

# SPANNING TREES

Eager prim'S ALGORITHM

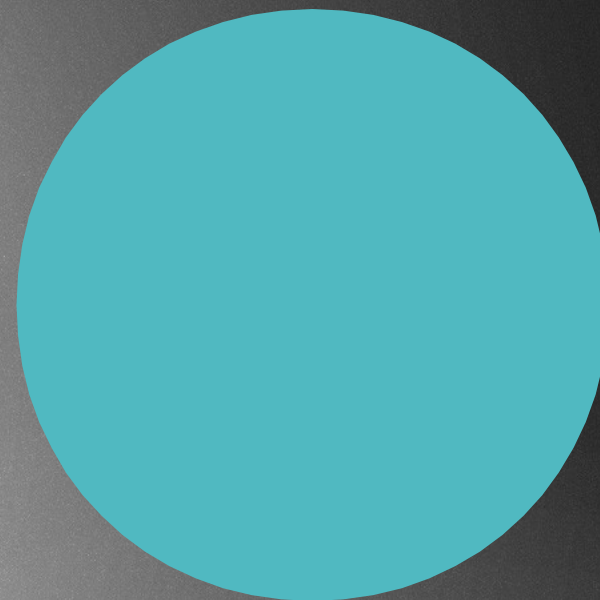
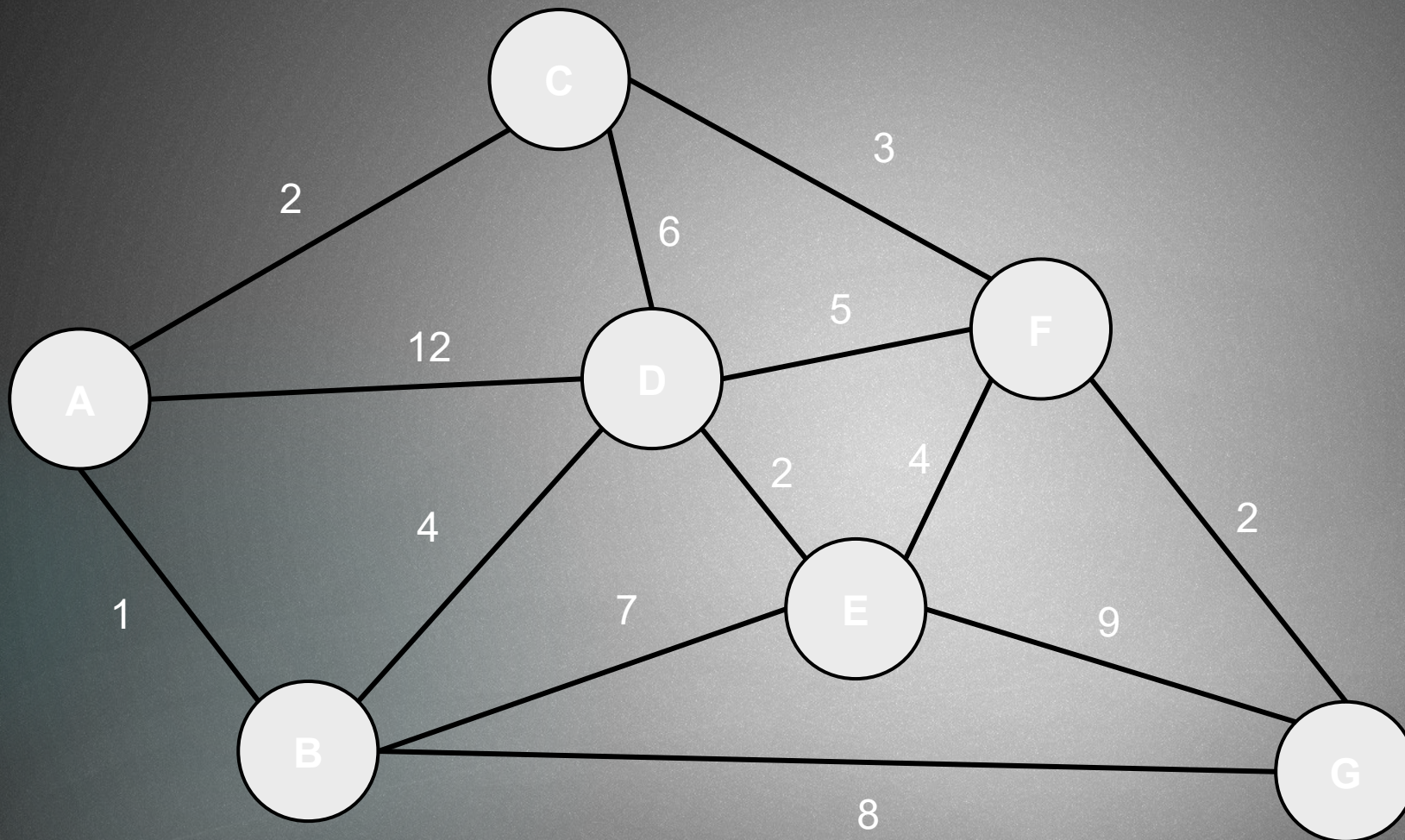




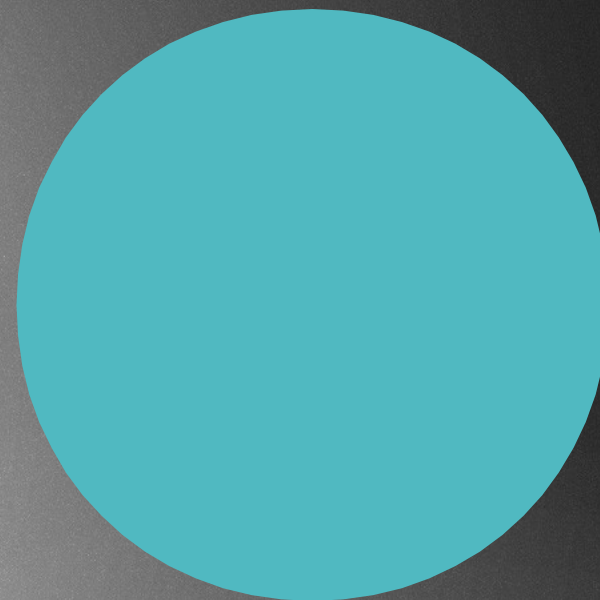
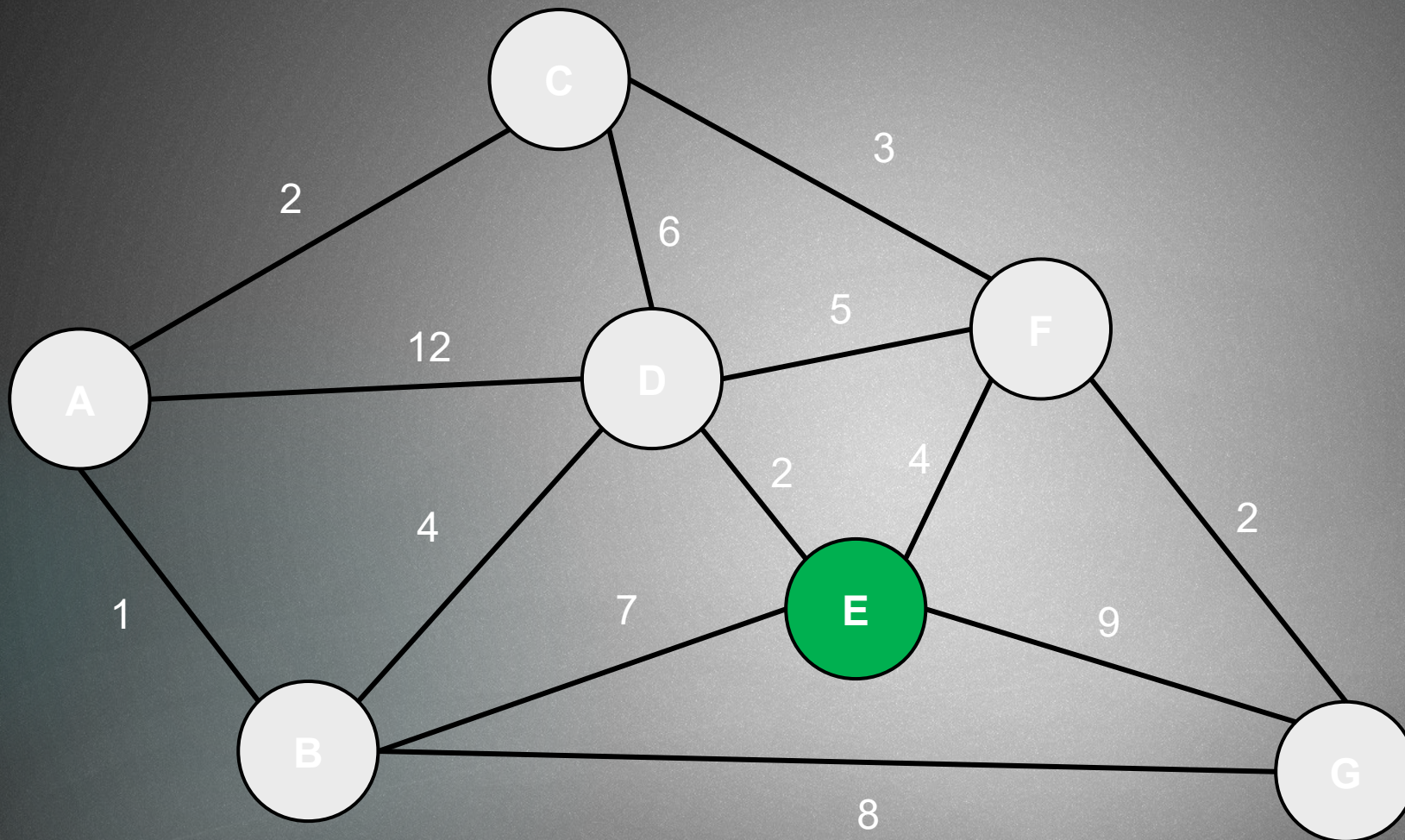
# Eager version

- ▶ The aim is the same: we want to construct a minimum spanning tree
- ▶ The lazy version → we use a priority queue ( heap ) in order to get the minimum edge weights + we insert all the edges to the heap without modifying the content !!!
- ▶ Eager version → we update the content of the heap if necessary
- ▶ On every iteration we check whether there is already a shorter path to the spanning tree

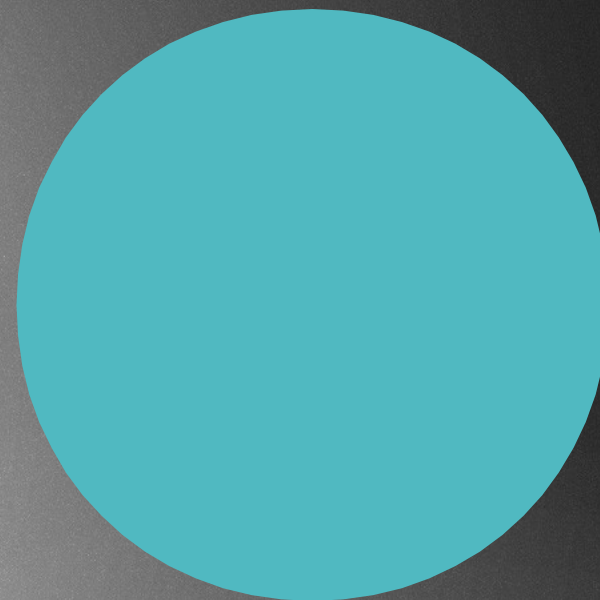
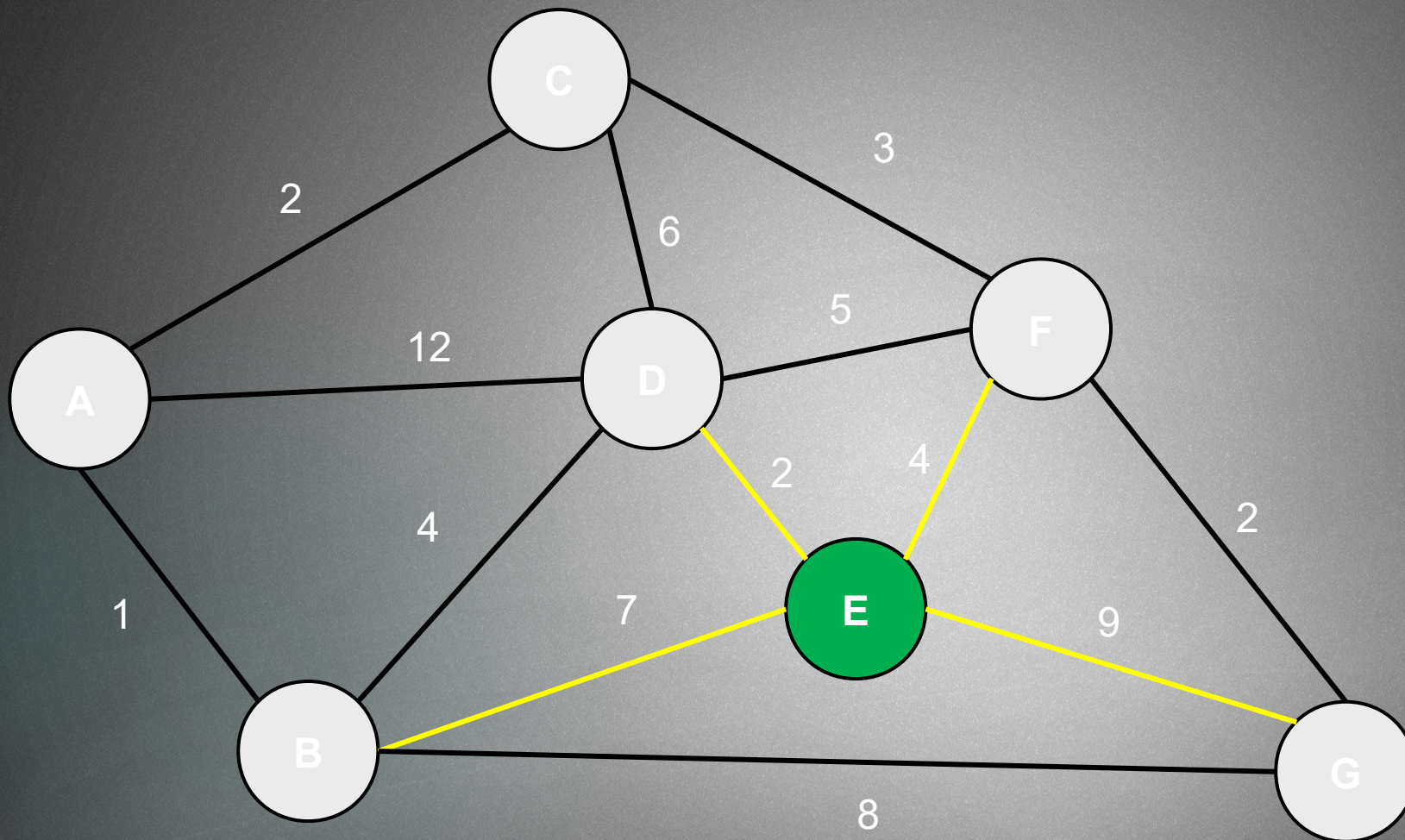




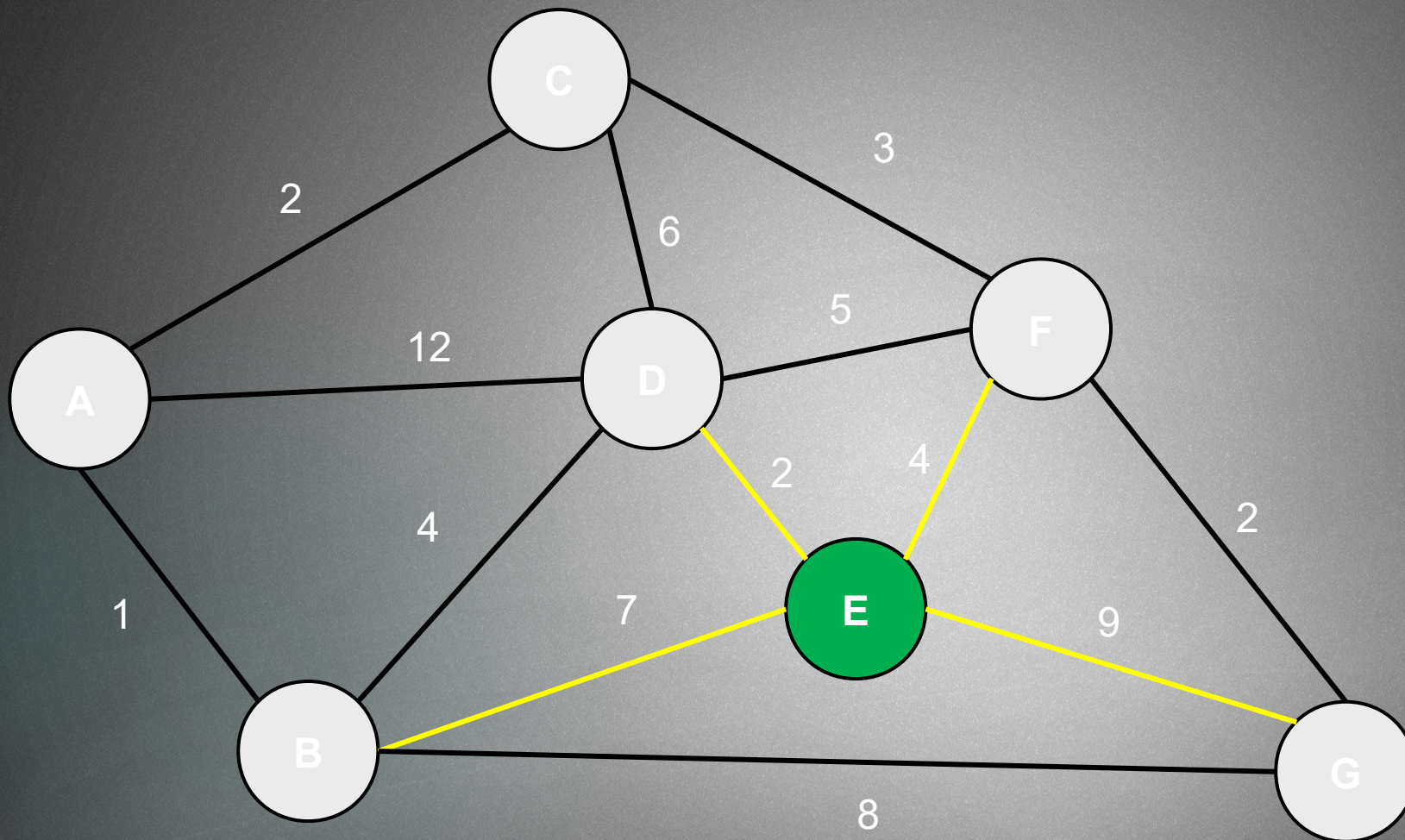








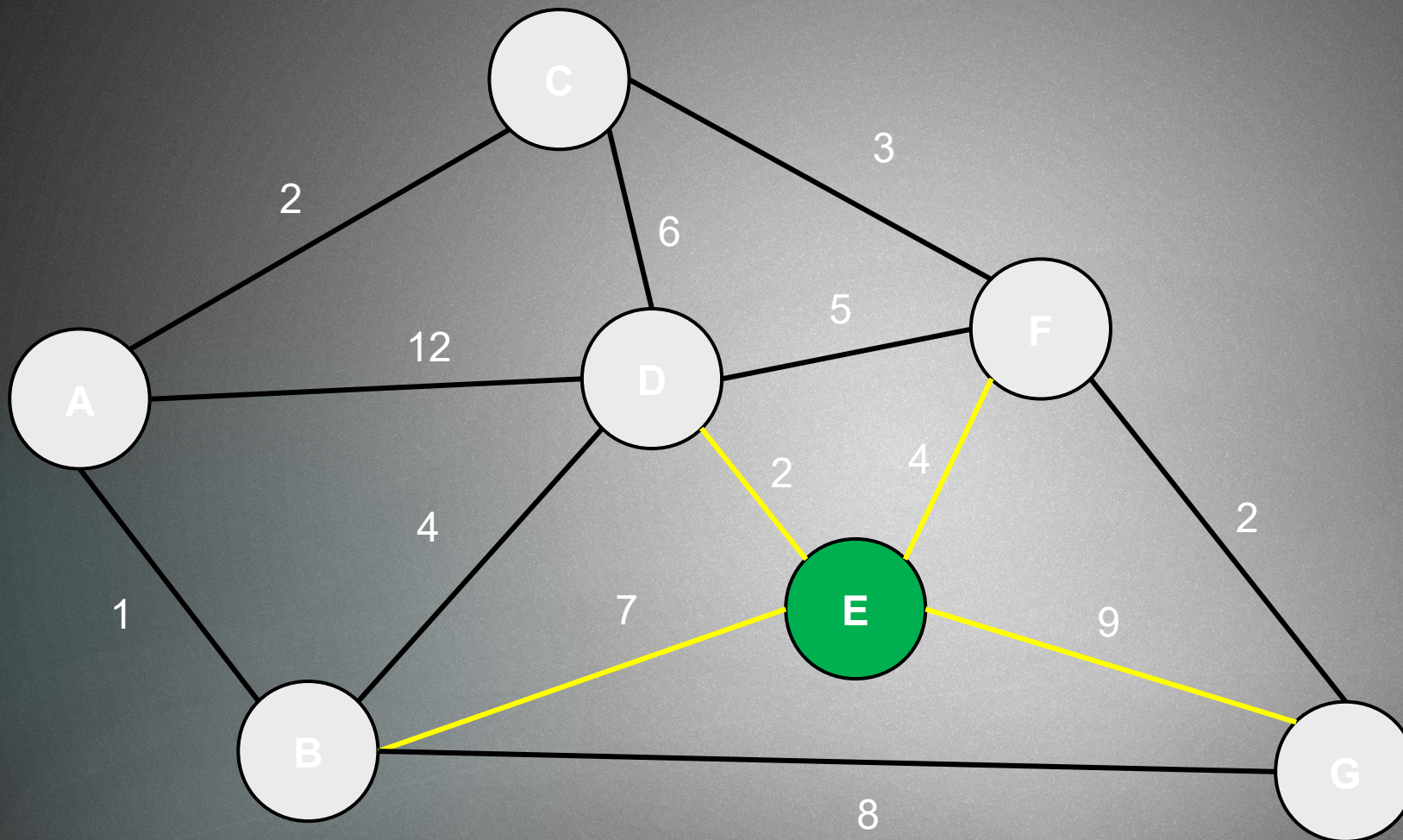




Heap

E-D 2  
E-F 4  
E-G 9  
E-B 7

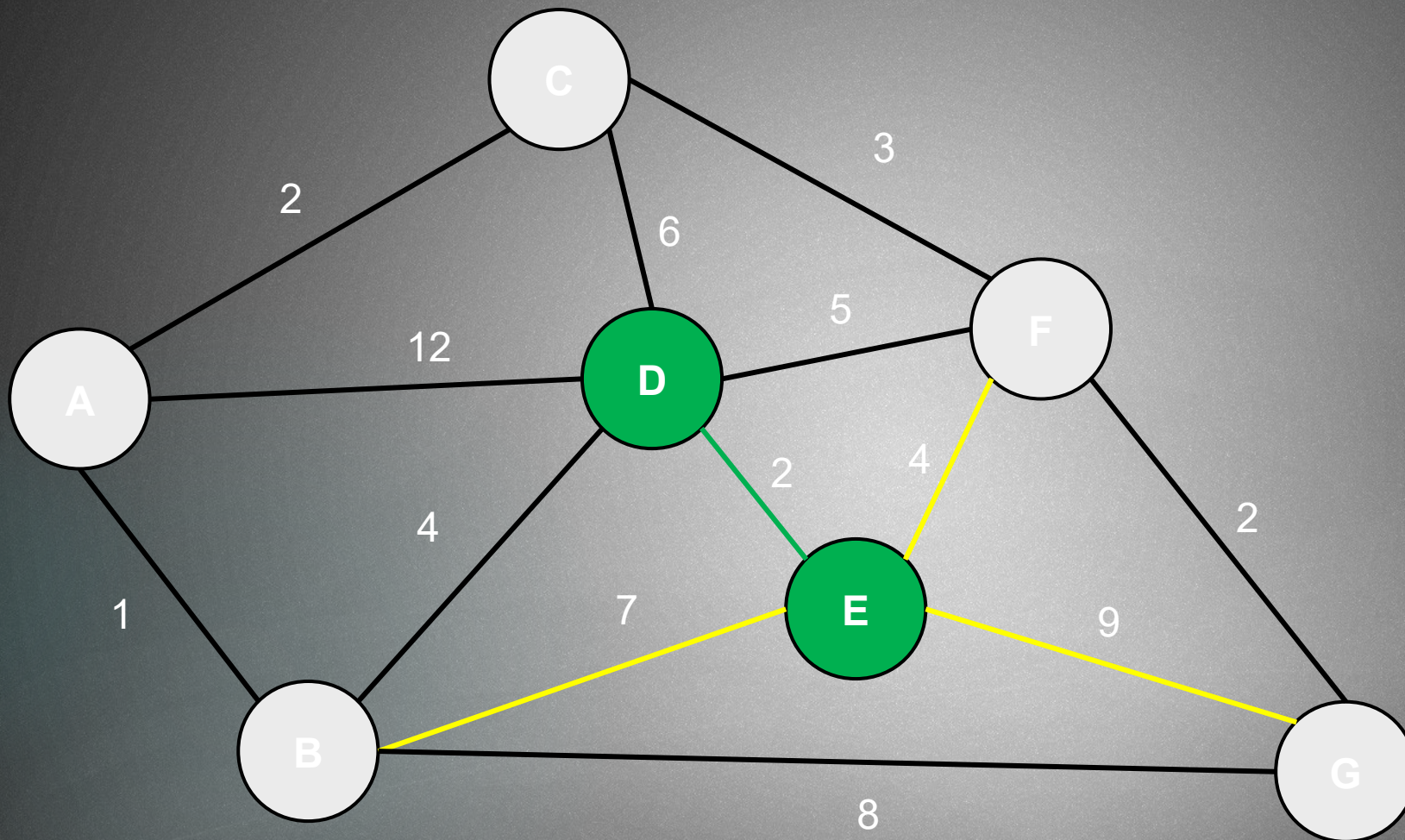




Heap

E-D 2  
E-F 4  
E-G 9  
E-B 7



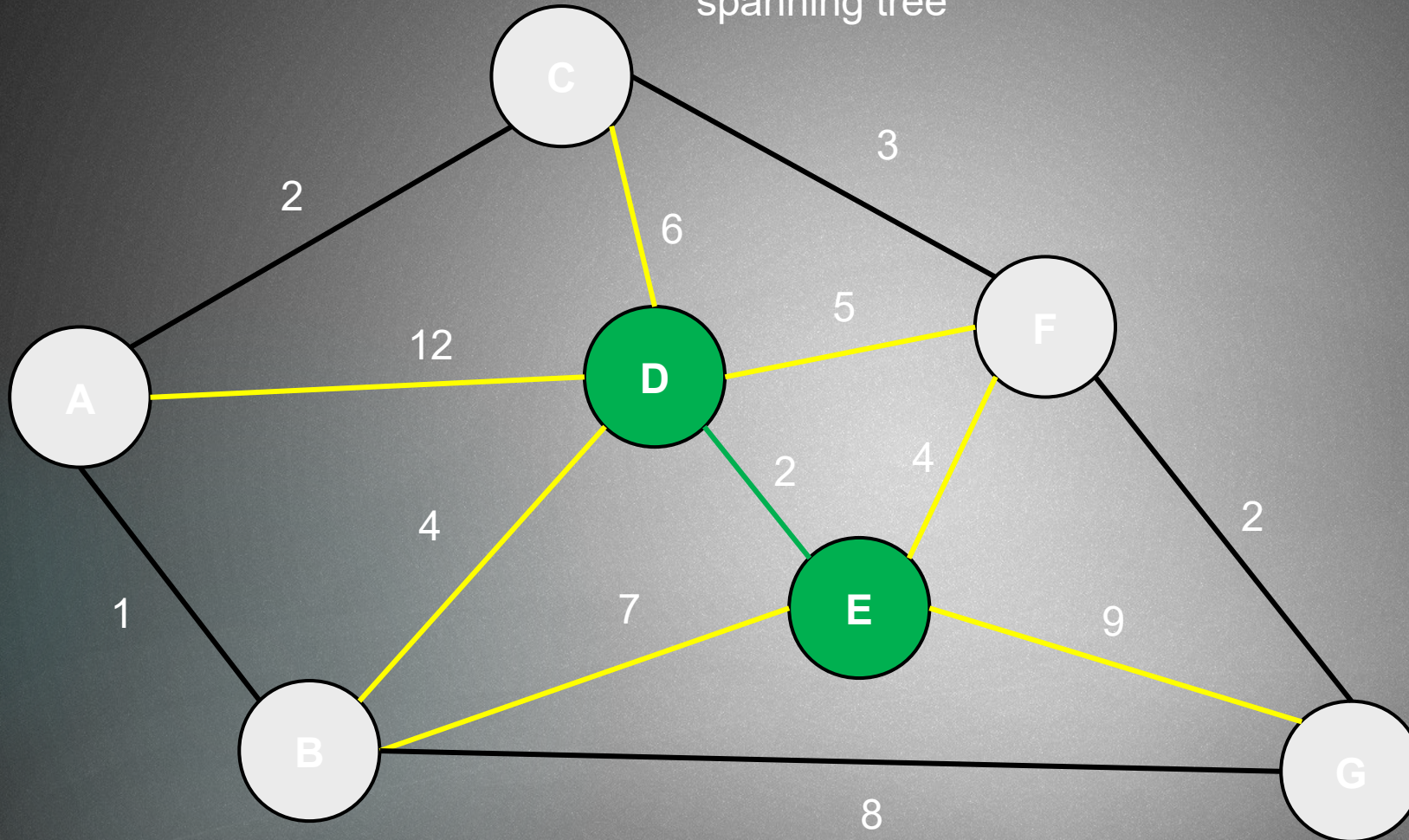


Heap

E-F 4  
E-G 9  
E-B 7



What about the D-F connection? We know that there is a cheaper path to the F node  $\rightarrow$  directly from vertex E  $\rightarrow$  so we exclude D-F as an option in the spanning tree

