Math 184A Final Exam

Fall 2015

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. You are free to make use of any result in the textbook or proved in class. You may use up to 12 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat indicated below.

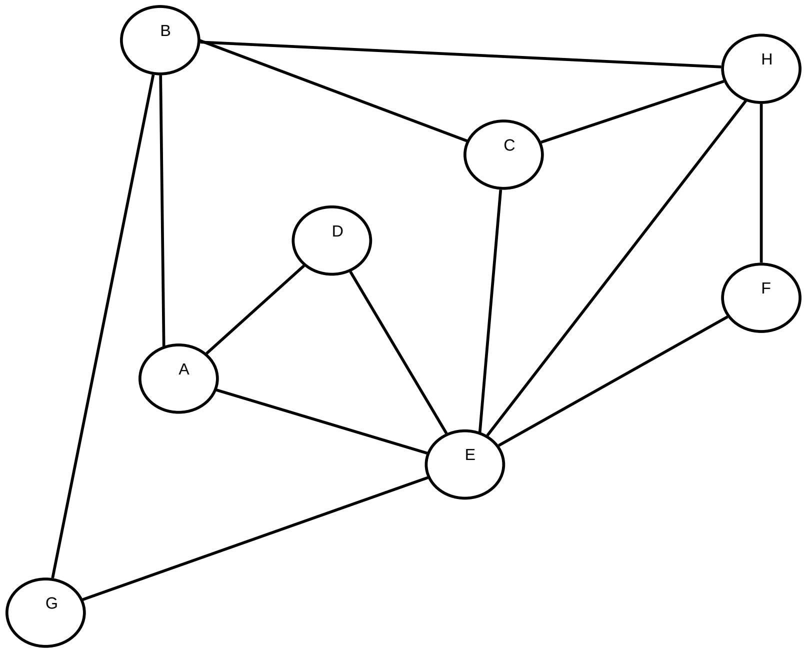
Name:

ID Number:

Seat:

Problem	1	2	3	4	5	6	Total
Score							

Question 1 (Eulerian Trail, 15 points). *Either exhibit an Eulerian trail (not necessarily closed) in the graph below or show that one does not exist:*



Question 2 (Restricted Permutation Counting, 15 points). *How many permutations* $\pi : [8] \to [8]$ *are there so that* $\pi(1), \pi(2), \pi(3)$ *are all at most* 5*?*

Question 3 (Stirling Number Computation, 15 points)**.** *Compute the unsigned Stirling number of the first kind* $c(6,3)$.

Question 4 (Near Integer Multiples, 15 points)**.** *Let x and y be real numbers and let n be an integer. Show that there is an integer m with $1 \leq m \leq n^2$ so that both mx and my are within $1/n$ of an integer.*

Hint: Find m, m' so that the fractional parts of mx and $m'x$ are close as are the fractional parts of my and $m'y$.

Question 5 (Generating Function Computation, 20 points). *(a) Let* a_n*,* $n \geq 0$ *be the sequence given by* $a_0 = 0$*,* $a_1 = 0$ *and* $a_{n+2} = a_{n+1} - a_n + 1$*. Find the ordinary generating function for this sequence.*

(b) Let a_n *be the number of* n*-letter strings using the* 26 *letters of the English alphabet so that each letter appears an even number of times. Find the exponential generating function for this sequence.*

Question 6 (Set Coloring, 20 points). *Let S be a finite set. Let $S_1, S_2, \dots, S_m$ be subsets of S. Let $p(n)$ be the number of ways that the points of S can be colored with n colors so that for no S_i are all the elements of S_i colored with the same color. Show that $p(n)$ is a polynomial in n.*