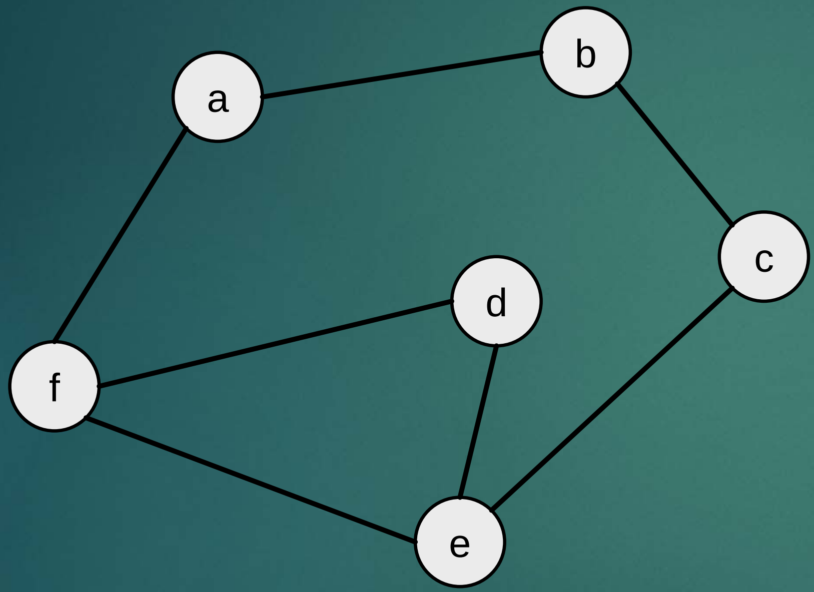
HAMILTONIAN PATH

BACKTRACKING

Hamiltonian cycle

G(V,E)

V: vertices in the graph
E: edges in the graph



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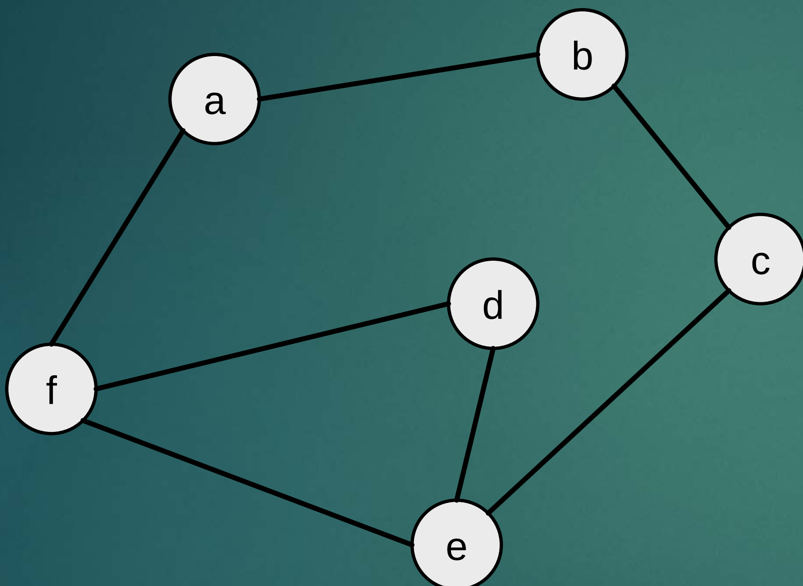

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & & & & & & \\
b & & & & & & \\
c & & & & & & \\
d & & & & & & \\
e & & & & & & \\
f & & & & & &
\end{array}
$$

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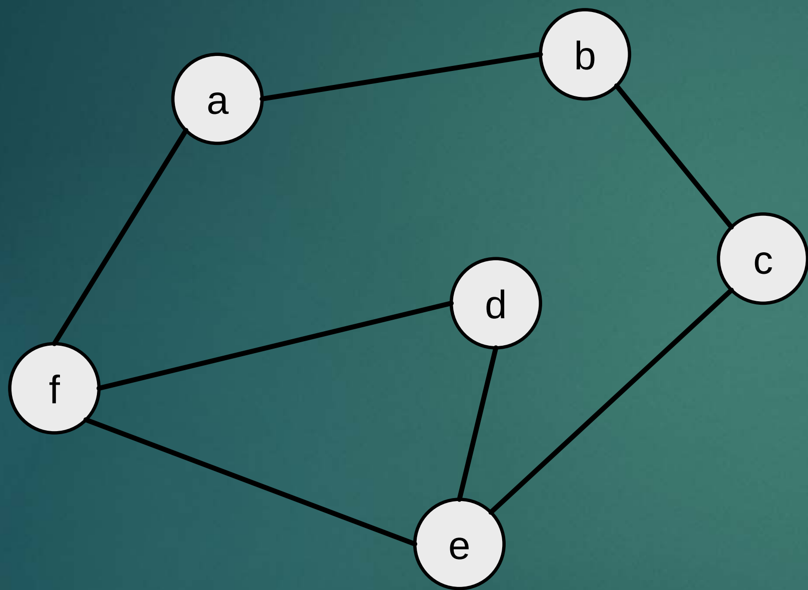

$$\begin{array}{c c} & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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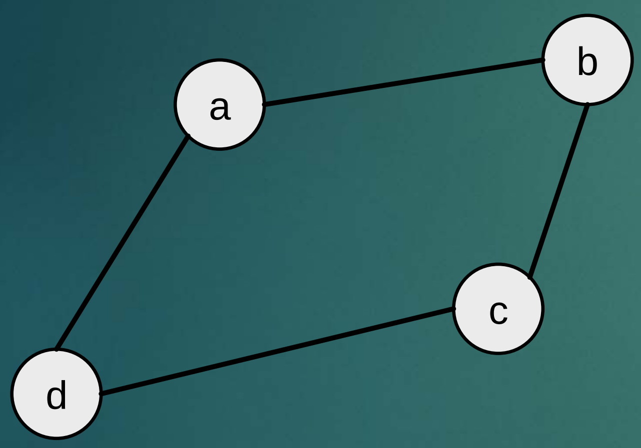
Hamiltonian path

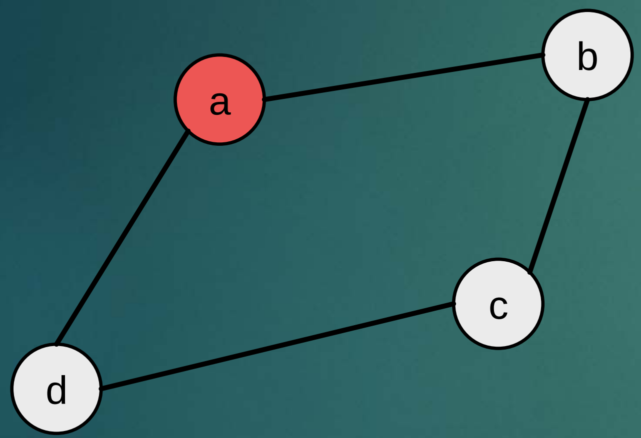


A hamiltonian path in an undirected graph is a path that visits every node exactly once !!!