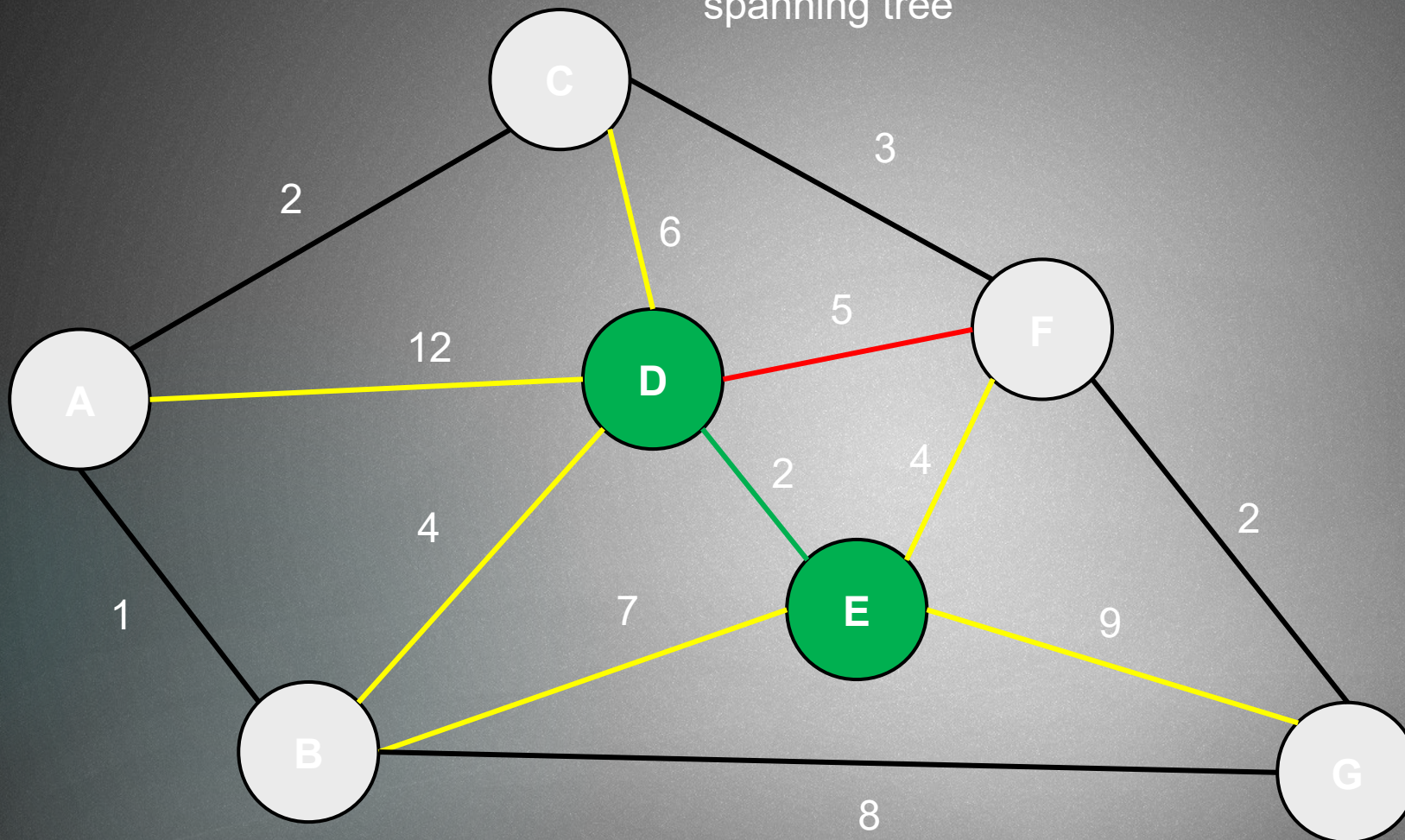


Heap

E-F 4  
E-G 9  
E-B 7



What about the D-F connection? We know that there is a cheaper path to the F node → directly from vertex E → so we exclude D-F as an option in the spanning tree

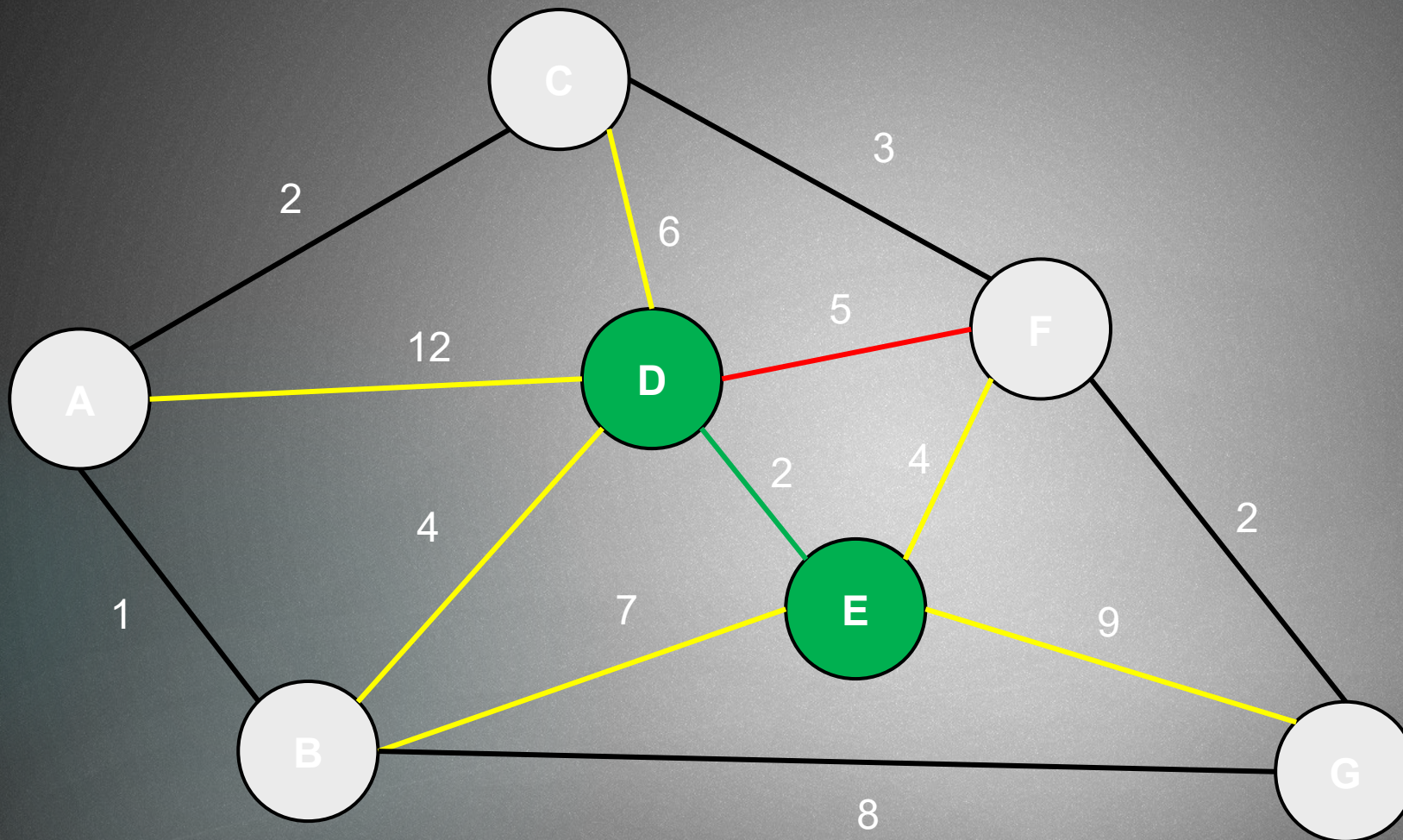


Heap

E-F 4  
E-G 9  
E-B 7



What about the D-C connection? It is the first time we consider vertex C so we add the D-C edge to the heap

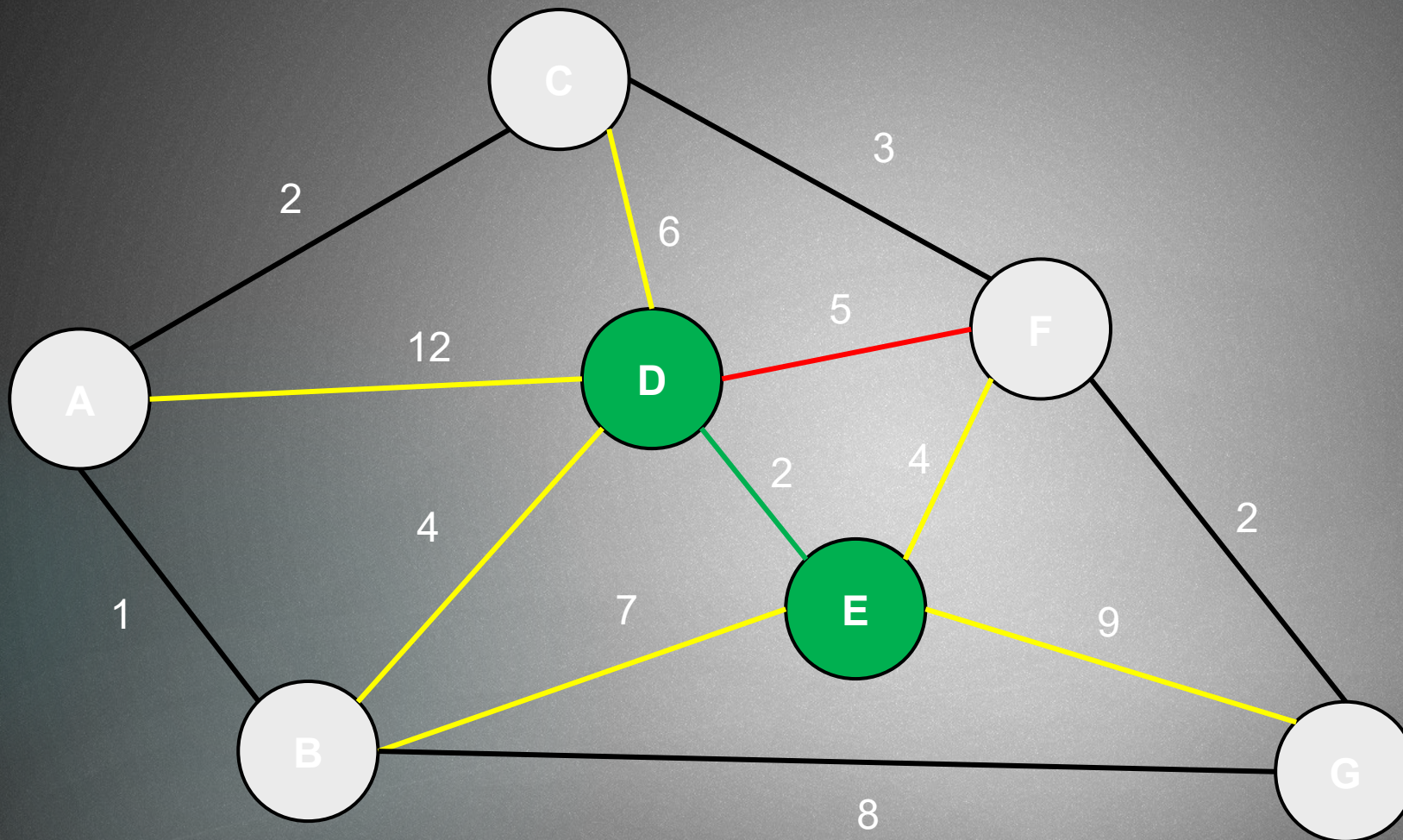


Heap

E-F 4  
E-G 9  
E-B 7



What about the D-C connection? It is the first time we consider vertex C so we add the D-C edge to the heap

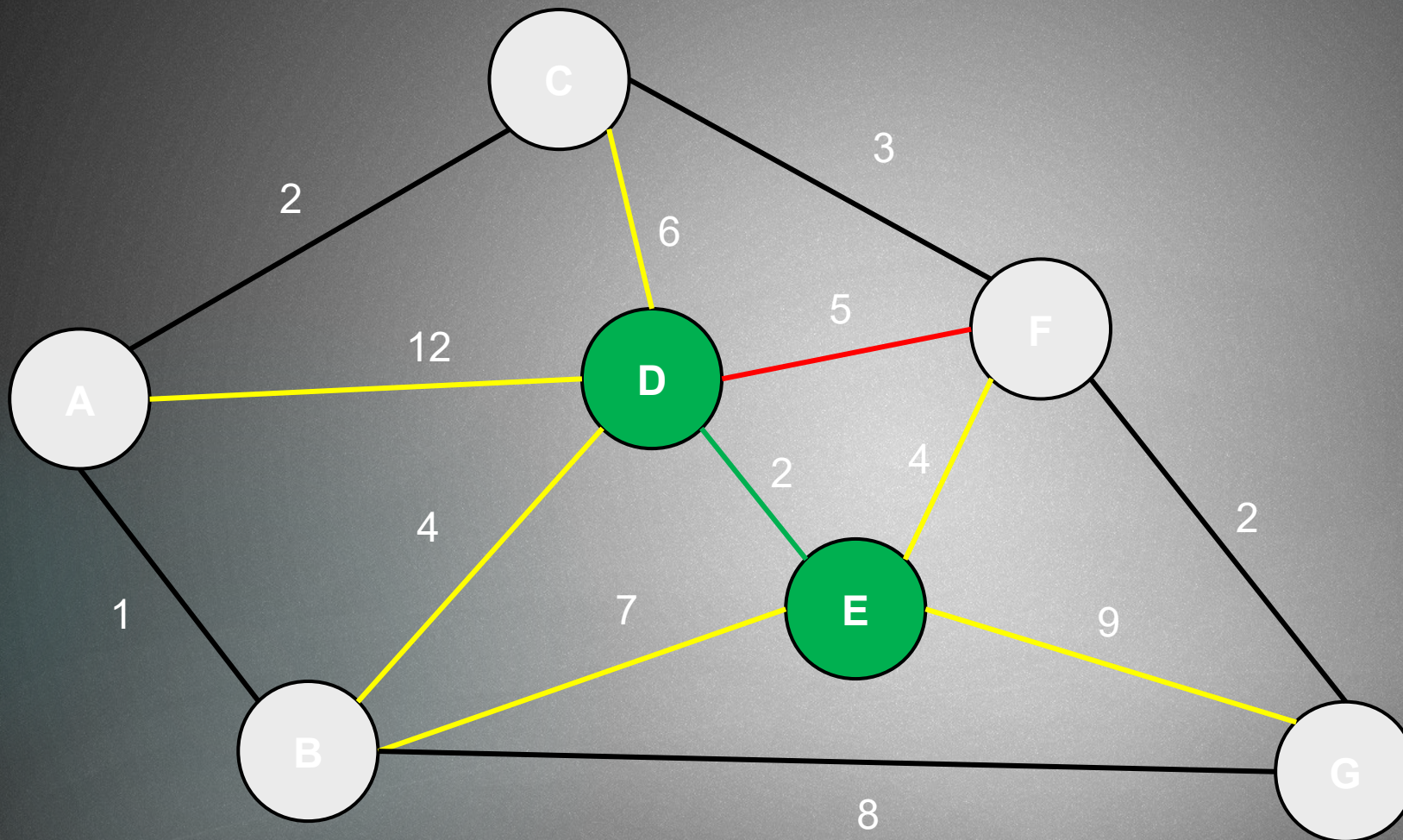


Heap

E-F 4  
E-G 9  
E-B 7  
D-C 6



What about the D-A connection? It is the first time we consider vertex A so we add the D-A edge to the heap

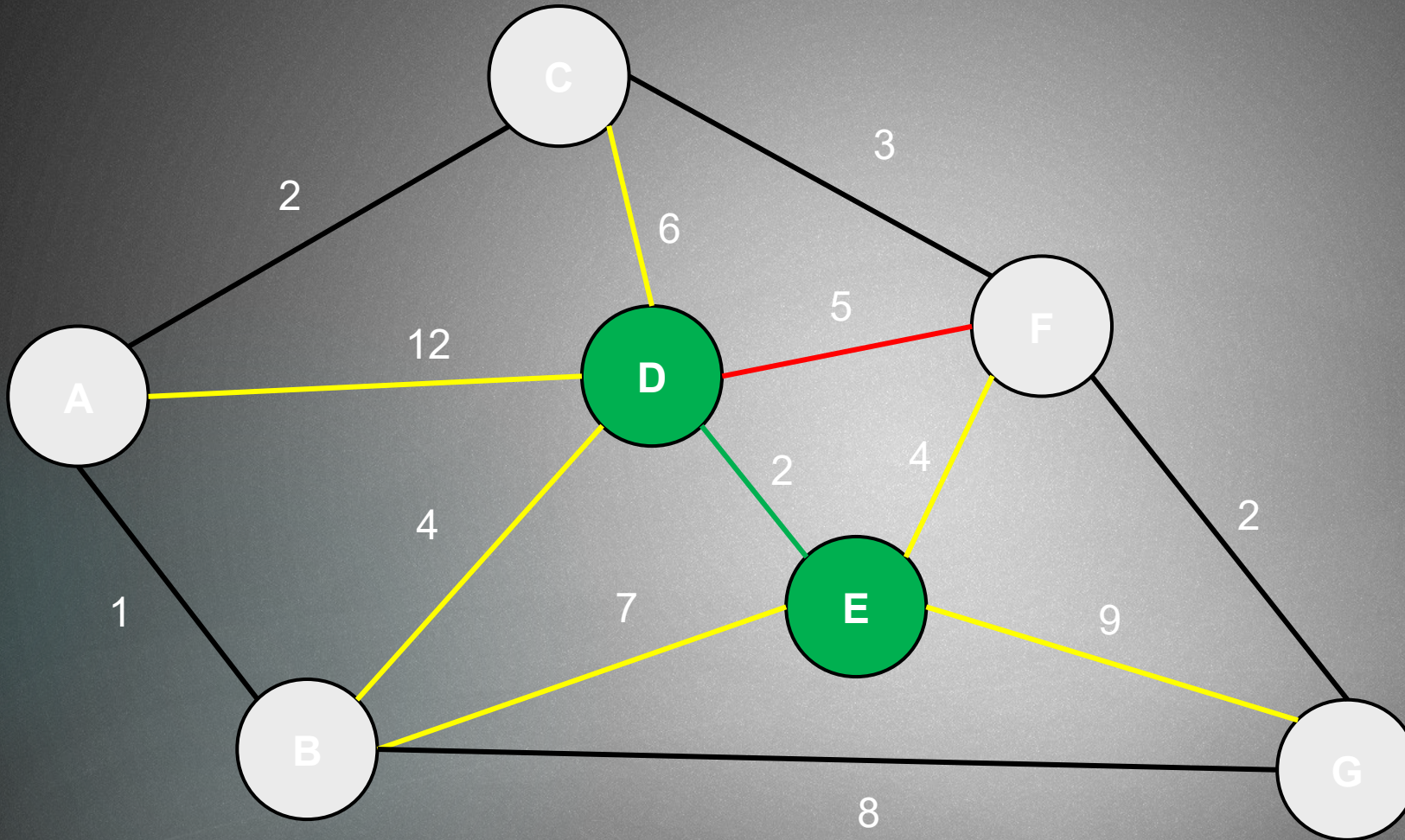


Heap

E-F 4  
E-G 9  
E-B 7  
D-C 6



What about the D-A connection? It is the first time we consider vertex A so we add the D-A edge to the heap

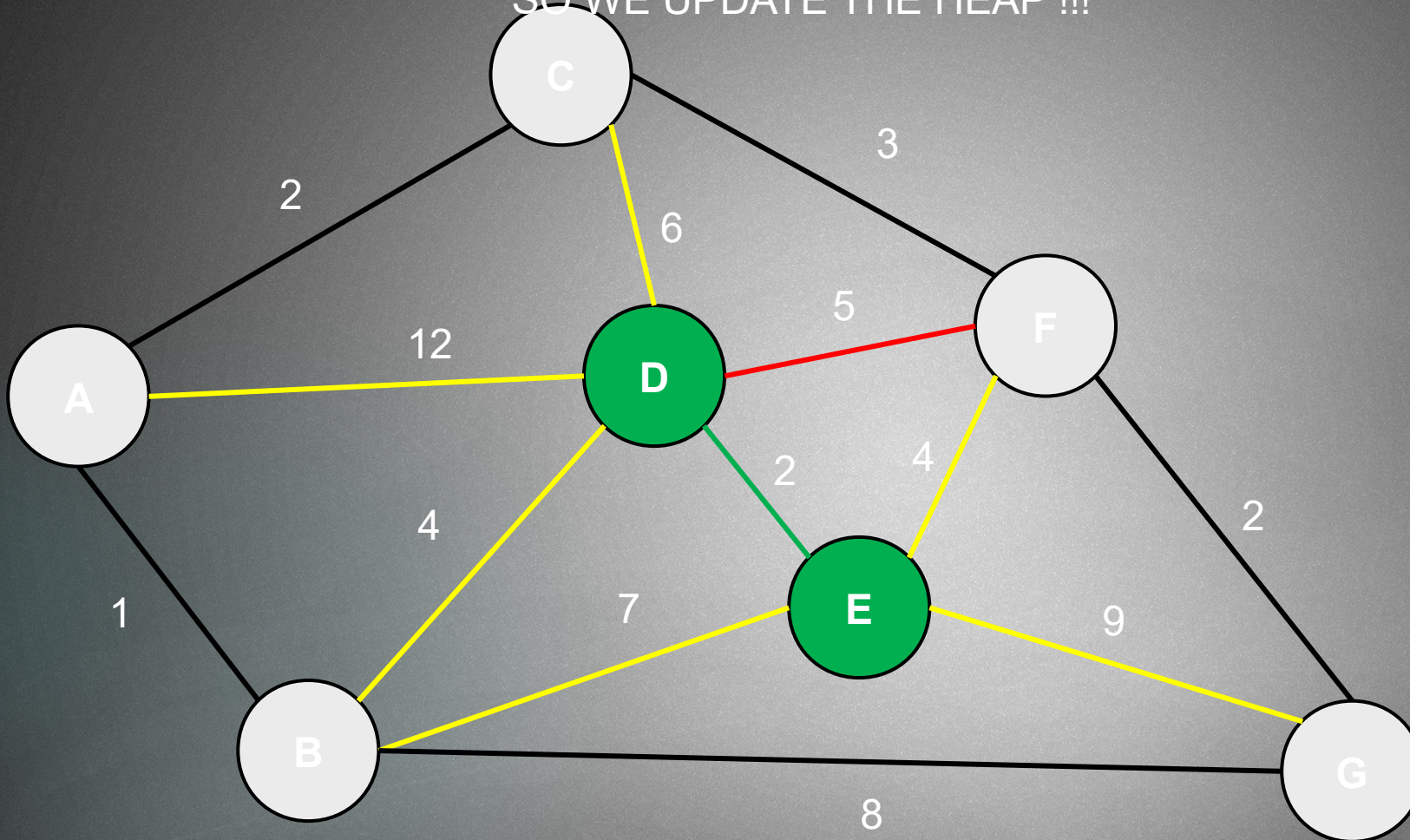


Heap

E-F 4  
E-G 9  
E-B 7  
D-C 6  
D-A 12



What about the D-B connection? We have considered a path to vertex B so far from vertex E. The cost from vertex E is 7, the cost from vertex D is 4  
SO WE UPDATE THE HEAP !!!

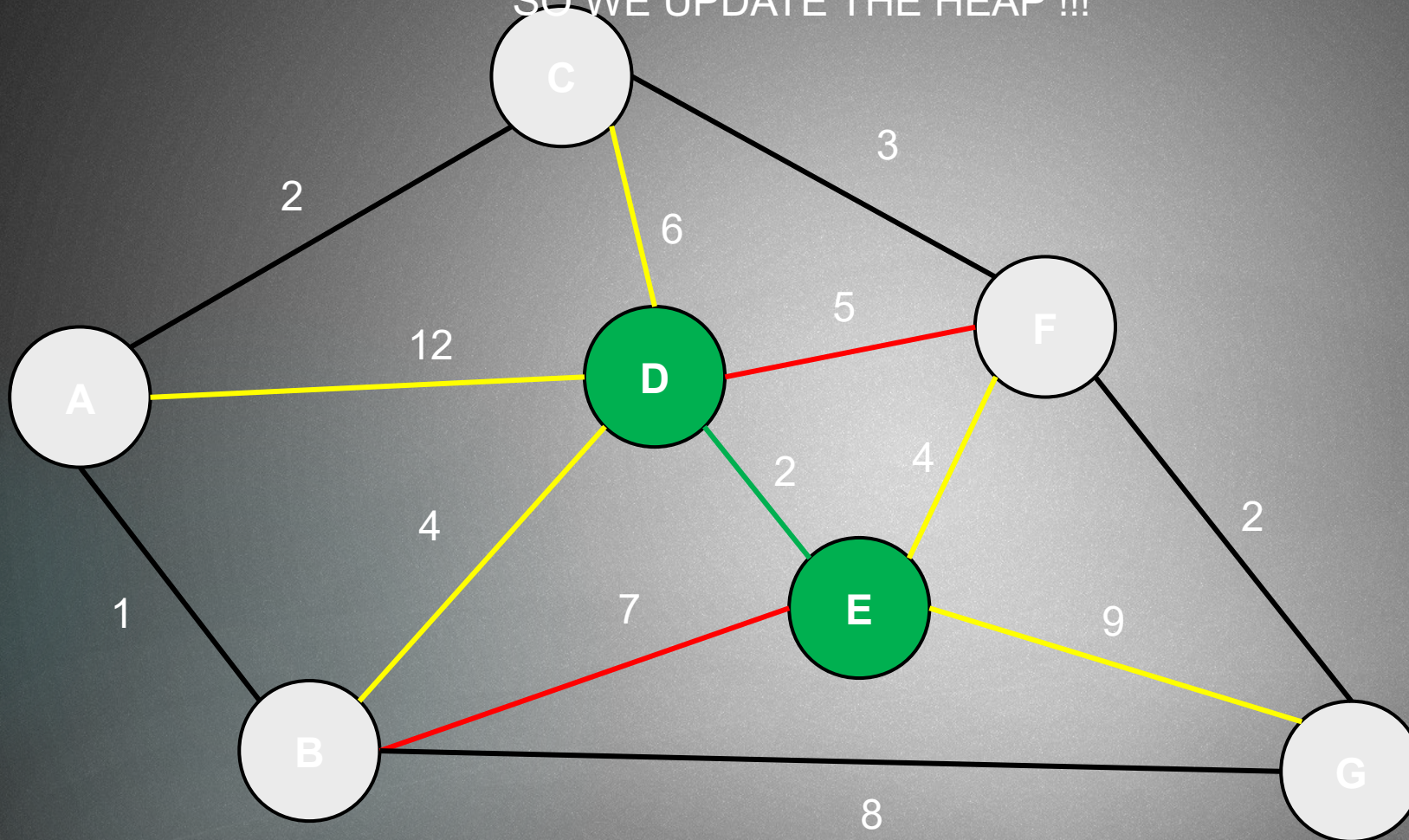


Heap

E-F 4  
E-G 9  
E-B 7  
D-C 6  
D-A 12



What about the D-B connection? We have considered a path to vertex B so far from vertex E. The cost from vertex E is 7, the cost from vertex D is 4  
SO WE UPDATE THE HEAP !!!

