Name: James Holden
TA Name: Alvaro Ruiz Martinez
Section: D07
Time: 8:00 PM

MatLab Assignment 3: Numerical Methods

Exercise 3.1

a) Run Euler's Method again, this time for h = 0.25. Plot the results in blue this time. (Use 'b' instead of 'r'.) Look at the arrays x and y side-by-side by typing in

>> [x,y]

Copy the long table of data that you just produced into your Word document. What estimates did the Euler Method come up with for y at x = 1 and x = 2 using this smaller value of h? Now run Euler's Method for h = 0.1 and again for h = 0.01, and plot the results in green ('g') and cyan ('c'), respectively. What are the estimates for y at x = 1 and x = 2 for these values of h? Write these down in your Word document, along with your graph with all four colored curves. (You need not include the data tables for these last two values of h.)

Command:

>> h = 0.25;

>> [x,y] = Euler(h, 0, 1, 2, f);

[x,y]

Output:

When h = 0.25:	
0	1.0000
0.2500	0.7500
0.5000	0.5156
0.7500	0.3281
1.000	0.2188
1.2500	0.2188
1.5000	0.3594
1.7500	0.6719
2.0000	1.1875

Answer:

When h = 0.25, x = 1, y = 0.2188, therefore the estimate is 0.2188

When h = 0.25, x = 2, y = 1.1875, therefore the estimate is 1.1875

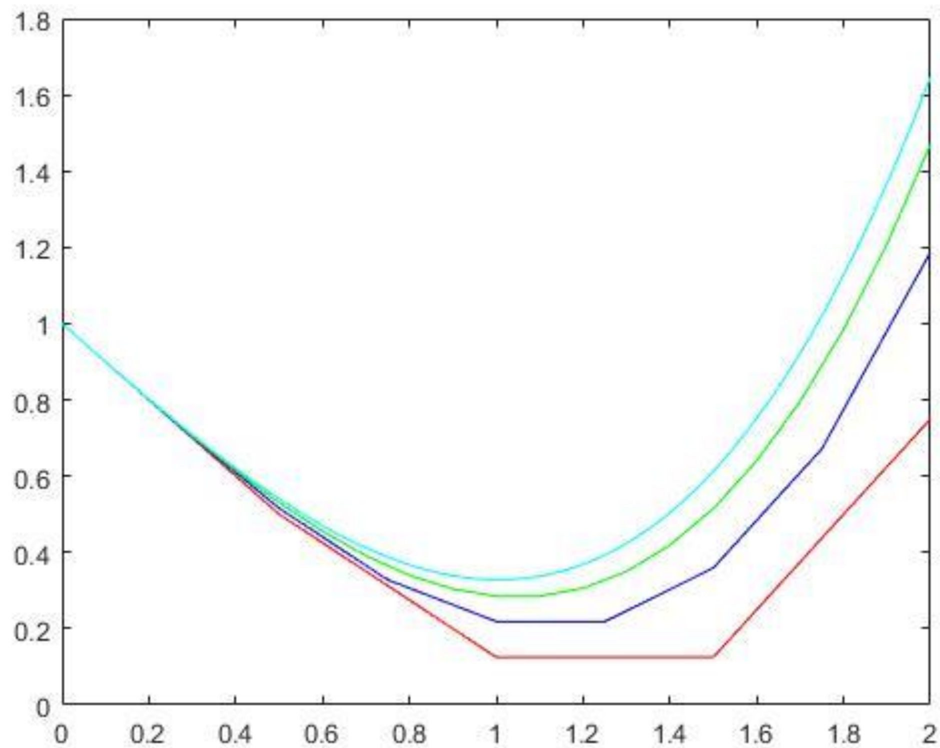
When h = 0.1, x = 1, y = 0.2850, therefore the estimate is 0.2850

When h = 0.1, x = 2, y = 1.470, therefore the estimate is 1.4700

When h = 0.01, x = 1, y = 0.3367, therefore the estimate is 0.3367

When h = 0.01, x = 2, y = 1.6467, therefore the estimate is 1.6467

Colored Graph:



b) The differential equation given in (2) is separable. Calculate the solution to the initial value problem by hand, and use MATLAB (or a calculator) to compute the actual values for y at x = 1 and x = 2. Make sure to include your manual work in your Word document. (You can simply type it.)

