# TOWERS OF HANOI

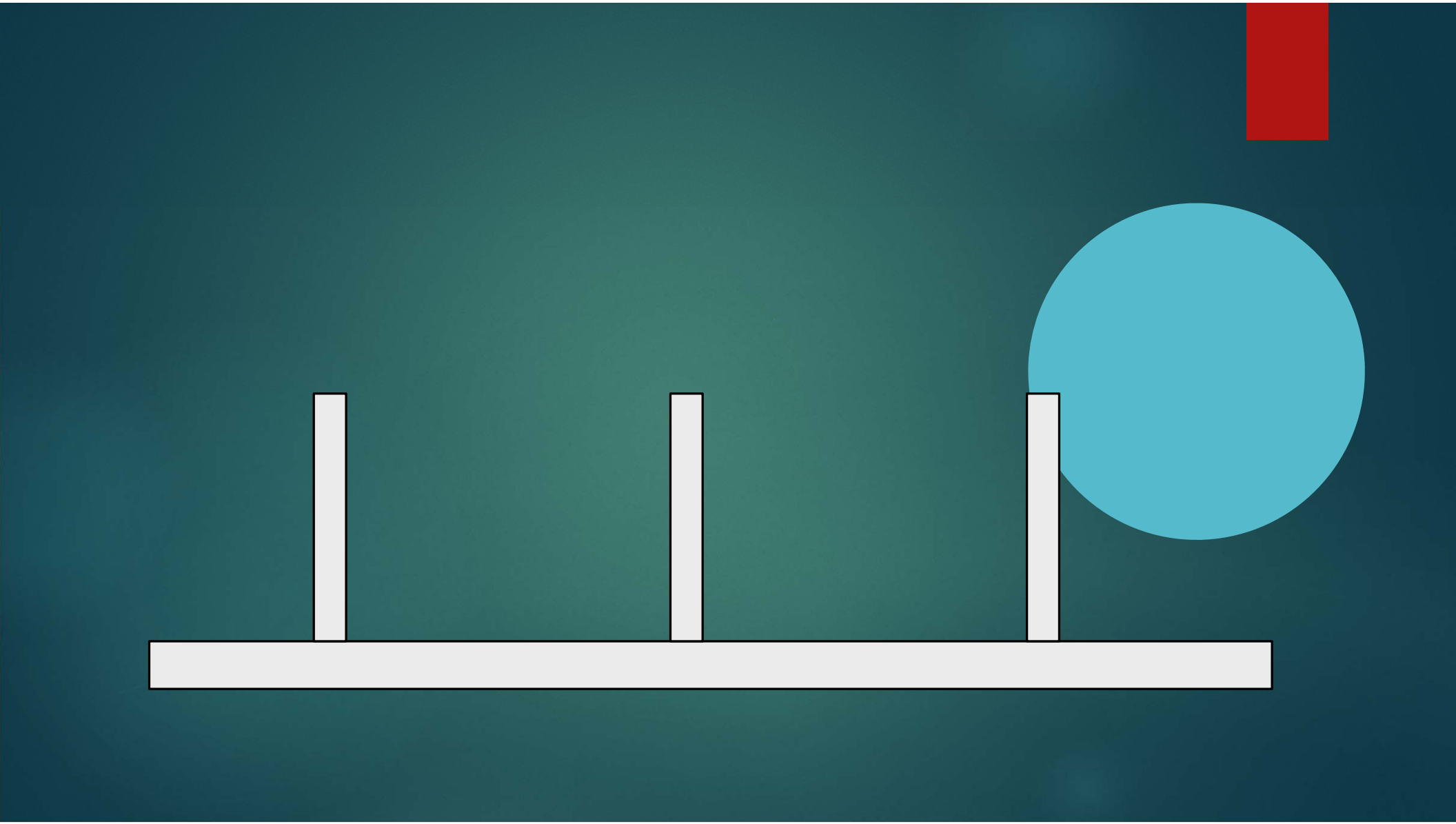
## RECURSION

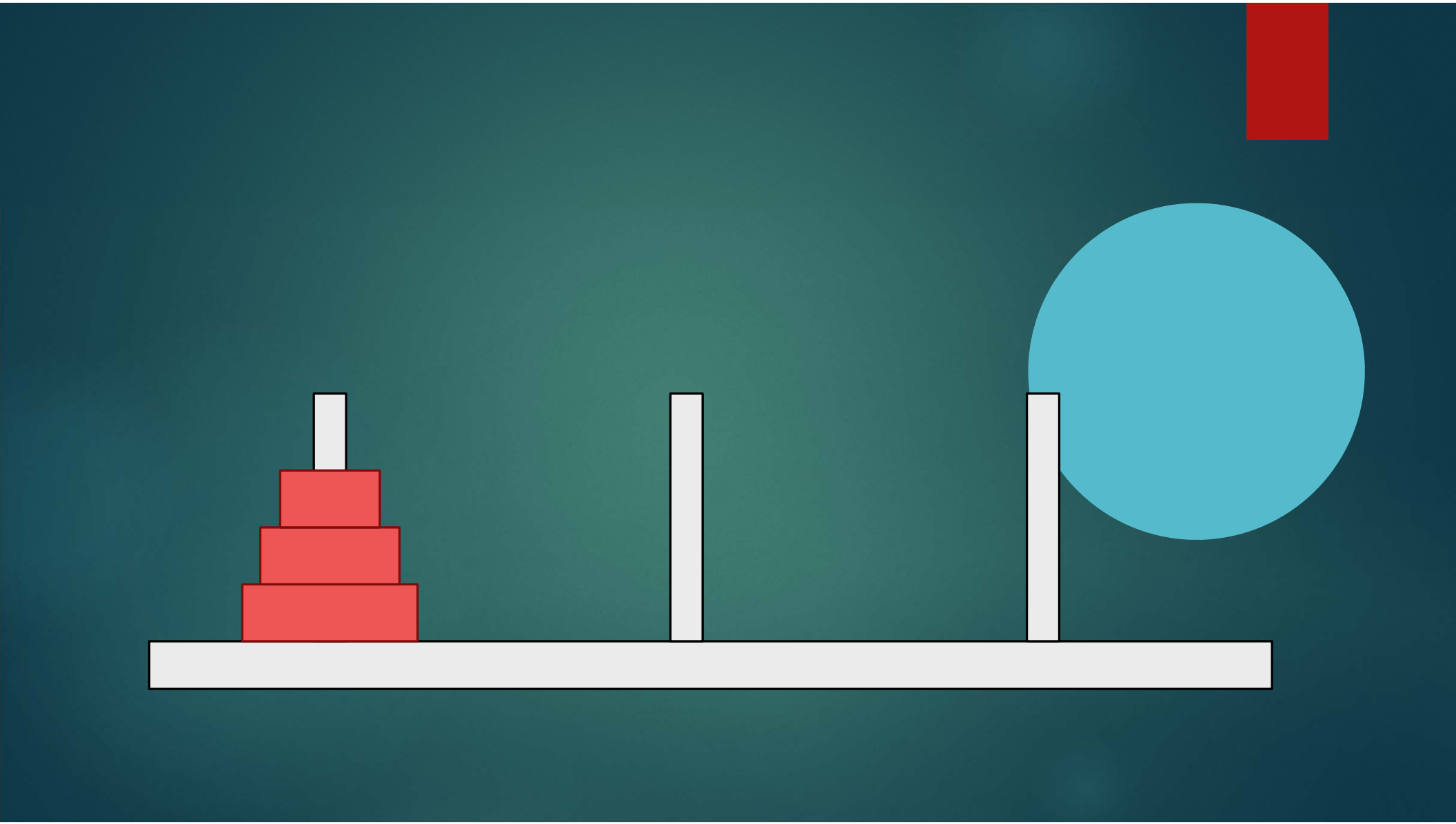
# Towers of Hanoi

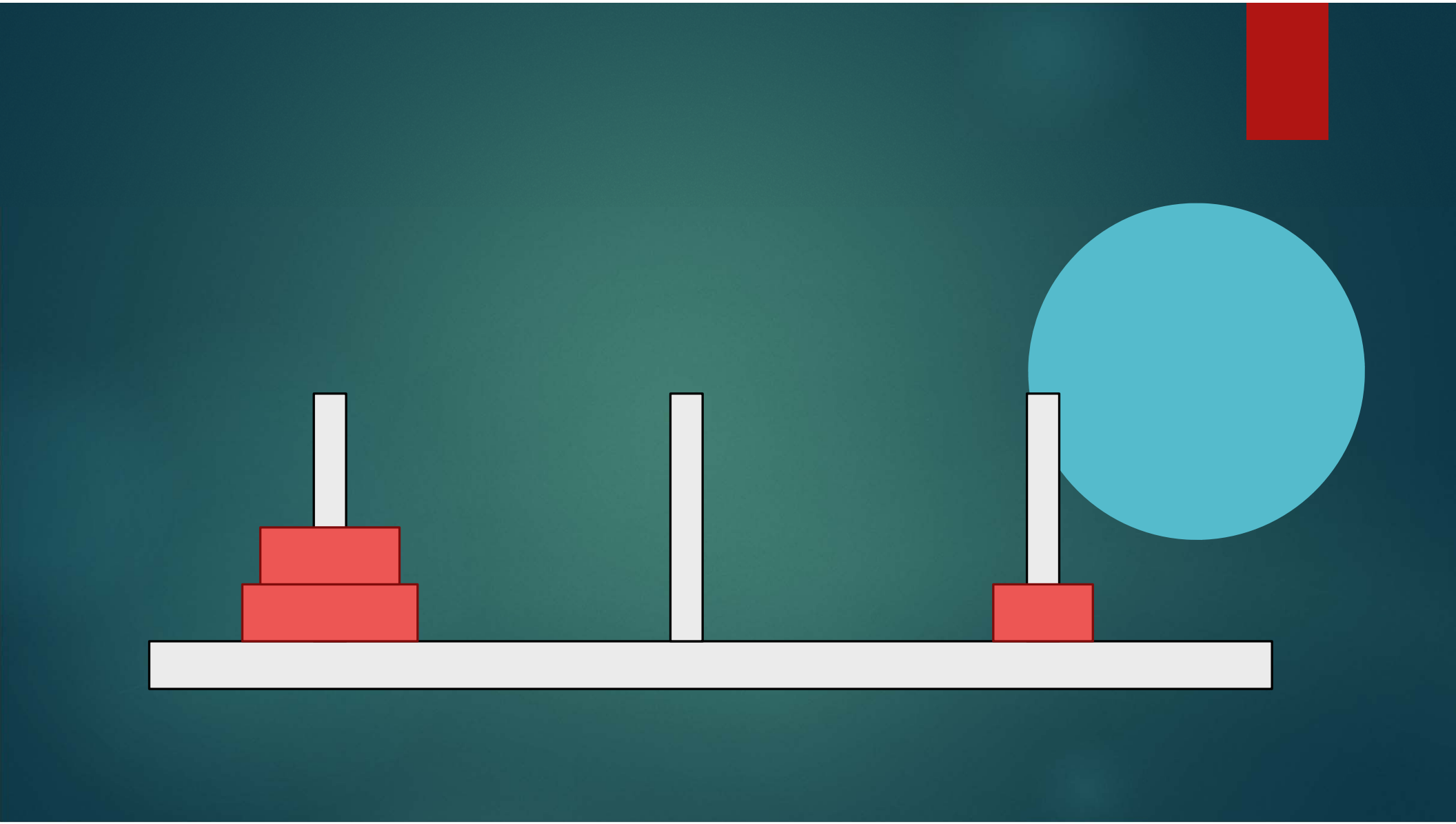
- It consists of three rods and number of disks of different sizes which can slide onto any rod
- The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape
- The minimum number of moves required to solve a Tower of Hanoi problem is $2^n - 1$ // $O(2^n)$ *exponential time complexity*
- We have some rules:
  - Only one disk can be moved at a time
  - Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack → a disk can only be moved if it is the uppermost disk on a stack
  - No disk may be placed on top of a smaller disk

