# Programming Report for Project1

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## The experiment design

The main idea in this experiment is applying FD methods to solve BVP.Here we're given two problem domains,the regular one and the irregular one,as well as three boundary conditions,Dirichlet,Neumann and mixed.Thus,we'll consider six combinations.However,the key point in the implementation of BVP solver related to these different situations lies in constructing the corresponding linear systems and solving them.

Compared to irregular domain,it may be less strenuous to obtain the linear system for the regular domain.Since we have talked all these three boundary conditions in one dimension,what we should do is transplant the argument to two dimension.Besides,we should pay attention to the use of ghost cell in the case of Neumann condition and mixed condition.

As required in programming assignments,we'll implement a Validity-Check function for the irregular domain,which can check whether $\Omega\backslash D$ cover at least four grid number and keeps connected.

To make our program more flexible,a input file,which stores user-specified parameters,is included in the project.Here we use json to support our input file.Besides problem domain and boundary condition,the center and the radius of the disk as well as parameters(they are $\alpha$ and $\beta$) in the mixed condition($\alpha u + \beta\frac{\partial u}{\partial n} = \sigma$) are also listed in the input.json.

In the experiment,we'll test three different function,they are $u_1 = e^{x^2+y^2}$,$u_2 = e^{sinx+y}$ and $u_3 = e^{x+y}$.

PS:All the statistic data will be kept in a file named result.txt.

## Experiment results

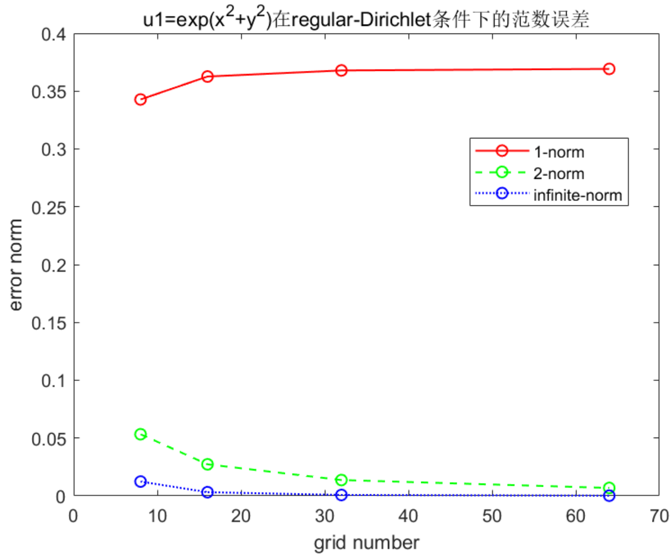
In this section,we're going to show 1-,2- and $\infty-$ norms of errors for different combinations of problem domain and boundary condition,and then get the corresponding convergence rates on the four grids.This procedure will be applied for these three functions.Besides,we'll also verify some analytic results in the next section based on the results in this section combined with the statistic data stored in result.txt.

Here we use $r_2$ to represent the convergence rate of 2-norm and $r_i$ the convergence of $\infty-$norm.We neglect the convergence rate of 1-norm,since we can conclude that 1-norm of errors diverges from following figures.We also use $||E_1||$ to represent error norm on n=8,and $||E_2||, ||E_3||, ||E_4||$ for n=16,32,64 respectively.