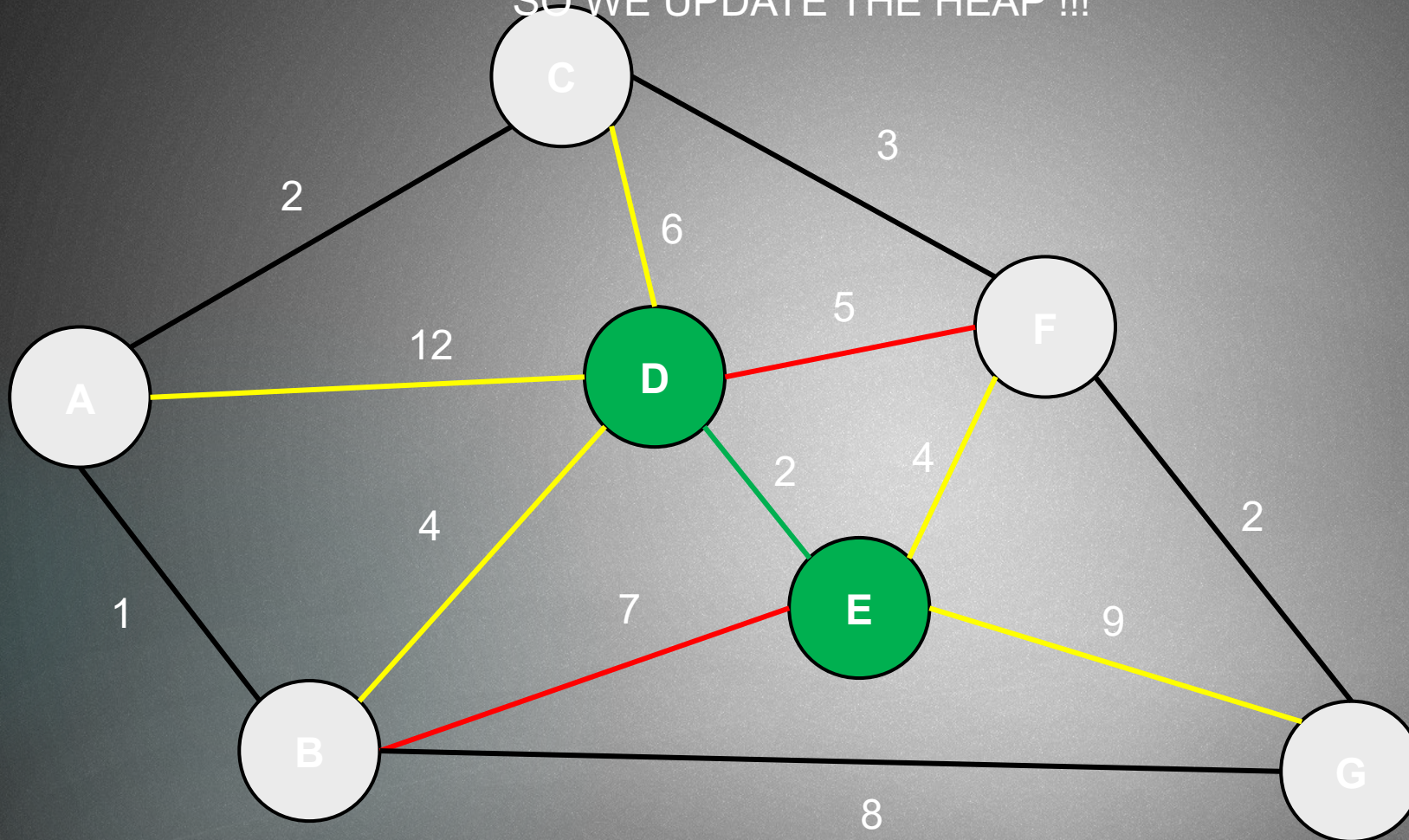


Heap

E-F 4  
E-G 9  
E-B 7  
D-C 6  
D-A 12



What about the D-B connection? We have considered a path to vertex B so far from vertex E. The cost from vertex E is 7, the cost from vertex D is 4  
SO WE UPDATE THE HEAP !!!

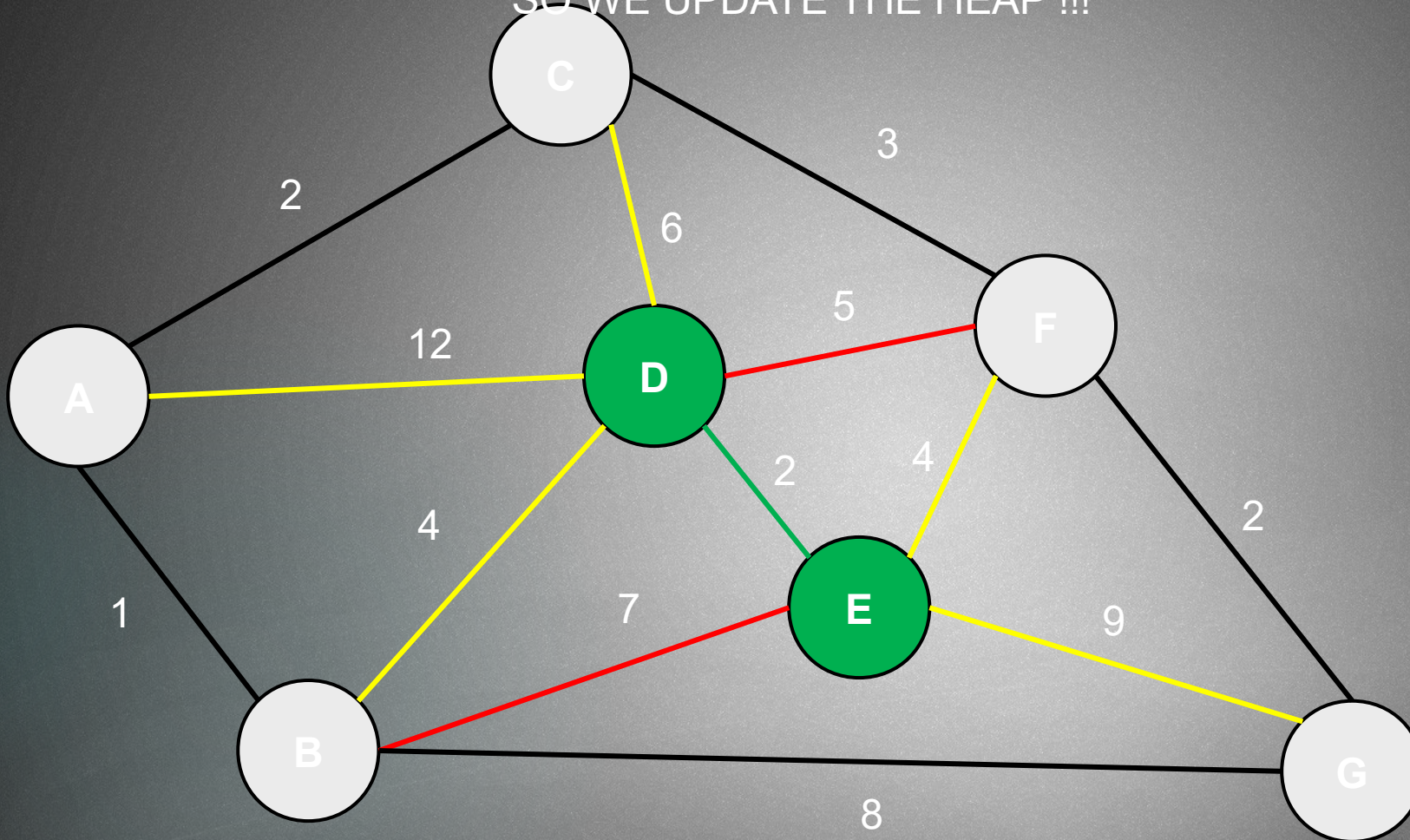


Heap

E-F 4  
E-G 9  
D-C 6  
D-A 12



What about the D-B connection? We have considered a path to vertex B so far from vertex E. The cost from vertex E is 7, the cost from vertex D is 4  
SO WE UPDATE THE HEAP !!!

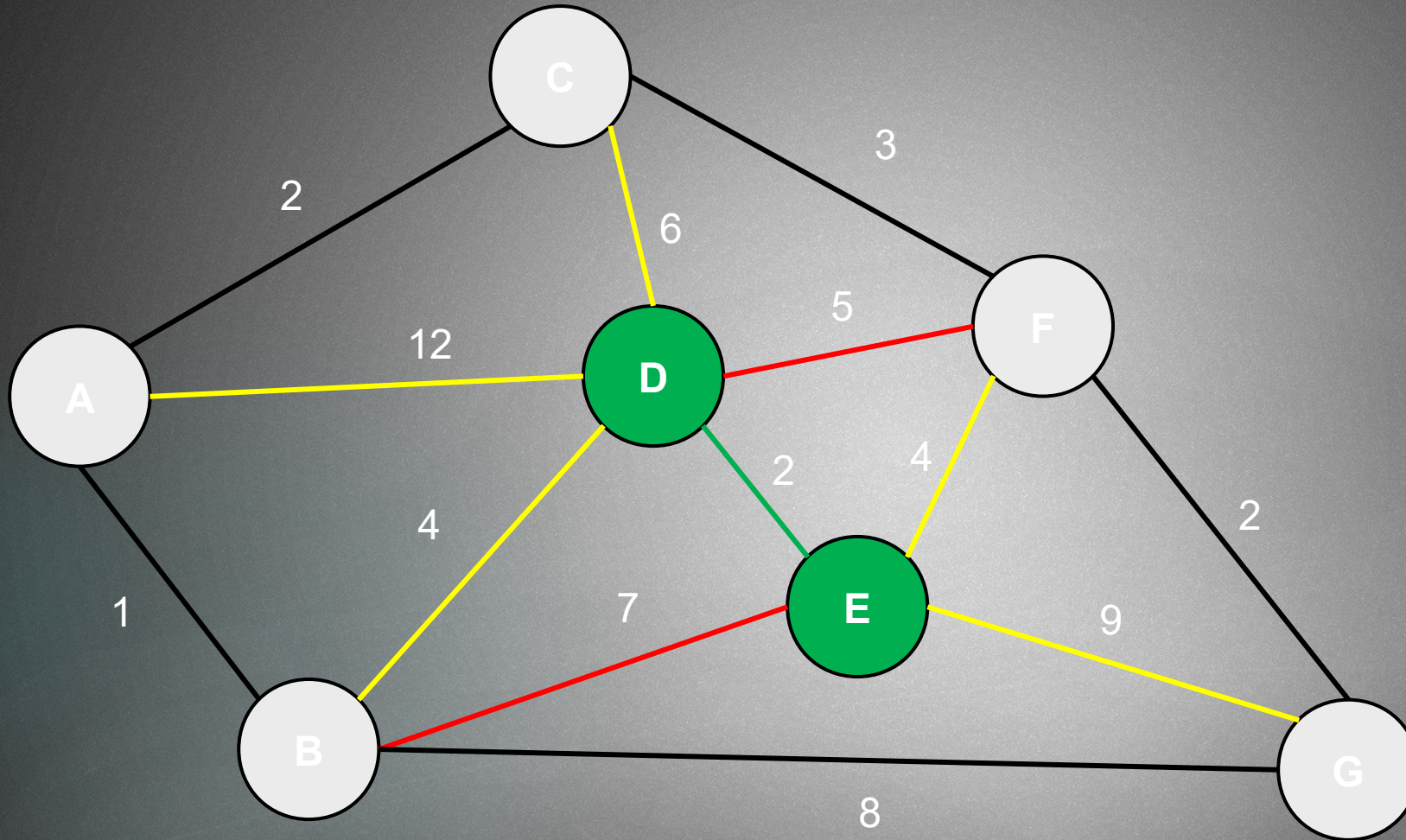


Heap

E-F 4  
E-G 9  
D-C 6  
D-A 12  
D-B 4



We iterate as far as we have not included all the vertexes in the spanning tree  
So we iterate until all the vertexes are green !!!

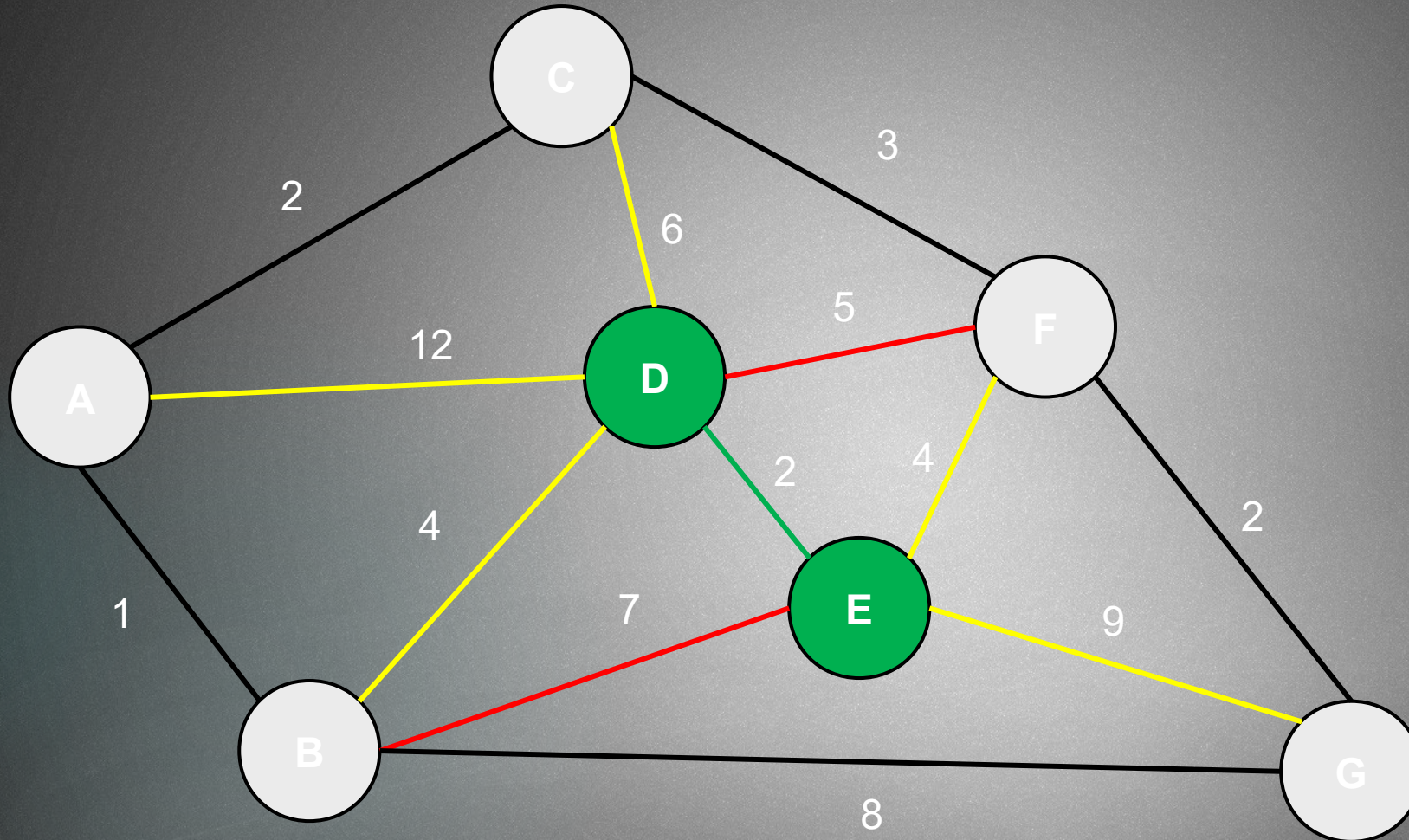


Heap

E-F 4  
E-G 9  
D-C 6  
D-A 12  
D-B 4



We iterate as far as we have not included all the vertexes in the spanning tree  
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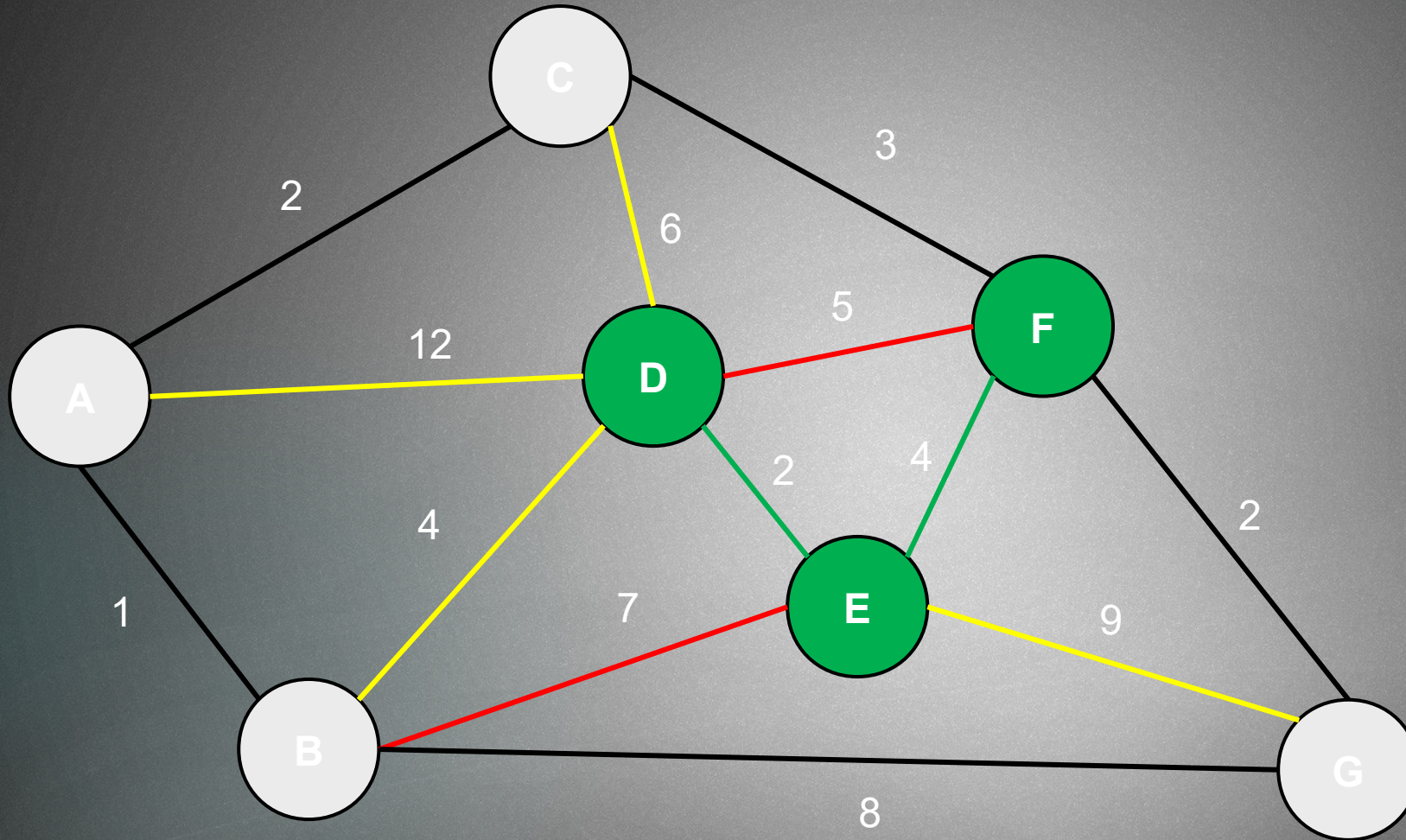


Heap

**E-F 4**  
E-G 9  
D-C 6  
D-A 12  
D-B 4



We iterate as far as we have not included all the vertexes in the spanning tree  
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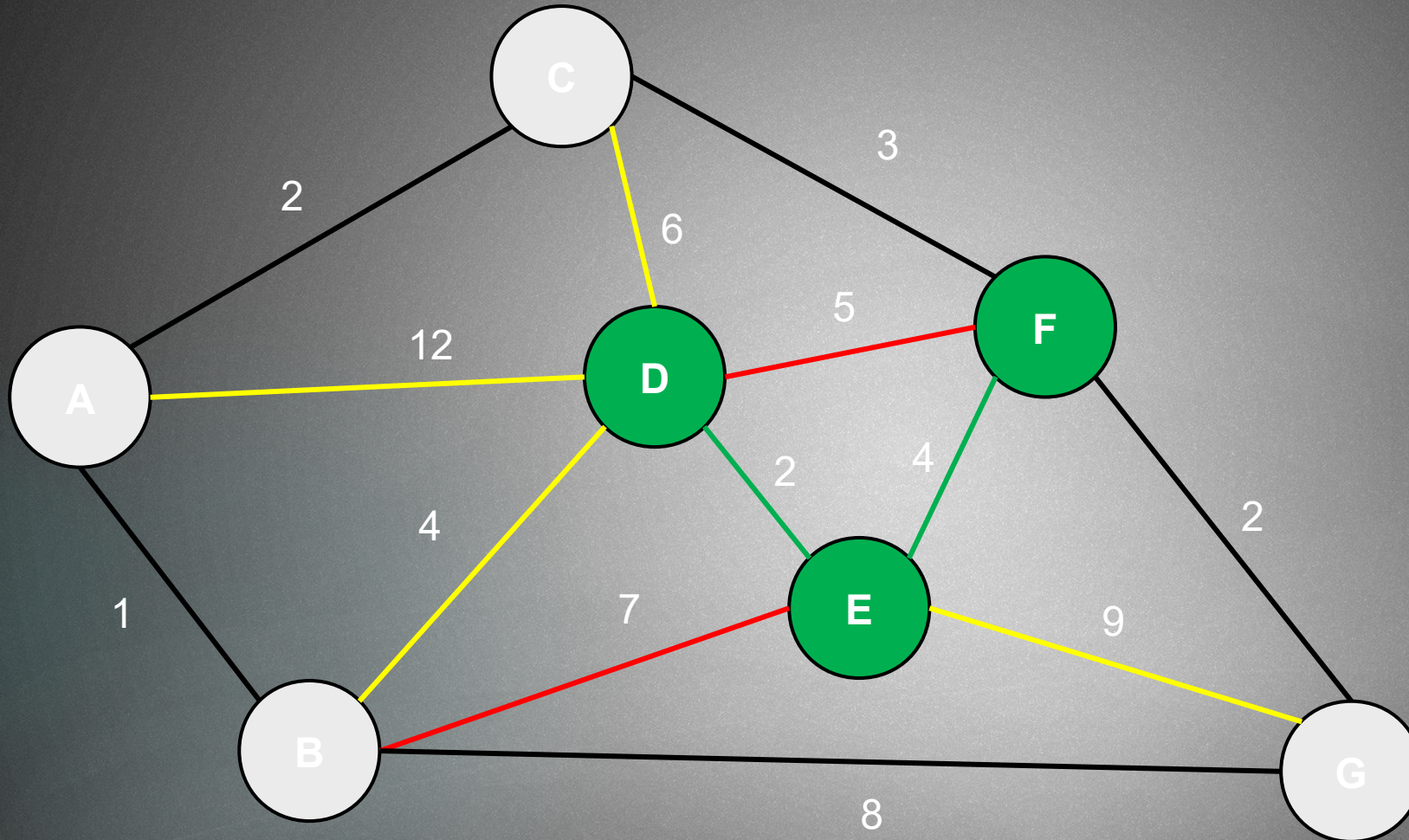


Heap

**E-F 4**  
E-G 9  
D-C 6  
D-A 12  
D-B 4



We iterate as far as we have not included all the vertexes in the spanning tree  
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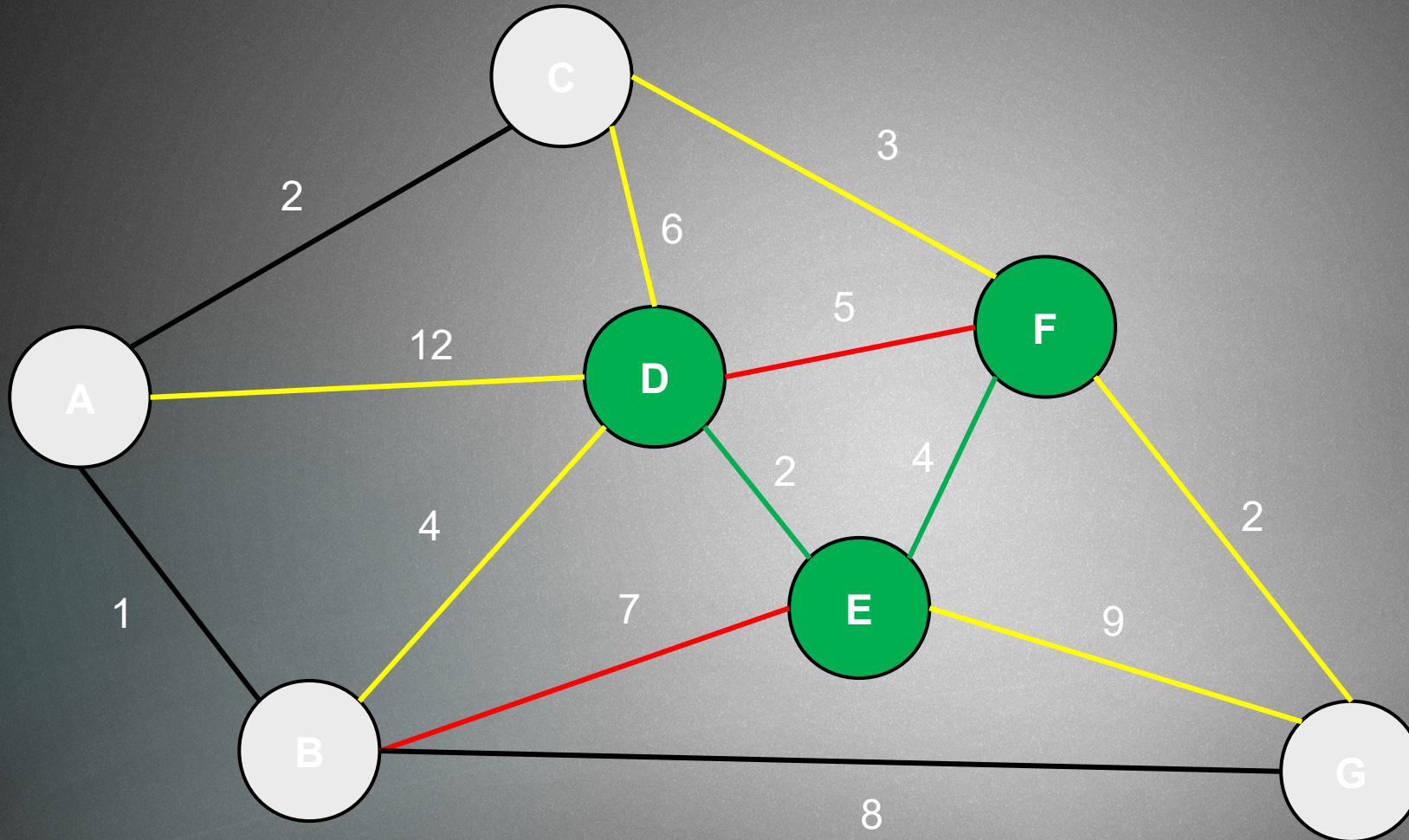


Heap

E-G 9  
D-C 6  
D-A 12  
D-B 4



We iterate as far as we have not included all the vertexes in the spanning tree  
So we iterate until all the vertexes are green !!!



