Math 184A Exam 1

Fall 2018

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please be sure to sit in the seat indicated below for the exam.

Name:

ID Number:

Discussion Section:

Seat:

Problem	1	2	3	Total
Score				

Question 1 (Conjugate Partitions, 30 points)**.** *What is the conjugate of the partition* $5+4+4+1$ *of* 14*?*

Question 2 (Semi-Sorted Permutations, 35 points)**.** *How many permutations* $\pi : \{1, 2, \ldots, 2n\} \to \{1, 2, \ldots, 2n\}$ *have* $\pi(1) < \pi(2), \pi(3) < \pi(4), \ldots, \pi(2n-1) < \pi(2n)$*? Justify your answer.*

Question 3 (Pile Splitting Game, 35 points)**.** *Locke and Sabetha are playing a game. The game starts with a single pile of N stones and the players take turns (starting with Locke) splitting every pile with more than one stone into two smaller piles. A player loses the game if at the start of their turn all piles have size 1.*

So for example, if $N = 5$, Locke might divide it into piles of size 2 and 3. Sabetha on her turn could split the pile of size 2 into two of size 1 and the pile of size 3 into one of size 2 and one of size 1. Locke would then split the pile of size 2 into two of size 1 (leaving the singleton piles alone), and Sabetha would lose on her next turn.

Show that Sabetha has a winning strategy in this game if and only if N is one less than a power of 2. [Hint: Show by induction that if the largest pile has size n that the player on move has a winning strategy unless $n + 1$ is a power of 2, in which case their opponent does.]