Hamiltonian cycle: the first node and the last node of the path are the same vertexes

StartingPoint == EndPoint

Hamiltonian path



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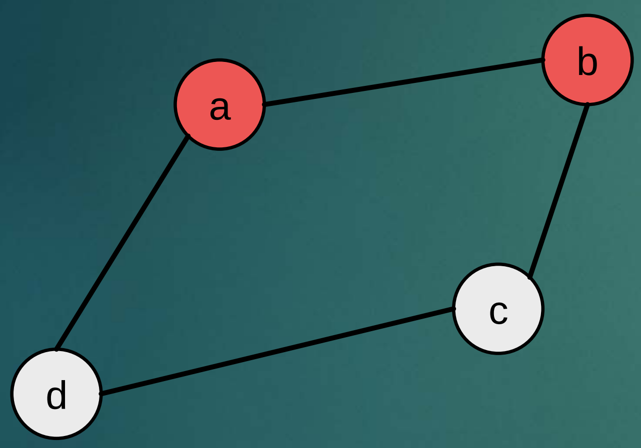

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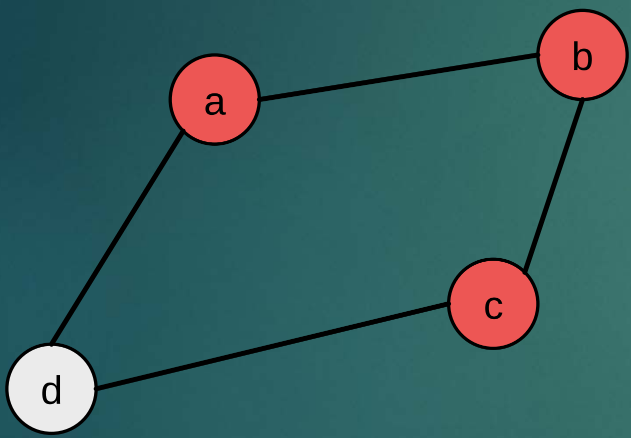

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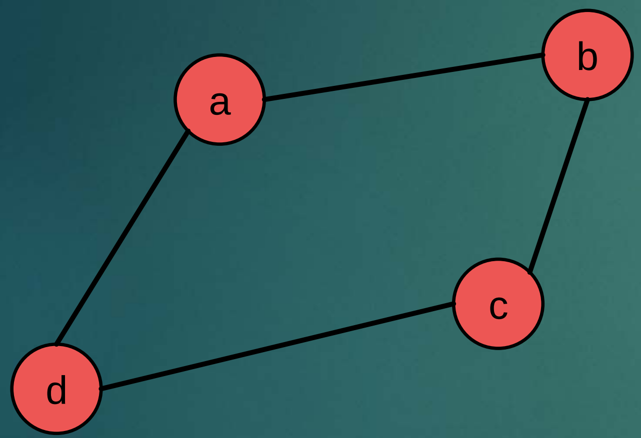

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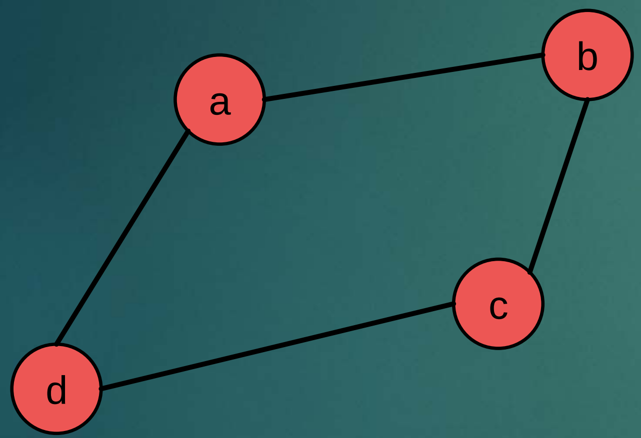

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Hamiltonian cycle: the first node and the last node of the path are the same vertexes

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A valid hamiltonian path is: **{ a b c d a }**

There may be several hamiltonian path in a given graph !!!

Hamiltonian problem

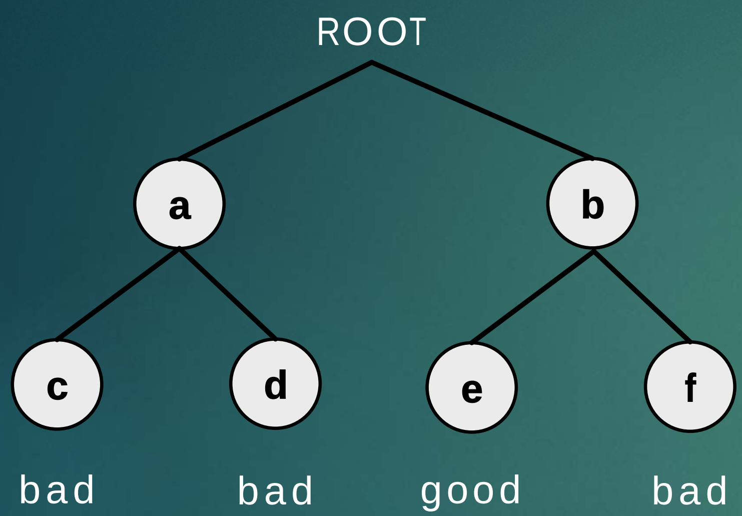
- Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem
- This is an **NP-complete** complete problem !!!
- Dirac-principle: a simple graph with **N** vertices is hamiltonian if every vertex has degree **N/2** or greater (degree is the number of edges of a vertex)
- Important fact: finding Hamiltonian path is **NP-complete**, but we can decide whether such path exists in linear time complexity with topological ordering

Solutions

- Naive approach:
- Generate all possible configurations of the vertices and print a configuration that satisfies the given constraints
- Problem → if the graph has **N** vertices, there are **N!** configurations, so the „solution space” is enormous
- Very very inefficient !!!

