Math 184A Homework 4

Fall 2015

This homework is due Monday November 2nd in discussion section. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeX is recommend though not required.
Optional Practice Problems: (do not turn in) Chapter 6 problems 2, 5, 7, 21, 25

Question 1 (Permutation Parity, 40 points). .

(a) *Show that for any $n > 1$ that the number of permutations of $[n]$ with an even number of cycles is equal to the number of permutations of $[n]$ with an odd number of cycles using identities relating to $c(n,k)$.[20 points]*

(b) *Find a bijection between the permutations of $[n]$ with an even number of cycles and those with an odd number of cycles. [Hint: If 1 and 2 are in different cycles, merge the cycles together, if they are in the same cycle, split the cycle in two.][20 points]*

Question 2 (Cycles and Powers, 20 points). *Let π be a permutation of $[n]$ consisting of only one cycle of length n. Let $r > 0$ be an integer. In terms of r and n, describe the cycle structure of π^r.*

Question 3 (Stirling Number Lower Bound, 30 points). .

(a) *Show that the number of permutations of $[n]$ with k cycles so that each of $1, 2, 3, \ldots, k$ is in a different cycle is $\frac{(n-1)!}{(k-1)!}$. Use this to show that*

$$c(n,k) \geq \frac{(n-1)!}{(k-1)!}.$$

[15 points]

(b) *Show this formula directly using the relation*

$$\sum_{k=1}^{n} c(n,k)x^k = x(x+1)\cdots(x+n-1).$$

[15 points]

Question 4 (Triplet Permutation, 10 points). *How many permutations of $[3n]$ have only cycles of length 3?*

Question 5 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*