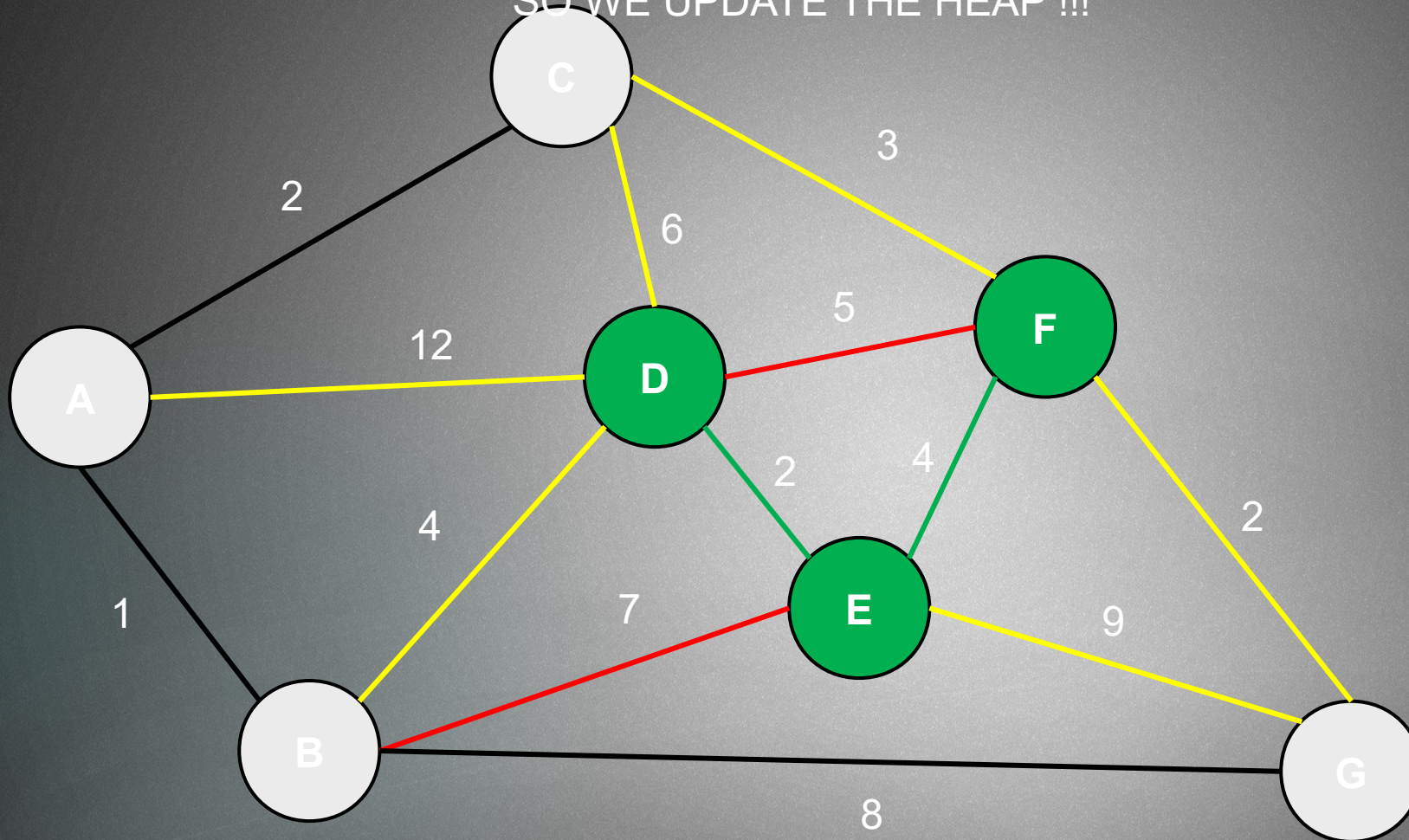
Heap

E-G 9  
D-C 6  
D-A 12  
D-B 4



What about the F-C connection? We have considered a path to vertex C so far from vertex D. The cost from vertex D is 6, the cost from vertex F is 3

SO WE UPDATE THE HEAP !!!

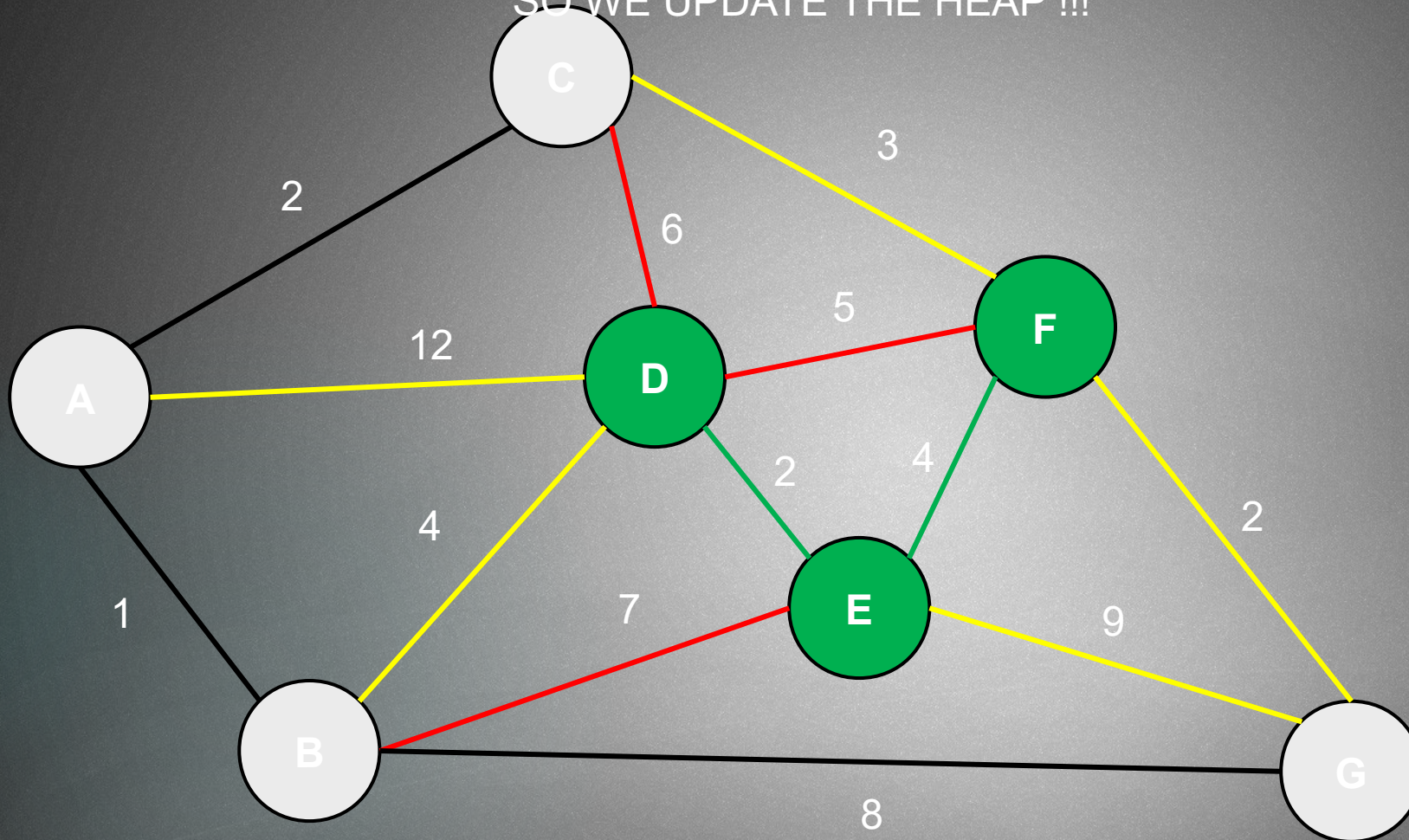


Heap

E-G 9  
D-C 6  
D-A 12  
D-B 4



What about the F-C connection? We have considered a path to vertex C so far from vertex D. The cost from vertex D is 6, the cost from vertex F is 3  
SO WE UPDATE THE HEAP !!!



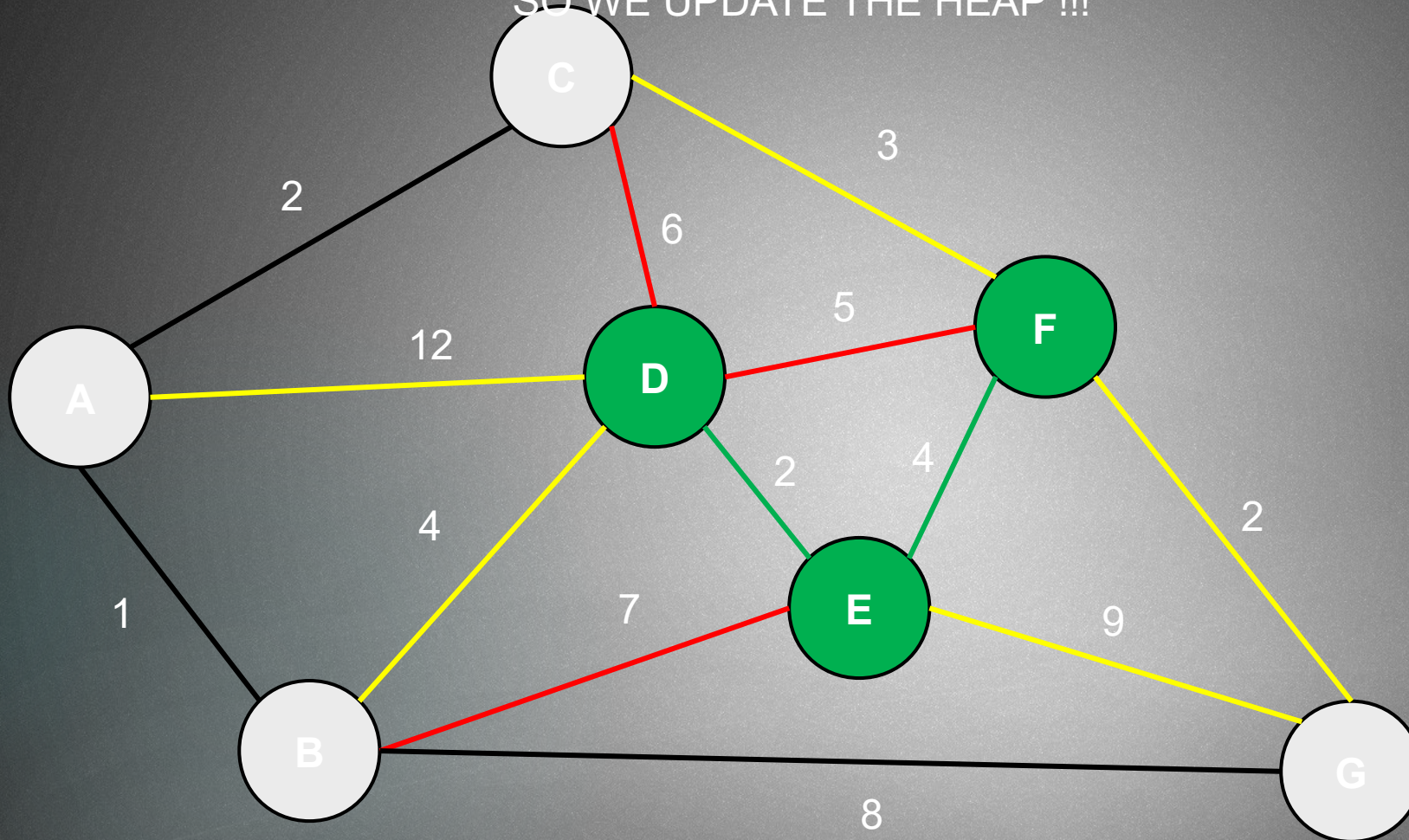
Heap

E-G 9  
**D-C 6**  
D-A 12  
D-B 4



What about the F-C connection? We have considered a path to vertex C so far from vertex D. The cost from vertex D is 6, the cost from vertex F is 3

SO WE UPDATE THE HEAP !!!

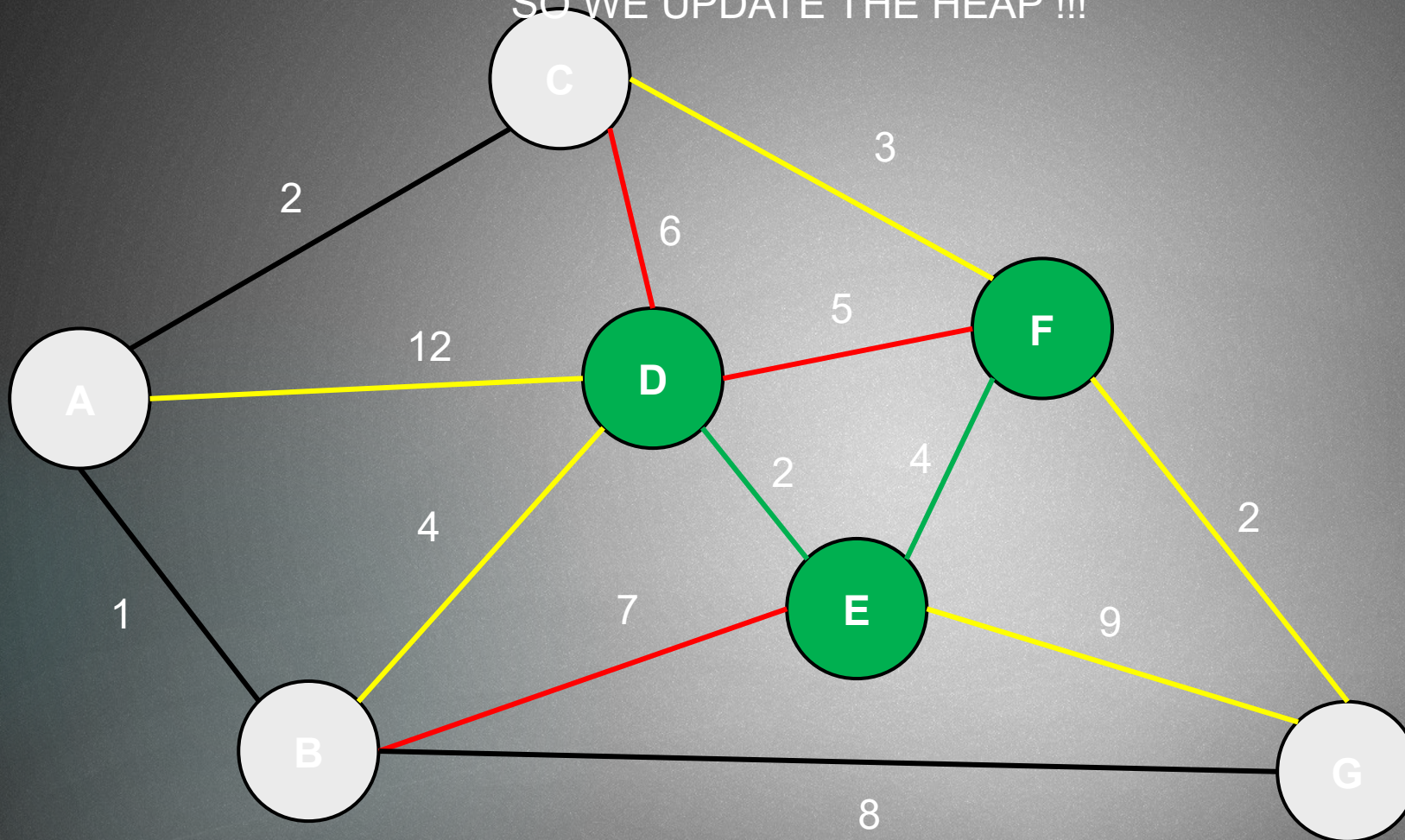


Heap

E-G 9  
D-A 12  
D-B 4



What about the F-C connection? We have considered a path to vertex C so far from vertex D. The cost from vertex D is 6, the cost from vertex F is 3  
SO WE UPDATE THE HEAP !!!



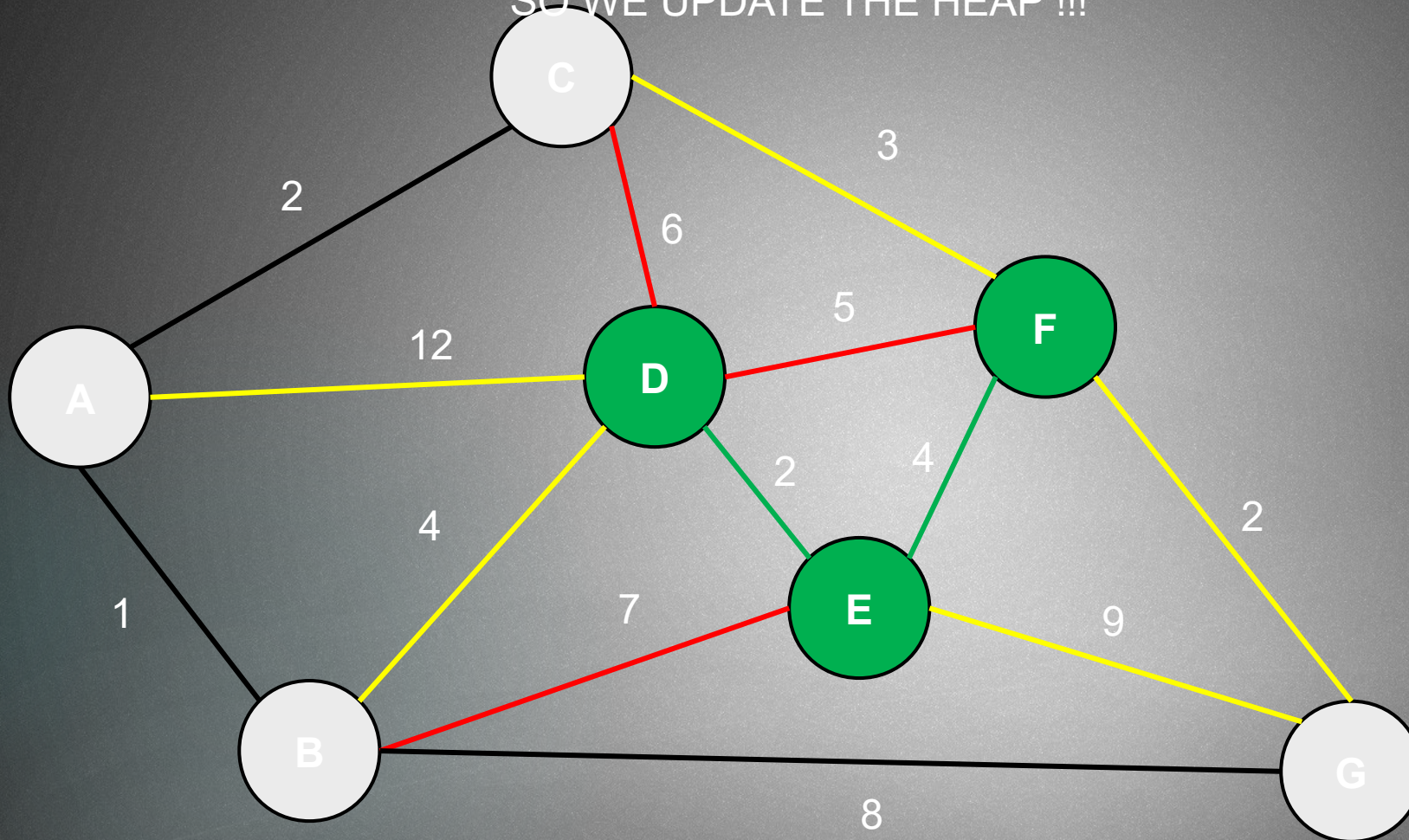
Heap

E-G 9  
D-A 12  
D-B 4  
F-C 3



What about the F-G connection? We have considered a path to vertex G so far from vertex E. The cost from vertex E is 9, the cost from vertex F is 2

SO WE UPDATE THE HEAP !!!



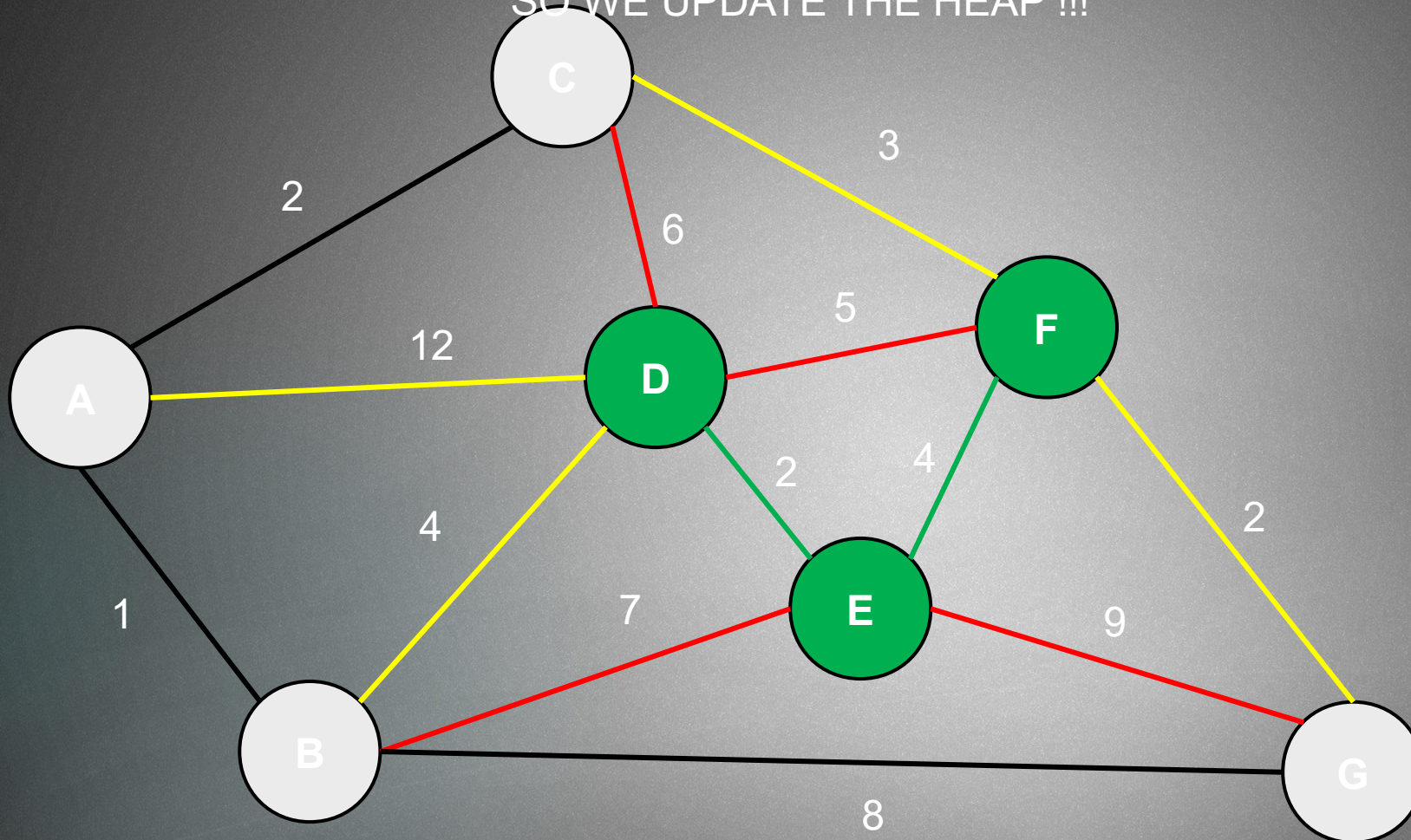
Heap

E-G 9  
D-A 12  
D-B 4  
F-C 3



What about the F-G connection? We have considered a path to vertex G so far from vertex E. The cost from vertex E is 9, the cost from vertex F is 2

SO WE UPDATE THE HEAP !!!



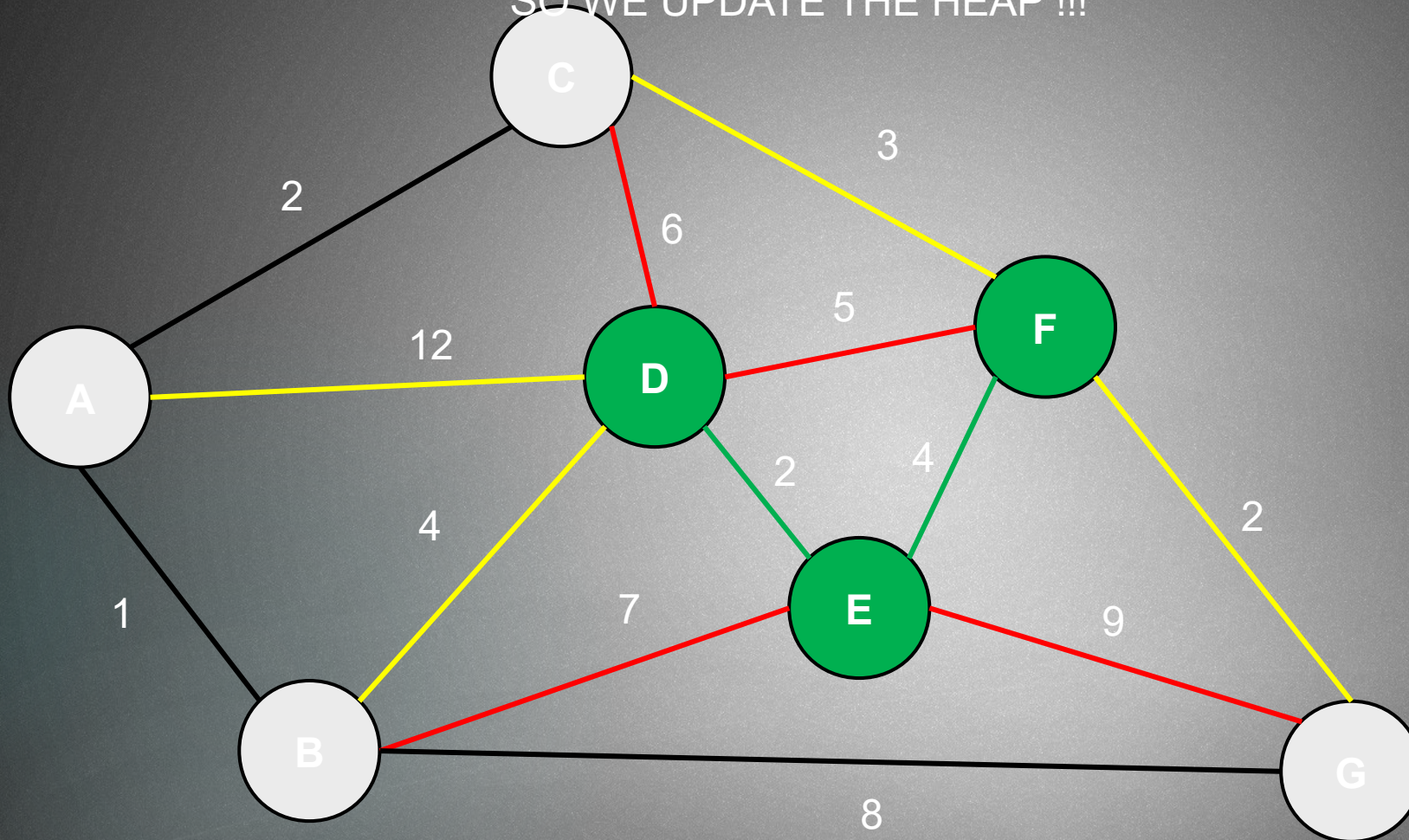
Heap

**E-G 9**  
D-A 12  
D-B 4  
F-C 3



What about the F-G connection? We have considered a path to vertex G so far from vertex E. The cost from vertex E is 9, the cost from vertex F is 2

SO WE UPDATE THE HEAP !!!