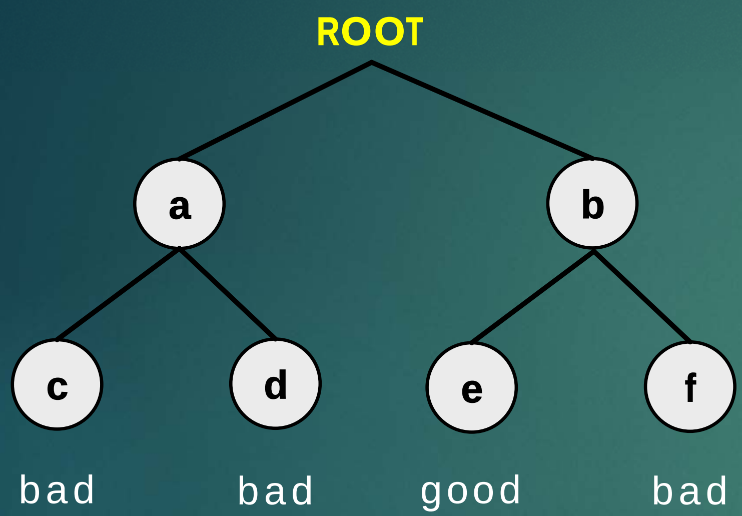
Solutions

- Constructing a tree:



We start at the root node, and
want to get to one of the good leaves

If we get to a bad leaf: we just „backtrack"
and keep moving on by revoking
our most recent choice



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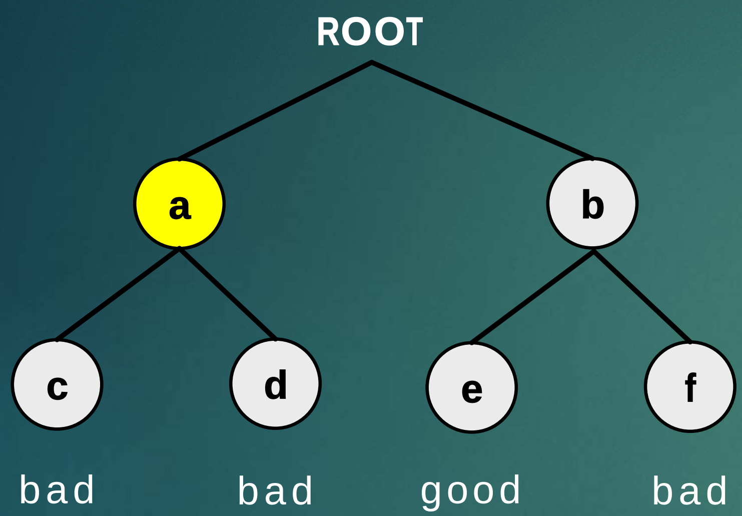

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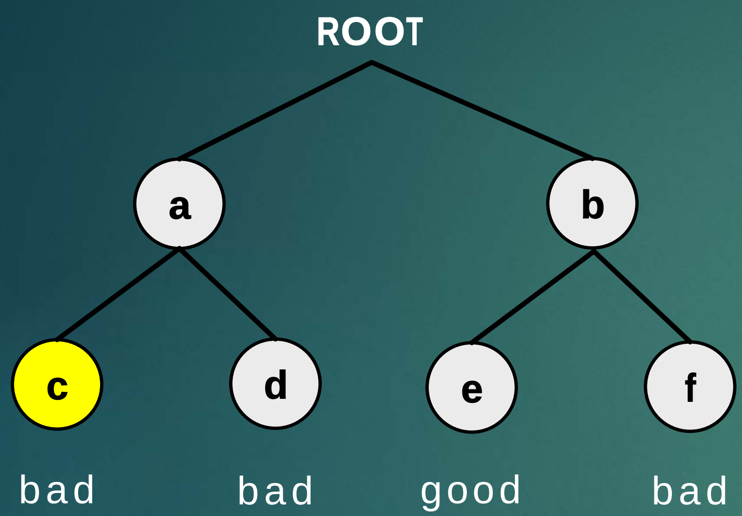

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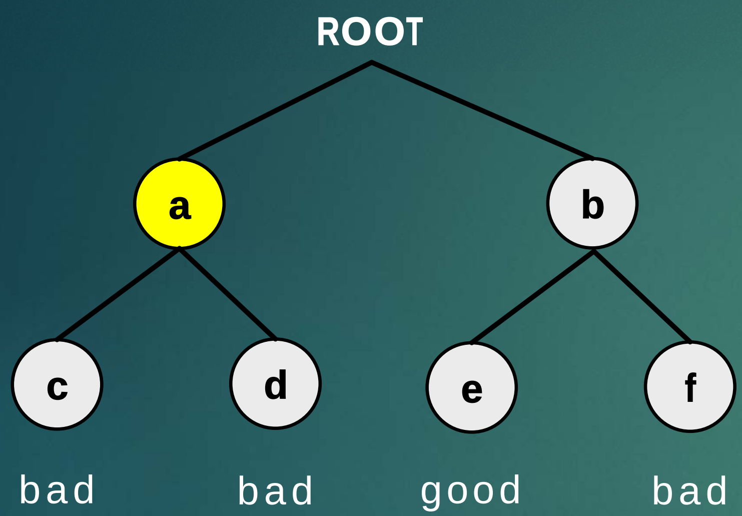

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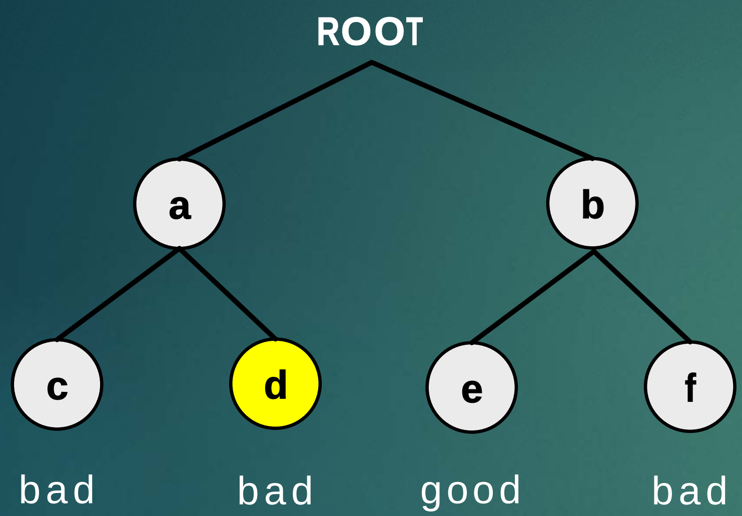

