Math 184A Exam 1

Fall 2015

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Name:

ID Number:

Problem	1	2	3	Total
Score				

Question 1 (TV Scheduling, 30 points)**.** *John's favorite TV show aired 60 episodes during its first calendar year on the air. Show that at least two of these episodes must have first aired within a week of each other.*

Question 2 (Stirling Recurrence, 35 points)**.** *Show that the following relation holds for all $n \geq k \geq 1$:*

$$S(n,k) = \sum_{m=0}^{n-k} \binom{n-1}{m} S(n-1-m, k-1).$$

[Hint: Count the number of partitions of $[n]$ into k parts where the element n ends up in a set of size $m+1$.]

Question 3 (Fibonacci Formula, 35 points)**.** *Define the Fibonacci numbers by the recurrence relation*

$$F_0 = 1,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2} \text{ for all } n \geq 2.$$

Show that

$$F_n = \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n-k}{k}$$

for all integers $n \geq 0$. (Recall that $\lfloor x \rfloor$ is the largest integer less than or equal to x).