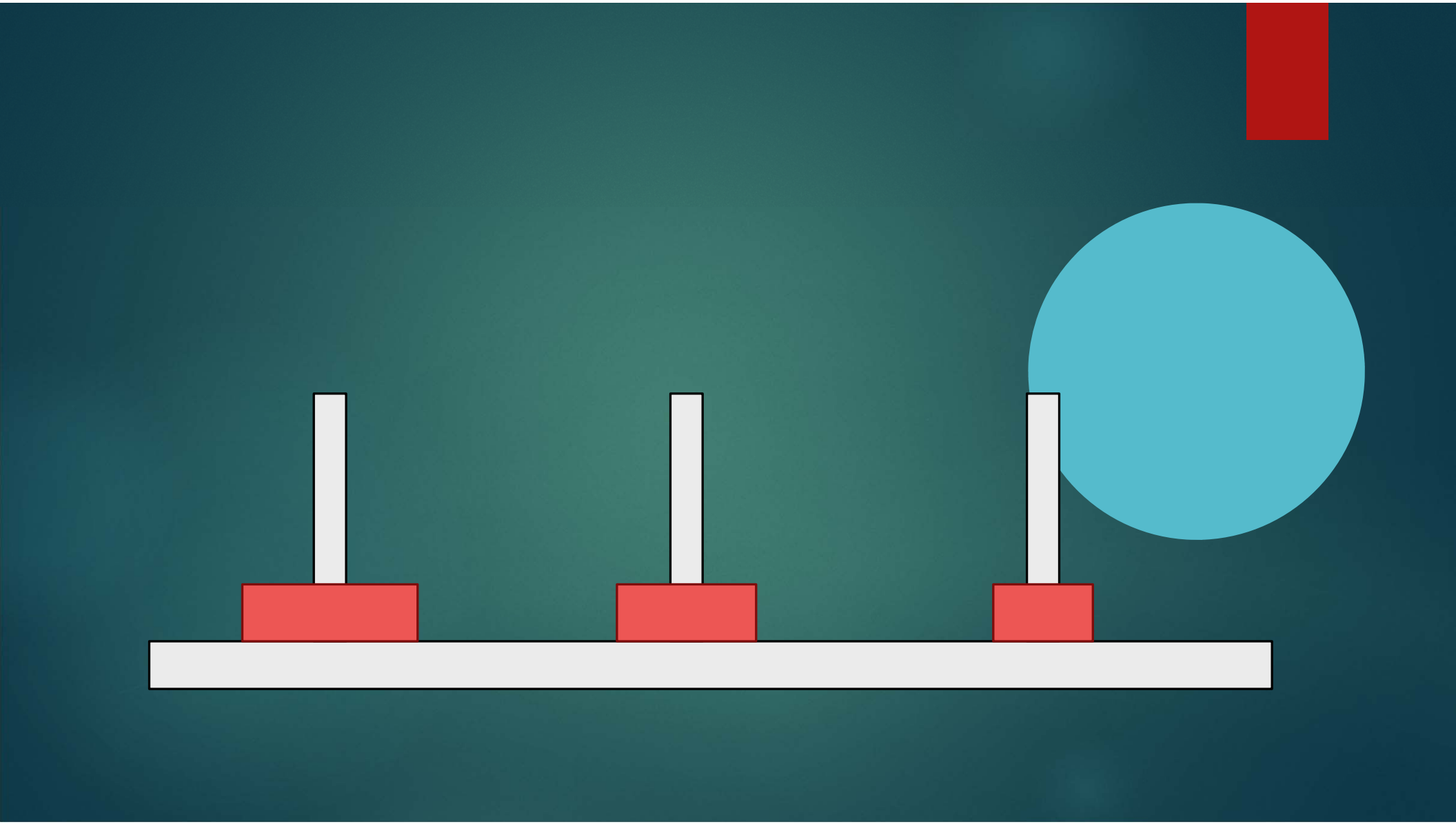
# Towers of Hanoi

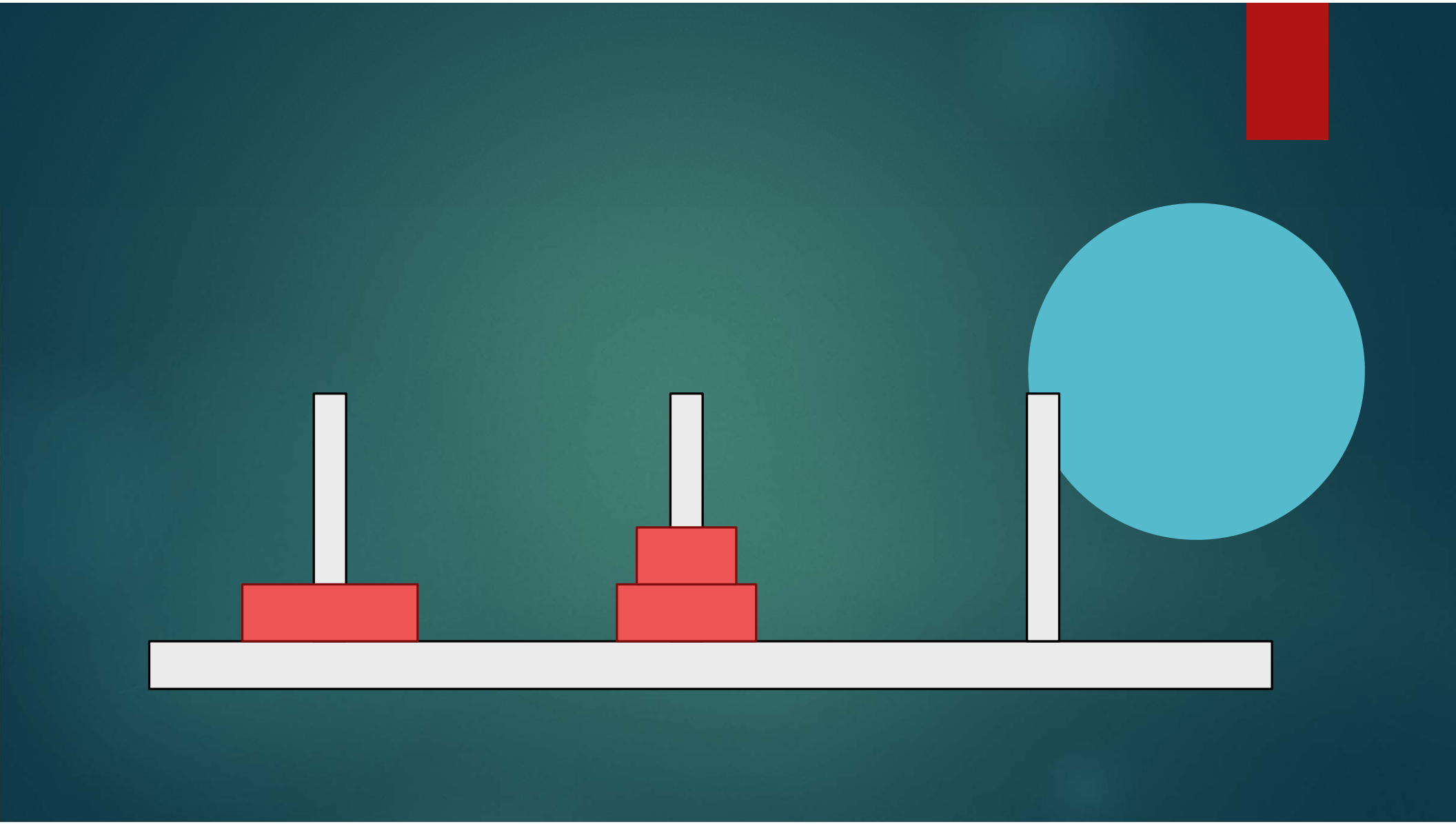
- The is legend concerning the *towers of Hanoi*
- Indian priests were to transfer a tower consisting **64** disks from one part of the temple to another
- One disk at a time + larger disk may never be placed upon a smaller one
- It is said when the priests complete their task → the world will come to an end !!!
- How many moves are there? $\mathbf{2^{64} - 1}$ moves **!!!**