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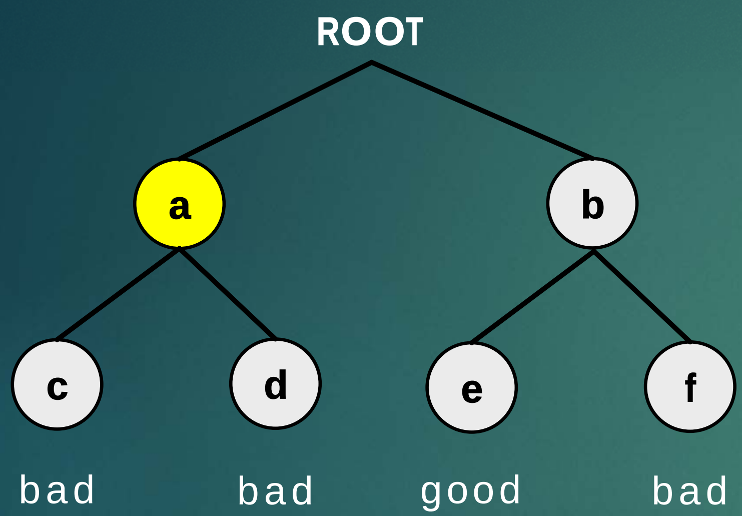

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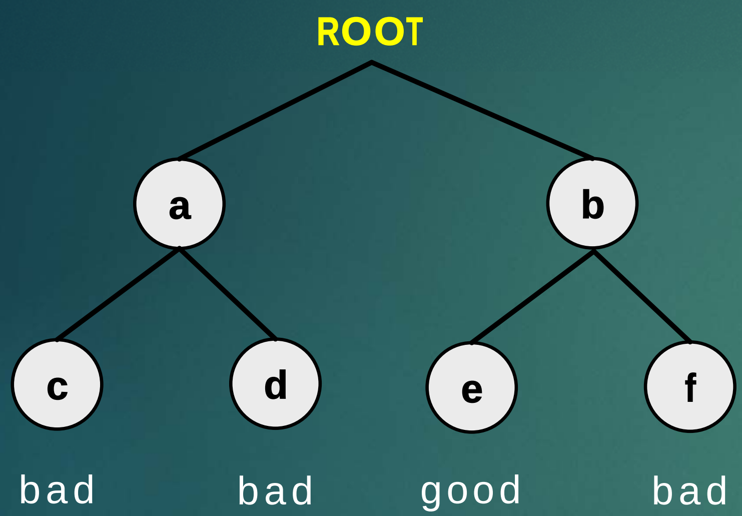

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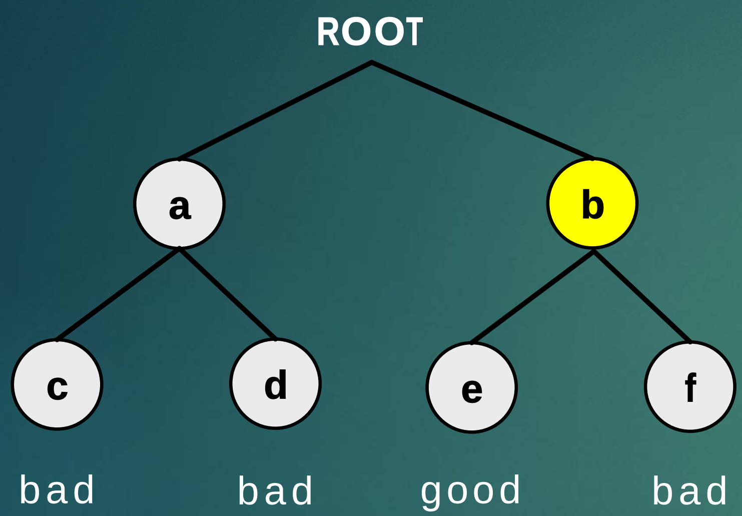

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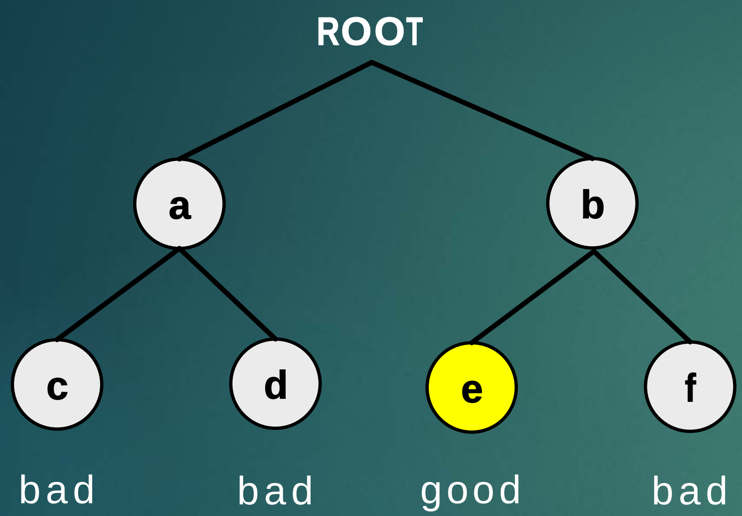

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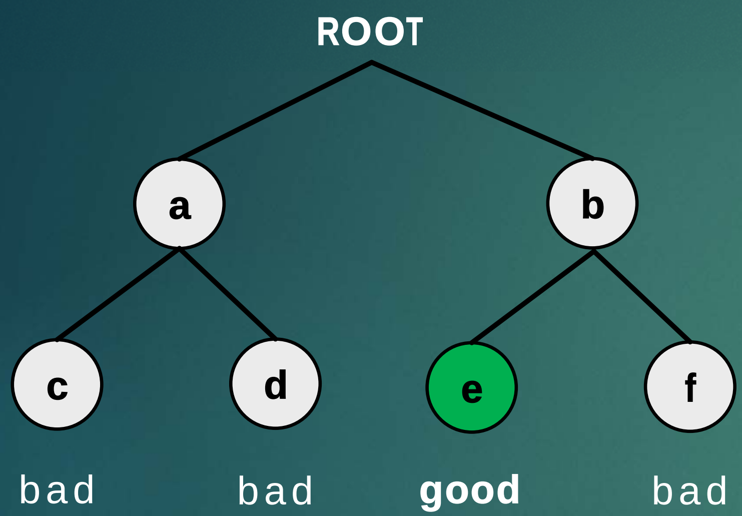

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The tree is an abstract model of the possible sequences of choices we could make
Here we do a depth-first search on the tree

Problem: hard to construct a tree if **N** is big !!!