1. **Regular domain**

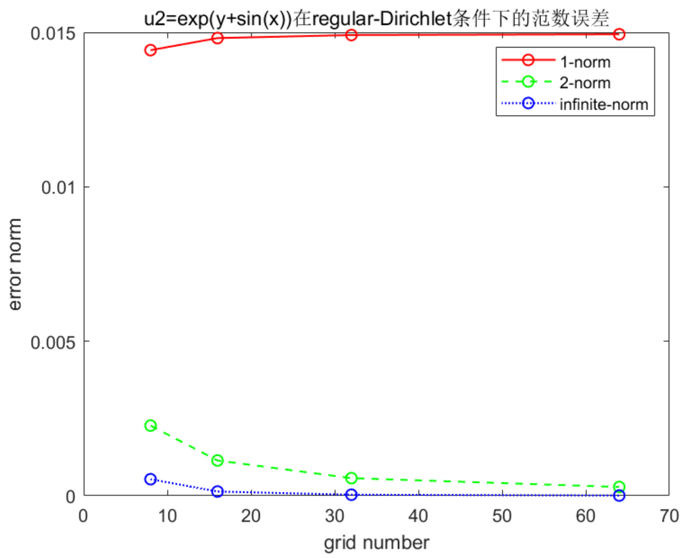
   (a) Dirichlet boundary condition

      i. $u_1 = e^{x^2+y^2}$



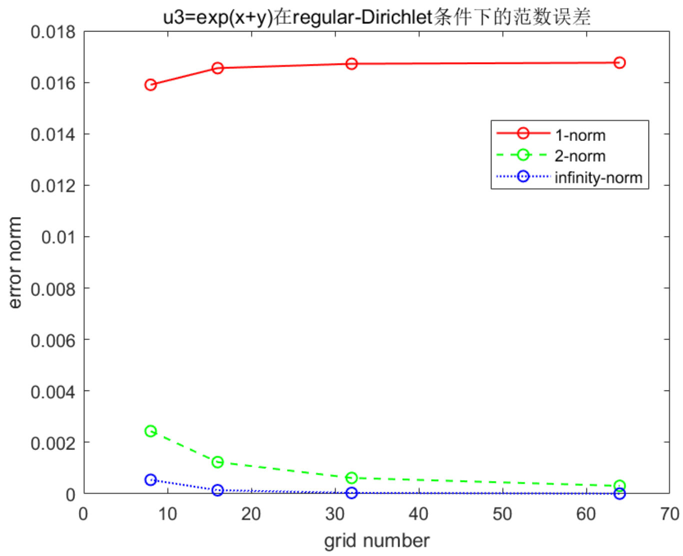

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5121299311 | 0.5029061782 | 0.5007180675 | $r_2 = 0.5$ |
| $\infty - norm$ | 0.2579288156 | 0.2520887977 | 0.2505580606 | $r_i = 0.25$ |

ii. $u_2 = e^{sinx+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.502283748 | 0.5005431545 | 0.5001347662 | $r_2 = 0.5$ |
| $\infty - norm$ | 0.2562381164 | 0.2503739235 | 0.2503364277 | $r_i = 0.25$ |

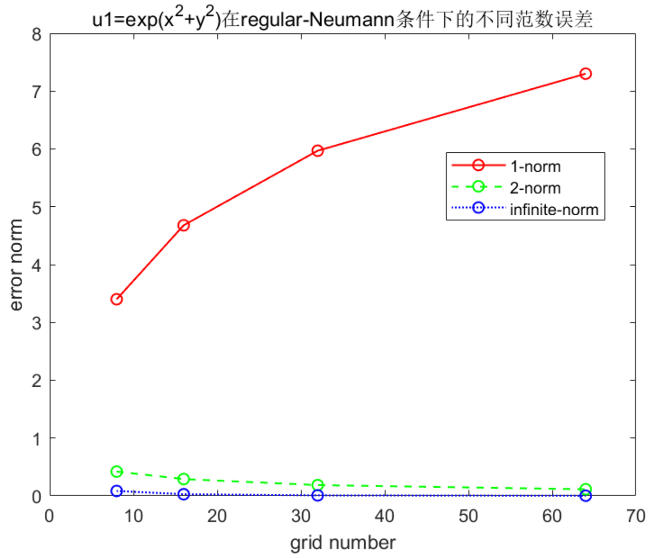
iii. $u_3 = e^{x+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5059917321 | 0.5014491931 | 0.5003596349 | $r_2 = 0.5$ |
| $\infty - norm$ | 0.2572442124 | 0.2506898646 | 0.2504071506 | $r_i = 0.25$ |

