Math 184A Homework 1

Fall 2016

This homework is due on gradescope by Friday October 7th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeX is recommend though not required.

Question 1 (Fibonacci Numbers, 30 points). *The Fibonacci Numbers is the sequence* $1, 1, 2, 3, 5, 8, 13 \ldots$ *defined as follows:*

$$F_1 = F_2 = 1.$$

$$F_{n+2} = F_{n+1} + F_n \text{ for all } n \geq 1.$$

(a) *Prove that* F_{n+2} *counts the number of strings of length* n *using letters* 'A' *and* 'B' *so that there are no two consecutive* $A's$. *So for example "$ABBAB''$ would be a valid string of length* 5, *but not "$AABBA''$. [15 points]*

(b) *Prove by induction that*

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right)$$

for all $n \geq 1$. *Hint: when using induction you will need to handle the case of* $n = 2$ *as a second base case. [15 points]*

Question 2 (Course Overlap, 30 points). *In Binomial University, there are a total of 32 classes taught each term and each student takes exactly 5 classes each term. Show that for any group of 45 students that some two of them must share a pair of classes in common. Hint: First show that there must be some class taken by at least 8 students.*

Question 3 (Counting Words, 40 points). *For the purposes of this problem, a word is simply a sequence of letters from* $\{a, b, \ldots, z\}$, *the vowels are a,e,i,o,u. How many words are there of each of the following types (you can give answers as unexpanded expressions, like* $\binom{26}{8}$, *but you should still justify your answers) [5 points each]:*

(a) *With either exactly 4 or exactly 5 letters.*

(b) *With at most 5 letters, all of which are the same.*

(c) *With exactly 5 letters, the first two of which are vowels.*

(d) *With exactly 5 letters, all of which are distinct.*

(e) *With exactly 5 letters, all distinct, appearing in alphabetical order.*

(f) *With exactly 5 letters, at least one of which is a vowel.*

(g) *With exactly 6 letters, exactly two of which are the same.*

(h) *With exactly 5 letters and with all the vowels in the word coming before all of the non-vowels.*

Question 4 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*