Solutions

- Backtracking:
- We use recursion to solve the problem
- Create an empty path array and add vertex **0** to it as the starting vertex
- Add other vertices, starting from the vertex **1**
- Before adding a vertex, check whether it is adjacent to the previously added vertex + make sure it is not already added
- If we find such a vertex → we add the vertex as part of the solution
- If we do not find a vertex → we return false

State-space tree

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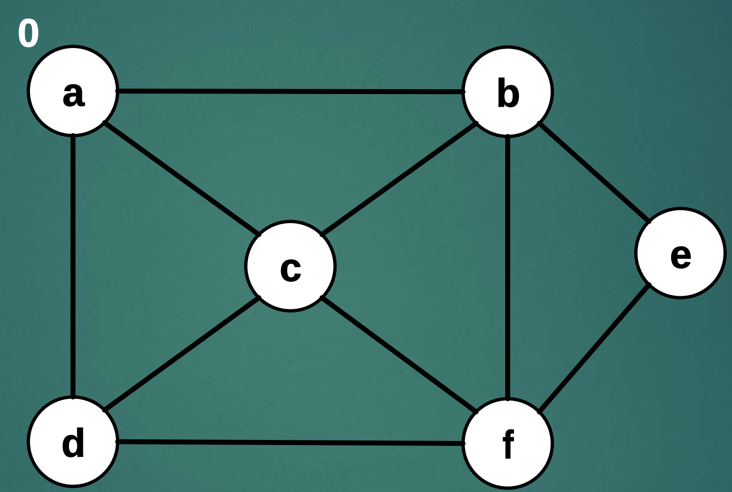

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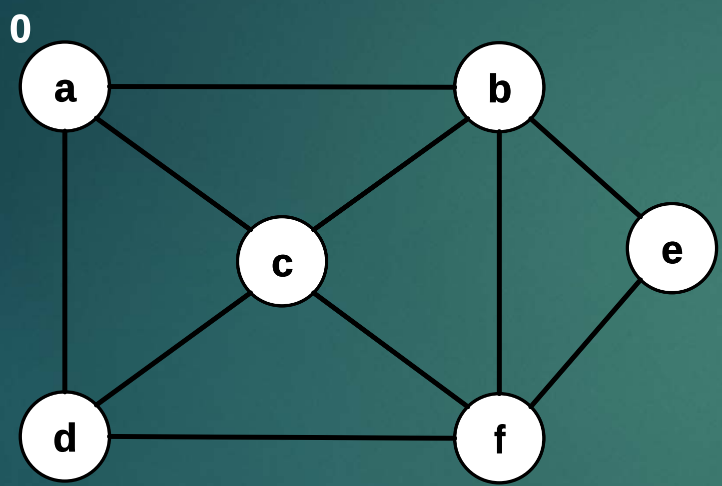

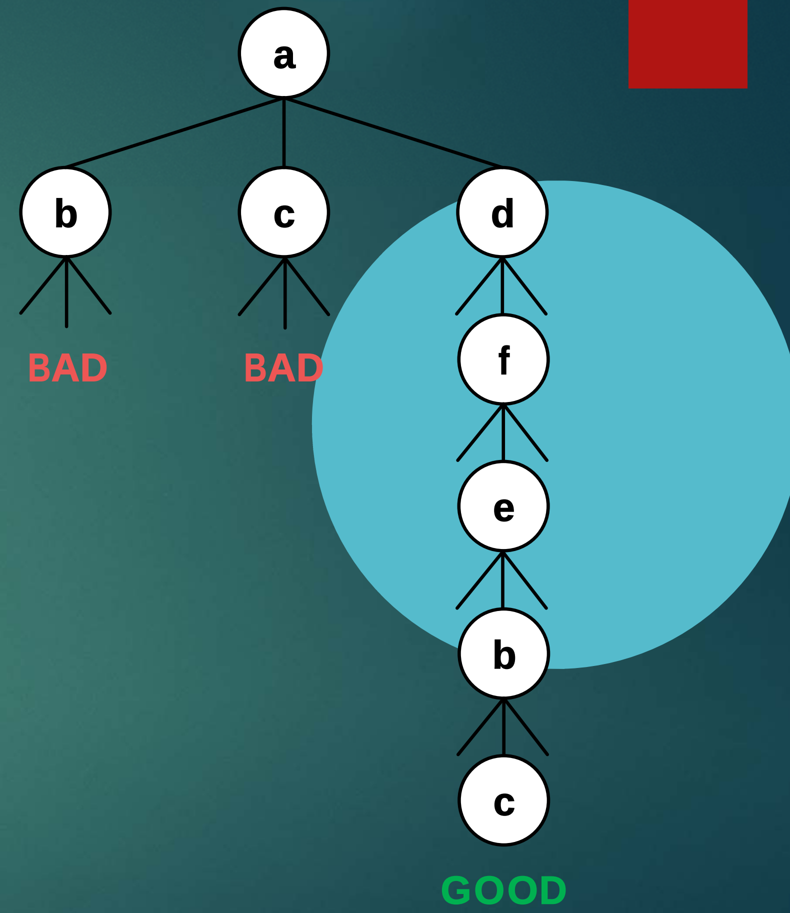

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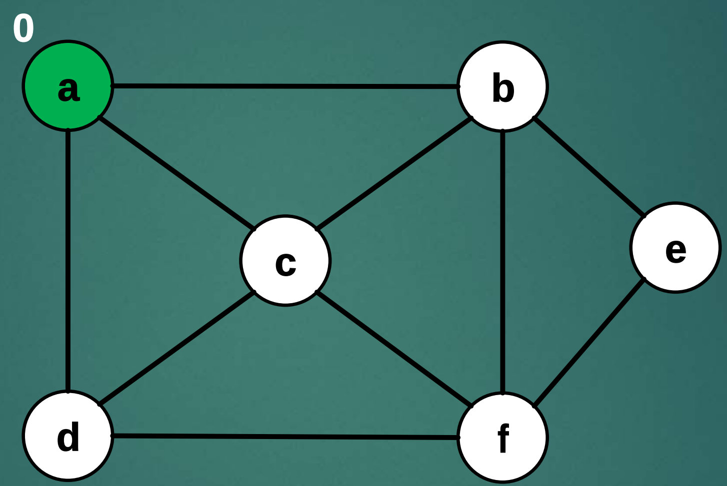

