{\alpha(\boldsymbol{z_n})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})} = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \frac{\alpha(\boldsymbol{z_{n-1}})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})} p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

$$c_n = p(\boldsymbol{x_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})$$

$$\frac{\dfrac{\alpha(\boldsymbol{z_n})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n})}}{\dfrac{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}, \boldsymbol{x_n})}} = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \frac{\alpha(\boldsymbol{z_{n-1}})}{p(\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})} p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

$$c_n \hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \hat{\alpha}(\boldsymbol{z_{n-1}}) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$



[FIG4] Shows the measurement of the location of the train at time $t = 1$ and the level of uncertainty in that noisy measurement, represented by the blue Gaussian pdf. The combined knowledge of this system is provided by multiplying these two pdfs together.

알려지지 않은 수량 z_n의 값을 노이즈가 포함되는 센서를 이용해서 측정해야 한다.

$$z_n = Az_{n-1} + noise$$

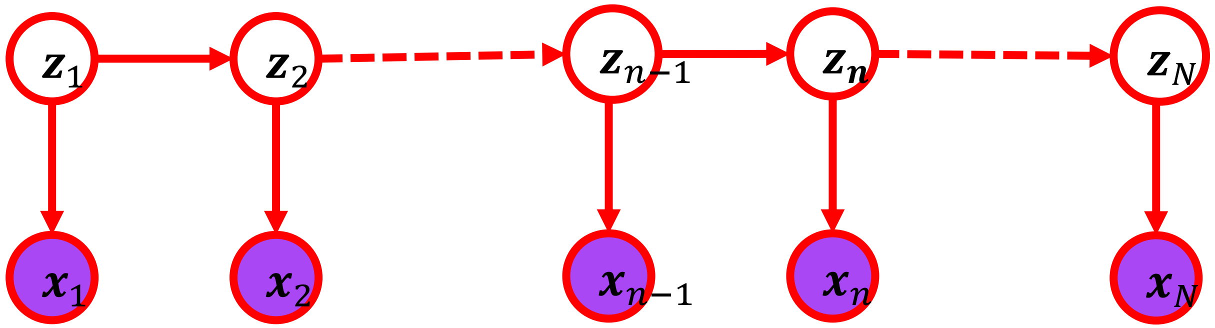
$z_n \rightarrow n$번째 추정값

$z_{n-1} \rightarrow n-1$번째 추정값

센서로부터 주어진 관측값 x_n은 z_n의 값에 0평균 가우시안 노이즈를 더한 것이다.

$$x_n = Cz_n + noise$$

$x_n \rightarrow n$번째 측정값



알려지지 않은 수량 $\boldsymbol{z_n}$의 값을 노이즈가 포함되는 센서를 이용해서 측정해야 한다.

$\boldsymbol{z_n} \to \boldsymbol{n}$번째 추정값

$$\boldsymbol{z_n = Az_{n-1} + noise}$$

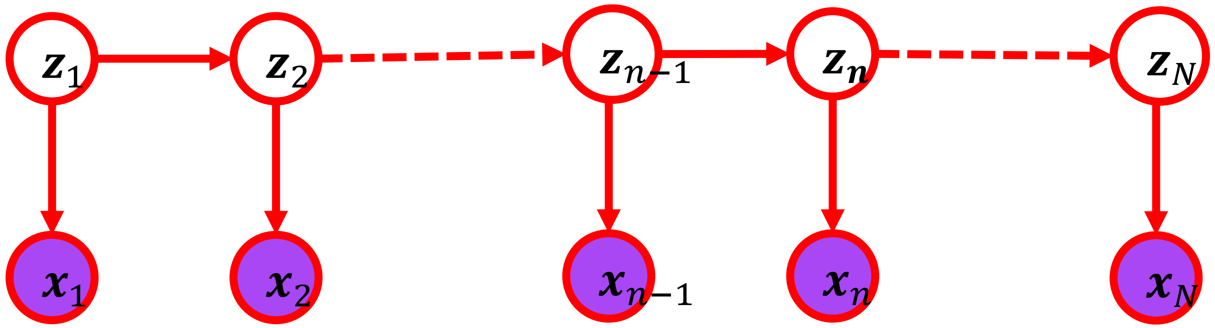
$\boldsymbol{z_{n-1}} \to \boldsymbol{n-1}$번째 추정값

