MATH 109 – PRACTICE PROBLEMS

(1) Prove that for all sets X and Y we have
$$X \cup Y = (X \setminus Y) \cup (Y \setminus X) \cup (X \cap Y).$$

(2) Bring the following logical proposition into a form where the quantifiers are in initial position:
$$\neg\, (\exists x \in X : \forall y \in Y : x = y \vee \forall p \in P(x, y) : Q(p))$$
Write down its negation.

(3) Bring the following logical proposition into a form where the quantifiers are in initial position:
$$\forall z \in C : \exists t \in U : (\forall q \in G(z, t) : W(q)) \wedge (\exists h \in H : J(t, h))$$
Write down its negation.

(4) How many even square numbers x satisfy $1 \leq x \leq 1000$?

(5) How many squares of prime numbers lie between 1 and 1000?

(6) Let x and y be irrational numbers. Show that $x + y$ is irrational or $x - y$ is irrational.

(7) Bring the following logical proposition into conjugate normal form:
$$(Z \wedge \neg X \wedge Y) \vee (X \wedge \neg Y) \vee Z$$

(8) Bring the following logical proposition into conjugate normal form:
$$(U \vee (V \wedge \neg W)) \wedge \neg\, (\neg V \wedge \neg W)$$

(9) How choices for positive integers x and y do we have to solve the equation
$$x + y = 10, \quad 3 < x < 100, \quad 5 < y < 50?$$

(10) Give three sets A, B, C such that the intersections $A \cap B$, $B \cap C$, $A \cap C$ contain exactly two elements each.

(11) Is the following set a power set of some other set?
$$A = \{\{1\}, \{2\}, \{1, 2\}\}?$$

(12) Does there exist a set that does not contain the empty set as a subset?