Homework due Thursday, November 29, at noon.

(1) Let G be a group and let $K \leq H \leq G$. Assume $(G : K)$ is finite. Prove that $(G : H)$ and $(H : K)$ are finite.

(2) Let G be a group and let H and K be two subgroups of G. Assume $(G : K)$ is finite. Prove that $(H : H \cap K)$ is finite.

(3) (Bonus Problem) Let G be a group and let H and K be two subgroups of G. Assume both $(G : H)$ and $(G : K)$ are finite. Prove that $(G : H \cap K)$ is finite.

(4) Let G be a group and let $H \leq G$ be a subgroup. Recall from HW7, problem 6 that $N_G(H) = \{g \in G | gHg^{-1} = H\}$. Assume $(G : H)$ is finite. Prove that there are finitely many elements $g_1, \ldots, g_n$ in G so that
   (a) $g_i H g_i^{-1} \neq g_j H g_j^{-1}$ for all $1 \leq i \neq j \leq n$, and
   (b) for every $g \in G$ there exists some $1 \leq i \leq n$ with $gHg^{-1} = g_i H g_i^{-1}$.

(5) Exercise 11 page 110: 1, 7, 13, 26, 29, 47, 50, 52, 53