SUBSET SUM PROBLEM

DYNAMIC PROGRAMMING

Subset sum problem

- One of the most important problems in complexity theory
- The problem: given an **S** set of integers, is there a non-empty subset whose **s** sum is zero or a given integer?
- For example: given the set **{5,2,1,3}** and **s=9** the answer is **YES** because the subset **{5,3,1}** sums to **9**
- The problem is **NP-complete** → we have efficient algorithms when the problem is small !!!
- Special case of knapsack-problem

Solutions

1.) Naive approach „brute force search”

- Generate all the subsets of the given set of integers
- **N** is the number of integers in the set **S**
- Check whether the sum of all subsets is equal to **s** or not
- Time complexity: exponential // **O($N * 2^N$)**

2.) Dynamic Programming: we want to avoid calculating the same problems over and over again ... we create a dynamic programming table and memoize

{1,2,3} → s = 5

{1,2,3,4} → s = 5 we can still solve this problem !!!

Of course if **j** can be constructed with the **i-1** integers → there must be a subset with sum **i** as well (**INCLUDE**)

Base cases

dpTable[i][j] =

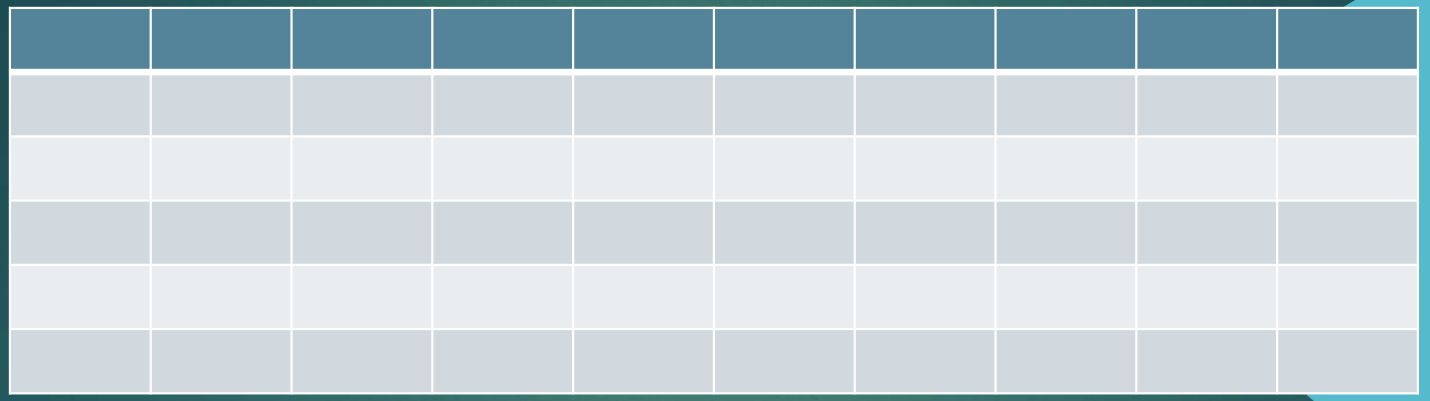
$$\begin{cases} \textbf{true if } j = 0 \textbf{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \textbf{ if } dpTable[i-1][j] \textbf{ is true} \\ dpTable[i][j] = dpTable[i-1][j - S[i-1]] \textbf{ else} \end{cases}$$

There is a non-empty subset of the first **i** integers that sums to **j**

If **j-actualInteger** can be constructed with the **i-1** integers (**EXCLUDE**)

S set of integers: **{5,2,1,3}**

s sum: **9**



S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0										
5										
2										
1										
3										

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	F	F	F	F	F	F	F	F	F	F
5										
2										
1										
3										

sub sums

integers

If sum **s** is not zero and subset is **0** so we consider no integers from subset **S** → no feasible solution ...

