

O'REILLY®

BINARYSEARCH

$O(\log n)$ Behavior



Problem Statement

- Does a collection contain a specific element
 - Assume collection is structured as a Python list
 - Python has **in** statement for this purpose
 - What is expected performance of **in**?

Does collection contain element

```
def contains(collection, target):  
    return target in collection
```

Best/Average/Worst Case Analysis

- Best case is when 1st element is target
- Worst case you have to check each one
- Average case
 - If element is random one from list, then must check $\frac{1}{2}$ of the elements; if not in list, must check each one
- Can you do better? Yes!
 - But only if you add structure the collection

Why Contains Functionality Is Important

- Fundamental functionality for a collection
- Precursor for insert request
 - In many situations you only want to insert a value if collection doesn't already contain it
 - Let's approach **in** with this concern in mind

```
if value not in collection:  
    col.append(value)
```

BINARYARRAYSEARCH Algorithm

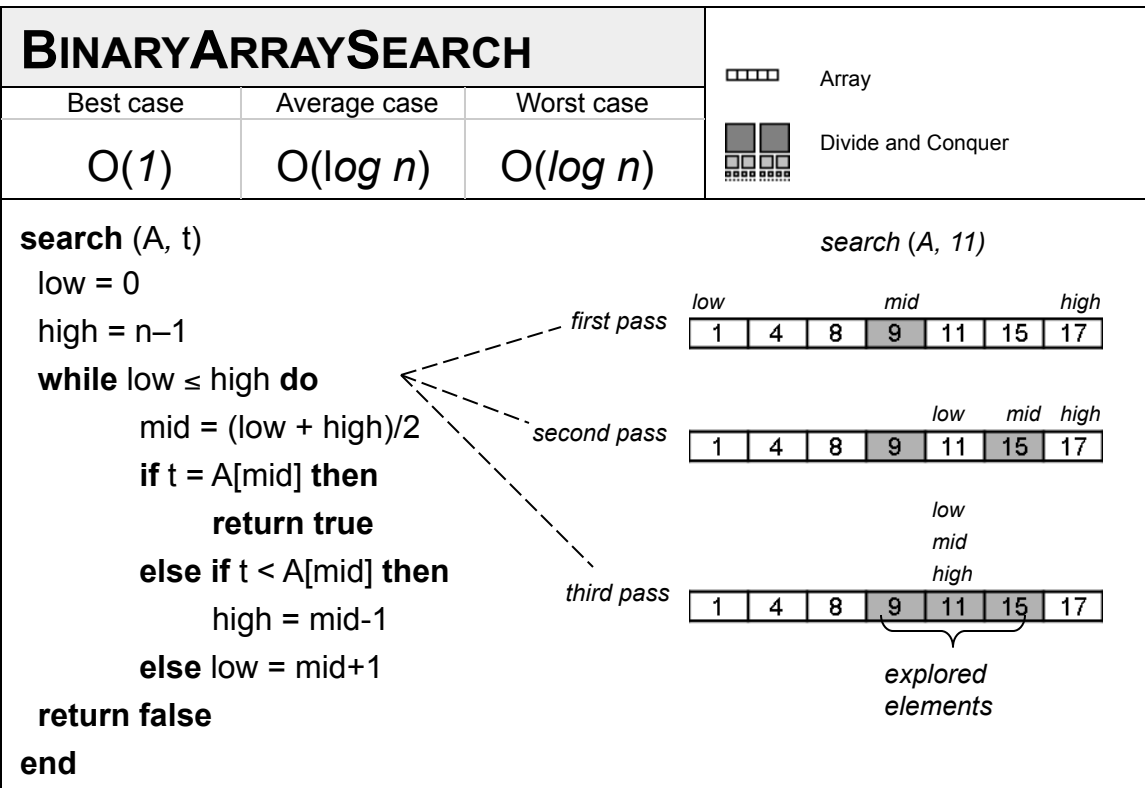
- A phone book with n entries is sorted by last name (and first name within last name)
 - Easy to locate a phone # for a given person
 - Hard to locate a person for a given phone #
- Observation on task difficulty
 - Searching through a phone book with 400 pages is not twice as hard as with a 200-page phone book

Algorithm Pseudocode

Assume A is sorted

To search through A, this algorithm cuts the problem size in half with each pass through the while loop