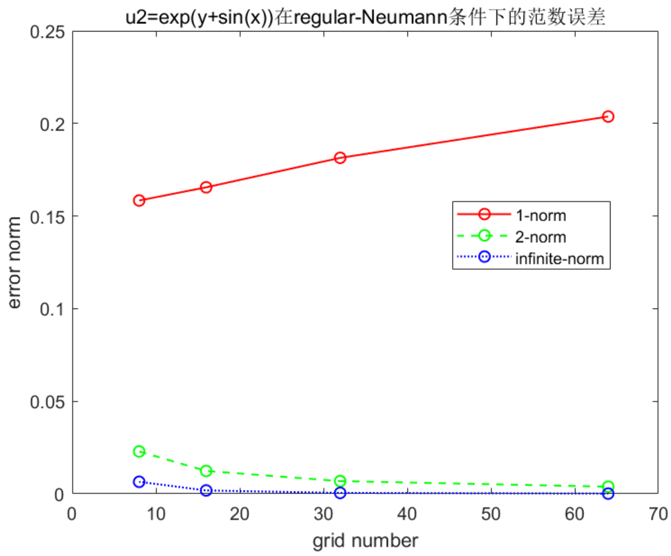
(b) Neumann boundary condition

i. $u_1 = e^{x^2+y^2}$



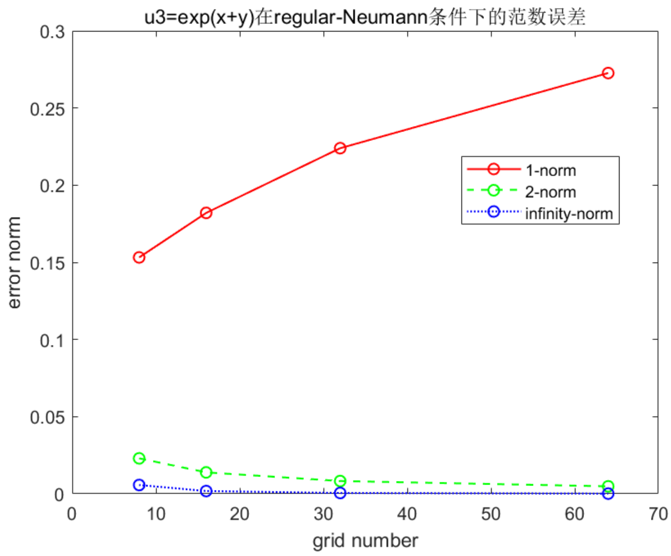

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.6914829902 | 0.6432717986 | 0.6145716453 | $r_2 \approx 0.6$ |
| $\infty - norm$ | 0.3406122871 | 0.3106513119 | 0.2968982732 | $r_i \approx 0.3$ |

ii. $u_2 = e^{sinx+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5400560934 | 0.5564410577 | 0.5615676036 | $r_2 \approx 0.55$ |
| $\infty - norm$ | 0.28098014 | 0.2757195556 | 0.2721561767 | $r_i \approx 0.27$ |