센서로부터 주어진 관측값 $\boldsymbol{x_n}$은 $\boldsymbol{z_n}$의 값에 0평균 가우시안 노이즈를 더한 것이다.

$$\boldsymbol{x_n = Cz_n + noise}$$

$\boldsymbol{x_n} \to \boldsymbol{n}$번째 측정값

선형 동적 시스템에서의 추론



$p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})$ → 전이 확률

$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma})$ → 방사확률

$p(\boldsymbol{z}_1) = \mathcal{N}(\boldsymbol{z}_1|\boldsymbol{\mu}_n, \mathbf{P}_0)$ → 초기값

알려지지 않은 수량 $\boldsymbol{z}_n$

$\Rightarrow$ $p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n)$

센서로부터 주어진 관측값 $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n$

$\boldsymbol{\theta} = \{\boldsymbol{A}, \boldsymbol{\Gamma}, \boldsymbol{C}, \boldsymbol{\Sigma}, \boldsymbol{\mu}_n, \mathbf{P}_0\}$

선형 동적 시스템에서의 추론

$p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}}) = \boldsymbol{\mathcal{N}}(\boldsymbol{z_n}|\boldsymbol{A}\boldsymbol{z_{n-1}}, \boldsymbol{\Gamma})$ → 전이 확률

$p(\boldsymbol{x_n}|\boldsymbol{z_n}) = \boldsymbol{\mathcal{N}}(\boldsymbol{x_n}|\boldsymbol{C}\boldsymbol{z_n}, \boldsymbol{\Sigma})$ → 방사확률

$p(\boldsymbol{z_1}) = \boldsymbol{\mathcal{N}}(\boldsymbol{z_1}|\boldsymbol{\mu_n}, \mathbf{P_0})$ → 초기값

센서로부터 주어진 관측값 $\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}$

알려지지 않은 수량 $\boldsymbol{z_n}$

$$\hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}) = \boldsymbol{\mathcal{N}}(\boldsymbol{z_n}|\boldsymbol{\mu_n}, \mathbf{V_n})$$

$$c_n = p(\boldsymbol{x_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})$$

$$c_n\hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \sum_{\boldsymbol{z_{n-1}}} \hat{\alpha}(\boldsymbol{z_{n-1}})p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})$$

$$c_n\hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \int \hat{\alpha}(\boldsymbol{z_{n-1}})p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}})\boldsymbol{dz_{n-1}}$$

선형 동적 시스템에서의 추론

$p(\mathbf{z}_n|\mathbf{z}_{n-1}) = \mathcal{N}(\mathbf{z}_n|\mathbf{A}\mathbf{z}_{n-1}, \boldsymbol{\Gamma}) \rightarrow$ 전이 확률

$p(\mathbf{x}_n|\mathbf{z}_n) = \mathcal{N}(\mathbf{x}_n|\mathbf{C}\mathbf{z}_n, \boldsymbol{\Sigma}) \rightarrow$ 방사확률

$p(\mathbf{z}_1) = \mathcal{N}(\mathbf{z}_1|\boldsymbol{\mu}_0, \mathbf{P}_0) \rightarrow$ 초기값

센서로부터 주어진 관측값 $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$

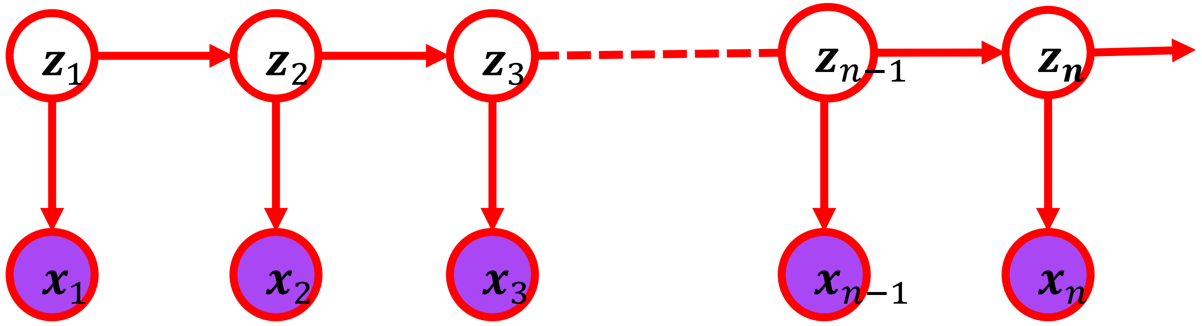
알려지지 않은 수량 $\mathbf{z}_n$

$$\hat{\alpha}(\mathbf{z}_n) = p(\mathbf{z}_n|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n) = \mathcal{N}(\mathbf{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n)$$

