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Section: D07
Time: 8:00 PM

MatLab Assignment 4: Systems of ODE's

Exercise 4.1

Let B be the matrix $\begin{bmatrix} 1.2 & 2.5 \\ 4 & 0.7 \end{bmatrix}$

a. Define the matrix B in MATLAB with the values above. Copy and paste the input and output from your command into your Word document.
b. Use the MATLAB commands to find the eigenvalues and eigenvectors for the matrix B. Copy and paste the input and output from your command into your Word document.

Now that we've seen how to use matrices in MATLAB, we should be ready to solve systems of equations such as (1) above.

Answer for A

```
>> B = [1.2, 2.5; 4, 0.7]

B =

    1.2000    2.5000

    4.0000    0.7000
```

Answer for B

```
>> [eigvec, eigval] = eig(B)

eigvec =

    0.6501   -0.5899

    0.7599    0.8075

eigval =

    4.1221         0

         0   -2.2221
```

Exercise 4.2

Consider the system of differential equations

(4) $dx/dt = 3x + 4y$

$dy/dt = -x - 2y$

a. When the system (4) above is put into the form $v' = Av$, what is the matrix A? Enter the matrix A into MATLAB, and record the input and output for this in your Word document
b. Use MATLAB to find the characteristic roots (eigenvalues) and characteristic vectors (eigenvectors) of your matrix A. Copy and paste all input and output from your command into your Word document.
c. Use the formula (2) above and the results from part (b) to write the general solution of our system (4). Write this solution in your document. What happens to the system as t gets large?
d. Use phaseplane and drawphase to create a phase diagram of the solutions. Make sure you draw at least six solution curves. (You may want to simply modify the code from the example.) Include this plot in your writeup. Does the plot support your answer to (c)?

Answer to A

The matrix A is:

```
>> A = [3, 4; -1, -2]

A =

     3     4

    -1    -2
```

Answer to B

```
>> [eigvec, eigval] = eig(A)

eigvec =

    0.9701   -0.7071

   -0.2425    0.7071

eigval =

     2     0

     0    -1
```

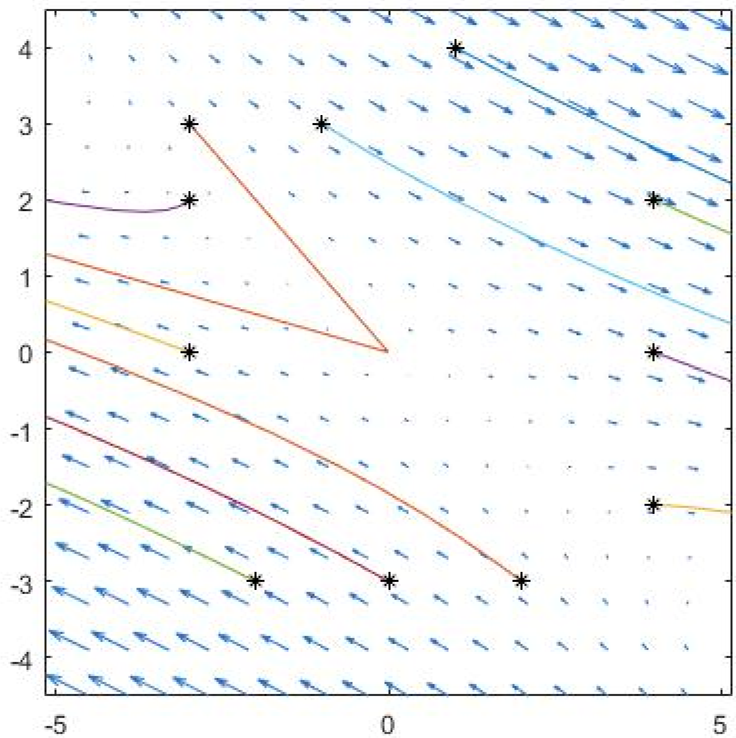
Answer to C

General solution:

v(t) = c1 * e^2*t * [0.9701, -0.2425] + c2 * e^-t * [-0.7071, 0.7071]

As t gets large, v(t) drifts toward infinity because the e^(2t) term dominates e^(-t)

Answer to D



Exercise 4.3

Consider the system of differential equations

(6) dx⁄dt = 2.7x - y

dy⁄dt = 4.1x + 3.7y

a. Put the system above in the form v′ = Av and enter the matrix A into MATLAB. Use MATLAB to find the eigenvalues and eigenvectors of A. Copy and paste all input and output from your command into your Word document.
b. Write the general solution of the system (6) in your Word document.
c. Use phaseplane and drawphase to create a phase diagram of the solution. Make sure to draw at least six different curves using drawphase. Include the resulting plot in your writeup. Which feature of the eigenvalues or eigenvectors causes the solutions to tend to infinity as t grows large?