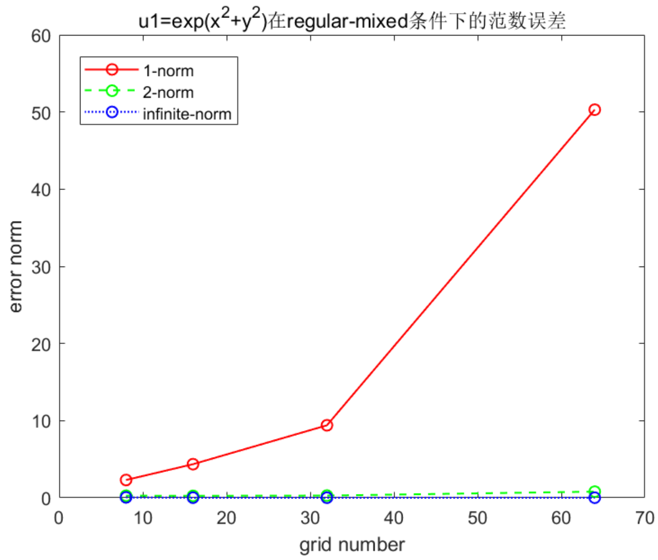
iii. $u_3 = e^{x+y}$



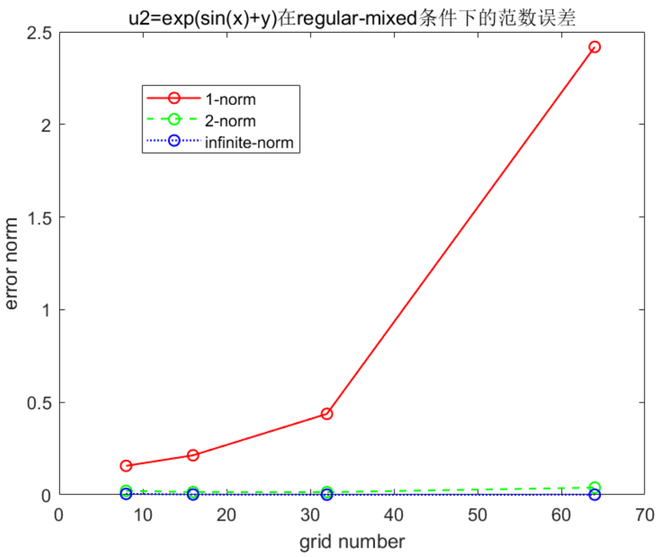

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.604912772 | 0.5968881473 | 0.5873033651 | $r_2 \approx 0.58$ |
| $\infty - norm$ | 0.3165868745 | 0.2962321112 | 0.2861720759 | $r_i \approx 0.28$ |

(c) mixed boundary condition

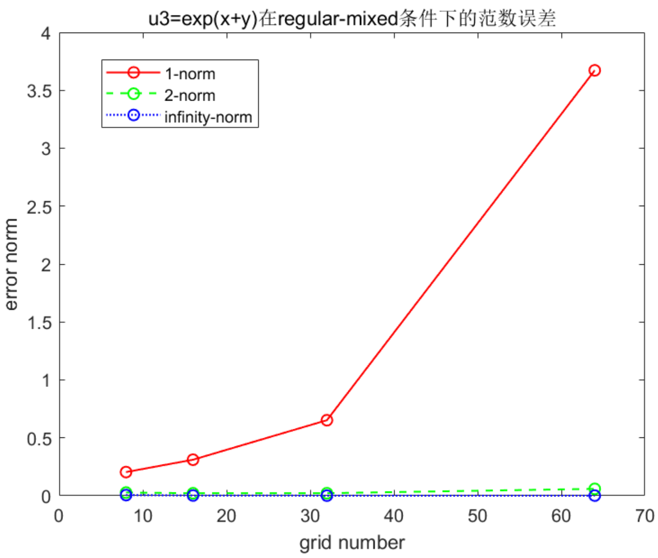
i. $u_1 = e^{x^2+y^2}$




ii. $u_2 = e^{sinx+y}$




iii. $u_3 = e^{x+y}$

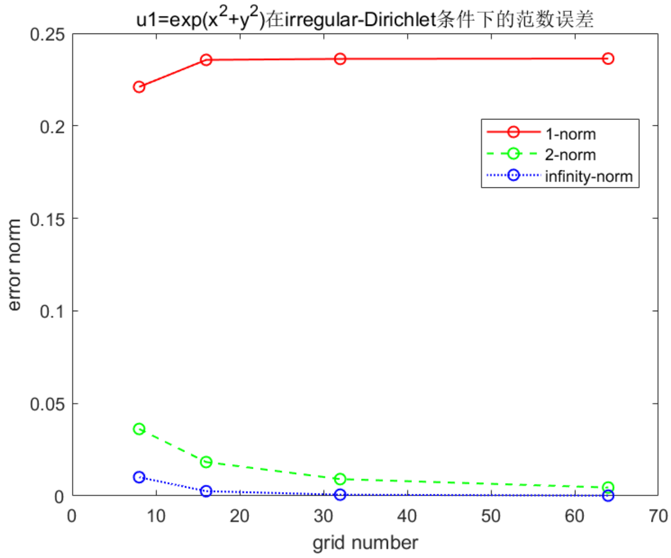


By our observation,in this case,all these three norms do not convergence.However,there is a interesting phenomenon,that is both 2-norm and $\infty - norm$ of errors oscillate a little bit around certain value.