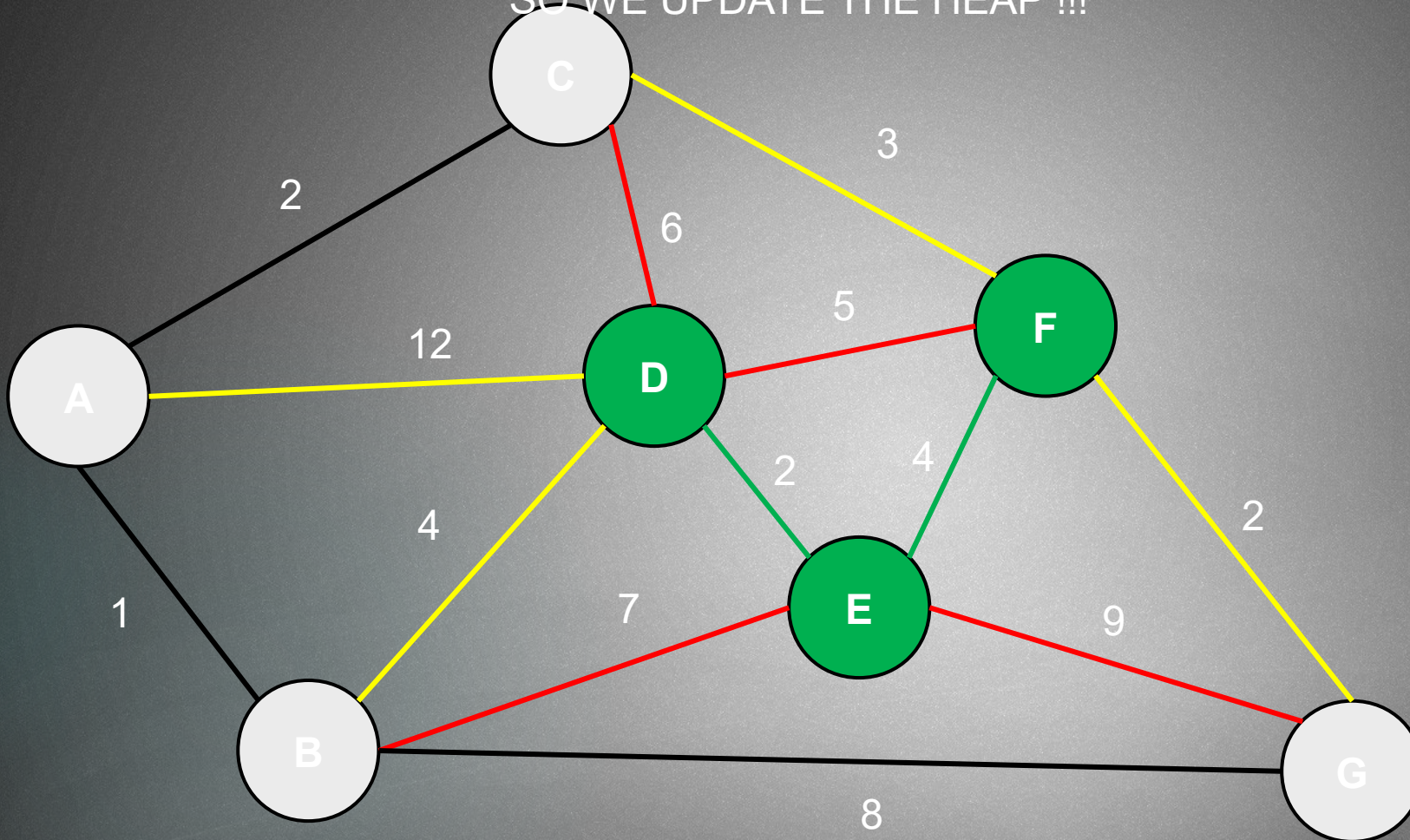


Heap

D-A 12  
D-B 4  
F-C 3



What about the F-G connection? We have considered a path to vertex G so far from vertex E. The cost from vertex E is 9, the cost from vertex F is 2  
SO WE UPDATE THE HEAP !!!

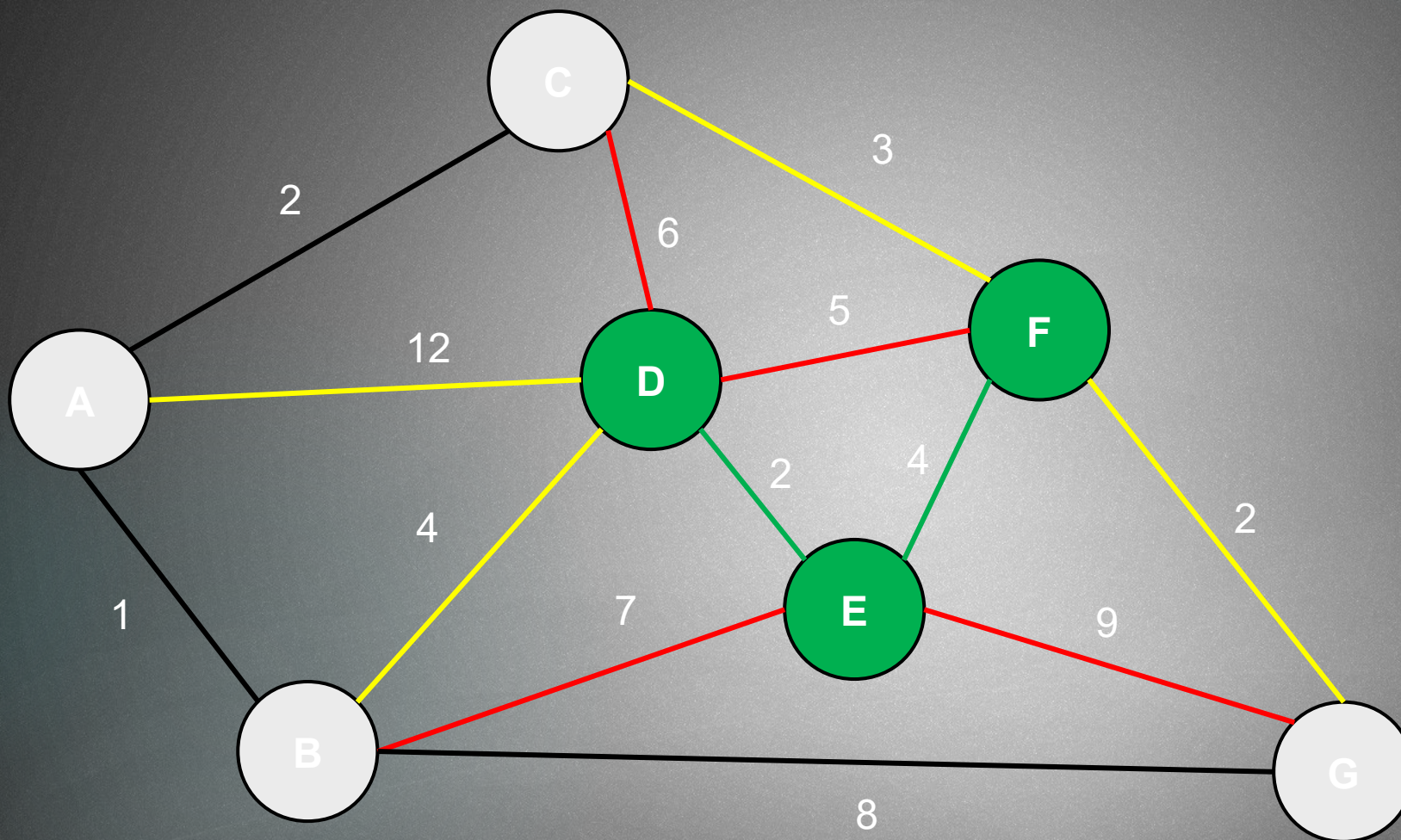


Heap

D-A 12  
D-B 4  
F-C 3  
F-G 2



We keep going because we do not have all the vertexes in the spanning tree

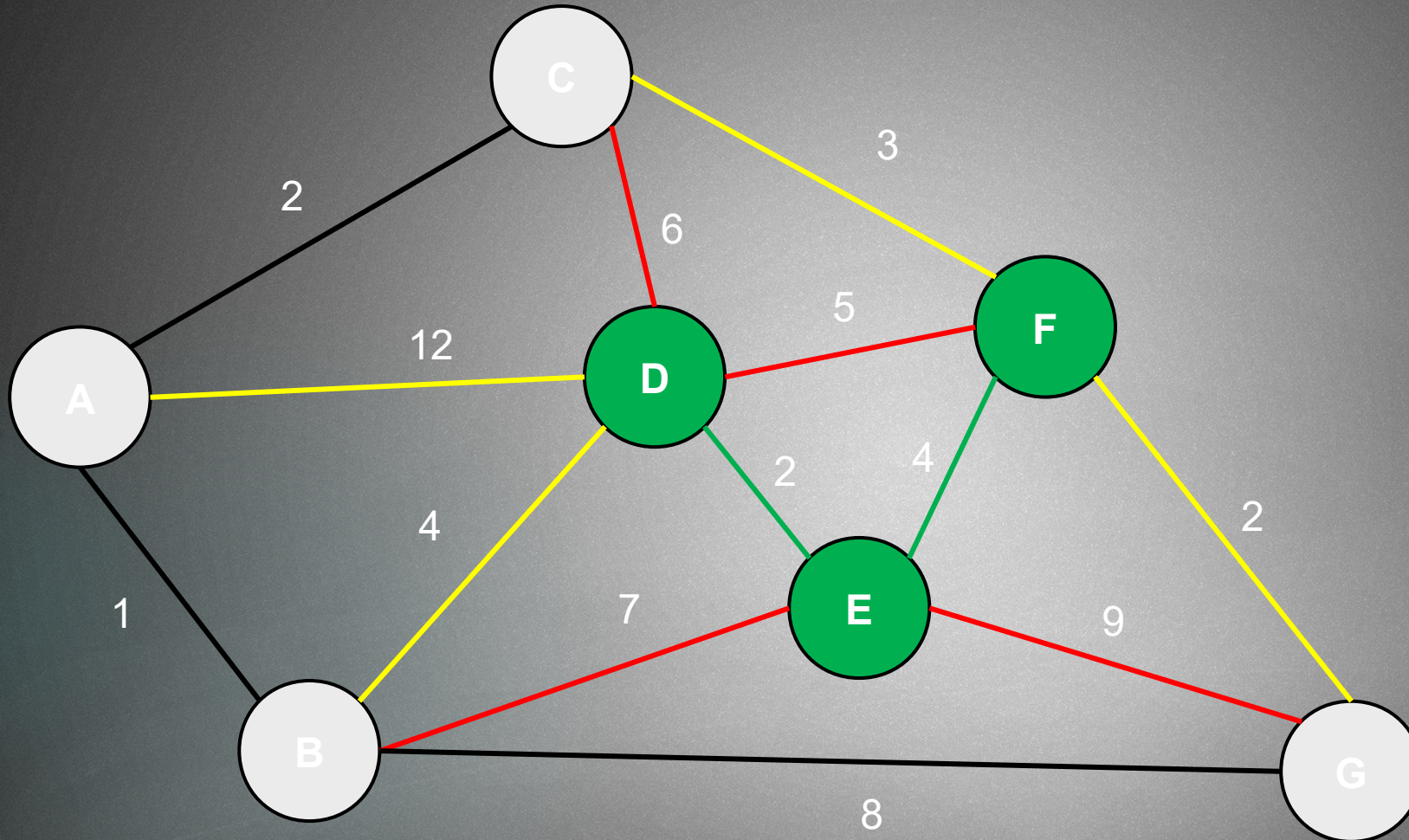


Heap

D-A 12  
D-B 4  
F-C 3  
F-G 2



We keep going because we do not have all the vertexes in the spanning tree

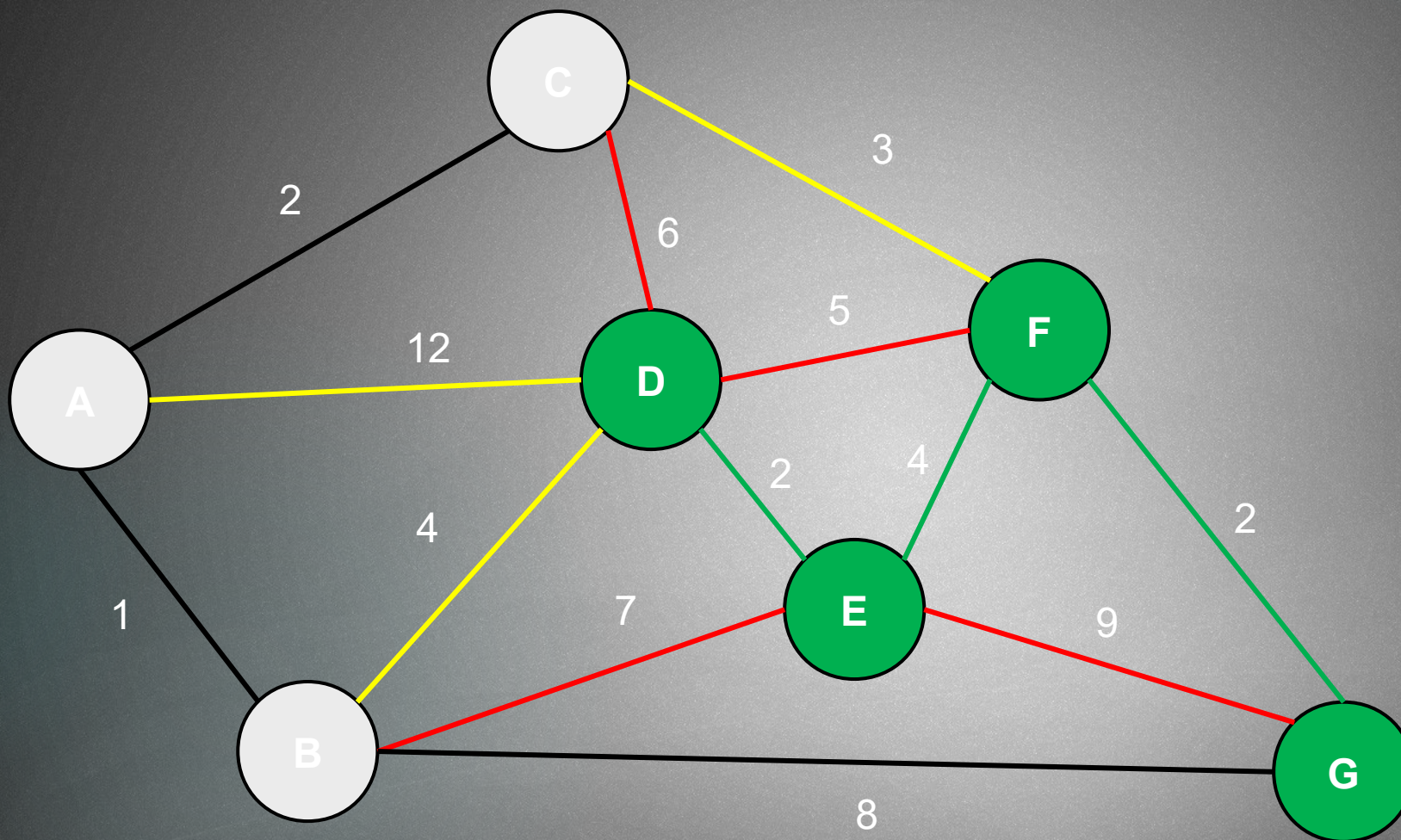


Heap

D-A 12  
D-B 4  
F-C 3  
**F-G 2**



We keep going because we do not have all the vertexes in the spanning tree

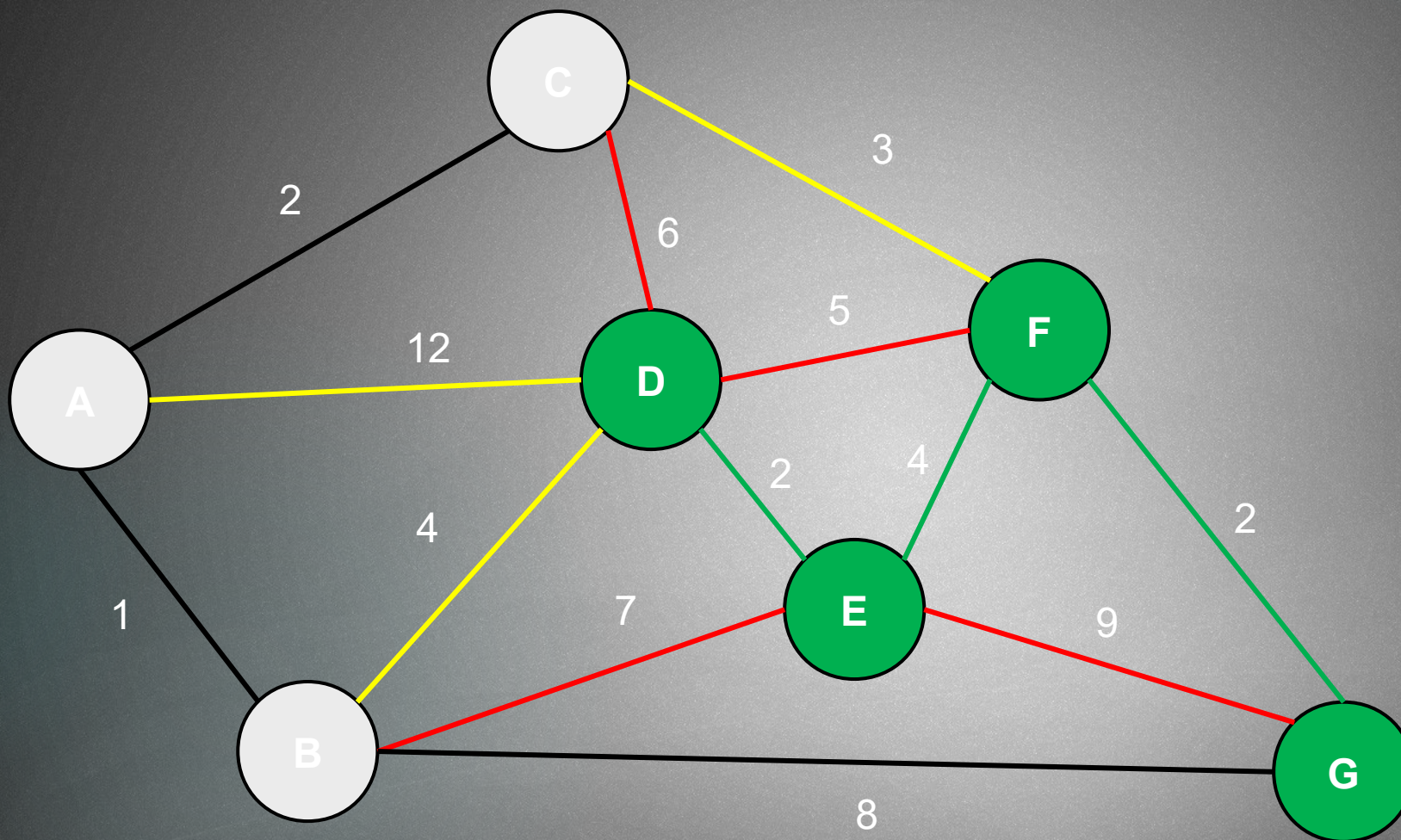


Heap

D-A 12  
D-B 4  
F-C 3  
**F-G 2**



We keep going because we do not have all the vertexes in the spanning tree

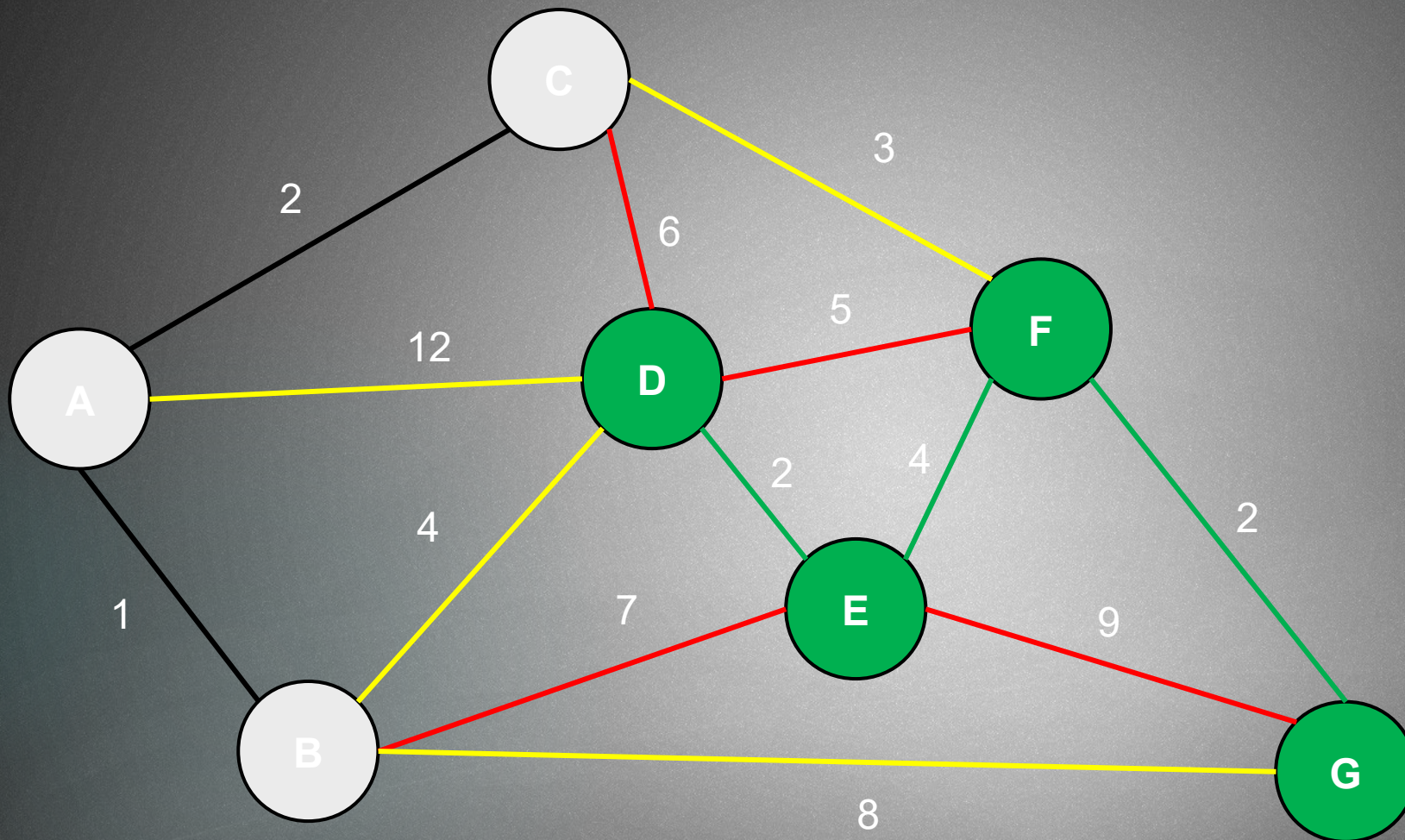


Heap

D-A 12  
D-B 4  
F-C 3



We keep going because we do not have all the vertexes in the spanning tree

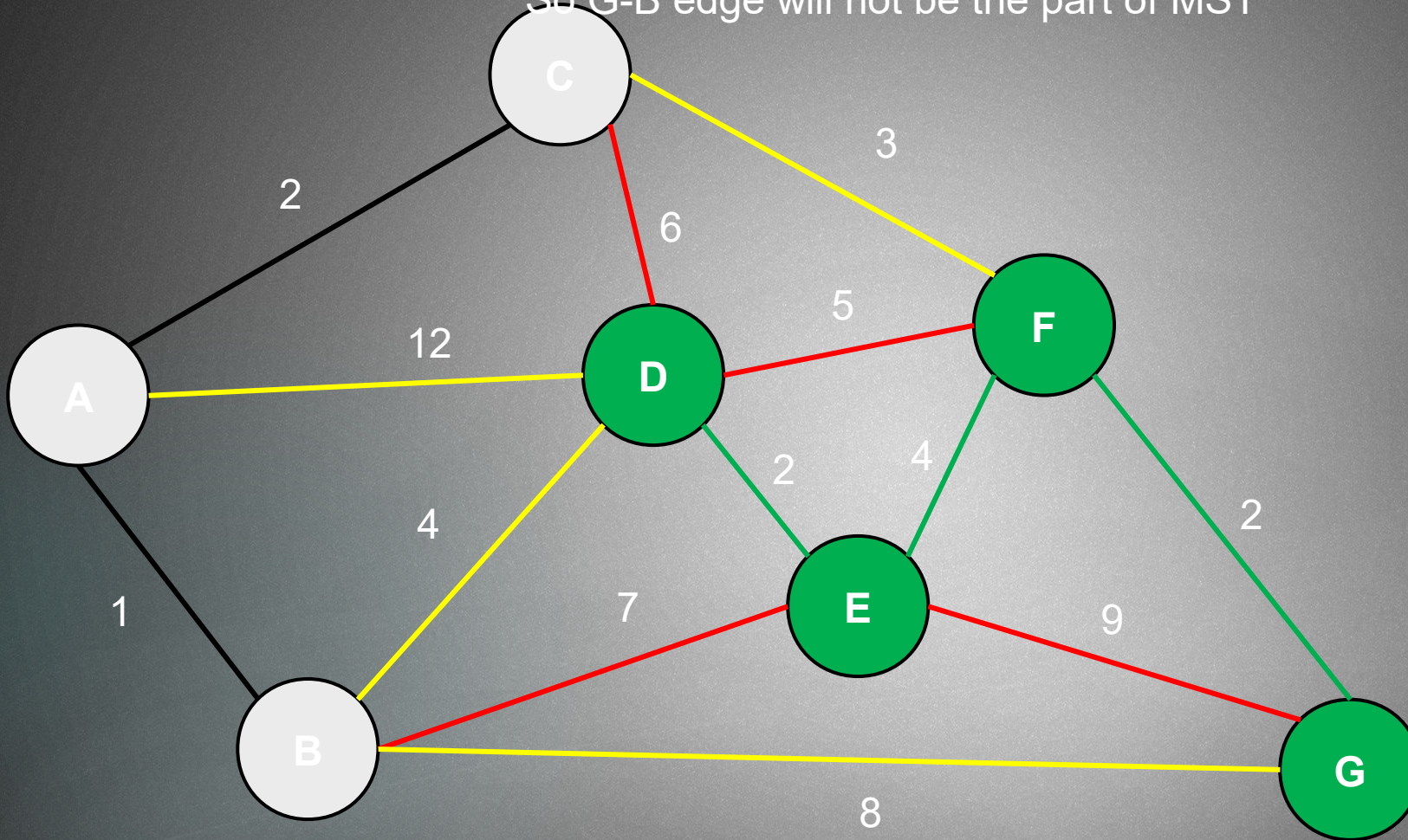


Heap

D-A 12  
D-B 4  
F-C 3



What about the G-B connection? We have considered a path to vertex B so far from vertex D. The cost from vertex D is 4 , the cost from vertex G is 8  
So G-B edge will not be the part of MST