$$c_n\hat{\alpha}(\mathbf{z}_n) = p(\mathbf{x}_n|\mathbf{z}_n)\int \hat{\alpha}(\mathbf{z}_{n-1})p(\mathbf{z}_n|\mathbf{z}_{n-1})\,d\mathbf{z}_{n-1}$$

$$c_n\mathcal{N}(\mathbf{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = \mathcal{N}(\mathbf{x}_n|\mathbf{C}\mathbf{z}_n, \boldsymbol{\Sigma})\int \mathcal{N}(\mathbf{z}_{n-1}|\boldsymbol{\mu}_{n-1}, \mathbf{V}_{n-1})\mathcal{N}(\mathbf{z}_n|\mathbf{A}\mathbf{z}_{n-1}, \boldsymbol{\Gamma})\,d\mathbf{z}_{n-1}$$

선형 동적 시스템에서의 추론



$$c_n \hat{\alpha}(\boldsymbol{z_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \int \hat{\alpha}(\boldsymbol{z_{n-1}}) p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}}) \boldsymbol{dz_{n-1}}$$

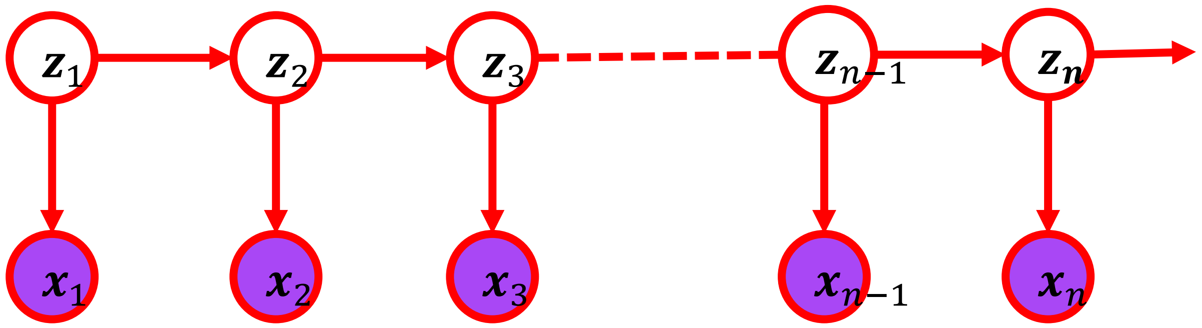
$$c_n p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \int p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}}) p(\boldsymbol{z_{n-1}}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}) \boldsymbol{dz_{n-1}}$$

$$\rightarrow p(\boldsymbol{z_{n-1}}, \boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}) = p(\boldsymbol{z_n}|\boldsymbol{z_{n-1}}) p(\boldsymbol{z_{n-1}}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})$$

$$c_n p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) \int p(\boldsymbol{z_{n-1}}, \boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}}) \boldsymbol{dz_{n-1}}$$

$$c_n p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_n}) = p(\boldsymbol{x_n}|\boldsymbol{z_n}) p(\boldsymbol{z_n}|\boldsymbol{x_1}, \boldsymbol{x_2}, \cdots, \boldsymbol{x_{n-1}})$$