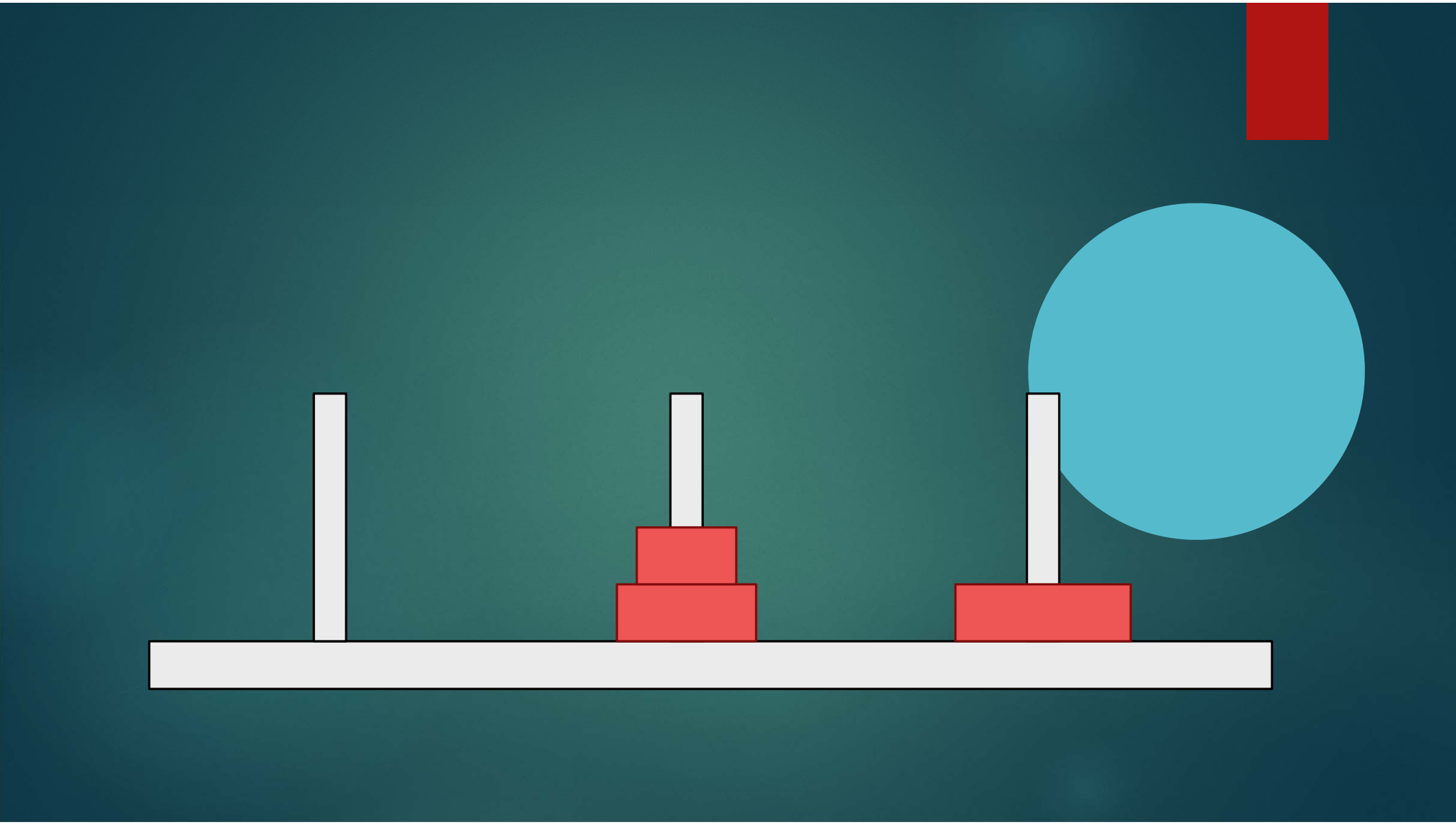


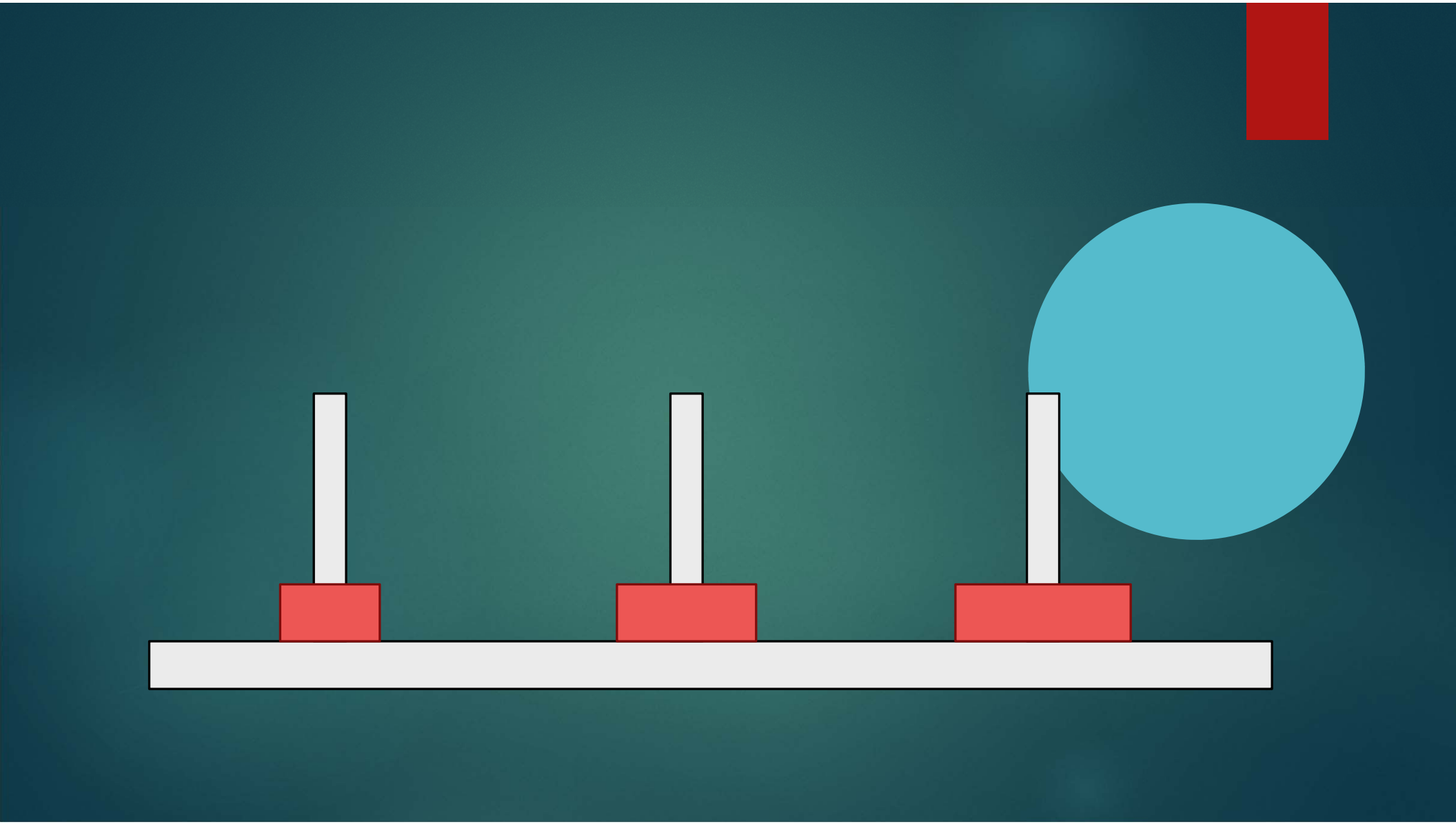


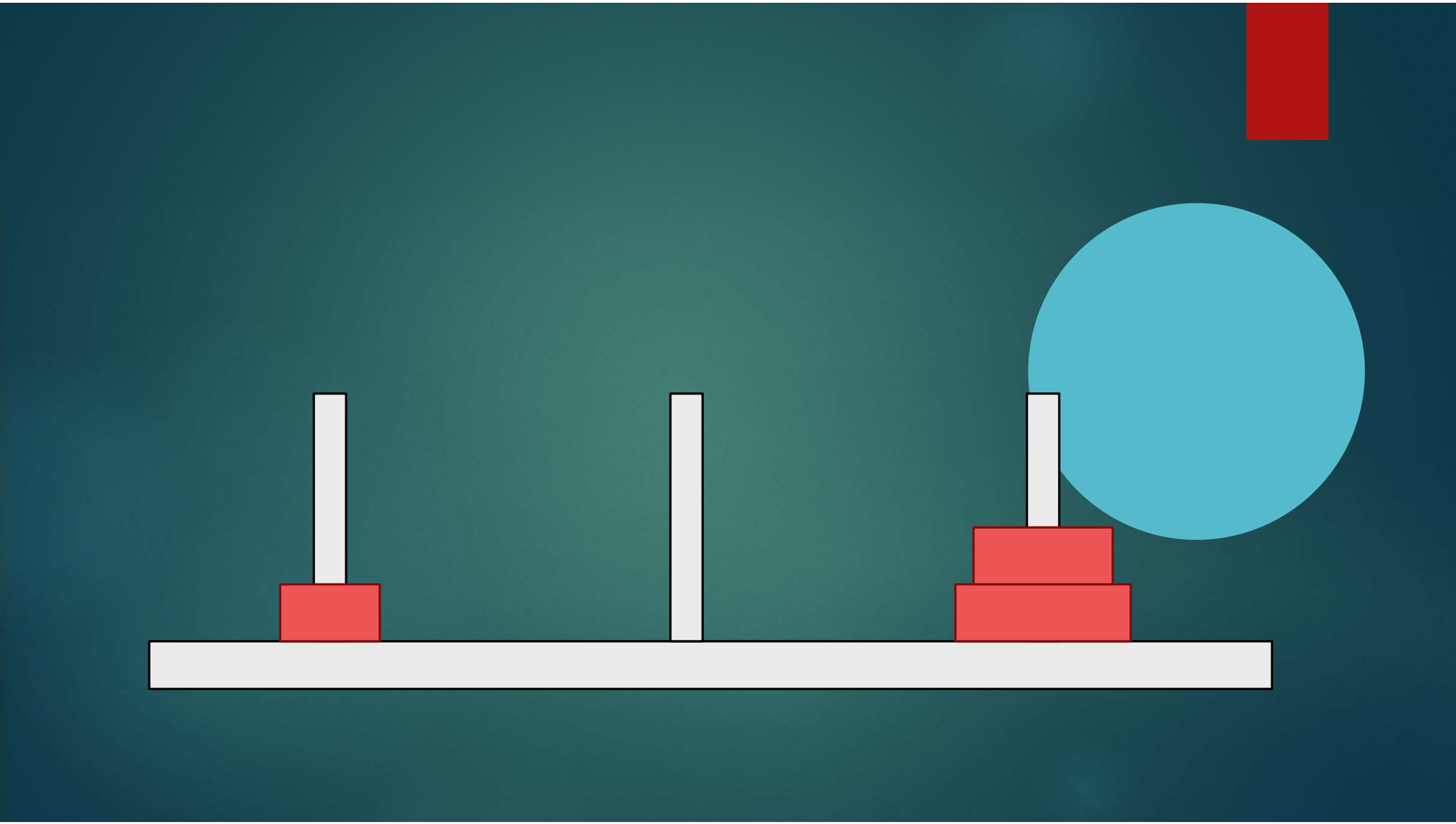