Divide problem into sub-problems half as large



BINARYARRAYSEARCH

Summary

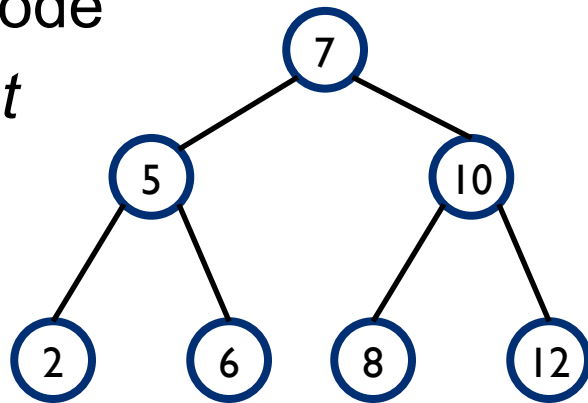
- Contiguous array storage inefficient
 - Block memory moves when inserting single values
- Consider using
 - You need to visit all elements in sorted order
 - Array doesn't change
- Evaluate each **in** use in your code
 - Easy performance gains

Binary Tree Data Structure

- Recursive data structure
 - Each **node** object has a value
 - A node may have a *left* or *right* child node
 - Topmost node in **tree** is called the *root*

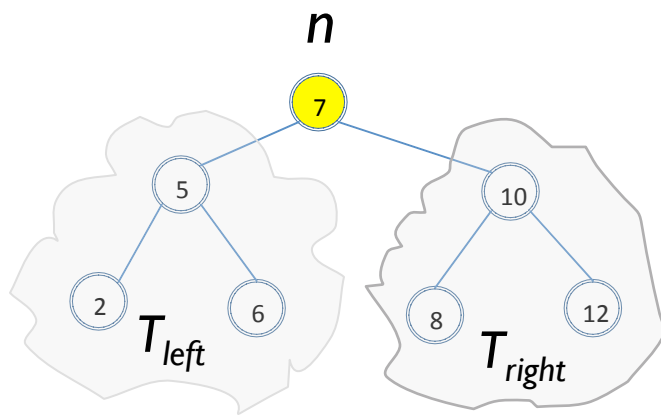
```
class BinaryNode {  
    int      value  
    BinaryNode left  
    BinaryNode right  
}
```

```
class BinaryTree {  
    BinaryNode root  
}
```



Binary Search Tree Structure

- Binary Search Tree Property
 - All values in T_{left} for n are $\leq n.value$
 - All values in T_{right} for n are $\geq n.value$
- Observation
 - Each node is root of a BST
 - Same Hierarchical property as used by BINARYARRAYSEARCH

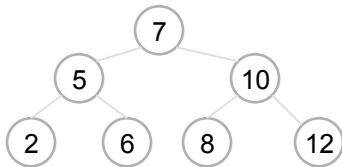


BST Issues

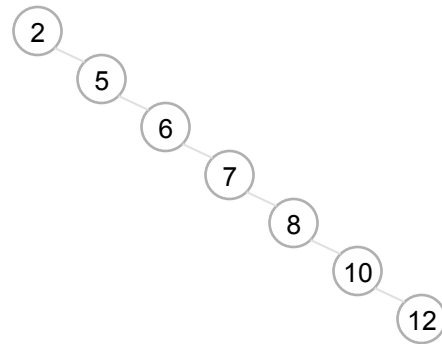
- Reasons to use Binary Search Tree
 - Input data size is unknown
 - Input data is highly dynamic, with significant number of insertions and deletions
- Problems that may arise
 - When a Binary Search Tree is constructed and modified, it may become *unbalanced*

BST Issues

- Use a BST implementation that self-balances
 - Otherwise...



