

## PRACTICE EXAM

- (1) List all the elements in the group  $D_4$  as permutations in  $S_4$ .
- (2) Let  $\sigma = (1, 2, 3, 4, 5, 6) \in S_7$ .
  - (a) Compute  $\sigma^{103}$ .
  - (b) Write  $\sigma^{14}$  as a product of disjoint cycles.
- (3) Let  $H = \{\sigma \in S_5 \mid \sigma(3) = 3\}$ . Show that  $H$  is a subgroup of  $S_5$ ; moreover, show that  $H$  is isomorphic to  $S_4$ .
- (4) Let  $G$  be a group and let  $H \leq G$  be a subgroup of  $G$ .
  - (a) Suppose  $(G : H) = 2$ . Prove  $gH = Hg$  for every  $g \in G$ .
  - (b) Prove or disprove by an example: suppose  $(G : H) = 3$ , then every right coset is a left coset.
- (5) Let  $G$  be a group. Show that  $G$  is isomorphic to a subgroup of  $S_G$ .
- (6) Let  $K \leq H \leq G$  and assume that  $(G : K)$  is finite. Prove that  $(G : H)$  and  $(H : K)$  are finite.