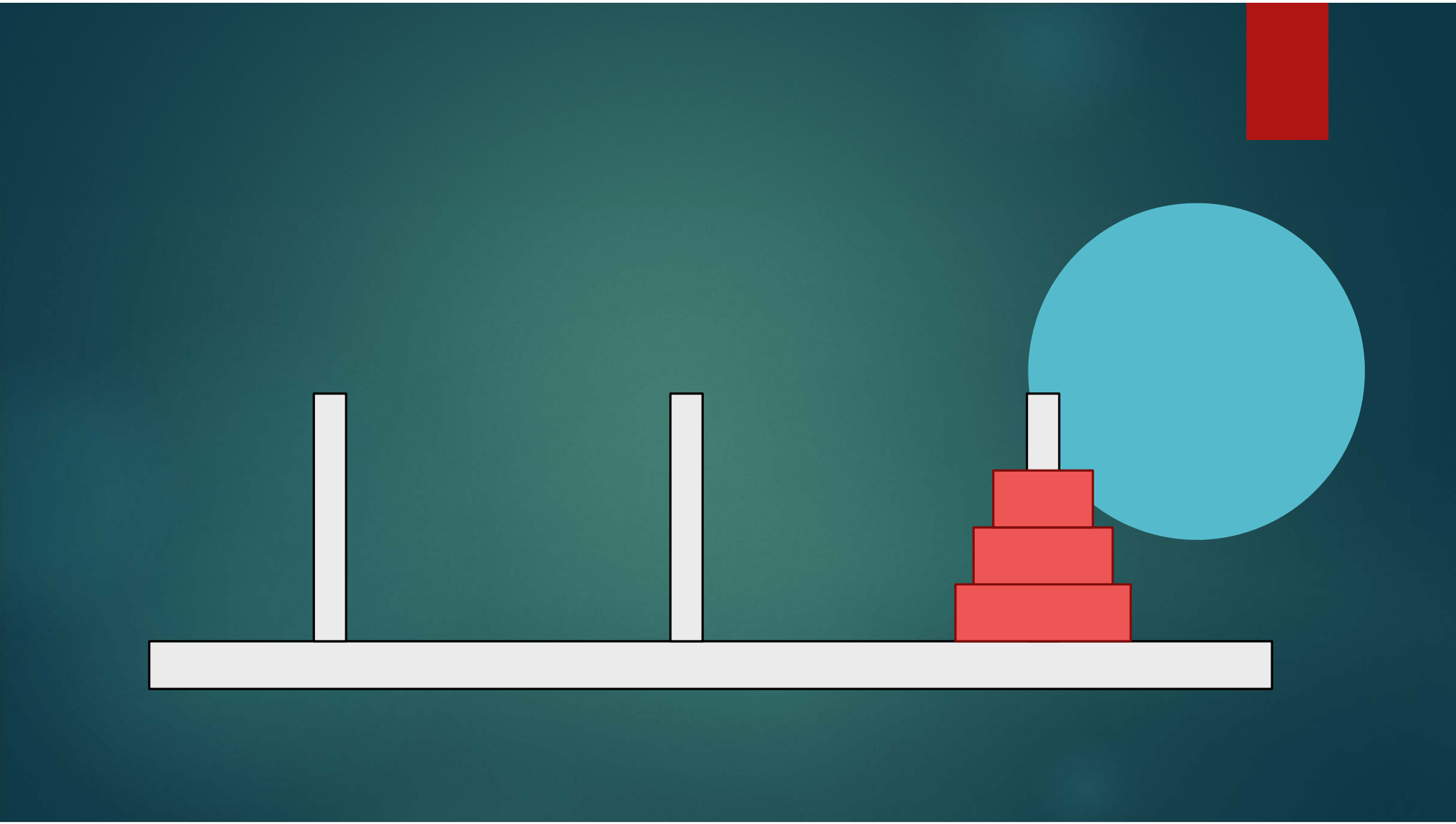




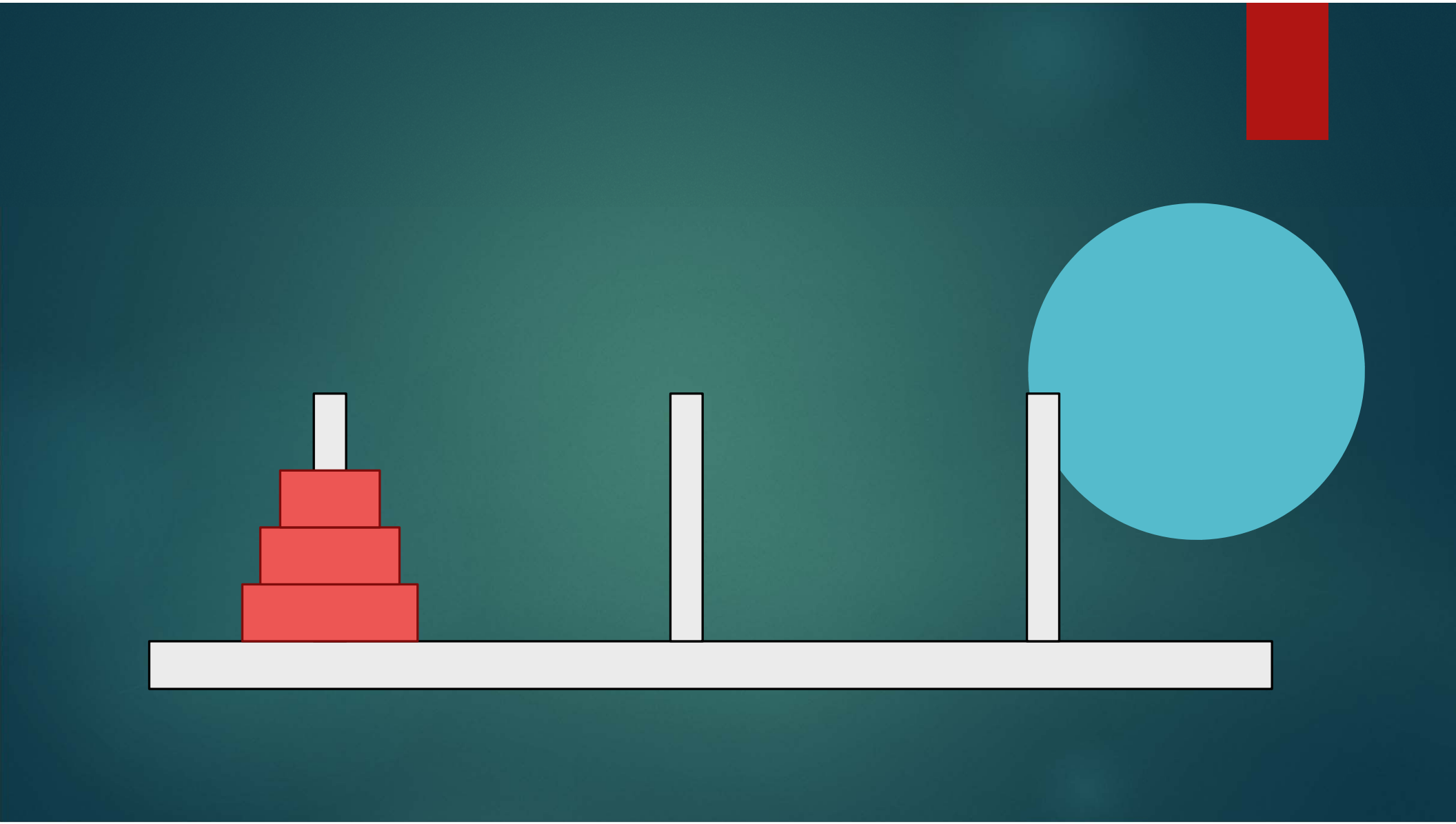


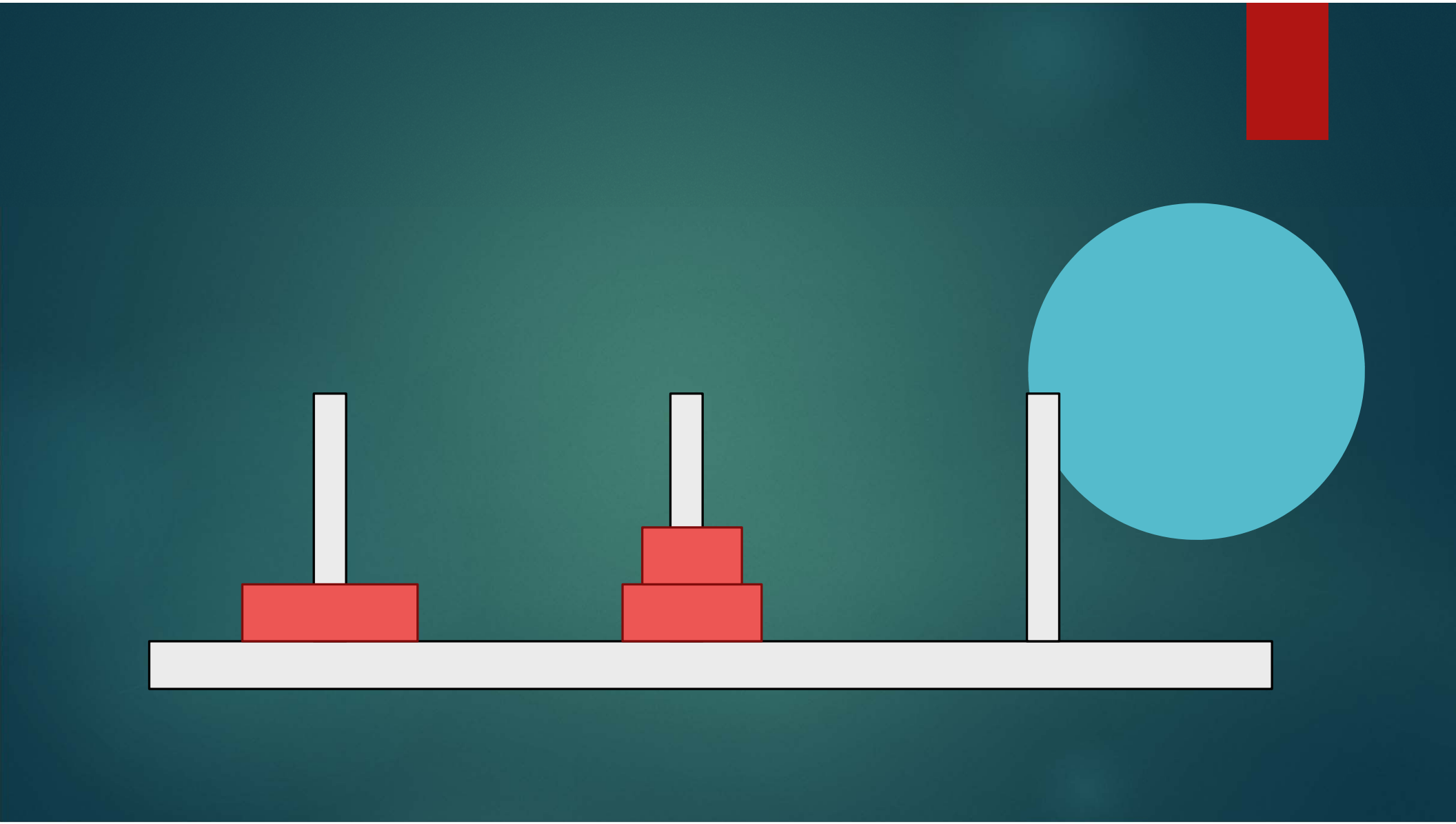