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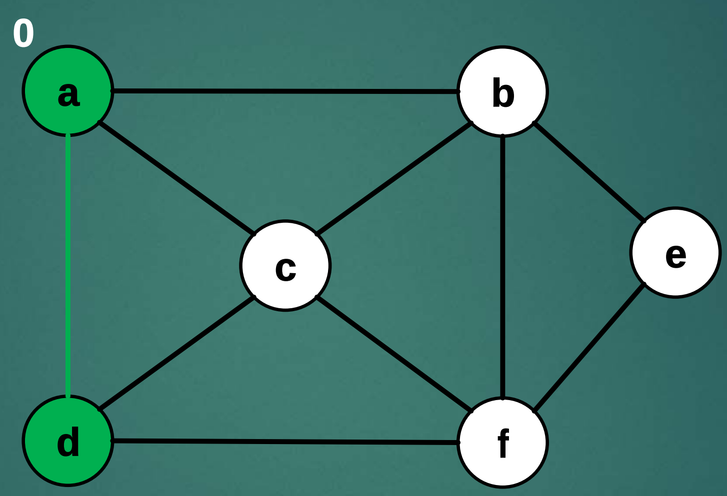

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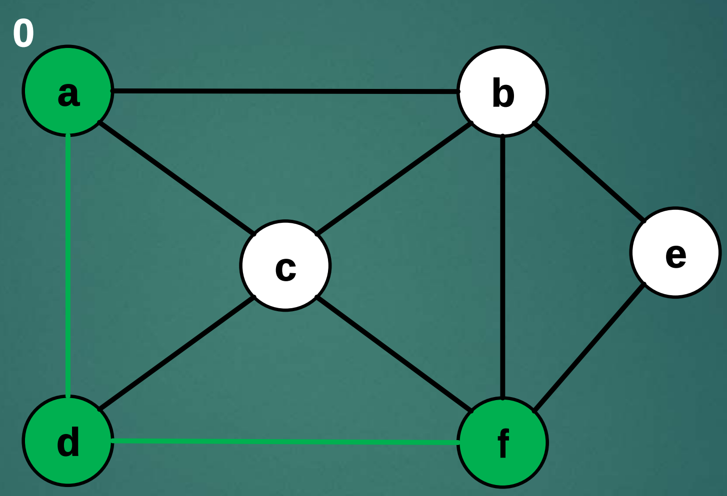

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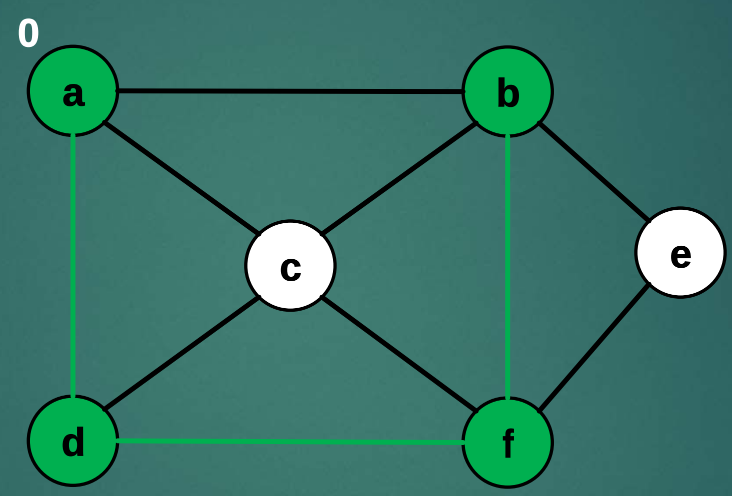

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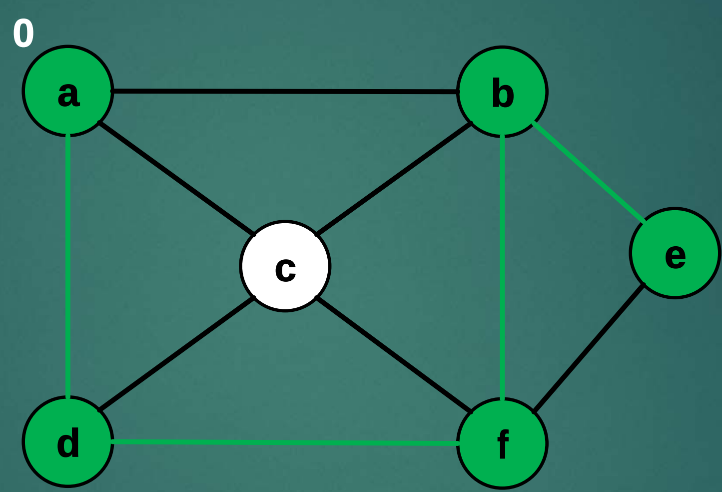

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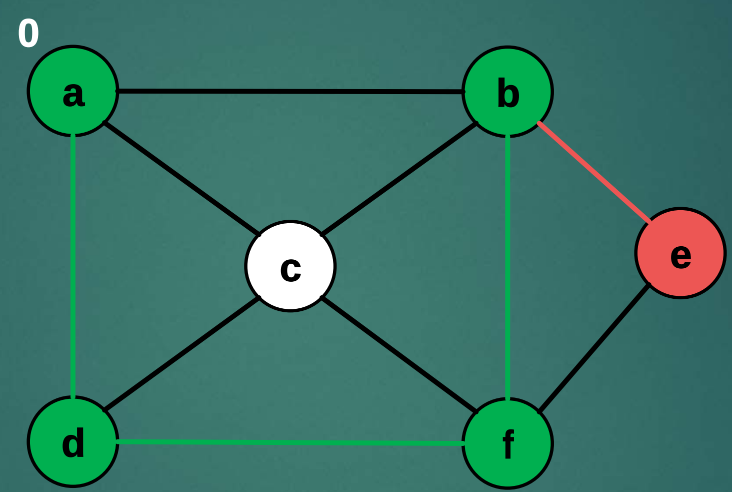