Tree is fully balanced for maximum efficiency

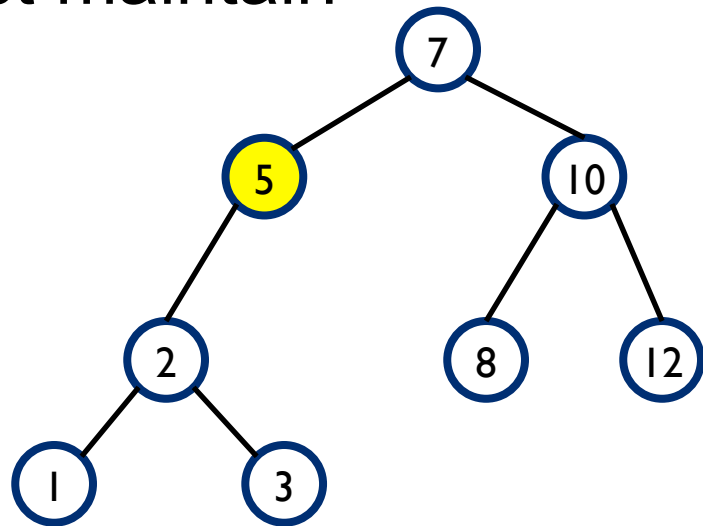


Tree looks more like a linked list, which leads to $O(n)$ search performance

BST

Removing A Node

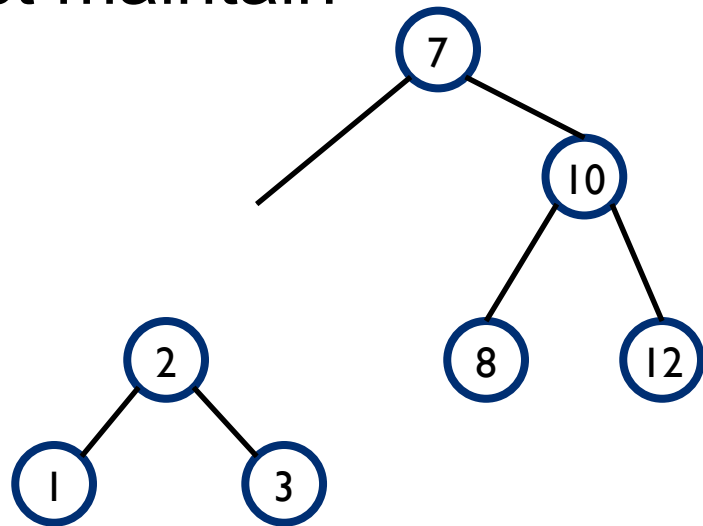
- To remove node from BST, must maintain Binary Search Tree Property
 - If leaf, just delete it
 - If has one child, replace self with sub-tree rooted at that child



BST

Removing A Node

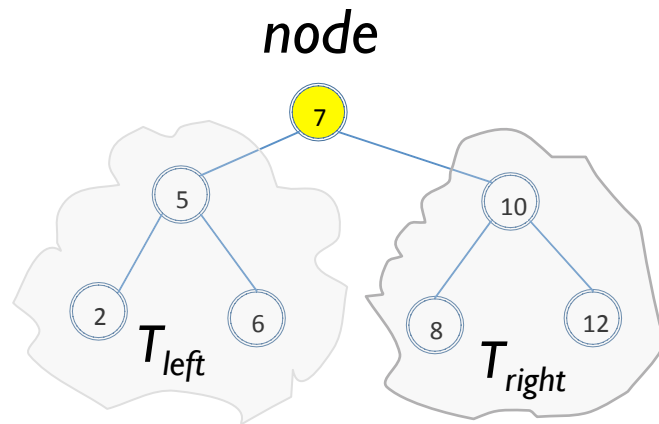
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BST

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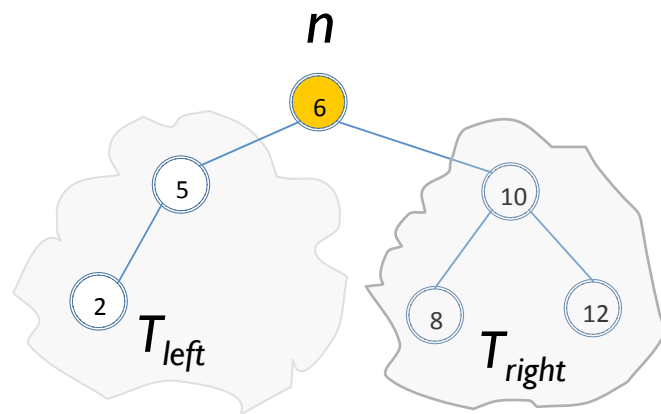
- If node has both children
 - Remove largest value M from T_{left}
 - Replace node's value with M
(in this case 6)



BST

Removing A Node

- If node has both children
 - Replace with largest value in left sub-tree (in this case 6)
 - Code describing delete case is provided for your review



BST Balancing

- Should you choose to use Binary Search Trees
 - Choose a balanced tree implementation
- Several choices
 - Red/Black Trees
 - AVL Trees (discovered in 1962)
- (Re)balance Tree after insert/delete
 - Insertions and deletions may unbalance tree

BST

Summary

- Efficient choice with dynamic behavior
 - Search, Insert, Delete all in $O(\log n)$ time
 - Iteration as traversal
- Balancing Required
 - Implementations exist (AVL and Red/Black trees)

BST Problem

- Create a balanced BST from a sorted collection?
 - Does the following image provide any suggestions?
 - Note how median is root?
 - And its two children are the medians of the left and right sub-arrays?

