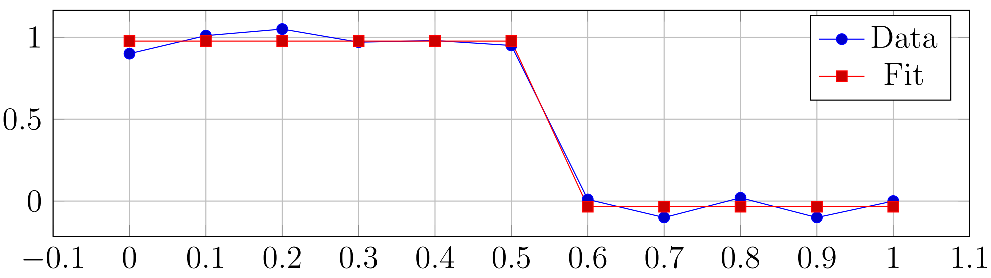

Figure 1: Piecewise constant fit to data in Problem 1.

PS 3

Due: Weds, Feb 11

Note: My solutions this week fit on the front and back of a page (typed), including the graph for problem 1 and problem text. Don't overthink these!

1: 6.4.1 We plot the data and the least squares fit in Figure 1. The break point is clearly between 0.5 and 0.6, though one cannot say more without additional information. In each half, the constant least squares fit is the mean of the data entries; the easiest way to see this is to note that the normal equations for fitting a data vector b to a constant function with value β is

$$e^T e\beta = e^T b$$

where e is the vector of all ones; this gives $\beta = e^T b/e^T e = \sum_j b_j/n$.

2: QR to SVD Suppose $A = QR$ is an economy QR factorization. Show that the singular values of A are the same as those of R.

Answer: If $A = QR$ and $R = U\Sigma V^T$, then $A = (QU)\Sigma V^T$ is an economy SVD of A.

3: Vector projector Suppose $A \in \mathbb{R}^{m\times n}$ where $m > n$ has full column rank. Given A and a vector b, write one line of MATLAB to compute the element c in the range space of A that is nearest to b (in the Euclidean norm).

Answer:

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c = A*(A\b);
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4: Generally speaking Often, we use least squares to construct models of the world. We assume that the "truth" is

$$Ax = b,$$

but what we measure is the first few rows of A and b (which we write as A_1 and b_1), and those measurements are corrupted by noise. Suppose we have A exactly, but only get the noisy partial right hand side $\hat{b}_1 = b_1 + e_1$, from which we form

$$\text{minimize } \|A_1\hat{x} - \hat{b}_1\|^2.$$

Our goal in this problem is to use the error analysis ideas in Section 6.2 to figure out the inherited error in the reconstruction of $\hat{b}_2 = A_2\hat{x}$.

1. Let $e_2 = \hat{b}_2 - b_2$. Argue *briefly* that $e_2 = A_2A_1^\dagger e_1$.
2. Show that
$$\frac{\|e_2\|}{\|b_2\|} \leq \kappa(A_2A_1^\dagger)\frac{\|e_1\|}{\|b_1\|}.$$

Things get somewhat more complicated if we also allow the entries of A to be contaminated by error, though the same basic ingredients come into play.

Answer:

1. Just subtract $b_2 = A_2A_1^\dagger b_1$ from $\hat{b}_2 = A_2A_1^\dagger b_2$.
2. Using the SVD, we have
$$\begin{aligned}\|e_2\| &= \|A_2A_1^\dagger e_1\| \leq \sigma_{\max}(A_2A_1^\dagger)\|e_1\| \\ \|b_2\| &= \|A_2A_1^\dagger b_1\| \geq \sigma_{\min}(A_2A_1^\dagger)\|e_1\|.\end{aligned}$$

   Dividing the two gives the desired result.