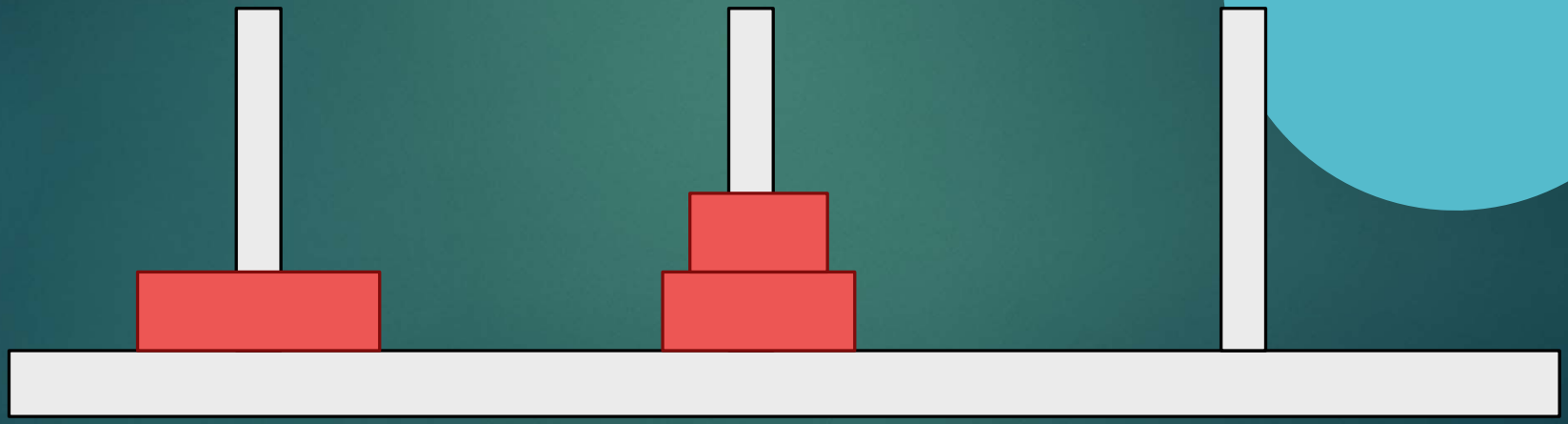


# Important subproblem

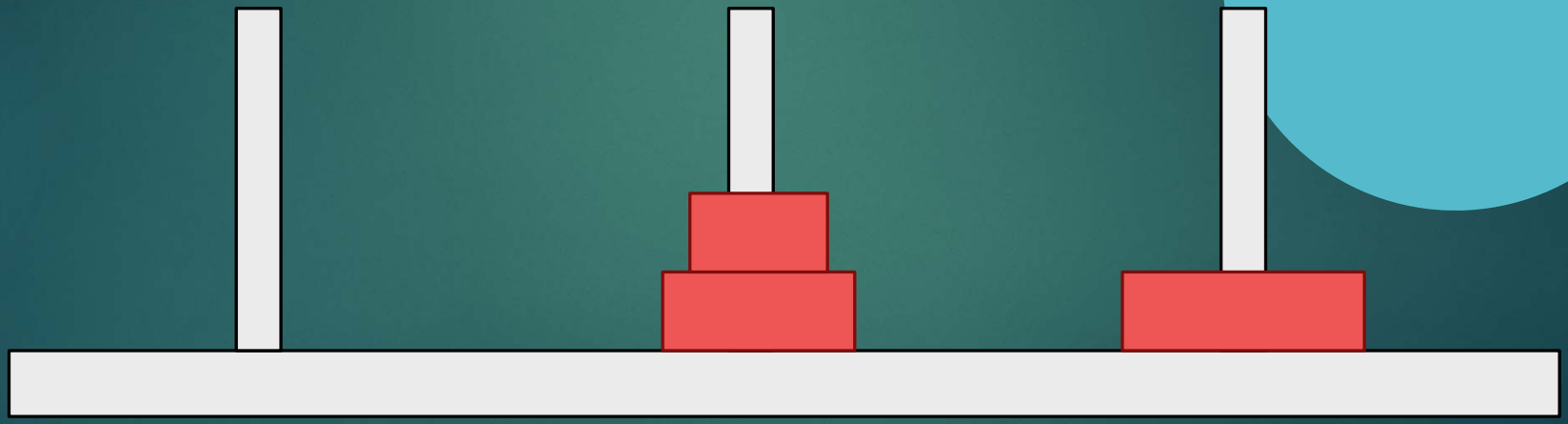




There will be always a situation like this: we have managed to shift **n-1** plates to the auxilary rod → we just have to put the largest to the last rod !!!