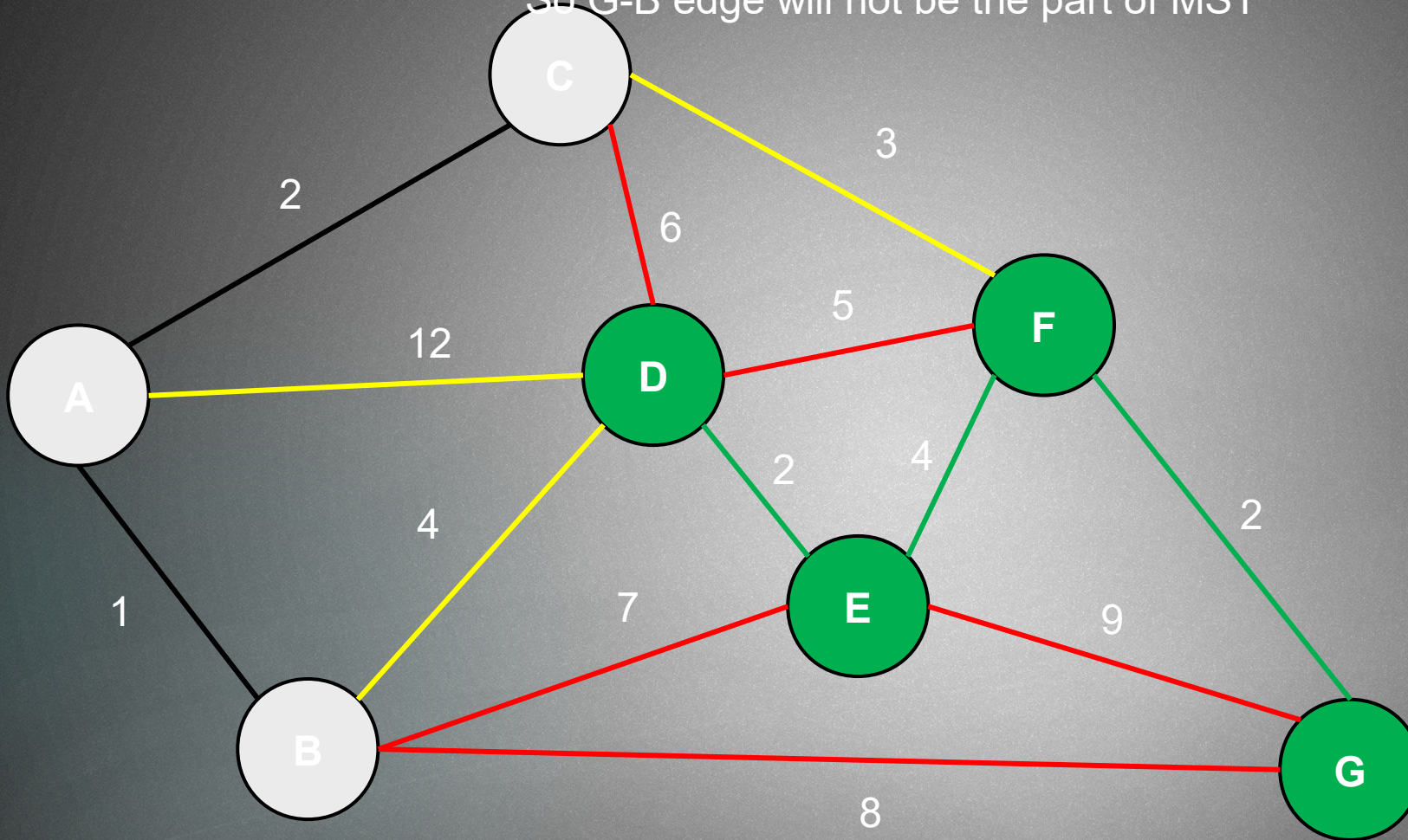


Heap

D-A 12  
D-B 4  
F-C 3



What about the G-B connection? We have considered a path to vertex B so far from vertex D. The cost from vertex D is 4 , the cost from vertex G is 8  
So G-B edge will not be the part of MST

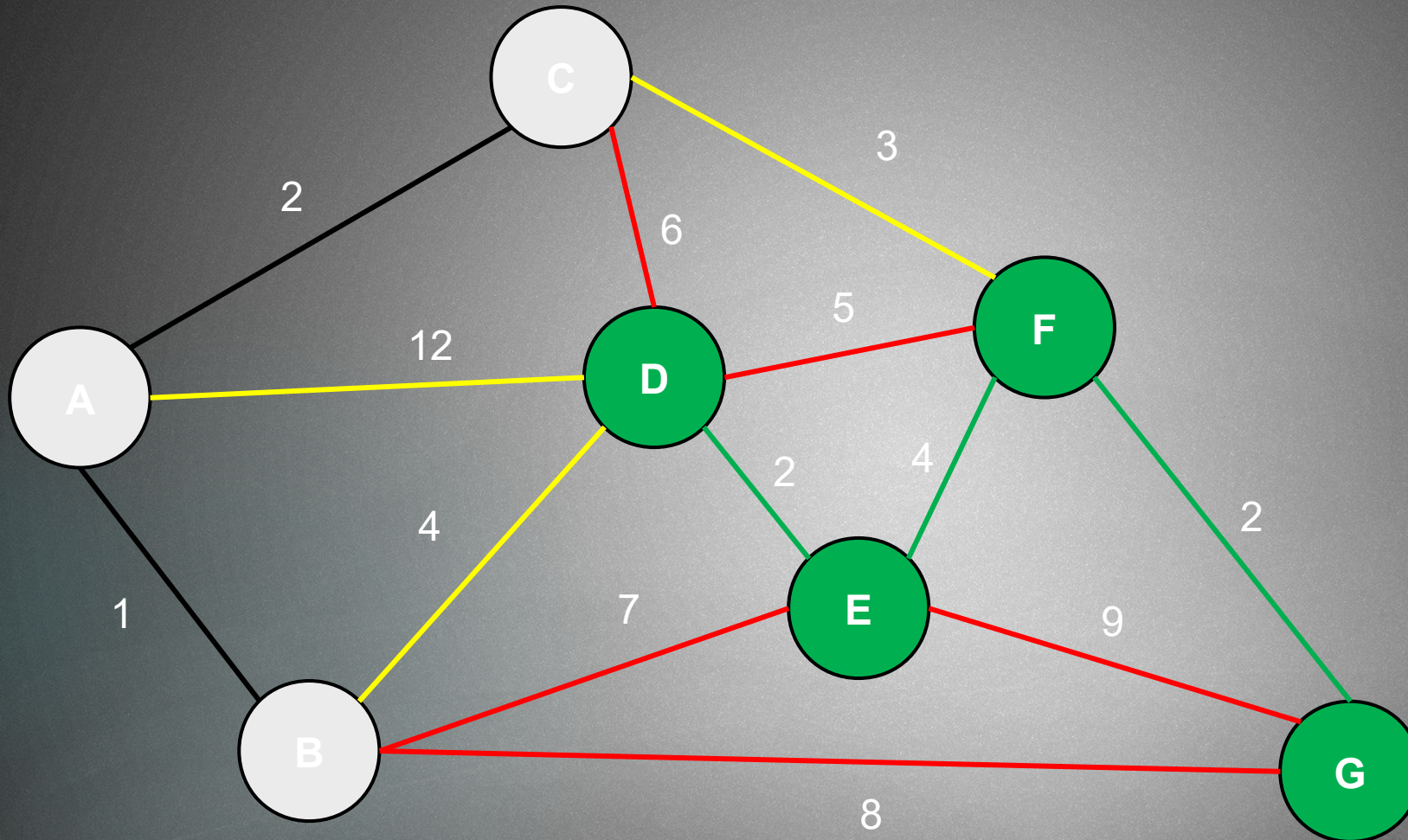


Heap

D-A 12  
D-B 4  
F-C 3



We keep going because not all the vertexes are included in the MST

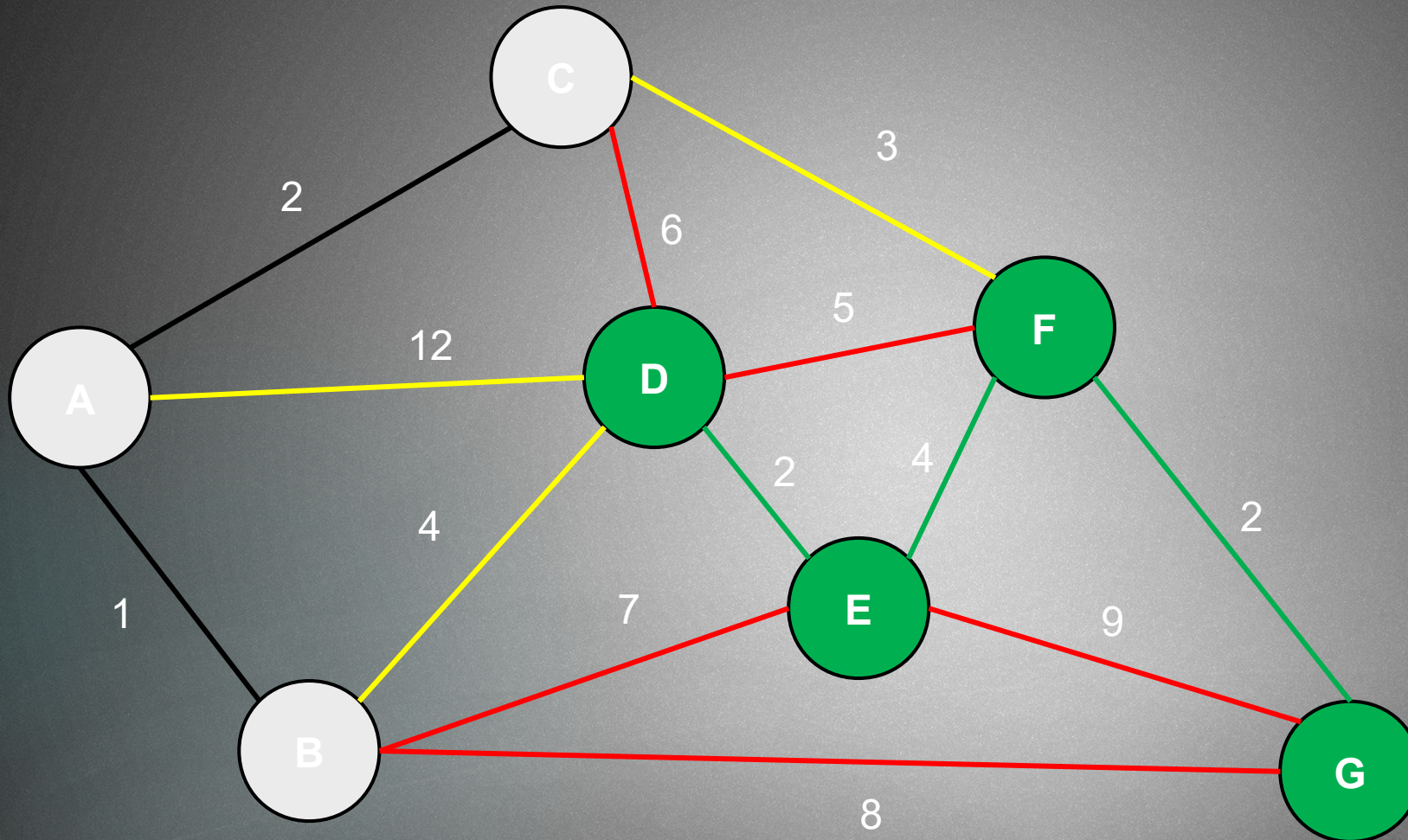


Heap

D-A 12  
D-B 4  
F-C 3



We keep going because not all the vertexes are included in the MST

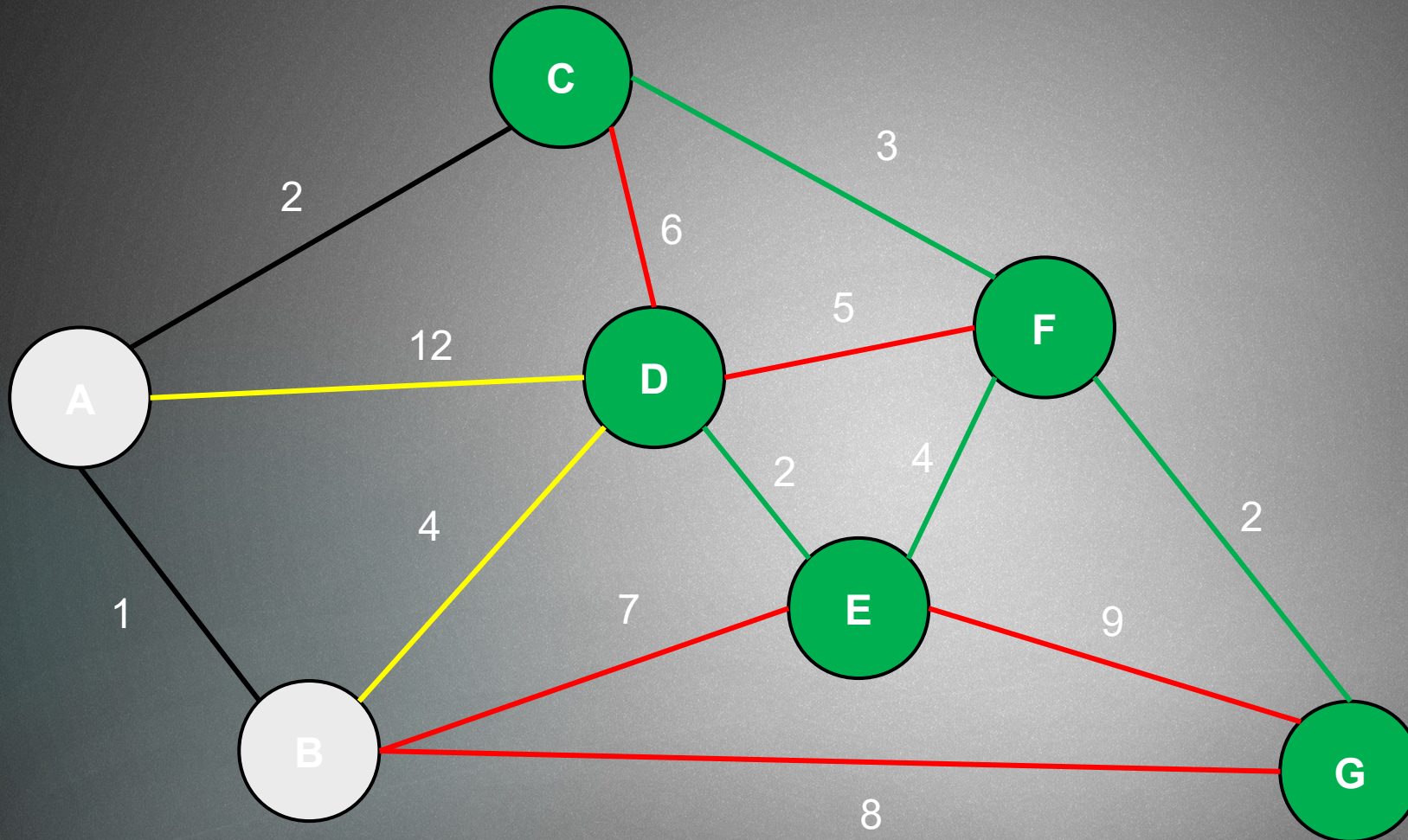


Heap

D-A 12  
D-B 4  
F-C 3



We keep going because not all the vertexes are included in the MST

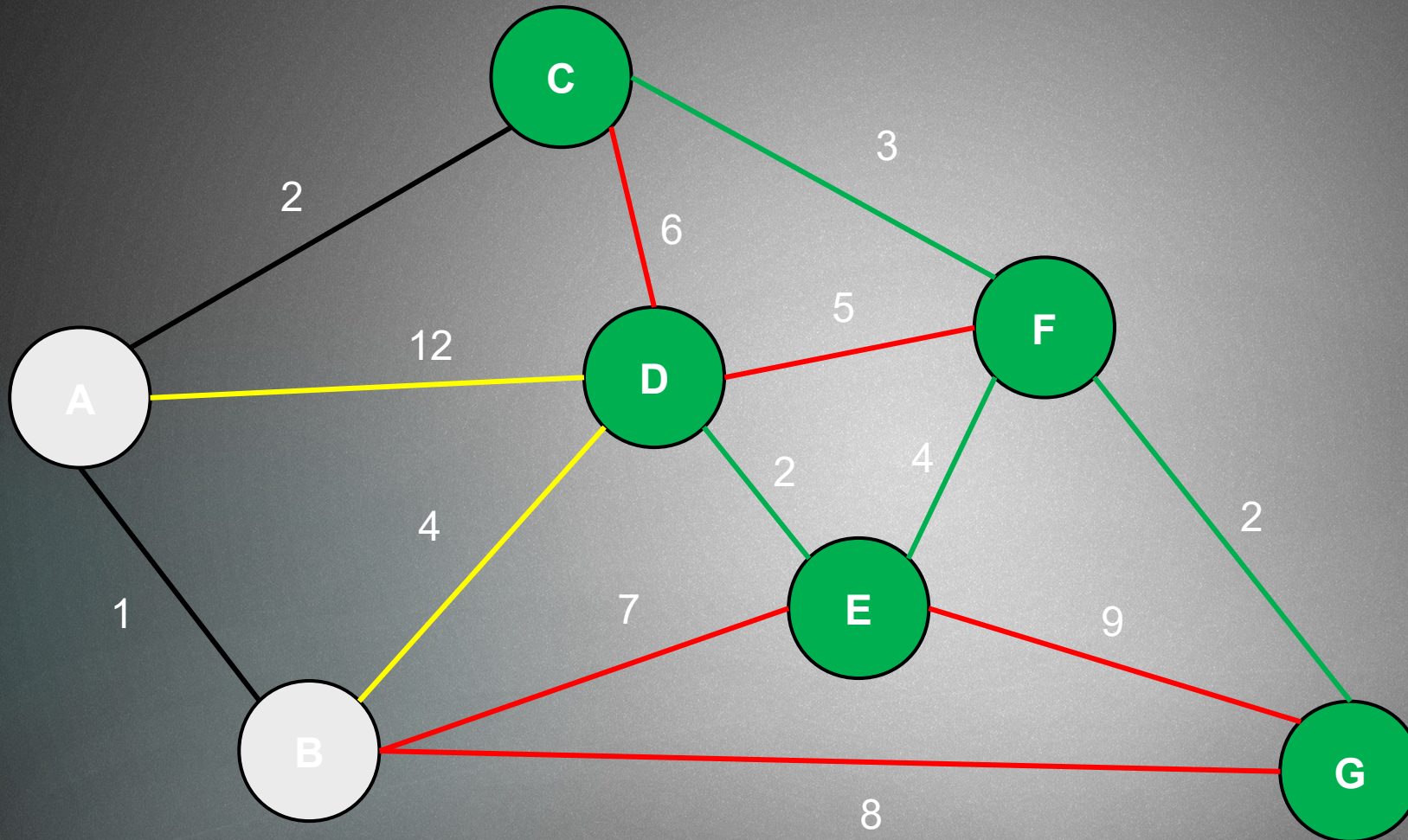


Heap

D-A 12  
D-B 4  
F-C 3



We keep going because not all the vertexes are included in the MST

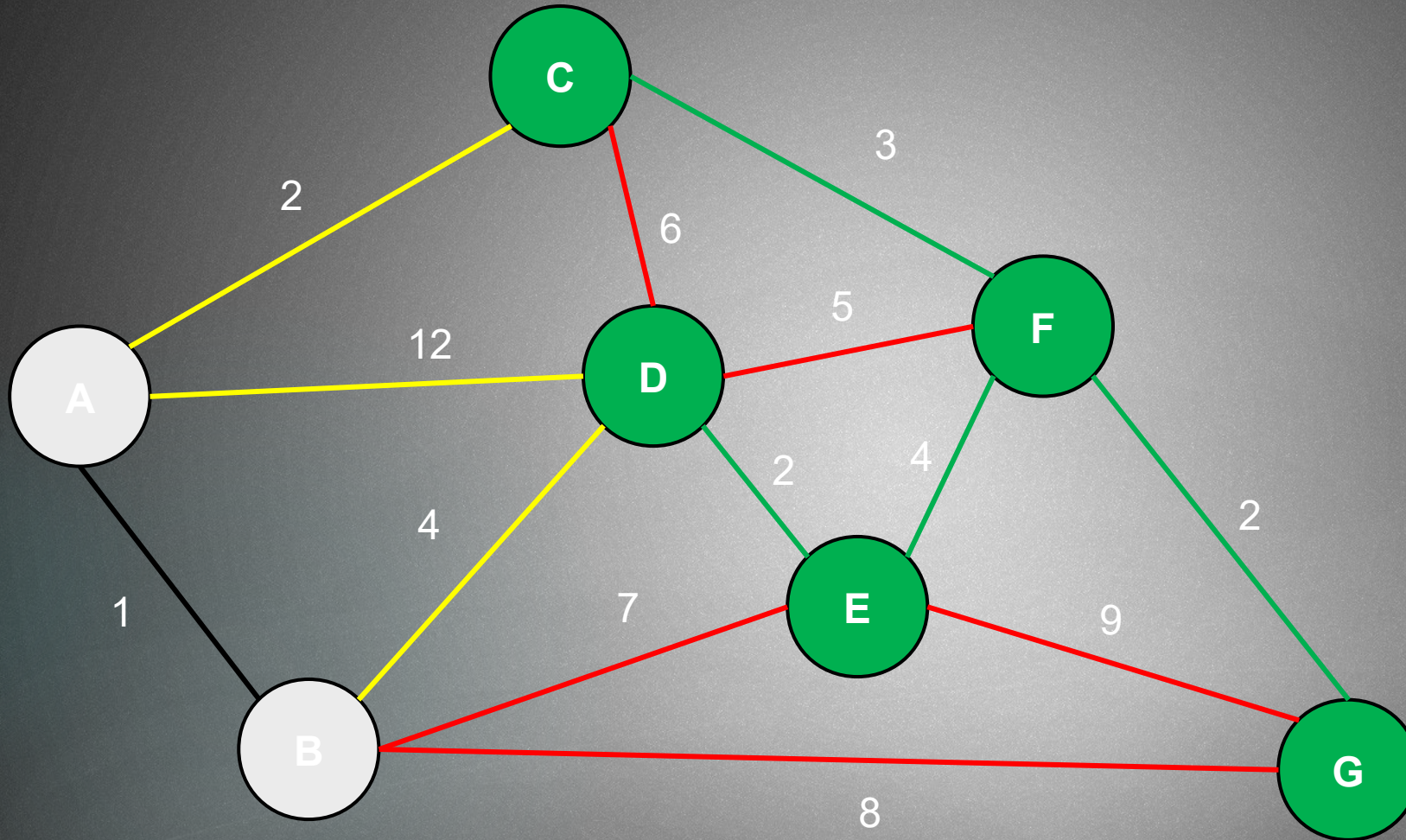


Heap

D-A 12  
D-B 4



We keep going because not all the vertexes are included in the MST

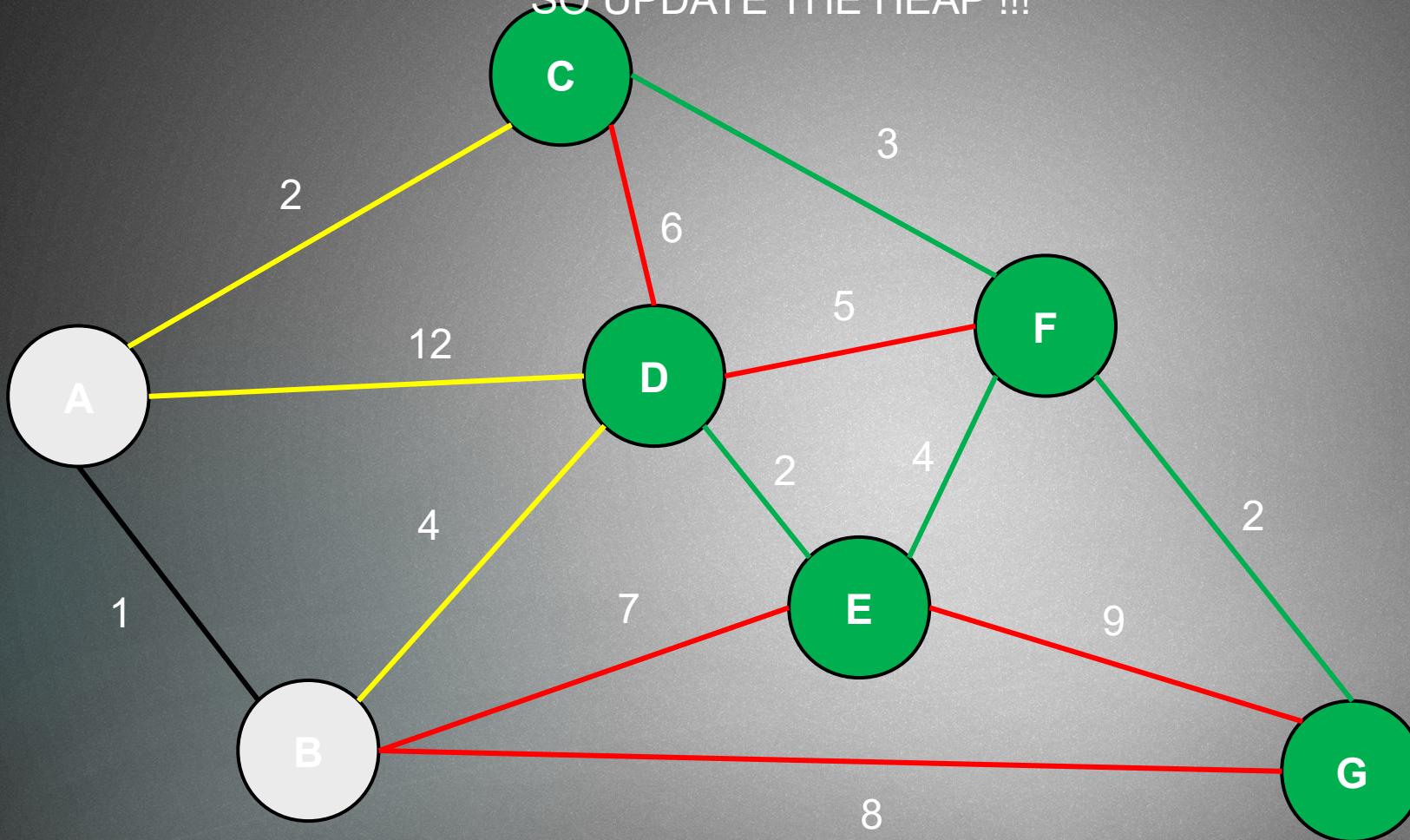


Heap

D-A 12  
D-B 4



What about the C-A connection? We have considered a path to vertex A so far from vertex D. The cost from vertex D is 12, the cost from vertex C is 2  
SO UPDATE THE HEAP !!!

