Math 184A Homework 6

Spring 2018

This homework is due on gradescope by Friday June 1st at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Limiting Exponent, 50 points)**.** .

(a) Show that for any permutation q and any positive integers n and m that

$$S_n(q)S_m(q) \leq S_{n+m}(q).$$

[25 points]

(b) Use this to prove that for any permutation q that

$$L(q) = \lim_{n\to\infty} \sqrt[n]{S_n(q)}$$

exists and is finite. [You may use the result mentioned in class that for each q there is a constant C_q so that $S_n(q) \leq C_q^n$] [25 points]

Question 2 (Avoiding 132 and 4321, 50 points)**.** *Let $S_n(132, 4321)$ be the number of permutations of $[n]$ that avoid both 132 and 4321. Show that*

$$S_n(132, 4321) = 2\binom{n}{4} + \binom{n+1}{3} + 1.$$

Question 3 (Extra credit, 1 point)**.** *Approximately how much time did you spend on this homework?*