

Math 184A Homework 5

Fall 2015

This homework is due Monday November 9th in discussion section. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L^AT_EX is recommended though not required. **Optional Practice Problems:** (do not turn in) Chapter 7 problems 4, 9, 10, 12, 13. If you want more practice on applying Inclusion-Exclusion, you can also try problems 16, 18, 19. These unfortunately, do not have solutions written in the book, but should be relatively easy.

Question 1 (Points in Separate Cycles, 25 points). *Show that the number of permutations of $[n]$ so that $1, 2, 3, \dots, k$ are in separate cycles is $\frac{n!}{k!}$. Note that this differs from the problem last week where we also required that there be exactly k cycles. [Hint: instead of $1, 2, \dots, k$ consider $n, n-1, \dots, n-k+1$. What does it mean for the canonical cycle notation for a permutation for these to be in different cycles?]*

Question 2 (Useless Poker Hands, 25 points). *How many four card poker hands have the “high card” valuation? That is hands of four cards so that*

- *No two cards in the hand have the same rank.*
- *Not all cards have the same suit (i.e. the hand is not a flush).*
- *Not all cards have different suits (i.e. the hand is not a flush).*
- *The ranks of the cards are not all consecutive (i.e. the hand is not a straight).*

Make sure that you don't double count anything. It is possible for a hand to be both a straight and a flush or both a straight and a flush. Please give your answer both in terms of a formula involving powers and binomial coefficients. Note that the numerical answer is 160,740. It is recommended that you use this to check your work.

Question 3 (Approximate Inclusion-Exclusion, 25 points). *Let A_1, A_2, \dots, A_n be finite sets. Show for even integers m that*

$$|A_1 \cup A_2 \cup \dots \cup A_n| \geq \sum_{k=1}^m (-1)^{k+1} \sum_{i_1 < i_2 < \dots < i_k} |A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}|,$$

and that for odd integers m

$$|A_1 \cup A_2 \cup \dots \cup A_n| \leq \sum_{k=1}^m (-1)^{k+1} \sum_{i_1 < i_2 < \dots < i_k} |A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}|.$$

Question 4 (No Repeated Letter Count, 25 points). *How many anagrams of ‘AABBCCDD’ (that is strings with 2 A’s, 2 B’s, 2 C’s and 2 D’s) have no repeated letters (that is letters that appear twice in a row)?*

Question 5 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*