2. **Irregular domain**

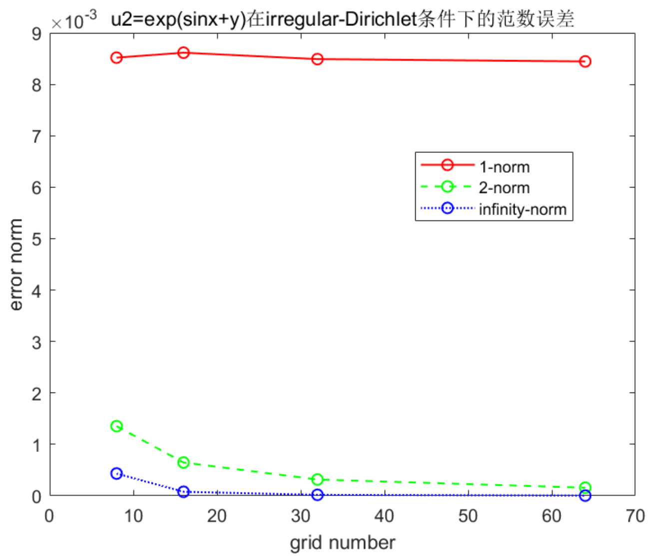
(a) Dirichlet boundary condition

i. $u_1 = e^{x^2+y^2}$



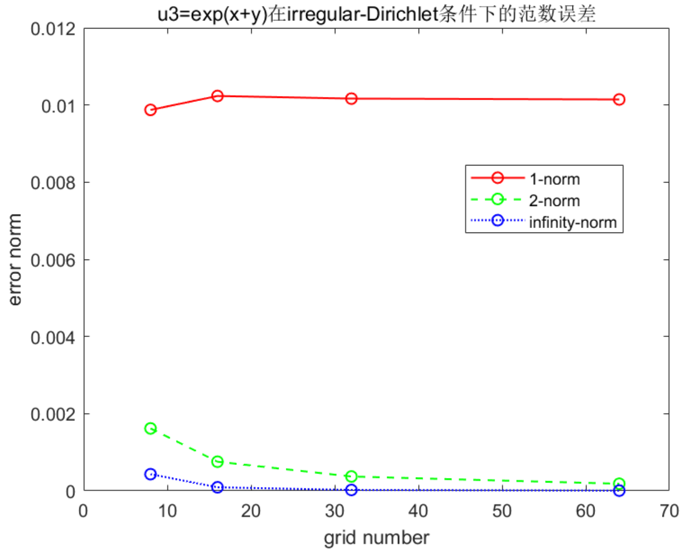

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5054488341 | 0.4968028711 | 0.4984764911 | $r_2 \approx 0.5$ |
| $\infty - norm$ | 0.2522879829 | 0.2499792015 | 0.2494788167 | $r_i \approx 0.25$ |

ii. $u_2 = e^{sinx+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.4783647567 | 0.4907180872 | 0.4999119796 | $r_2 \approx 0.5$ |
| $\infty - norm$ | 0.1816433812 | 0.2476863022 | 0.256708827 | $r_i \approx 0.25$ |

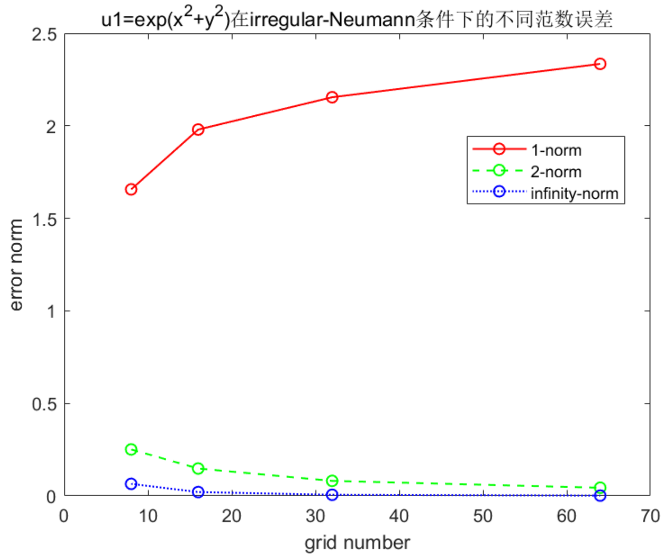
iii. $u_3 = e^{x+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.466324052 | 0.4926711987 | 0.4945885961 | $r_2 \approx 0.5$ |
| $\infty - norm$ | 0.2125576421 | 0.2463335795 | 0.2439666111 | $r_i \approx 0.25$ |