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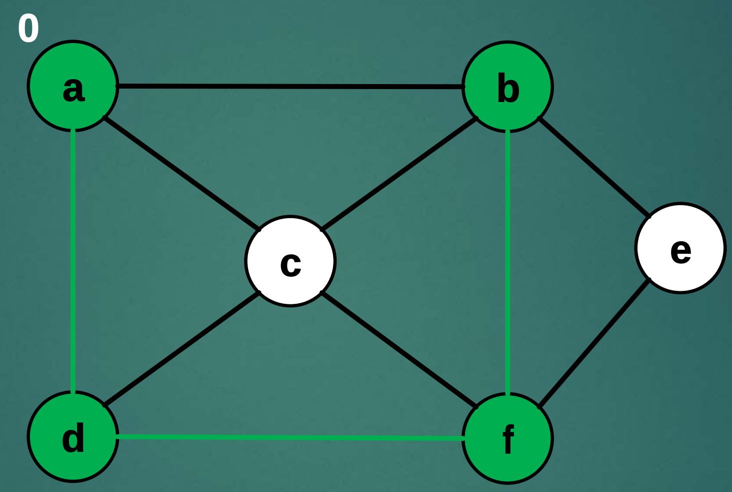

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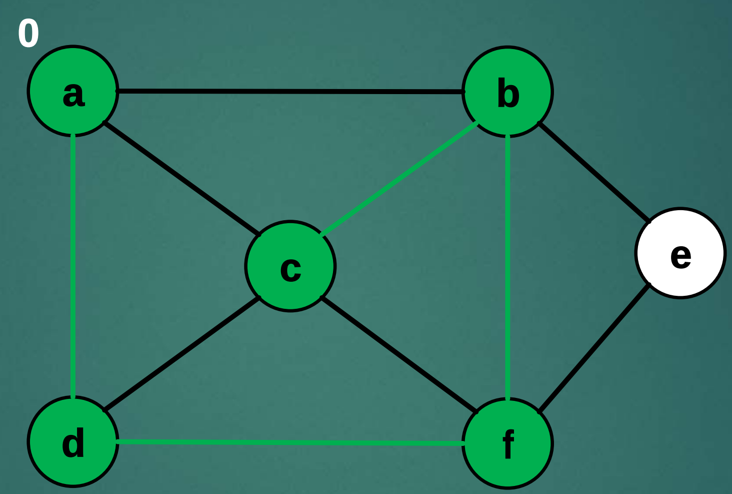

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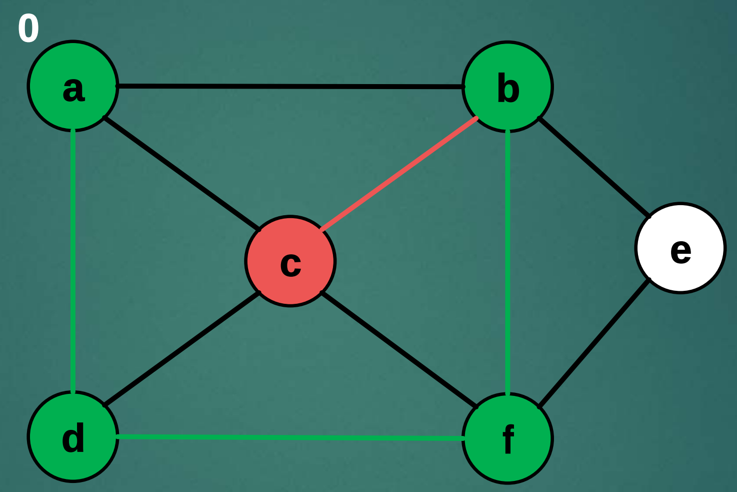

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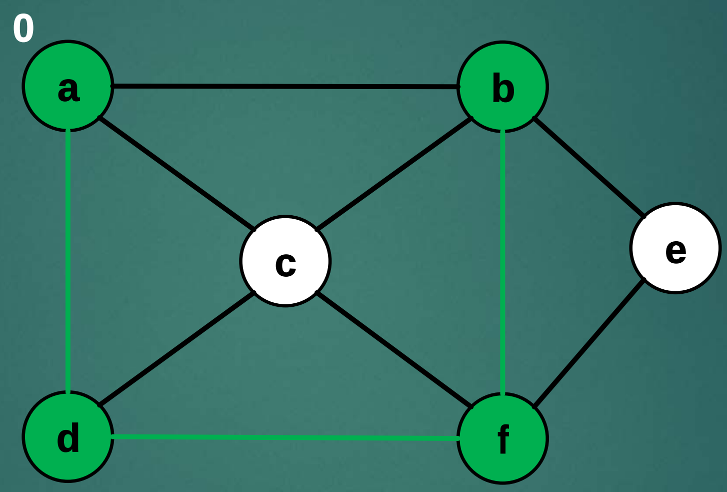

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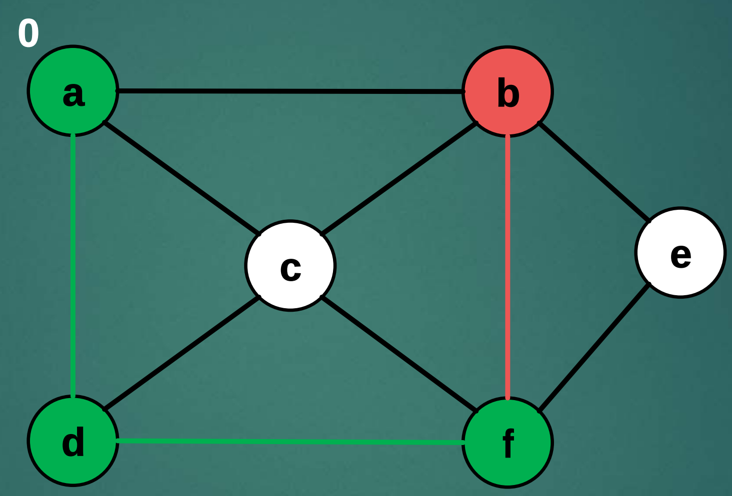

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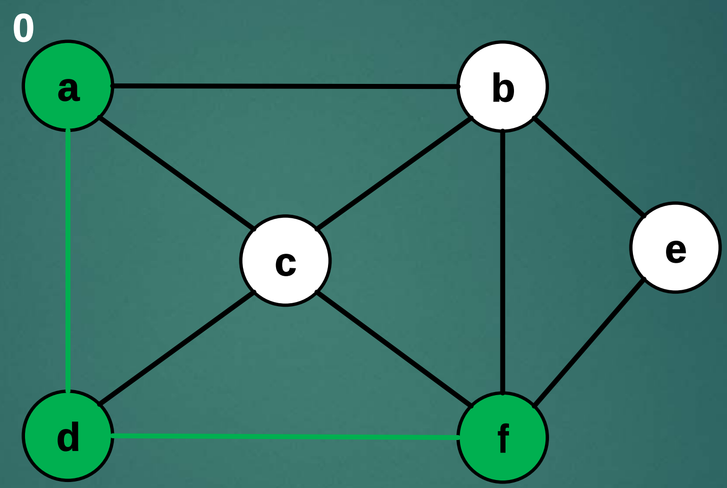

