Artificial Intelligence HW 2 - 2020085014 강준서

1. (a) $\sigma(x) = 1/(1+e^{-x})$

$$\sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \underline{\sigma(x)(1-\sigma(x))}$$

(b) Let $h_w(x) = 1/(1+e^{-w\cdot x})$.

w^* : updated weight , α : learning rate

$$w^* = w + \alpha \sum_{i=1}^{n} (y_i - h_w(x_i)) \cdot h_w'(x_i) \cdot x_i$$

$$= \underline{w + \alpha \sum_{i=1}^{n} (y_i - h_w(x_i)) \cdot h_w(x_i)(1-h_w(x_i)) \cdot x_i}$$

2.1 $P(Y=1 \mid X_1=1, X_2=1) = \alpha P(Y=1) \prod_i P(X_i=1 \mid Y=1)$

$= \alpha P(Y=1) P(X_1=1 \mid Y=1) P(X_2=1 \mid Y=1)$

$= \alpha q P_1 P_2$ .... (1)

$P(Y=0 \mid X_1=1, X_2=1) = \alpha P(Y=0) \prod_i P(X_i=1 \mid Y=0)$

$= \alpha P(Y=0) P(X_1=1 \mid Y=0) P(X_2=1 \mid Y=0)$

$= \alpha (1-q)(1-P_1)(1-P_2)$ .... (2)

(1) + (2) = 1

$$\therefore \alpha = \frac{1}{qP_1P_2 + (1-q)(1-P_1)(1-P_2)}$$

$$\therefore P(Y=1 \mid X_1=1, X_2=1) = \underline{\frac{qP_1P_2}{qP_1P_2 + (1-q)(1-P_1)(1-P_2)}}$$

2.2 (a) Total : 13

Long : 7 Medium : 2 Short : 4

$$H(\text{target}) = -\left(\frac{7}{13}\log_2\frac{7}{13} + \frac{2}{13}\log_2\frac{2}{13} + \frac{4}{13}\log_2\frac{4}{13}\right)$$

$= \underline{0.8956...}$ calculated by WolframAlpha

(b) Hour = 8 : Long 3

Hour = 9 : Long 3, Medium 2

Hour = 10 : Long 1, Short 4

$$H^*(\text{target}) = -\left[\frac{3}{13}\times 0 + \frac{5}{13}\times\left(\frac{3}{5}\log_2\frac{3}{5} + \frac{2}{5}\log_2\frac{2}{5}\right) + \frac{5}{13}\left(\frac{1}{5}\log_2\frac{1}{5} + \frac{4}{5}\log_2\frac{4}{5}\right)\right]$$

$= 0.4108...$

Gain (Hour) = $H(\text{target}) - H^*(\text{target})$ = 0.8956... - 0.4108...

= <u>0.4848...</u>

(c) if Weather :

Weather = Sunny : Long 3, Medium 1, Short 2

Weather = Cloudy : Long 3, Short 1

Weather = Rainy : Long 1, Medium 1, Short 1

Gain (Weather) = 0.8956... - 0.8132... = <u>0.0824...</u>

if Accident :

Accident = No : Long 2, Medium 2, Short 4

Accident = Yes : Long 5

Gain (Accident) = 0.8956... - 0.5824... = 0.3132...

if Stall :

Stall = No : Long 4 , Medium 2, Short 4

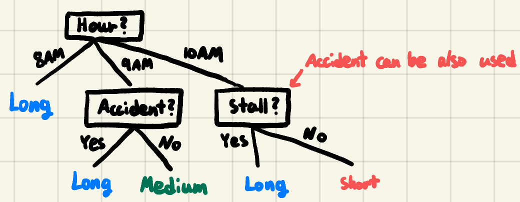
Stall = Yes : Long 3

Gain (Stall) = 0.8956... - 0.7386... = 0.1570...

$\text{argmax}_{\text{gain}} = \text{Gain (Hour)}$

∴ Hour must be selected as the first criterion.

(d)



3. (a) $y = \max(0, mx + b)$

(b) $$\frac{\partial Loss}{\partial a} = \frac{\partial Loss}{\partial y} \cdot \frac{\partial y}{\partial a} = 2(y - y^*) \cdot \frac{\partial y}{\partial a}$$

if $a < 0$: $\frac{\partial y}{\partial a} = 0$ $\qquad$ if $a > 0$: $\frac{\partial y}{\partial a} = 1$

$\frac{\partial Loss}{\partial a} = 0$ $\qquad$ $\frac{\partial Loss}{\partial a} = 2(y - y^*)$

(c) $$\frac{\partial Loss}{\partial m} = \frac{\partial Loss}{\partial y} \cdot \frac{\partial y}{\partial a} \cdot \frac{\partial a}{\partial m} \qquad \frac{\partial a}{\partial m} = x$$

if $a < 0$: $\frac{\partial Loss}{\partial m} = 0$

if $a > 0$: $\frac{\partial Loss}{\partial m} = 2(y - y^*)x$

(d) $$\frac{\partial Loss}{\partial b} = \frac{\partial Loss}{\partial y} \cdot \frac{\partial y}{\partial a} \cdot \frac{\partial a}{\partial b} \qquad \frac{\partial a}{\partial b} = 1$$

if $a < 0$: $\frac{\partial Loss}{\partial m} = 0$

if $a > 0$: $\frac{\partial Loss}{\partial m} = 2(y - y^*)$

(e) α : learning rate

$m \leftarrow m - 2\alpha(y - y^*)x$

$b \leftarrow b - 2\alpha(y - y^*)$

(if $a < 0$, update doesn't occur in both m, b)