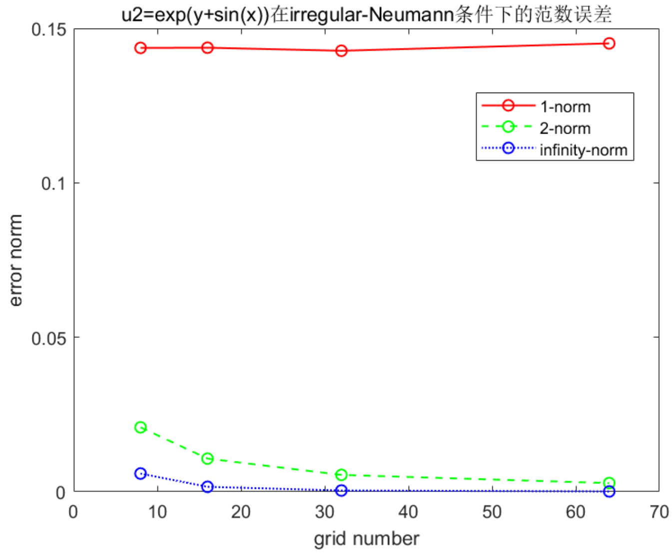
(b) Neumann boundary condition

i. $u_1 = e^{x^2+y^2}$



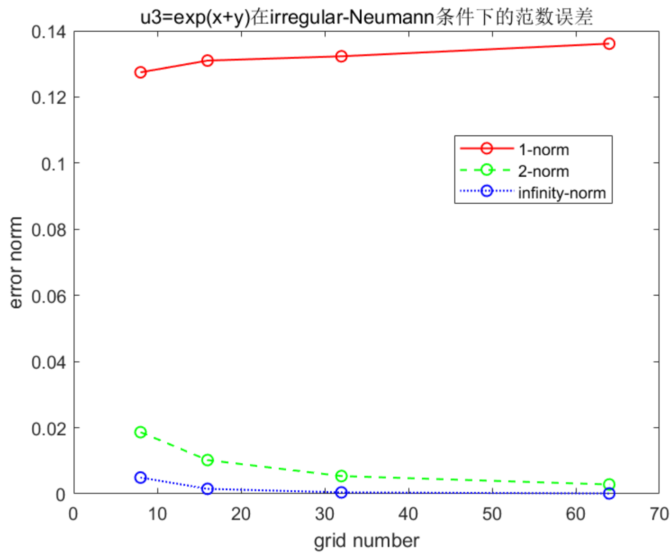

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5892594038 | 0.5483341893 | 0.5423100482 | $r_2 \approx 0.54$ |
| $\infty - norm$ | 0.3192409224 | 0.2904224118 | 0.2794476615 | $r_i \approx 0.29$ |

ii. $u_2 = e^{sinx+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5148743174 | 0.5087258946 | 0.5150658203 | $r_2 \approx 0.5$ |
| $\infty - norm$ | 0.2718046004 | 0.2608189795 | 0.2589987361 | $r_i \approx 0.26$ |

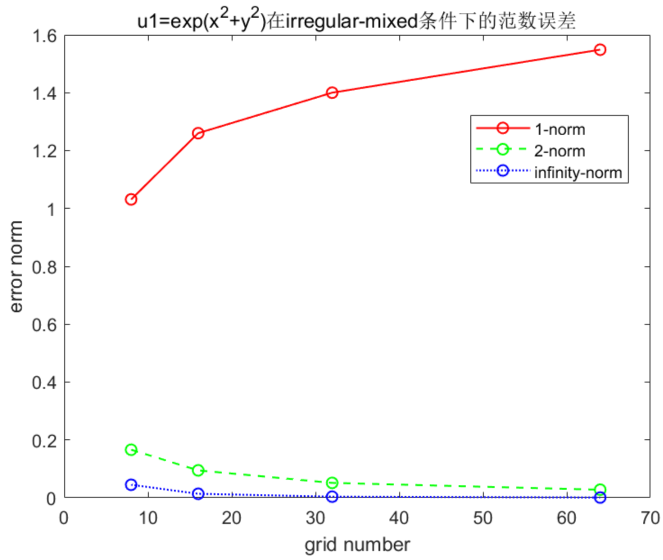
iii. $u_3 = e^{x+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5479145659 | 0.5261260291 | 0.5263944269 | $r_2 \approx 0.52$ |
| $\infty - norm$ | 0.3057717718 | 0.2827130255 | 0.2744805241 | $r_i \approx 0.27$ |