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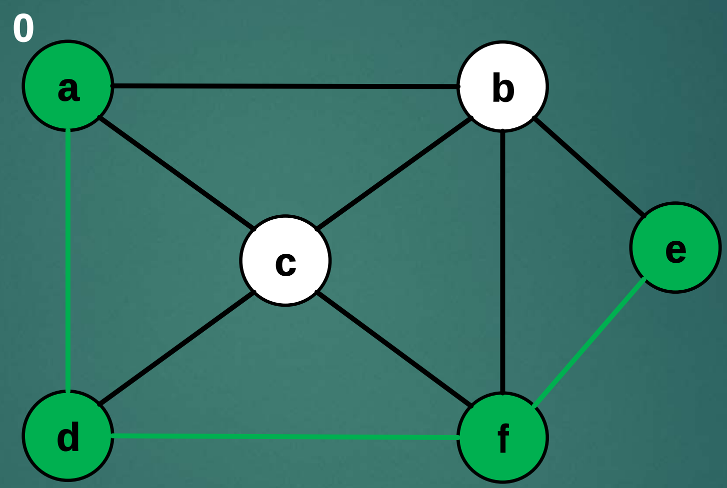

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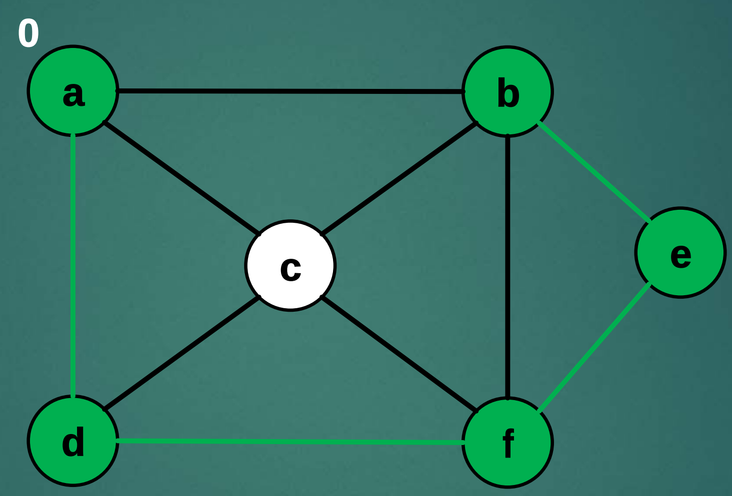

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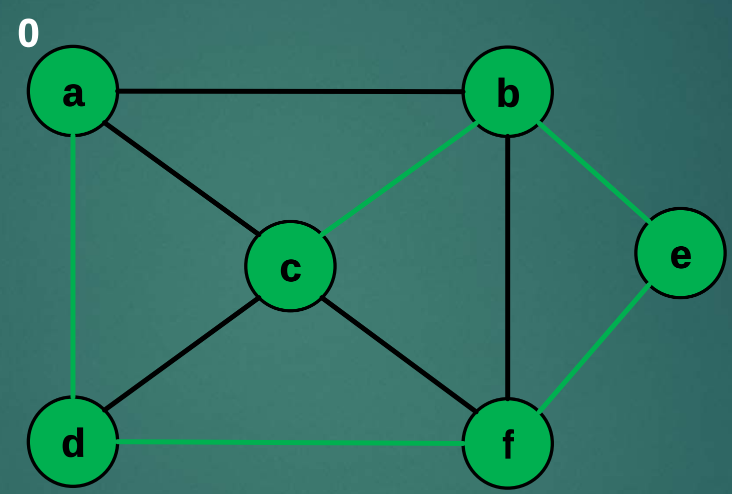

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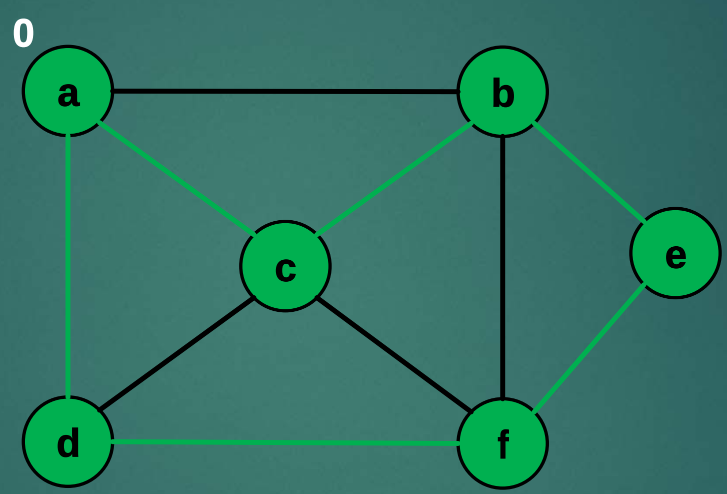

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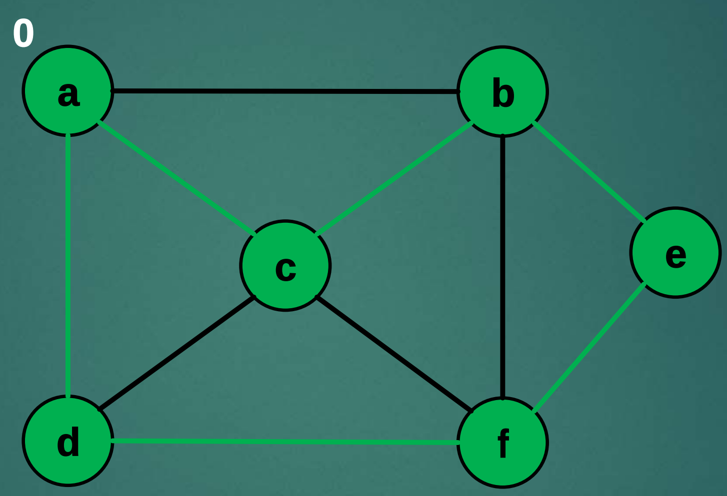

We have found the Hamiltonian-cycle in this graph

{ a , d , f , e , b , c , a }

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