Name ____________________________

PID ____________________________

Section ____________________________

Math 103A - MIDTERM II

Fall 2018

This examination booklet contains 6 problems and a Bonus Problem.

Do all of your work in this booklet, show all your computations and justify/explain your answers. Electronic devices are NOT allowed.

Problem	Possible score	Your score
1	10	
2	10	
3	10	
4	8	
5	7	
6	15	
Total	60	

(1) (a) (5 pts) Write $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 4 & 5 & 1 & 2 & 7 \end{pmatrix}$ as a product of disjoint cycles.

(b) (5 pts) Let $\sigma = (1,2,3,4)(4,5,6)$ (we assume $\sigma \in S_6$). Write σ as a product of disjoint cycles.

(2) Let $\sigma = (1, 2, 3, 5, 9)$ and $\tau = (4, 6, 7, 8, 10)$.

(a) (5 pts) Compute σ^{2018}.

(b) (5 pts) Find a permutation $\mu \in S_{10}$ so that $\sigma = \mu\tau\mu^{-1}$.

(3) Recall that D_4 is the group of symmetries of a square (the vertices are labeled $\{1, 2, 3, 4\}$). Let $H = A_4 \cap D_4$.

(a) (5 pts) Write down all the elements in H.

(b) (5 pts) Write down all the left cosets of H in A_4.

(4) (8 pts) Prove that every permutation $\sigma \in S_n$ is a product of disjoint cycles.

(5) (7 pts) Let G be a finite group of order n and let $e \in G$ denote the identity element. Prove that $g^n = e$ for all $g \in G$.

(6) Let G be a group and let $H \leq G$ be a subgroup.

(a) (8 pts) Suppose $(G : H) = 2$. Prove that $gHg^{-1} = H$ for all $g \in G$.

(b) (7 pts) Prove or provide a counter example: Suppose $(G : H) = 3$. Then $gHg^{-1} = H$ for all $g \in G$.

(Bonus Problem) Let G be a group and let $H \leq G$ be a subgroup with $(G : H) = n$. Prove that for every $g \in G$ there exists some $1 \leq k \leq n$ so that $g^k \in H$.