선형 동적 시스템에서의 추론



$$p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = p(\boldsymbol{z}_n | \boldsymbol{z}_{n-1}, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) p(\boldsymbol{z}_{n-1} | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

$$\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1} \perp\!\!\!\perp \boldsymbol{z}_n | \boldsymbol{z}_{n-1}$$

$$\rightarrow p(\boldsymbol{z}_{n-1}, \boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = p(\boldsymbol{z}_n | \boldsymbol{z}_{n-1}) p(\boldsymbol{z}_{n-1} | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

$$c_n p(\boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = p(\boldsymbol{x}_n | \boldsymbol{z}_n, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) p(\boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

$$\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1} \perp\!\!\!\perp \boldsymbol{x}_n | \boldsymbol{z}_n$$

$$c_n p(\boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = p(\boldsymbol{x}_n | \boldsymbol{z}_n) p(\boldsymbol{z}_n | \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

선형 동적 시스템에서의 추론

$p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})$ → 전이 확률

$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma})$ → 방사확률

$p(\boldsymbol{z}_1) = \mathcal{N}(\boldsymbol{z}_1|\boldsymbol{\mu}_n, \mathbf{P}_0)$ → 초기값

센서로부터 주어진 관측값 $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n$

알려지지 않은 수량 $\boldsymbol{z}_n$

$$\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n)$$

$$c_n\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{x}_n|\boldsymbol{z}_n)\int \hat{\alpha}(\boldsymbol{z}_{n-1})p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})d\boldsymbol{z}_{n-1}$$

$$c_n\mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma})\int \mathcal{N}(\boldsymbol{z}_{n-1}|\boldsymbol{\mu}_{n-1}, \mathbf{V}_{n-1})\mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})d\boldsymbol{z}_{n-1}$$

식 2.113~2.115 참고

선형 동적 시스템에서의 추론

식 2.113 $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}^{-1})$

$$p(\boldsymbol{z}_{n-1}|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \mathcal{N}(\boldsymbol{z}_{n-1}|\boldsymbol{\mu}_{n-1}, \mathbf{V}_{n-1})$$

식 2.114 $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|\mathbf{A}\boldsymbol{x}, \mathbf{L}^{-1})$

$$p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})$$

식 2.115 $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|\mathbf{A}\boldsymbol{\mu}, \mathbf{L}^{-1} + \mathbf{A}\boldsymbol{\Lambda}^{-1}\mathbf{A}^T)$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \mathbf{P}_{n-1})$$

$$\mathbf{P}_{n-1} = \boldsymbol{\Gamma} + \boldsymbol{A}\mathbf{V}_{n-1}\boldsymbol{A}^T$$

$$p(\boldsymbol{y}) = \int p(\boldsymbol{y}|\boldsymbol{x})\, p(\boldsymbol{x}) dx$$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \int p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) p(\boldsymbol{z}_{n-1}|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) \boldsymbol{d}\boldsymbol{z}_{n-1}$$

선형 동적 시스템에서의 추론

$p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|A\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})$ → 전이 확률

$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|C\boldsymbol{z}_n, \boldsymbol{\Sigma})$ → 방사확률

$p(\boldsymbol{z}_1) = \mathcal{N}(\boldsymbol{z}_1|\boldsymbol{\mu}_n, \mathbf{P}_0)$ → 초기값

센서로부터 주어진 관측값 $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n$

알려지지 않은 수량 $\boldsymbol{z}_n$

$$\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n)$$

$$c_n\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{x}_n|\boldsymbol{z}_n)\int \hat{\alpha}(\boldsymbol{z}_{n-1})p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})d\boldsymbol{z}_{n-1}$$

$$c_n\mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = \mathcal{N}(\boldsymbol{x}_n|C\boldsymbol{z}_n, \boldsymbol{\Sigma})\int \mathcal{N}(\boldsymbol{z}_{n-1}|\boldsymbol{\mu}_{n-1}, \mathbf{V}_{n-1})\mathcal{N}(\boldsymbol{z}_n|A\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma})d\boldsymbol{z}_{n-1}$$

식 2.113~2.115 참고

$$c_n\mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = \mathcal{N}(\boldsymbol{x}_n|C\boldsymbol{z}_n, \boldsymbol{\Sigma})\ \mathcal{N}(\boldsymbol{z}_n|A\boldsymbol{\mu}_{n-1}, \mathbf{P}_{n-1})$$

$$\mathbf{P}_{n-1} = \boldsymbol{\Gamma} + A\mathbf{V}_{n-1}A^T$$

선형 동적 시스템에서의 추론

$$\mathbf{P}_{n-1} = \boldsymbol{\Gamma} + \boldsymbol{A}\mathbf{V}_{n-1}\boldsymbol{A}^{T}$$