Answer:

Handwritten Solution for problem: dy/dx = x^2 – 1 y(x) = (1/3) * (x^3) – x + C where C is an arbitrary contant	Matlab solution to problem: >> dsolve('Dy=(x^2) - 1','y(0)=1') ans = t*(x^2 - 1) + 1
Solve for y when x = 1: >> dsolve('Dy=(1^2) - 1','y(0)=1') ans = 1	Solve for y when x = 2: >> dsolve('Dy=(1^2) - 1','y(0)=1') ans = 2

c) Does Euler's estimate appear to give better or worse estimates for the solution as h decreases? Give a brief reasoning for why this might be the case. Does Euler's estimate appear to give better or worse estimates for the actual solution as you move farther from the initial point?

Answer:

Euler's estimate seems to give better estimates as h decreases, as you can see from the graph above becoming smoother and smoother. This means that the estimated solution more closely represents the real solution. From the graph, it would seem the further from the initial point, the worse the estimates become.

<u>Exercise 3.2</u>

In your Word document, briefly explain what is happening in each remaining line of the M-File Euler.m.

Answer:

Code line 5: Sets x to an initial value x0

Code line 6: Sets y to an initial value y0

Code line 7. For loop command that increments i

Code line 8: y += (stepsize * derivative at that point)

Code line 9: size x += stepsize

Code line 10: end the loop

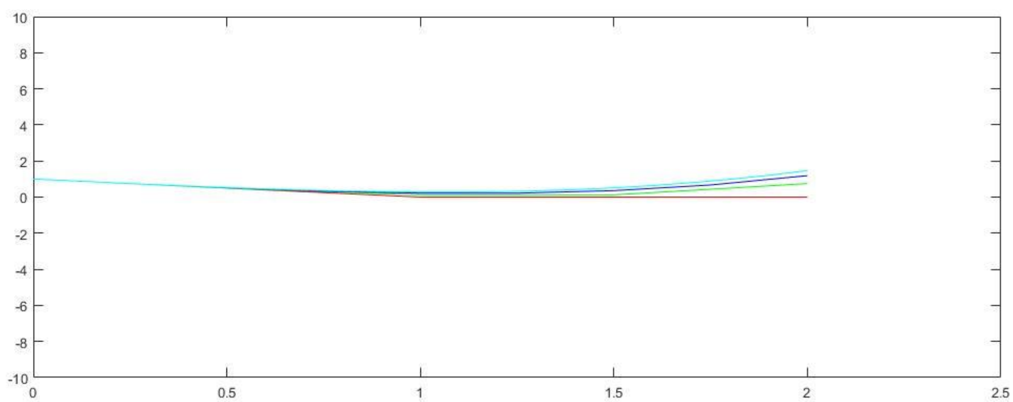
Exercise 3.3

a) Run Euler's Method in MATLAB for the initial value problem (1) on the interval $0 \leq x \leq 4$. Use different colors to plot the estimates for the following values of h: 1, 0.5, 0.25, and 0.1. Remember to use hold on! Some of the colors you can use with MATLAB's plot command are 'r', 'g', 'b', 'c', 'm', 'y', and 'k'.

Notice that the y-axis is very large. Rescale the y-axis to range from -10 to 10. (Assignment 1 showed us how to do this.) Given the results of the plot, what can you say about the accuracy of the h = 0.1 case? You do not need to include the actual data in the table [x,y] in your write-up, only the plot and your written answer.