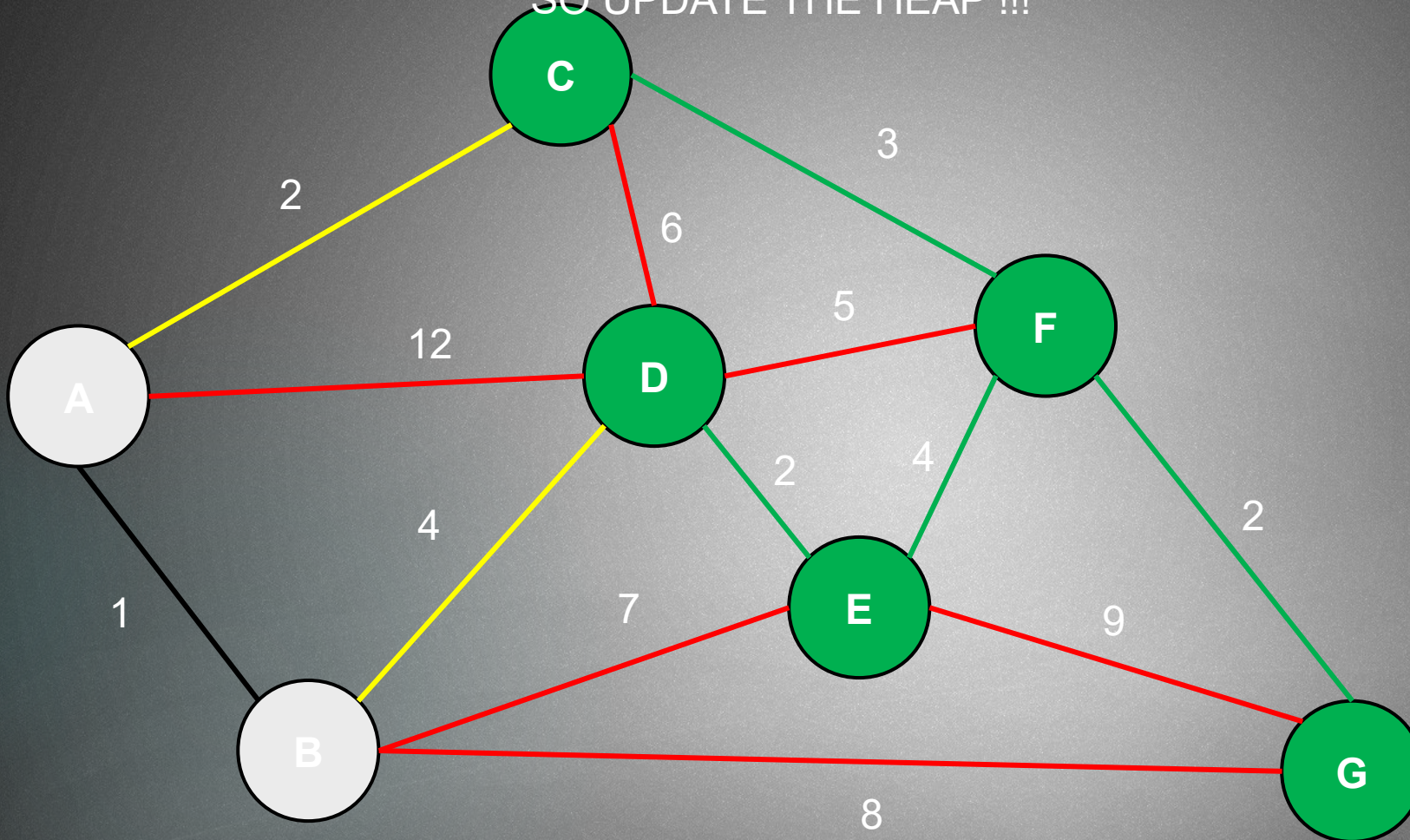


Heap

D-A 12  
D-B 4



What about the C-A connection? We have considered a path to vertex A so far from vertex D. The cost from vertex D is 12, the cost from vertex C is 2  
SO UPDATE THE HEAP !!!

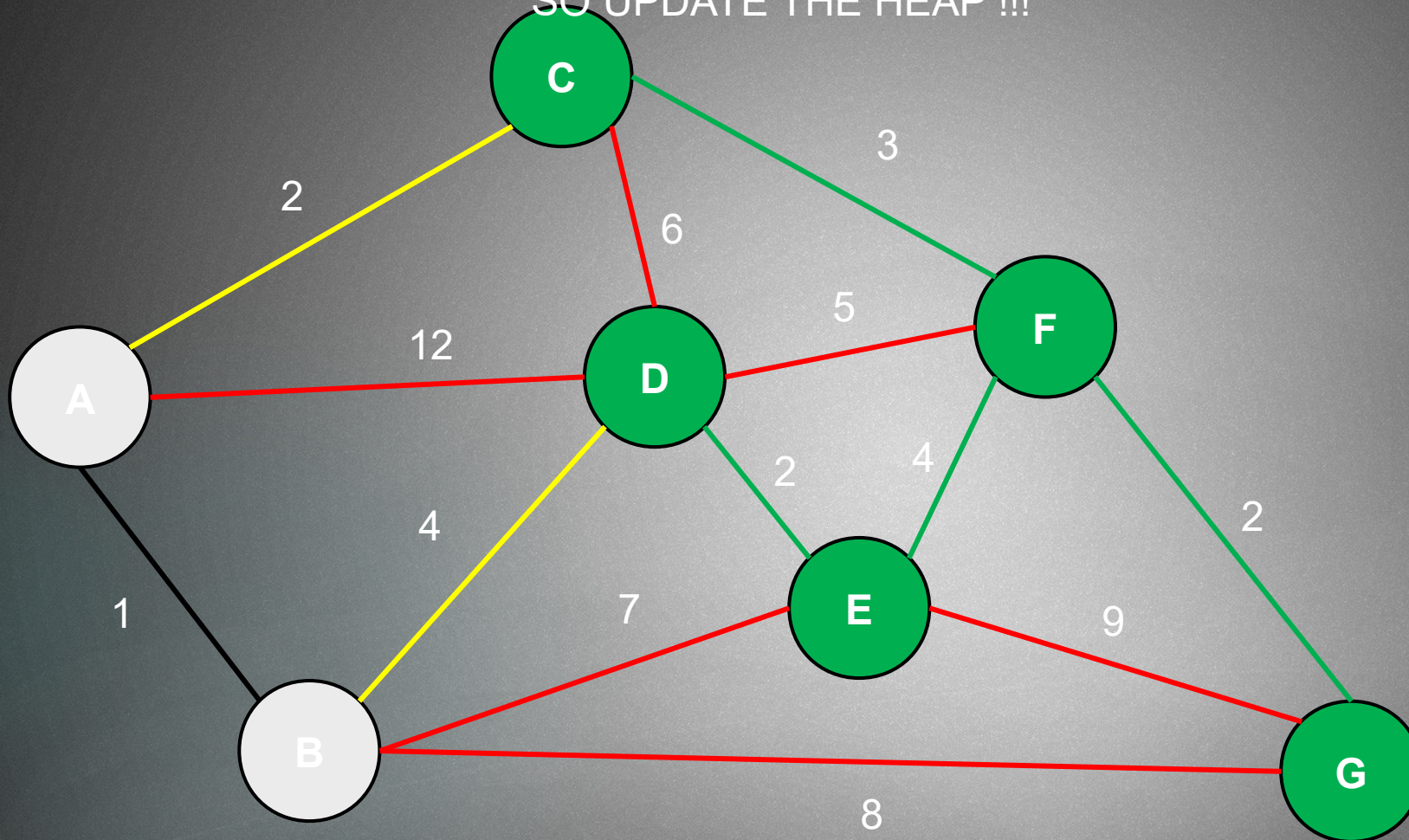


Heap

D-A 12  
D-B 4



What about the C-A connection? We have considered a path to vertex A so far from vertex D. The cost from vertex D is 12, the cost from vertex C is 2  
SO UPDATE THE HEAP !!!

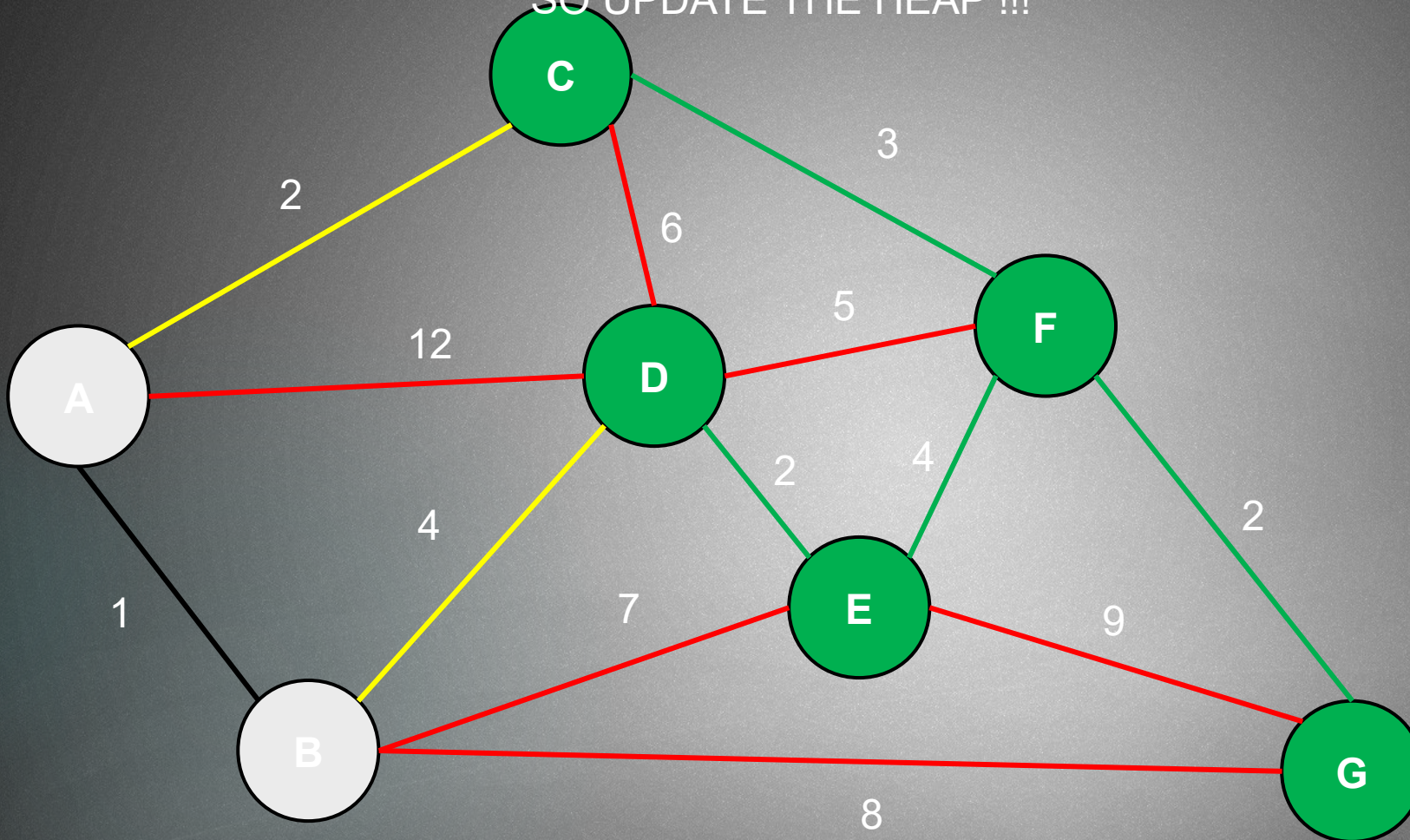


Heap

D-B 4



What about the C-A connection? We have considered a path to vertex A so far from vertex D. The cost from vertex D is 12, the cost from vertex C is 2  
SO UPDATE THE HEAP !!!

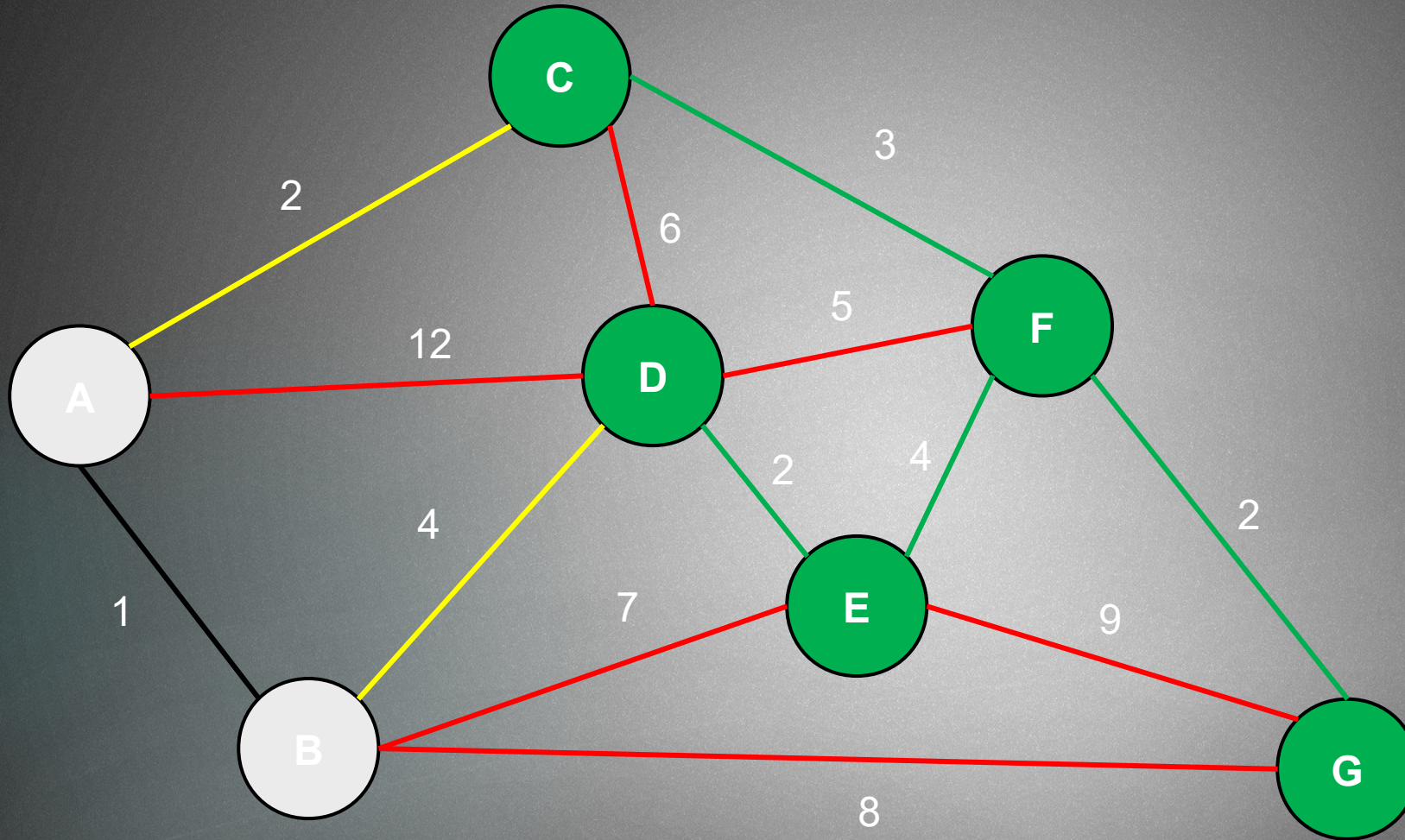


Heap

D-B 4  
C-A 2



Keep going !!!

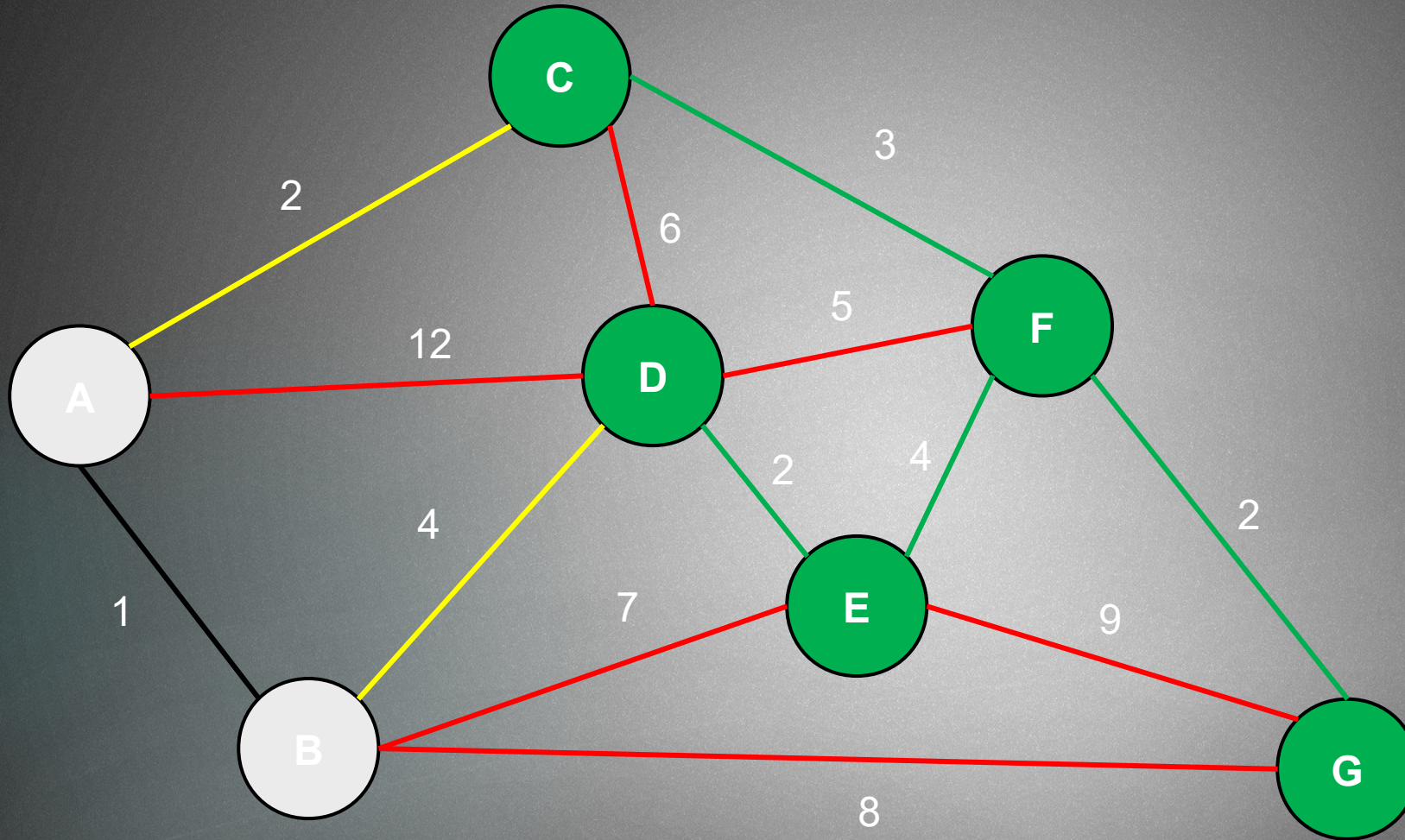


Heap

D-B 4  
C-A 2



Keep going !!!

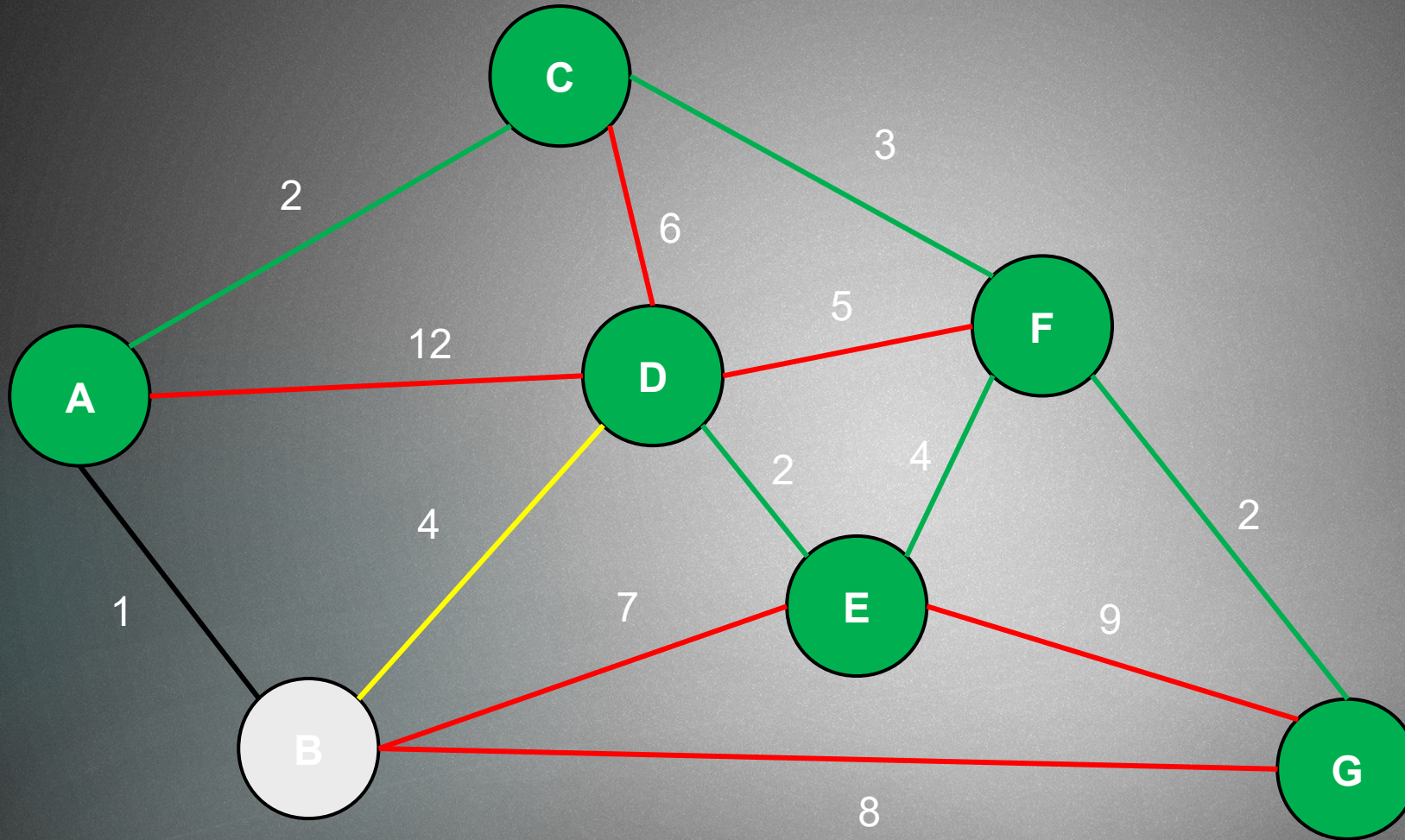


Heap

D-B 4  
C-A 2



Keep going !!!

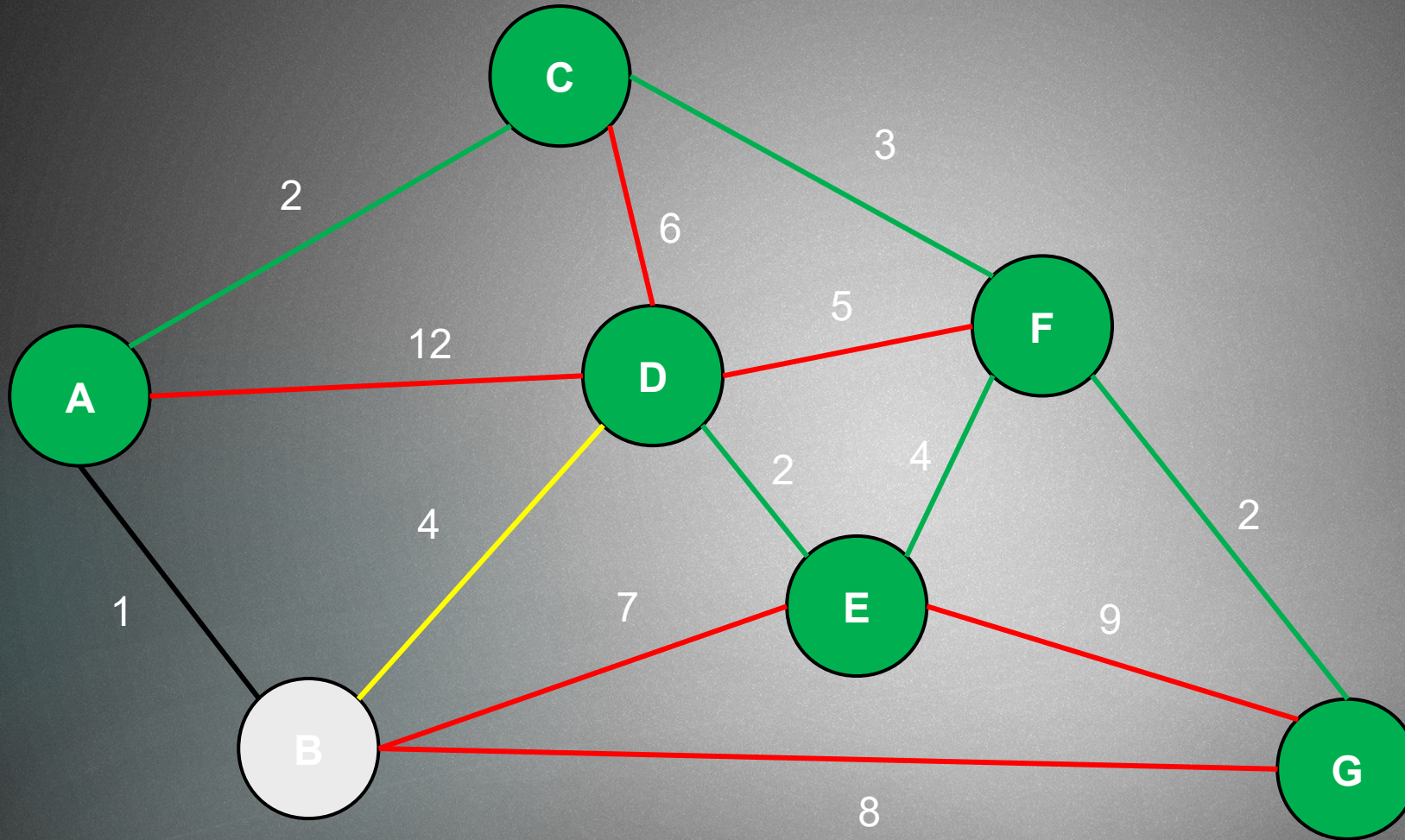


Heap

D-B 4  
C-A 2



Keep going !!!

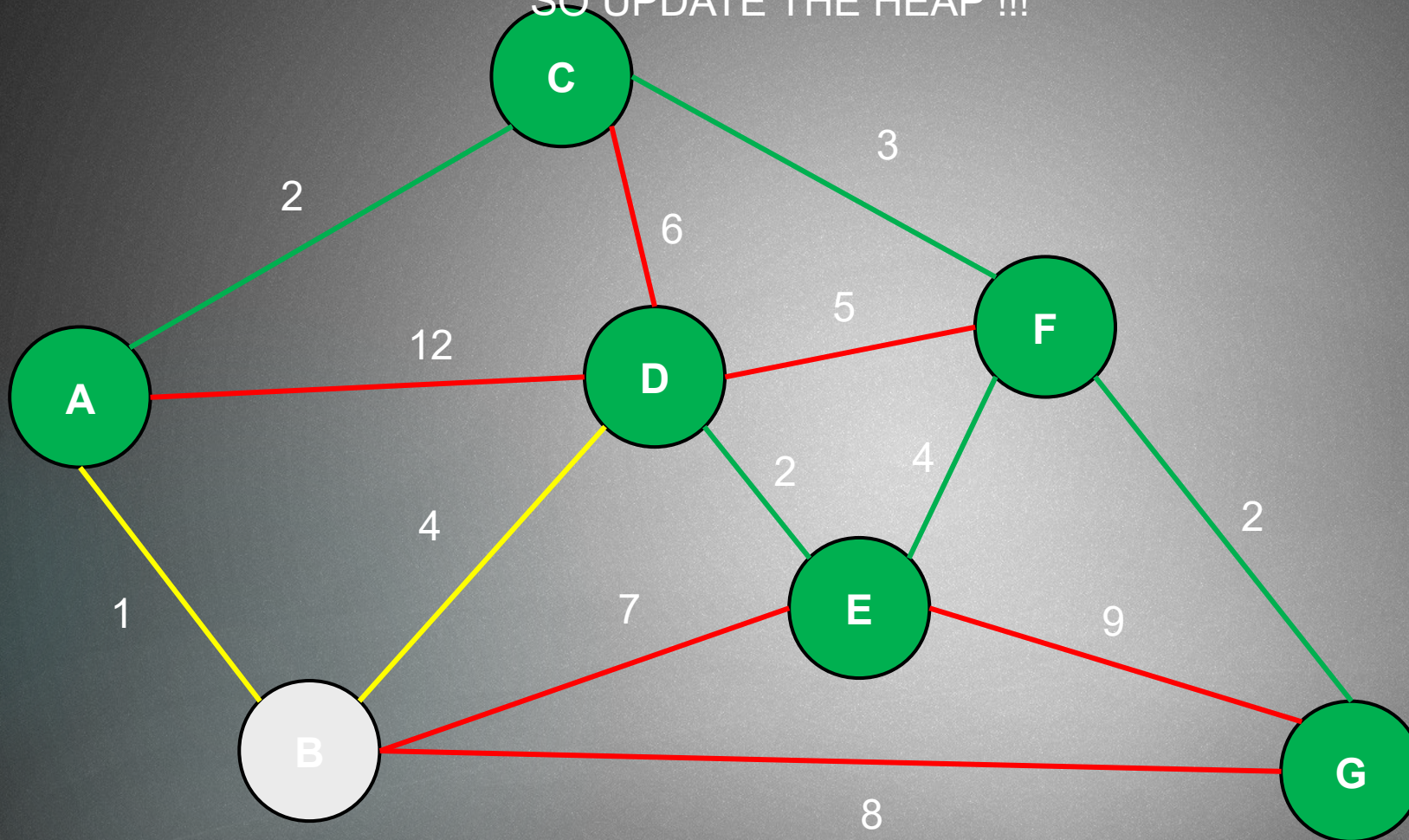


Heap

D-B 4



What about the A-B connection? We have considered a path to vertex B so far from vertex D. The cost from vertex D is 4, the cost from vertex A is 1  
SO UPDATE THE HEAP !!!

