Math 184A Homework 7

Spring 2018

This homework is due on gradescope by Friday June 8th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Avoidance Bounds, 20 points)**.** *From the book we know that* $S_n(1432) \leq 9^n$*. Find a constant* C *so that* $S_n(321456987) \leq C^n$ *for all* n*.*

Question 2 (Hill Avoidance, 40 points)**.** *Let a* k*-hill in a permutation be a subsequence of* $2k-1$ *of the entries the first* k *of which are in increasing order and the last* k *of which are in decreasing order. Note that a* k*-hill is not a single pattern. For example, a* 2*-hill is either an instance of the pattern* 132 *or an instance of the pattern* 231*.*

(a) Show that the number of permutations of $[n]$ *with no* 2*-hill is* 2^{n-1}*. [15 points]*

(b) Show that the number of permutations of $[n]$ *with no* k*-hill is at most* $(4(k-1)^2)^n$*. [Hint: try to find a decreasing sequence among elements that are the largest of a* k*-term increasing subsequence.] [25 points]*

Question 3 (Marriage Lemma, 40 points)**.** *The Marriage Lemma states that if you are given two sets* S *and* T *of size* n *and a set* E *of pairs of one element of each set, then there is a matching between* S *and* T *(namely a set of* n *pairs from* E *using each element of* S *and each element of* T *exactly once) unless there is some subset* $S' \subset S$ *so that the total number of elements of* T *that pair with some element of* S' *is less than* $|S'|$*.*

Prove the Marriage Lemma using Dilworth's Theorem.

Question 4 (Extra credit, 1 point)**.** *Free point!*