Graph:

h = 1 –> red; h = 0.5 -> green; h = 0.25 -> blue; h = 0.1 -> cyan



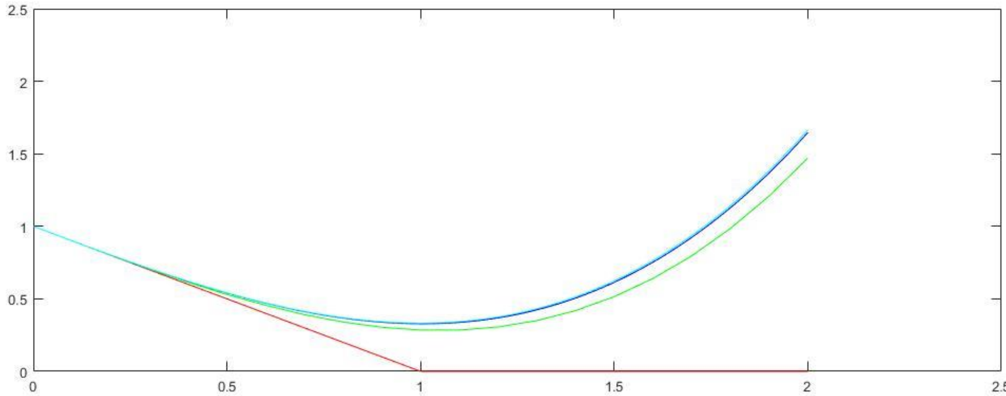
Answer:

Because it steps in smaller increments, when h = 0.1, the estimate graph holds more accuracy than the estimate graph when h = 1, h= 0.5, or h = 0.25.

b) Enter hold off so that MATLAB will overwrite your previous plot, and this time, run Euler's Method with the h values 1, 0.1, 0.01, and 0.001. Plot the results on one graph using colors of your choice, and rescale so that y ranges from 0 to 2.5. Paste this plot into your write-up.

Did the additional shrinking of h change the results significantly? What does this suggest about the accuracy of your solution for each step size? Comment on the consistency of your plot with the direction field shown at the top of the page, and also describe any inconsistencies between different step sizes.

Graph:



Answer:

When h = 0.1 or below, solution curves are extremely similar. I would say that the difference in solution curves is negligible, meaning that additionally shrinking h would not change the results significantly. The curve for h = 0.001 (graphed in cyan) looks very similar to the direction field.

Exercise 3.4

a) As can be easily verified, the solution to the initial value problem in Example 3.3 is

h(x) = -cos(x) + 2

Now with the solution of the above differential equation, we can compare the results obtained by using ode45 to the actual values. Type the following commands into MATLAB:

```
>> h = @(x) -cos(x) + 2;
[x, y, h(x), abs(y - h(x))]
```

Include this code and the output in your write-up. The second and third columns give us the estimate that we found using ode45 and the actual function value at each specified value of x. How do the values in the two columns compare? The fourth column gives the absolute value of the difference between the function and our approximation; this is the error of our approximation.

b) Now let us compare the results that ode45 gives us with the results we would get from using Euler's method. Enter the following commands into MATLAB:

```
>> [x, z] = Euler(0.25, 0, 1, 10, g);
[y, z, h(x), abs(y - h(x)), abs(z - h(x))]
```

Copy the input and output to your Word document. In the first column, we have the results of ode45; in the second, the results of our Euler's Method routine; and in the third, the values at x of the real solution to our differential equation. Columns four and five give the error of ode45 and of Euler's Method, respectively. Which method seems to give more accurate answers in this situation?

Problem (a) code: h = @(x) -cos(x) + 2; [x, y, h(x), abs(y - h(x))]	Problem (b): >> [x, z] = Euler(0.25, 0, 1, 10, g); [y, z, h(x), abs(y - h(x)), abs(z - h(x))]
0 1.0000 1.0000 0 0.2500 1.0311 1.0311 0.0000 0.5000 1.1224 1.1224 0.0000 0.7500 1.2683 1.2683 0.0000 1.0000 1.4597 1.4597 0.0000 1.2500 1.6848 1.6847 0.0001 1.5000 1.9293 1.9293 0.0000 1.7500 2.1782 2.1782 0.0001 2.0000 2.4161 2.4161 0.0000 2.2500 2.6282 2.6282 0.0001 2.5000 2.8012 2.8011 0.0000 2.7500 2.9243 2.9243 0.0000 3.0000 2.9900 2.9900 0.0000 3.2500 2.9941 2.9941 0.0000 3.5000 2.9365 2.9365 0.0000	1.0000 1.0000 1.0000 0 0 1.0311 1.0000 1.0311 0.0000 0.0311 1.1224 1.0619 1.1224 0.0000 0.0606 1.2683 1.1817 1.2683 0.0000 0.0866 1.4597 1.3521 1.4597 0.0000 0.1076 1.6848 1.5625 1.6847 0.0001 0.1222 1.9293 1.7997 1.9293 0.0000 0.1295 2.1782 2.0491 2.1782 0.0001 0.1291 2.4161 2.2951 2.4161 0.0000 0.1210 2.6282 2.5224 2.6282 0.0001 0.1057 2.8012 2.7169 2.8011 0.0000 0.0842 2.9243 2.8666 2.9243 0.0000 0.0577 2.9900 2.9620 2.9900 0.0000 0.0280 2.9941 2.9973 2.9941 0.0000 0.0031 2.9365 2.9702 2.9365 0.0000 0.0338