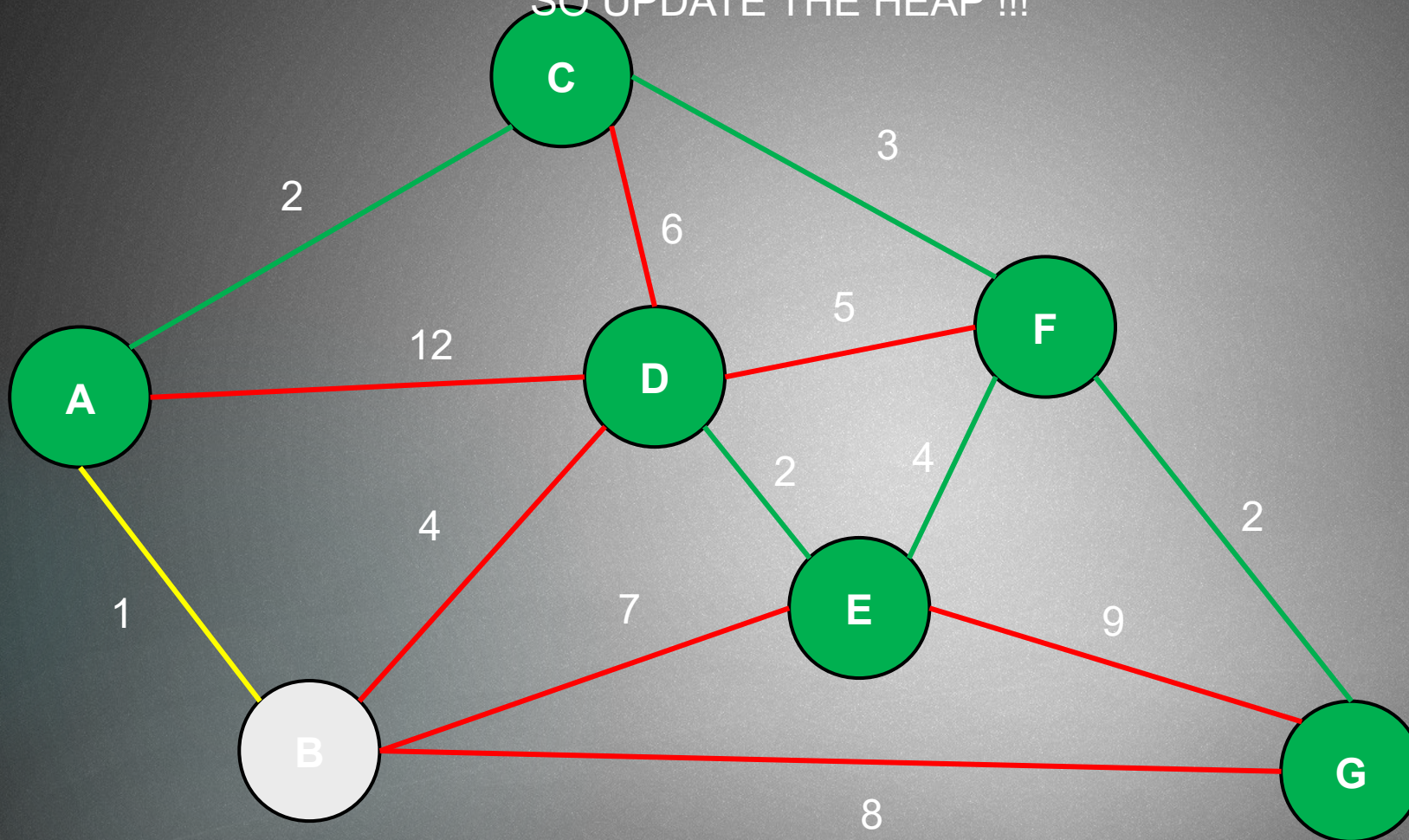


Heap

D-B 4



What about the A-B connection? We have considered a path to vertex B so far from vertex D. The cost from vertex D is 4, the cost from vertex A is 1  
SO UPDATE THE HEAP !!!

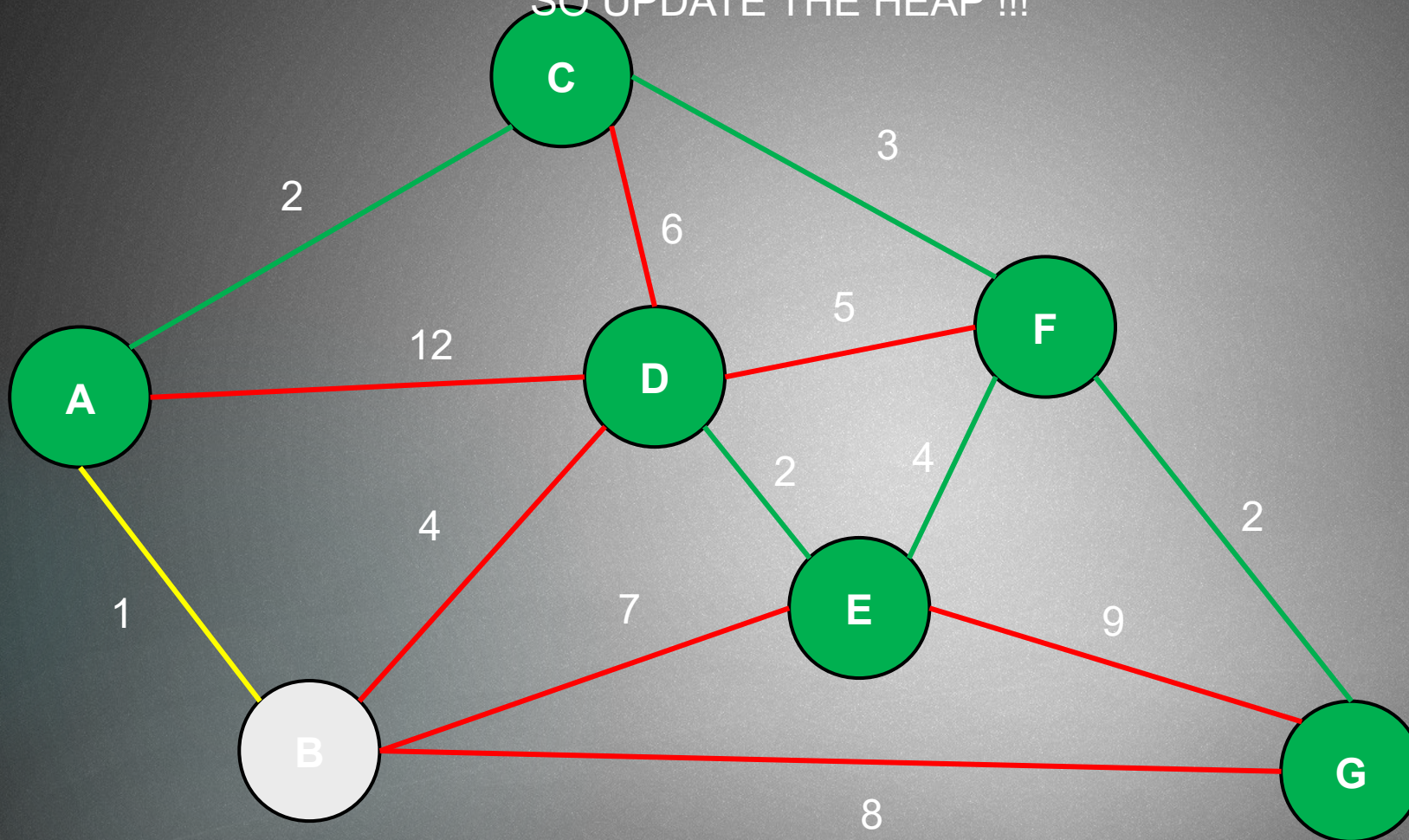


Heap

D-B 4



What about the A-B connection? We have considered a path to vertex B so far from vertex D. The cost from vertex D is 4, the cost from vertex A is 1  
SO UPDATE THE HEAP !!!

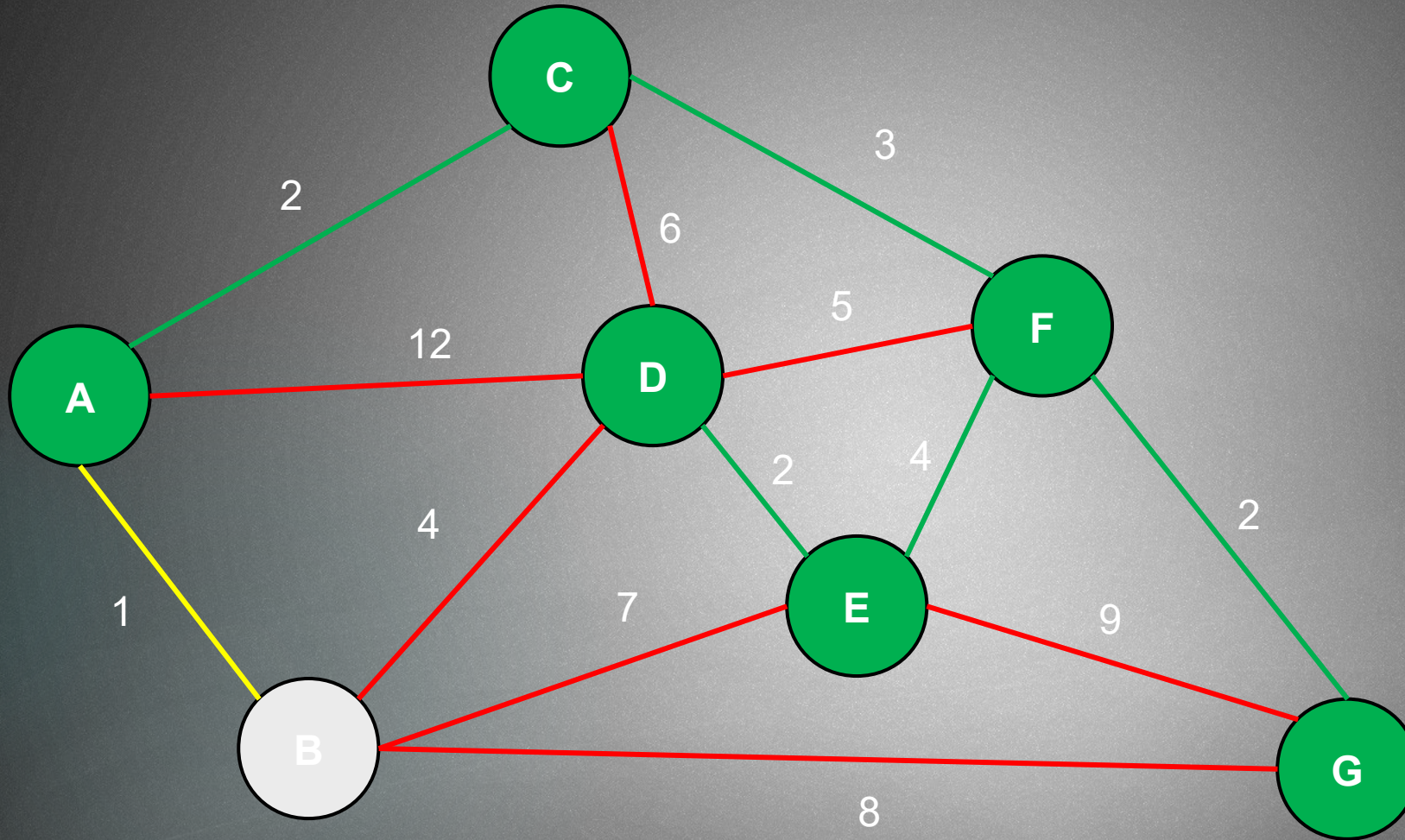


Heap

A-B 1



Keep going !!!

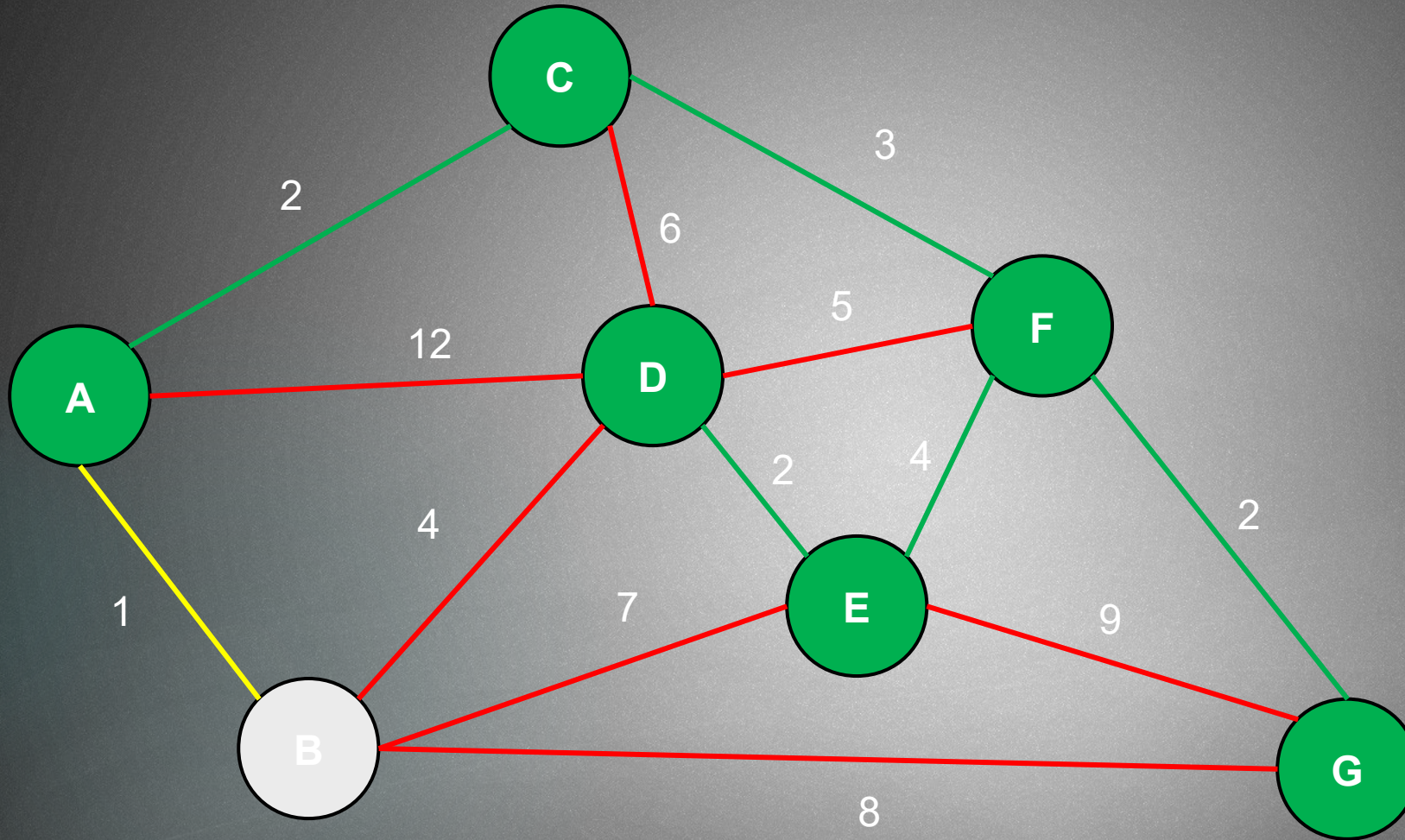


Heap

A-B 1



Keep going !!!

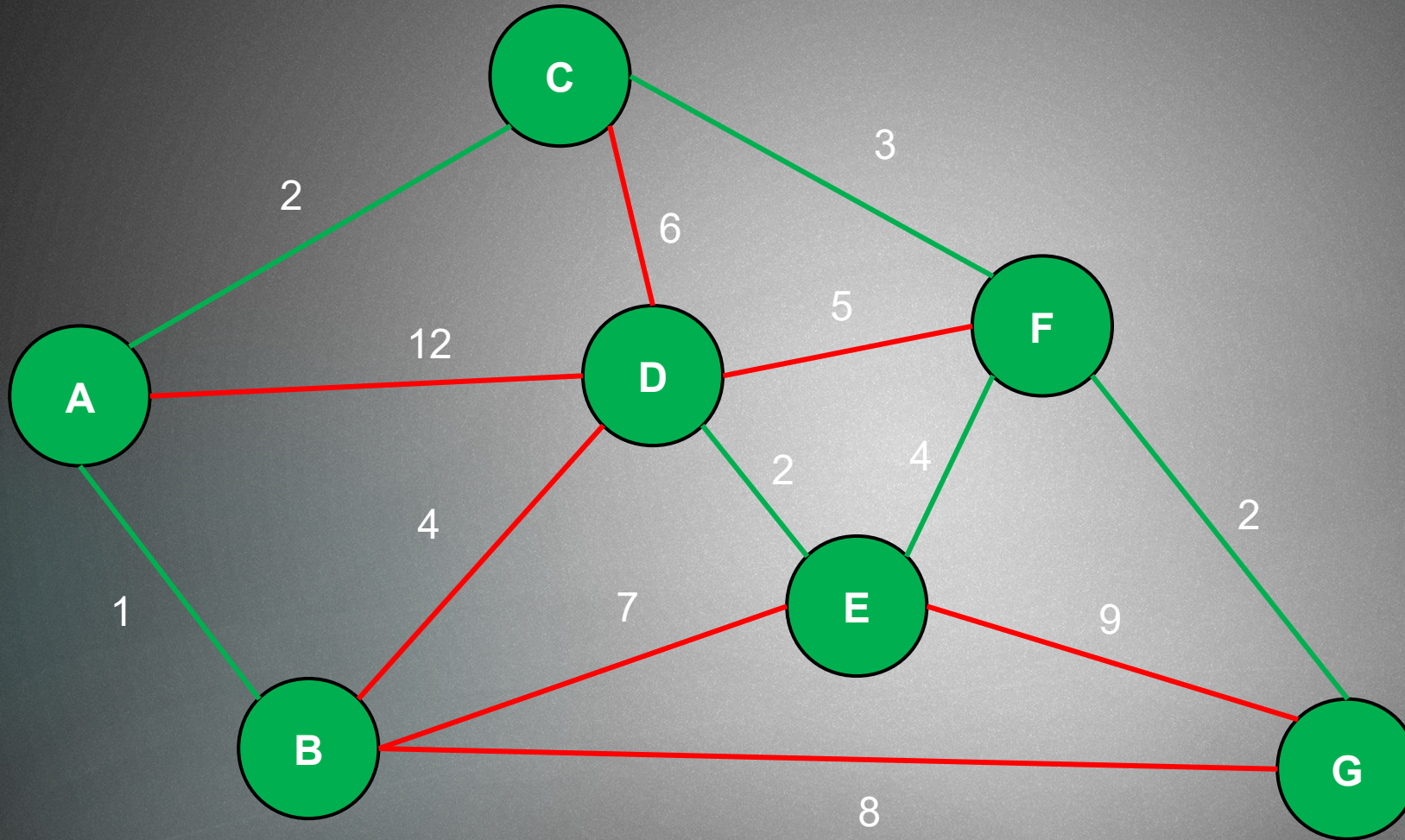


Heap

A-B 1



Keep going !!!

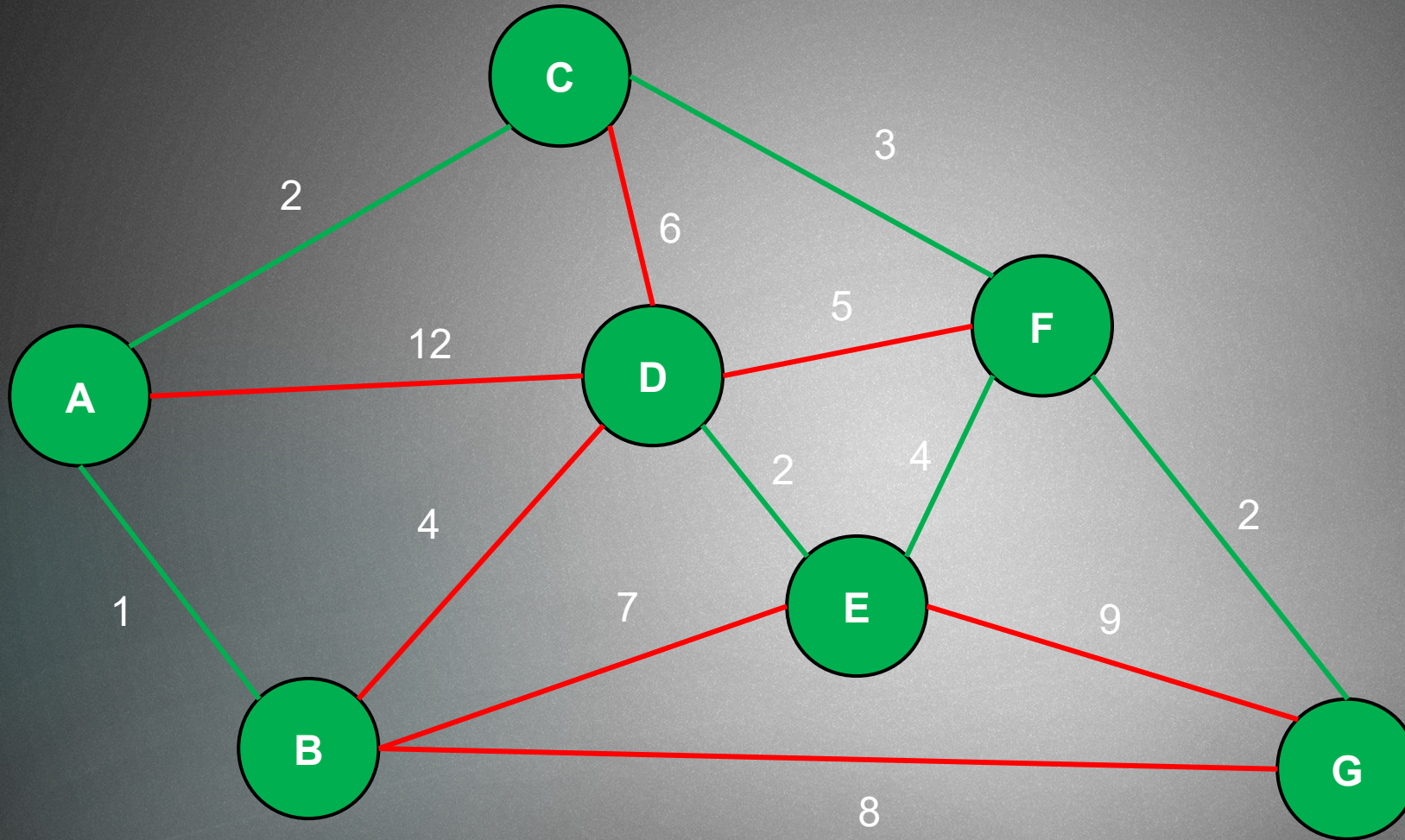


Heap

A-B 1



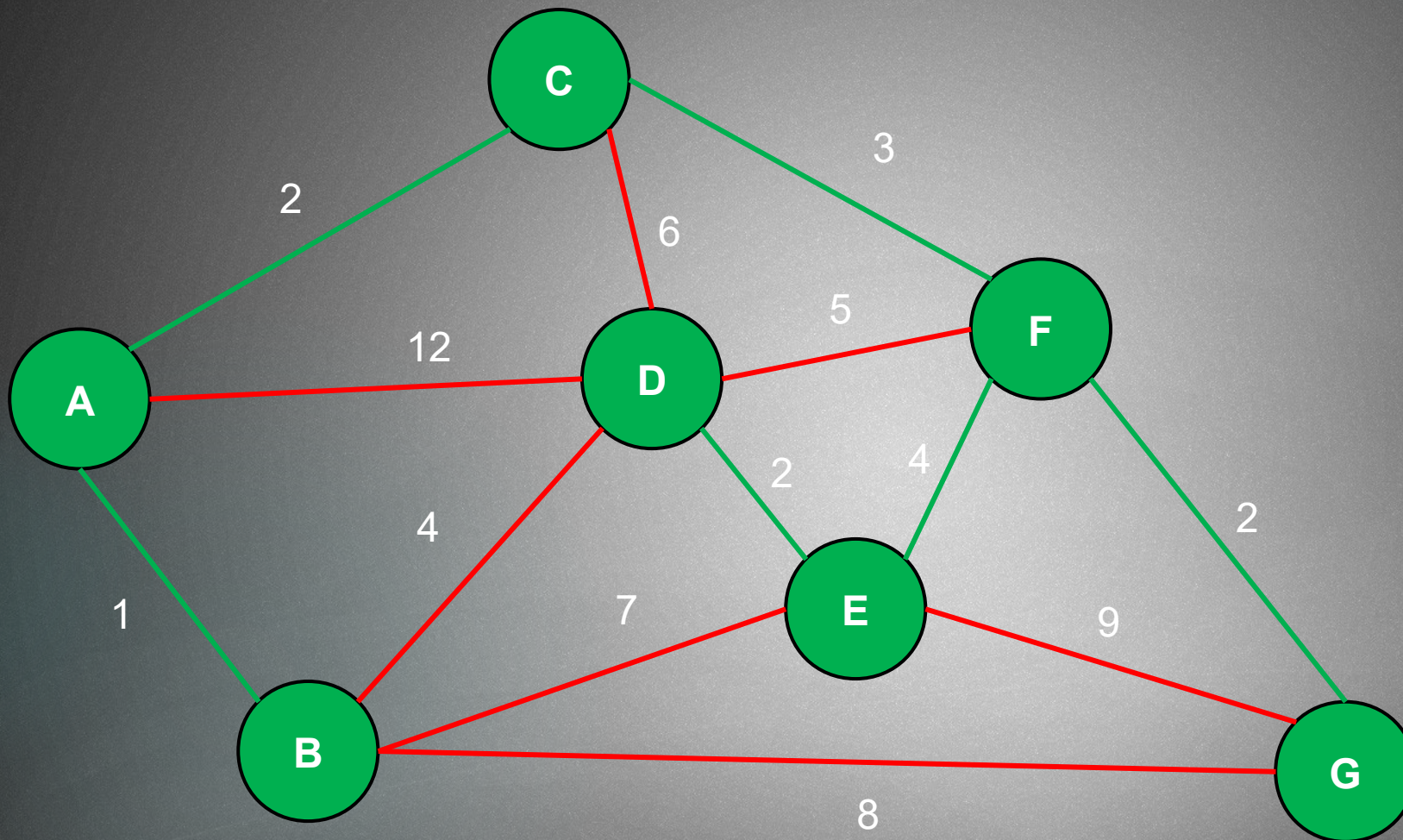
Keep going !!!



Heap



We have managed to include all the vertexes in the MST  
So the algorithm terminates  $\rightarrow$  overall cost = 14



Heap



We have managed to include all the vertexes in the MST  
So the algorithm terminates  $\rightarrow$  overall cost = 14

