

# Math 184A Exam 2

Fall 2015

**Instructions:** Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

**Name:**

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**ID Number:**

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Problem	1	2	3	Total
Score				

**Question 1** (Permutation Notation, 30 points). Consider the string  $S = 425613$ .

(a) If  $S$  is interpreted as a permutation of  $[6]$  in the standard notation, how would you write this permutation in canonical cycle form? [15 points]

(b) If  $S$  is interpreted as a permutation of  $[6]$  in canonical cycle form, how would you write this permutation in the standard notation? [15 points]

**Question 2** (Binomial Identity, 35 points). *Prove the following identity for all integers  $n \geq k \geq 0$ .*

$$2^k \binom{n}{k} = \sum_{i=0}^k \binom{n}{n-k, i, k-i}.$$

*Recall here that  $\binom{n}{n-k, i, k-i}$  is the multinomial coefficient.*

**Question 3** (Unique Inclusion-Exclusion, 35 points). Let  $A_1, A_2, \dots, A_n$  be finite sets. Let  $S$  be the set of elements  $x$  that are in exactly one of the  $A_i$ . So for example if  $A_1 = \{1, 2, 3\}$  and  $A_2 = \{1, 3, 5\}$ , then  $S = \{2, 5\}$ , since 1, 3 are in more than one of the  $A$ 's. Show that

$$|S| = \sum_{k=1}^n (-1)^{k+1} k \sum_{1 \leq i_1 < i_2 < \dots < i_k \leq n} |A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}|.$$