식 2.113 $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}^{-1})$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \mathbf{P}_{n-1})$$

식 2.114 $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|\mathbf{A}\boldsymbol{x}, \mathbf{L}^{-1})$

$$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma})$$

식 2.115 $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|\mathbf{A}\boldsymbol{\mu}, \mathbf{L}^{-1} + \mathbf{A}\boldsymbol{\Lambda}^{-1}\mathbf{A}^{T})$

$$c_n = ?$$

식 2.116 $p(\boldsymbol{x}|\boldsymbol{y}) = \mathcal{N}(\boldsymbol{x}|\Sigma(\mathbf{A}^{\mathrm{T}}\mathbf{L}\mathbf{y} + \mathbf{A}\boldsymbol{\mu}), \Sigma)$

$$\Sigma = (\boldsymbol{\Lambda} + \mathbf{A}^{\mathrm{T}}\mathbf{L}\mathbf{A})^{-1}$$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = ?$$

$$p(\boldsymbol{y})p(\boldsymbol{x}|\boldsymbol{y}) = p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x})$$

$$c_n p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = p(\boldsymbol{x}_n|\boldsymbol{z}_n)p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

선형 동적 시스템에서의 추론

$$\mathbf{P}_{n-1} = \boldsymbol{\Gamma} + \boldsymbol{A}\mathbf{V}_{n-1}\boldsymbol{A}^T$$

식 2.113 $p(\boldsymbol{x}) = \mathcal{N}\left(\boldsymbol{x}\middle|\boldsymbol{\mu}, \boldsymbol{\Lambda}^{-1}\right)$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \mathbf{P}_{n-1})$$

식 2.114 $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}\left(\boldsymbol{y}\middle|\mathbf{A}\boldsymbol{x}, \mathbf{L}^{-1}\right)$

$$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma})$$

식 2.115 $p(\boldsymbol{y}) = \mathcal{N}\left(\boldsymbol{y}\middle|\mathbf{A}\boldsymbol{\mu}, \mathbf{L}^{-1} + \mathbf{A}\boldsymbol{\Lambda}^{-1}\mathbf{A}^T\right)$

$$c_n = \mathcal{N}\left(\boldsymbol{x}_n\middle|\boldsymbol{C}\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \boldsymbol{\Sigma} + \boldsymbol{C}\mathbf{P}_{n-1}\boldsymbol{C}^T\right)$$

식 2.116 $p(\boldsymbol{x}|\boldsymbol{y}) = \mathcal{N}\left(\boldsymbol{x}\middle|\boldsymbol{\Sigma}\left(\mathbf{A}^{\mathrm{T}}\mathbf{L}\mathbf{y} + \boldsymbol{\Lambda}\boldsymbol{\mu}\right), \boldsymbol{\Sigma}\right)$

$$\boldsymbol{\Sigma} = \left(\boldsymbol{\Lambda} + \mathbf{A}^{\mathrm{T}}\mathbf{L}\mathbf{A}\right)^{-1}$$

$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) \rightarrow$$

$$\mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n) = \mathcal{N}\left(\boldsymbol{z}_n\middle|\mathbf{V}_n\left(\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}_n + \mathbf{P}_{n-1}^{-1}\boldsymbol{A}\boldsymbol{\mu}_{n-1}\right), \mathbf{V}_n\right)$$

$$\mathbf{V}_n = \left(\mathbf{P}_{n-1}^{-1} + \mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{C}\right)^{-1}$$

선형 동적 시스템에서의 추론

$$\boldsymbol{\mu}_n = \mathbf{V}_n\left(\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}_n + \mathbf{P}_{n-1}^{-1}\boldsymbol{A}\boldsymbol{\mu}_{n-1}\right)$$

$$(P^{-1} + B^T R^{-1} B)^{-1} B^T R^{-1} = PB^T (BPB^T + R)^{-1} \quad (\text{식 } C.5)$$

$$\mathbf{V}_n = \left(\mathbf{P}_{n-1}^{-1} + \mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{C}\right)^{-1}$$

$$(A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(D + CA^{-1}B)^{-1}CA^{-1} \quad (\text{식 } C.7)$$