And we have to put the plates from the auxilary rod
to the top of the biggest plate !!!
**BUT** it is the same problem again so we can use recursion



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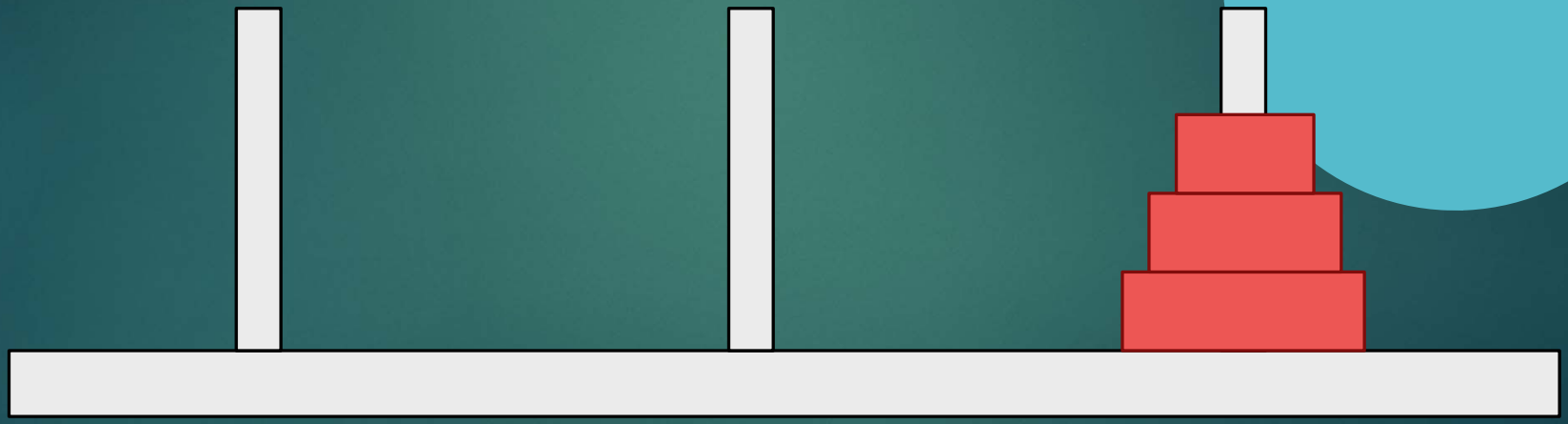

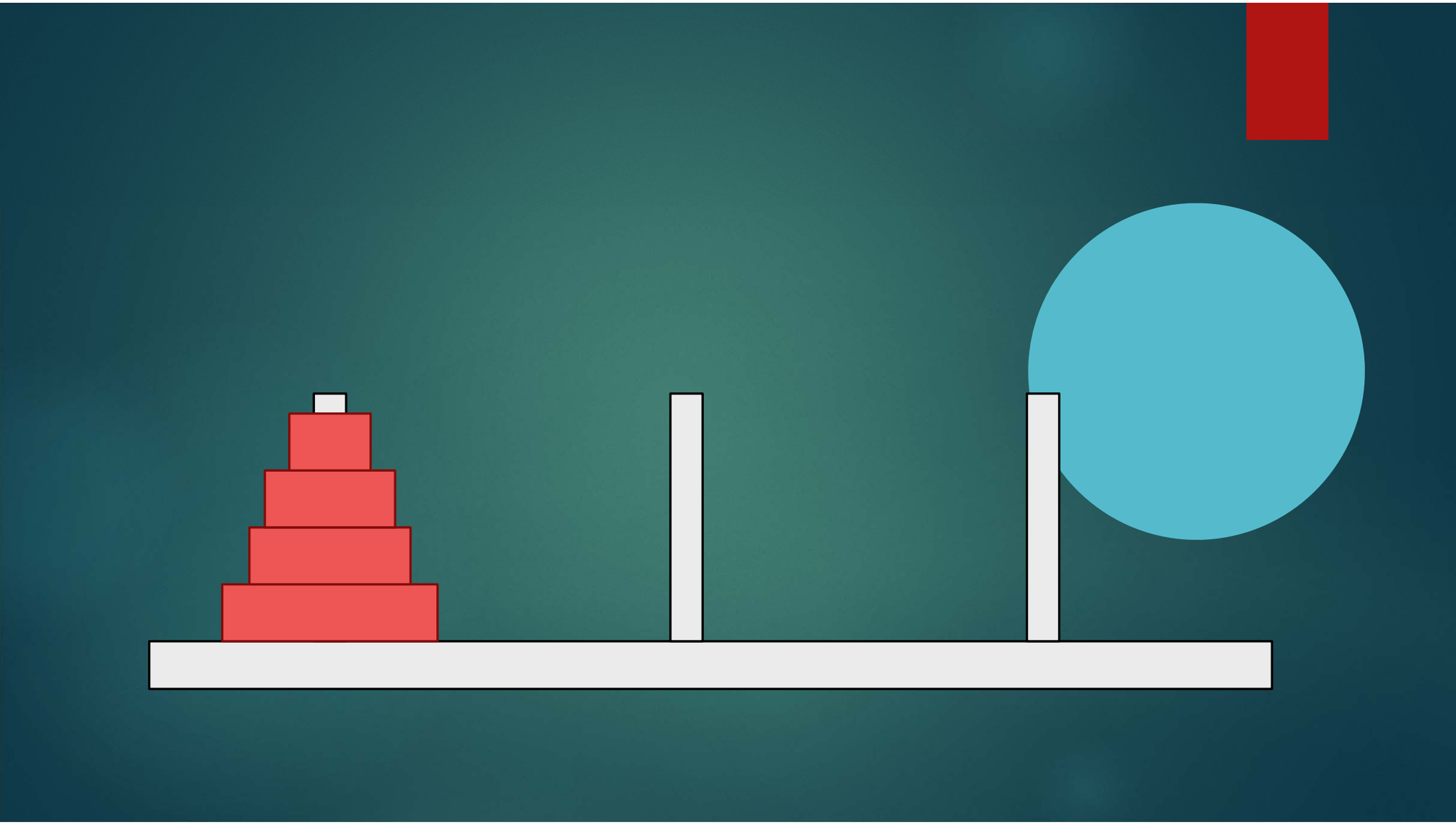

The same situation again: we have the largest plate alone → just move it to the last rod