

# Math 184A Homework 3

Fall 2015

This homework is due Monday October 26th in discussion section. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in L<sup>A</sup>T<sub>E</sub>X is recommended though not required.

**Question 1** (Different Types of Compositions, 40 points). .

(a) How many compositions of  $n$  are there into  $k$  parts each of size at least 2? [20 points]

(b) How many compositions of  $n$  are there into  $k$  odd parts? [20 points]

**Question 2** (Number of Parts in a Set Partition, 20 points). Let  $P_n$  be the sum over all set partitions of the set  $[n]$  of the number of parts in the partition. For example, since the set partitions of  $[3]$  are  $\{1, 2, 3\}$ ,  $\{1\}\{2, 3\}$ ,  $\{1, 2\}\{3\}$ ,  $\{1, 3\}\{2\}$ ,  $\{1\}\{2\}\{3\}$ , we have that  $P_3 = 1 + 2 + 2 + 2 + 3 = 10$ . Show that

$$P_n = B(n+1) - B(n).$$

[Hint: Either write both sides in terms of Stirling numbers of the second kind, or find a way to relate set partitions of  $[n+1]$  to set partitions of  $[n]$  with a specified part.]

**Question 3** (Partitions into Distinct Parts, 20 points). Show that the number of integer partitions of  $n$  into  $k$  distinct parts is equal to  $p_k(n - k(k-1)/2)$ . [Hint: find numbers that you can add to the parts of an arbitrary partition into  $k$  parts to ensure that they are distinct.]

**Question 4** (Average Number of Fixed Points, 20 points). What is the average number of fixed points that a permutation of  $[n]$  has? [Hint: count the number of pairs  $(\pi, x)$  of a permutation  $\pi : [n] \rightarrow [n]$  and an  $x \in [n]$  so that  $\pi(x) = x$ .]

**Question 5** (Extra credit, 1 point). Approximately how much time did you spend working on this homework?