$$c_n = \mathcal{N}\left(\boldsymbol{x}_n \middle| \boldsymbol{C}\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \boldsymbol{\Sigma} + \boldsymbol{C}\mathbf{P}_{n-1}\boldsymbol{C}^T\right)$$

$$\mathbf{V}_n = \left(\mathbf{P}_{n-1}^{-1} + \mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{C}\right)^{-1} = \mathbf{P}_{n-1} - \mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}}\left(\boldsymbol{\Sigma} + \mathbf{C}\mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}}\right)^{-1}\mathbf{C}\mathbf{P}_{n-1} \quad (\text{식 } C.7) \text{ 이용}$$

$$= \mathbf{P}_{n-1} - \mathbf{K}_n\mathbf{C}\mathbf{P}_{n-1}$$

$$\mathbf{V}_n = (\mathbf{I} - \mathbf{K}_n\mathbf{C})\mathbf{P}_{n-1}$$

$$\mathbf{K}_n = \mathbf{P}_{n-1}\,\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}} + \boldsymbol{\Sigma}\right)^{-1}$$

선형 동적 시스템에서의 추론

$$\boldsymbol{\mu}_n = \mathbf{V}_n\left(\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}_n + \mathbf{P}_{n-1}^{-1}\mathbf{A}\boldsymbol{\mu}_{n-1}\right)$$

$$(P^{-1} + B^T R^{-1} B)^{-1} B^T R^{-1} = PB^T(BPB^T + R)^{-1} \text{ (식 } C.5)$$

$$\mathbf{V}_n = (\mathbf{I} - \mathbf{K}_n\mathbf{C})\mathbf{P}_{n-1}$$

$$(A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(D + CA^{-1}B)^{-1}CA^{-1} \text{ (식 } C.7)$$

$$\mathbf{K}_n = \mathbf{P}_{n-1}\,\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}} + \boldsymbol{\Sigma}\right)^{-1}$$

$$c_n = \mathcal{N}\left(\boldsymbol{x}_n \middle| \boldsymbol{CA\mu}_{n-1}, \boldsymbol{\Sigma} + \boldsymbol{C}\mathbf{P}_{n-1}\boldsymbol{C}^{T}\right)$$

$$\mathbf{V}_n\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1} = \left(\mathbf{P}_{n-1}^{-1} + \mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1} = \mathbf{P}_{n-1}\,\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}} + \boldsymbol{\Sigma}\right)^{-1}$$

$$\begin{aligned}\boldsymbol{\mu}_n &= \mathbf{V}_n\mathbf{C}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}_n + \mathbf{V}_n\mathbf{C}\mathbf{A}\boldsymbol{\mu}_{n-1} \\ &= \mathbf{K}_n\boldsymbol{x}_n + \mathbf{A}\boldsymbol{\mu}_{n-1} - \mathbf{K}_n\mathbf{C}\mathbf{A}\boldsymbol{\mu}_{n-1} \\ &= \mathbf{A}\boldsymbol{\mu}_{n-1} + \mathbf{K}_n(\boldsymbol{x}_n - \mathbf{K}_n\mathbf{C}\mathbf{A}\boldsymbol{\mu}_{n-1})\end{aligned}$$

$$\mathbf{V}_n\mathbf{P}_{n-1}^{-1}\mathbf{A}\boldsymbol{\mu}_{n-1} = (\mathbf{I} - \mathbf{K}_n\mathbf{C})\mathbf{P}_{n-1}\mathbf{P}_{n-1}^{-1}\mathbf{A}\boldsymbol{\mu}_{n-1}$$

선형 동적 시스템에서의 추론

$$\boldsymbol{\mu}_n = \mathbf{A}\boldsymbol{\mu}_{n-1} + \mathbf{K}_n(\boldsymbol{x}_n - \mathbf{C}\mathbf{A}\boldsymbol{\mu}_{n-1})$$

$$\mathbf{V}_n = (\mathbf{I} - \mathbf{K}_n\mathbf{C})\mathbf{P}_{n-1}$$

$$\mathbf{K}_n = \mathbf{P}_{n-1}\,\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}_{n-1}\mathbf{C}^{\mathrm{T}} + \boldsymbol{\Sigma}\right)^{-1}$$

$$\mathbf{P}_{n-1} = \boldsymbol{\Gamma} + \mathbf{A}\mathbf{V}_{n-1}\mathbf{A}^{T}$$

$$\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{\mu}_n, \mathbf{V}_n)$$

$$c_n\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{x}_n|\boldsymbol{z}_n)\int \hat{\alpha}(\boldsymbol{z}_{n-1})p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})d\boldsymbol{z}_{n-1}$$

