Math 184A Final Exam

Spring 2018

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. You are free to make use of any result in the textbook or proved in class. You may use up to 12 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat indicated below.

Name:

ID Number:

Seat:

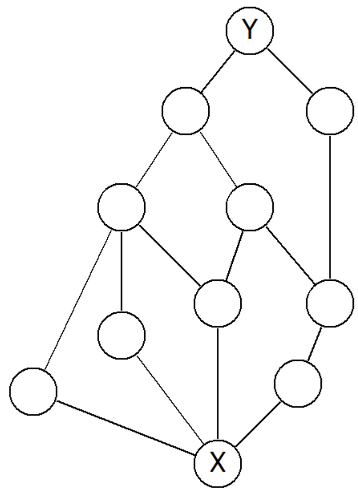
Problem	1	2	3	4	5	6	Total
Score							

Question 1 (Recurrence, 15 points). *Let a_n be given by the recurrence relation*

$$a_n = \begin{cases} 1 & \textit{if } n = 1 \\ 5 & \textit{if } n = 2 \\ a_{n-1} + 2a_{n-2} & \textit{otherwise.} \end{cases}$$

Show that $a_n = 2^n + (-1)^n$.

Question 2 (Mobius Computation, 15 points)**.** *Bellow is the Hasse diagram for a partial order. Compute* $\mu(X, Y)$.



Question 3 (Tile Coloring, 15 points)**.** *Jean has a sequence of n tiles lined up in a row. He would like to paint them all red, blue or green with the added requirement that all of the green colored tiles should be to the left of all blue colored tiles. In how many different ways can Jean accomplish this? For full credit, you should give an explicit formula.*

Question 4 (Elements in the Same Cycle and not, 15 points)**.** *Let $n > k > 0$ be integers. Show that the number of permutations of $[n]$ with exactly k cycles and with 1 and 2 in the same cycle is the same as the number of permutations of $[n]$ into $k+1$ cycles and 1 and 2 in different cycles.*

Question 5 (Generating Functions, 20 points)**.** *Let $a(n)$ be the number of triples of integers n_1, n_2, n_3 with $n = n_1 + n_2 + n_3$, $n_1 \geq n_2 \geq 0$, and n_3 positive and odd. Give an explicit formula for the generating function $\sum_{n=0}^{\infty} a(n)x^n$.*

Question 6 (Separated Partitions, 20 points)**.** *Show that the number of set partitions of* $[n]$ *with* $1, 2,$ *and* 3 *in distinct parts is* $B(n) - 3B(n-1) + 2B(n-2)$.