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T									
2	T									
1	T									
3	T									

sub sums

integers

If sum **s** is **0** (the first column) → we can make the empty subset to make sum **0** ... so there is always a trivial solution for this subproblem

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F								
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F							
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F						
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F					
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T				
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F			
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F		
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T									
1	T									
3	T									

sub sums

integers

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F								
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T							
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][2] = dpTable[1][2-2] = dpTable[1][0] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F						
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][3] = dpTable[1][3-2] = dpTable[1][1] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F					
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][4] = dpTable[1][4-2] = dpTable[1][2] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T				
1	T									
3	T									

sub sums

$$\textbf{dpTable[i][j]} = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \text{ if } dpTable[i-1][j] \text{ is true} \\ dpTable[i][j] = dpTable[i-1][j-S[i-1]] \text{ else} \end{cases}$$

dpTable[2][5] = dpTable[1][5] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F			
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][6] = dpTable[1][6-2] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T		
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][7] = dpTable[1][7-2] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	
1	T									
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][8] = dpTable[1][8-2] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T									
3	T									

sub sums

$$\textbf{dpTable[i][j]} = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[2][9] = dpTable[1][9-2] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T								
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][1] = dpTable[2][1-1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T							
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][1] = dpTable[2][1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T						
3	T									

sub sums

dpTable[i][j] =

$$\begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \text{ if } dpTable[i-1][j] \text{ is true} \\ dpTable[i][j] = dpTable[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][3] = dpTable[2][3-1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F					
3	T									

sub sums

$$\textbf{dpTable[i][j]} = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \text{ if } dpTable[i-1][j] \text{ is true} \\ dpTable[i][j] = dpTable[i-1][j-S[i-1]] \text{ else} \end{cases}$$

dpTable[3][4] = dpTable[2][4-1] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T				
3	T									

sub sums

$$\textbf{dpTable[i][j]} = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \text{ if } dpTable[i-1][j] \text{ is true} \\ dpTable[i][j] = dpTable[i-1][j-S[i-1]] \text{ else} \end{cases}$$

dpTable[3][5] = dpTable[2][5] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T			
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][6] = dpTable[2][6-1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T		
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][7] = dpTable[2][7] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][8] = dpTable[2][8-1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T									

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[3][9] = dpTable[2][9-1] = F

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T								

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][1] = dpTable[3][1] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T							

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][2] = dpTable[3][2] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T						

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][3] = dpTable[3][3] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T					

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][4] = dpTable[3][4-3] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T				

sub sums

$$\textbf{dpTable[i][j]} = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ dpTable[i][j] = dpTable[i-1][j] \text{ if } dpTable[i-1][j] \text{ is true} \\ dpTable[i][j] = dpTable[i-1][j-S[i-1]] \text{ else} \end{cases}$$

dpTable[4][5] = dpTable[3][5] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T			

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][6] = dpTable[3][6] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T		

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][7] = dpTable[3][7] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][8] = dpTable[3][8] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

$$\text{dpTable}[i][j] = \begin{cases} \text{true if } j = 0 \text{ and false if } i = 0 \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j] \text{ if dpTable}[i-1][j] \text{ is true} \\ \text{dpTable}[i][j] = \text{dpTable}[i-1][j - S[i-1]] \text{ else} \end{cases}$$

dpTable[4][9] = dpTable[3][9-3] = T

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

True: which means there is a feasible solution...so **9** can be constructed from the **S** set of integers

OK, but what are these integers?

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

Starting from the last cell ... if the **T** is not coming from above, it means it is in the solution set

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

Starting from the last cell ... if the T is not coming from above, it means it is in the solution set

Decrement the rowIndex (se we go up) and go as many steps to the left as the included integer from set **S**

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

Starting from the last cell ... if the T is not coming from above, it means it is in the solution set

Decrement the rowIndex (se we go up) and go as many steps to the left as the included integer from set **S**

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
5	T	F	F	F	F	T	F	F	F	F
2	T	F	T	F	F	T	F	T	F	F
1	T	T	T	T	F	T	T	T	T	F
3	T	T	T	T	T	T	T	T	T	T

sub sums

Starting from the last cell ... if the T is not coming from above, it means it is in the solution set

Decrement the rowIndex (se we go up) and go as many steps to the left as the included integer from set **S**

S set of integers: **{5,2,1,3}**

s sum: **9**

	0	1	2	3	4	5	6	7	8	9	sub sums
0	T	F	F	F	F	F	F	F	F	F	
5	T	F	F	F	F	T	F	F	F	F	
2	T	F	T	F	F	T	F	T	F	F	
1	T	T	T	T	F	T	T	T	T	F	
3	T	T	T	T	T	T	T	T	T	T	

WE BUMP INTO COLUMN 0 → so we terminate the algorithm

Solution set: **{5, 1, 3}**