$p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1}) = \mathcal{N}(\boldsymbol{z}_n|\boldsymbol{A}\boldsymbol{z}_{n-1}, \boldsymbol{\Gamma}) \rightarrow$ 전이 확률

$p(\boldsymbol{x}_n|\boldsymbol{z}_n) = \mathcal{N}(\boldsymbol{x}_n|\boldsymbol{C}\boldsymbol{z}_n, \boldsymbol{\Sigma}) \rightarrow$ 방사확률

$p(\boldsymbol{z}_1) = \mathcal{N}(\boldsymbol{z}_1|\boldsymbol{\mu}_n, \mathbf{P}_0) \rightarrow$ 초기값

센서로부터 주어진 관측값 $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n$

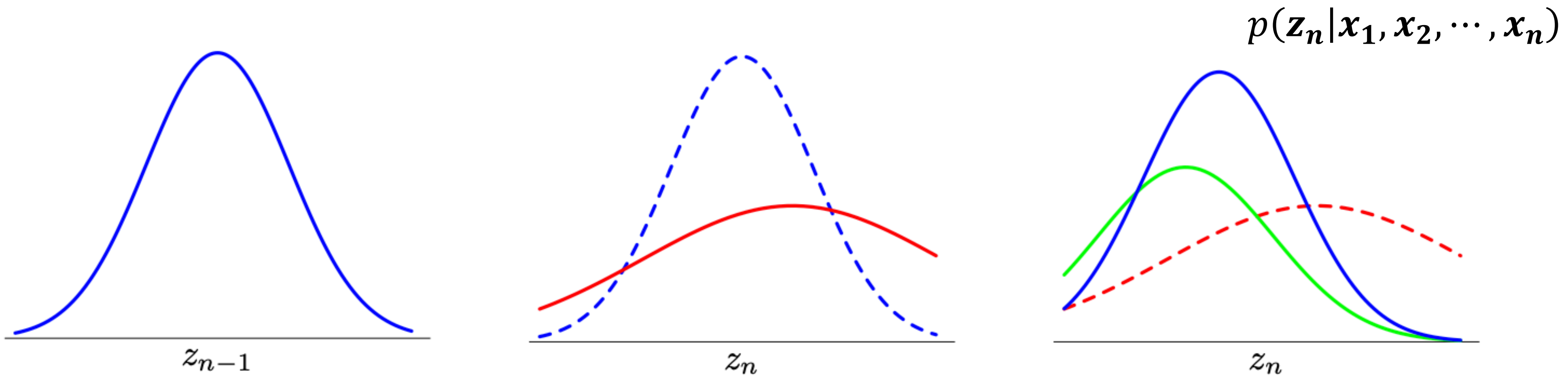
알려지지 않은 수량 $\boldsymbol{z}_n$

$$c_n = p(\boldsymbol{x}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$

$$\hat{\alpha}(\boldsymbol{z}_n) = p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n)$$

$$c_n = \mathcal{N}\left(\boldsymbol{x}_n\middle|\boldsymbol{C}\boldsymbol{A}\boldsymbol{\mu}_{n-1}, \boldsymbol{\Sigma} + \boldsymbol{C}\mathbf{P}_{n-1}\boldsymbol{C}^{T}\right)$$

선형 동적 시스템에서의 추론



$$p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1}) = \int p(\boldsymbol{z}_n|\boldsymbol{z}_{n-1})p(\boldsymbol{z}_{n-1}|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})\mathrm{d}\boldsymbol{z}_{n-1}$$

$$p(\boldsymbol{x}_n|\boldsymbol{z}_n)$$

$$c_n p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n) = p(\boldsymbol{x}_n|\boldsymbol{z}_n)p(\boldsymbol{z}_n|\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n-1})$$