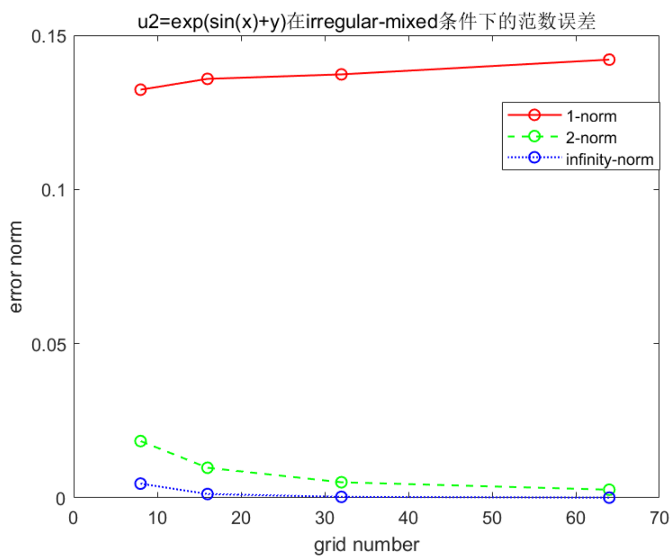
(c) mixed boundary condition

i. $u_1 = e^{x^2+y^2}$



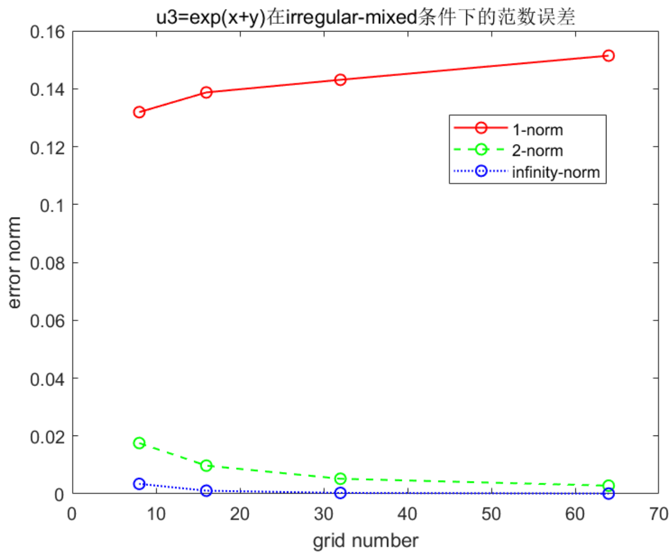

| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.570932554 | 0.5440702816 | 0.5423870559 | $r_2 \approx 0.54$ |
| $\infty - norm$ | 0.3100026111 | 0.2847776897 | 0.2753919478 | $r_i \approx 0.27$ |

ii. $u_2 = e^{sinx+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.530446321 | 0.5191408732 | 0.5251904756 | $r_2 \approx 0.52$ |
| $\infty - norm$ | 0.271546676 | 0.2627844797 | 0.2718567933 | $r_i \approx 0.27$ |

iii. $u_3 = e^{x+y}$




| | $\frac{\|\|E_2\|\|}{\|\|E_1\|\|}$ | $\frac{\|\|E_3\|\|}{\|\|E_2\|\|}$ | $\frac{\|\|E_4\|\|}{\|\|E_3\|\|}$ | r |
|---|---|---|---|---|
| 2-norm | 0.5553088038 | 0.533888192 | 0.5419447485 | $r_2 \approx 0.54$ |
| $\infty - norm$ | 0.3148841036 | 0.2895421518 | 0.286515883 | $r_i \approx 0.28$ |

# Analytic results based on experiments

1. As we can see from figures showed above,the FD method is convergent in both 2-norm and max-norm(infinity-norm).And in the same condition,the FD method converges faster in the max-norm than in the 2-norm.

2. The FD method usually don't converge in the 1-norm.However,in some circumstances,the 1-norm of error oscillates a little bit around certain value,in other words,it is stable.

3. For both regular and irregular domain,the FD method usually converges faster in the Dirichlet boundary condition than in the Neumann and mixed boundary conditions.