Homework due Thursday, November 8, at noon.

(1) Let A and B be two nonempty sets. Let $f : A \to B$ be a bijection. Define $\phi : S_A \to S_B$ by $\phi(\sigma) = f \circ \sigma \circ f^{-1}$. Prove that ϕ is an isomorphism between S_A and S_B.

(2) Let $n \in \mathbb{N}$ be a positive integer. Define

$$R_n = \begin{bmatrix} \cos\frac{2\pi}{n} & -\sin\frac{2\pi}{n} \\ \sin\frac{2\pi}{n} & \cos\frac{2\pi}{n} \end{bmatrix} \text{ and } X = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Also recall that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ denotes the identity matrix. Let

$$G_n = \{I, R, R^2, \ldots, R^{n-1}, X, RX, R^2X, \ldots, R^{n-1}X\}.$$

(a) (Bonus problem) Prove that G_n is a group.

(b) Let $n = 4$. Prove that G_4 is isomorphic to D_4 (the dihedral group of order 8). (Hint: Observe that R_4 is a rotation with angle $\pi/2$)

(3) Exercise 8 page 83: 2, 8, 12, 21, 47, 49

(4) Exercise 9 page 94: 2, 9, 13, 34