

# Math 184A Homework 2

Fall 2015

This homework is due Monday October 12th in discussion section. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L<sup>A</sup>T<sub>E</sub>X is recommended though not required.

**Question 1** (Equal Sum Subsets, 20 points). *Let  $S$  be a set of 10 positive integers each at most 100. Show that there exist two different subsets  $A \subseteq S$  and  $B \subseteq S$  so that the sum of the elements of  $A$  equals the sum of the elements of  $B$ .*

**Question 2** (Counting Poker Hands, 80 points). *Recall that a standard deck of cards has cards with 13 different ranks 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A and 4 different suits  $\clubsuit, \diamondsuit, \heartsuit, \spadesuit$  for a total of  $4 \times 13 = 52$  total cards. We say that a  $k$ -card hand is an (unordered) collection of  $k$  cards from this deck. How many hands are there of each of the following types?*

*For each part you should justify your answer in addition to giving a number. The number is allowed to be given in terms of standard operations and binomial coefficients (for example, if you got an answer of  $\binom{13}{6} \cdot 10 - 4^7$ , you may leave it like that rather than computing the actual value of this number). Each part is worth 10 points.*

- (a) *How many hands are there with at most 5 cards?*
- (b) *How many 5 card hands have no two cards of the same rank?*
- (c) *How many 5 card hands are a flush, that is have all cards of the same suit?*
- (d) *How many 5 card hands are a full house, that is have three cards of one rank and two cards of another rank?*
- (e) *How many 5 card hands are a straight, that is have all five cards with five consecutive ranks (for this purpose consider the ranks to be ordered in a 13-long sequence)?*
- (f) *How many 6 card hands have 3 pairs, that is have three different ranks with two cards in the hand of each rank?*
- (g) *How many 4 card hands are a flash, that is have four cards of different suits?*
- (h) *How many 4 card hands are a 2 pair, flash, that is have four cards of different suits and have two ranks each with two cards?*

**Question 3** (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*