3.7500 2.8206 2.8206 0.0000 4.0000 2.6536 2.6536 0.0000 4.2500 2.4460 2.4461 0.0001 4.5000 2.2108 2.2108 0.0000 4.7500 1.9625 1.9624 0.0001 5.0000 1.7163 1.7163 0.0000 5.2500 1.4879 1.4879 0.0001 5.5000 1.2913 1.2913 0.0000 5.7500 1.1388 1.1388 0.0000 6.0000 1.0398 1.0398 0.0000 6.2500 1.0006 1.0006 0.0000 6.5000 1.0234 1.0234 0.0000 6.7500 1.1070 1.1070 0.0000 7.0000 1.2461 1.2461 0.0000 7.2500 1.4321 1.4321 0.0001 7.5000 1.6534 1.6534 0.0000 7.7500 1.8961 1.8962 0.0001 8.0000 2.1455 2.1455 0.0000 8.2500 2.3858 2.3857 0.0001 8.5000 2.6020 2.6020 0.0000 8.7500 2.7808 2.7808 0.0000 9.0000 2.9111 2.9111 0.0000 9.2500 2.9848 2.9848 0.0000 9.5000 2.9972 2.9972 0.0000 9.7500 2.9476 2.9476 0.0000 10.0000 2.8391 2.8391 0.0000	2.8206 2.8825 2.8206 0.0000 0.0620 2.6536 2.7396 2.6536 0.0000 0.0860 2.4460 2.5504 2.4461 0.0001 0.1043 2.2108 2.3267 2.2108 0.0000 0.1159 1.9625 2.0823 1.9624 0.0001 0.1199 1.7163 1.8325 1.7163 0.0000 0.1161 1.4879 1.5927 1.4879 0.0001 0.1048 1.2913 1.3780 1.2913 0.0000 0.0867 1.1388 1.2016 1.1388 0.0000 0.0628 1.0398 1.0745 1.0398 0.0000 0.0347 1.0006 1.0047 1.0006 0.0000 0.0041 1.0234 0.9964 1.0234 0.0000 0.0270 1.1070 1.0502 1.1070 0.0000 0.0568 1.2461 1.1627 1.2461 0.0000 0.0834 1.4321 1.3269 1.4321 0.0001 0.1051 1.6534 1.5327 1.6534 0.0000 0.1207 1.8961 1.7672 1.8962 0.0001 0.1290 2.1455 2.0159 2.1455 0.0000 0.1296 2.3858 2.2632 2.3857 0.0001 0.1226 2.6020 2.4938 2.6020 0.0000 0.1082 2.7808 2.6935 2.7808 0.0000 0.0874 2.9111 2.8497 2.9111 0.0000 0.0615 2.9848 2.9527 2.9848 0.0000 0.0321 2.9972 2.9962 2.9972 0.0000 0.0010 2.9476 2.9774 2.9476 0.0000 0.0298 2.8391 2.8975 2.8391 0.0000 0.0584
Answer: The second and third columns of the output are almost exactly the same. The fourth column represents the difference between the ode45 function and the real solution. Since the differences are minuscule, the ode45 function actually does a pretty good job.	Answer: In this test ode45 is more accurate that Euler's method when compared to the real solution. The differences between the ode45 function and the solution are very small. On the other hand, the difference between the Euler method's approximations and the real solutions are far larger and vary wildly.