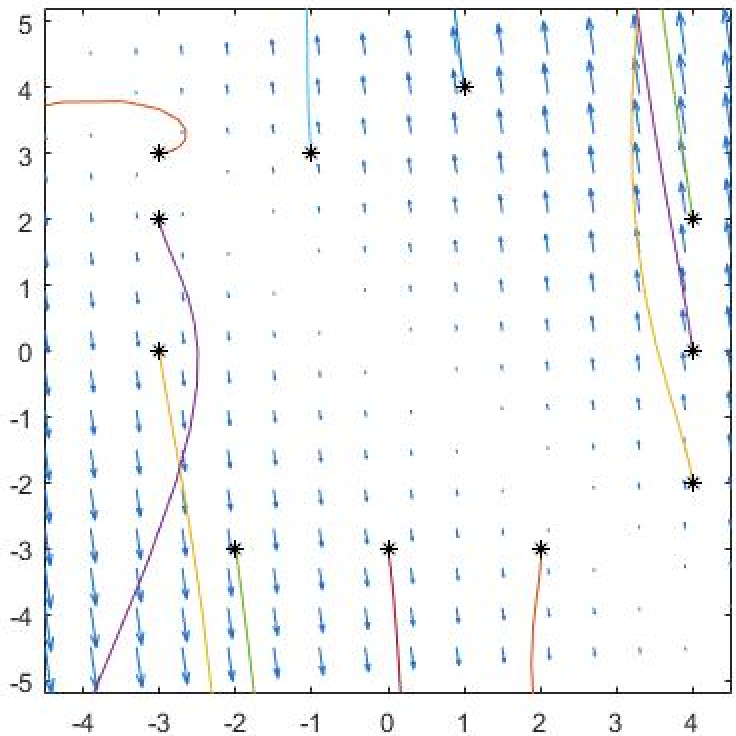
Answer to A

```
>> A = [2.7, -1; 4.1, 3.7]

A =

   2.7000   -1.0000

   4.1000    3.7000

>> [eigvec, eigval] = eig(A)

eigvec =

  -0.1093 + 0.4291i  -0.1093 - 0.4291i

   0.8966 + 0.0000i   0.8966 + 0.0000i

eigval =

   3.2000 + 1.9621i   0.0000 + 0.0000i

   0.0000 + 0.0000i   3.2000 - 1.9621i
```

Answer to B

Y(t) = c1 * e^(3.2000 + 1.9621i) * [-0.1093 + 0.4291i , 0.8966 + 0.0000i] + c2 * e^(3.2000 - 1.9621i) * [-0.1093 - 0.4291i, 0.8966 + 0.0000i]

Answer to C



<u>**Exercise 4.4**</u>

Consider the 3×3 system

(8) $dx/dt = 1.25x - 0.97y + 4.6z$

$dy/dt = -2.6x - 5.2y - 0.31z$

$dz/dt = 1.18x - 10.3y + 1.12z$

a. Define the matrix A in MATLAB with the values above, using A = [1.25, -0.97, 4.6; -2.6, -5.2, -0.31; 1.18, -10.3, 1.12], and calculate the eigenvalues of A. Copy and paste the input and output from MATLAB.
b. Using your answer from part (a), is the system in (8) stable? Justify your answer.

Answer to A

```
>> A = [1.25, -0.97, 4.6; -2.6, -5.2, -0.31; 1.18, -10.3, 1.12]

A =

    1.2500   -0.9700    4.6000

   -2.6000   -5.2000   -0.3100

    1.1800  -10.3000    1.1200

>> [eigvec, eigval] = eig(A)

eigvec =

   0.7351 + 0.0000i   0.4490 + 0.2591i   0.4490 - 0.2591i

  -0.1961 + 0.0000i  -0.3375 + 0.2242i  -0.3375 - 0.2242i

   0.6490 + 0.0000i  -0.7530 + 0.0000i  -0.7530 + 0.0000i

eigval =

   5.5698 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i

   0.0000 + 0.0000i  -4.1999 + 2.6606i   0.0000 + 0.0000i

   0.0000 + 0.0000i   0.0000 + 0.0000i  -4.1999 - 2.6606i
```

Answer to B

The system is unstable because for its eigenvalues, one real part is positive. This means that the solutions are unbounded.

Exercise 4.5

Define the matrices A and B in MATLAB:

```
>> A = [-0.0558 -0.9968 0.0802 0.0415;
        0.598 -0.115 -0.0318 0;
        -3.05 0.388 -0.465 0;
        0 0.0805 1 0]
>> B = [0.01; -0.175; 0.153; 0]
```

a. What are the eigenvalues and eigenvectors of the matrix A? Include your input commands and your output in your Word document.
b. According to the mathematical definitions, is the system x′ = Ax stable, asymptotically stable, or unstable? Why?
c. One of the eigenvalues you obtained is very close to zero. Look at the corresponding eigenvector. Which component is biggest? Based on that, which type of rotation is this eigenvector most closely associated with: yaw, roll, or pitch?

Answer to A

```
>> [eigvec, eigval] = eig(A)

eigvec =

   0.1994 - 0.1063i   0.1994 + 0.1063i  -0.0172 + 0.0000i   0.0067 + 0.0000i
  -0.0780 - 0.1333i  -0.0780 + 0.1333i  -0.0118 + 0.0000i   0.0404 + 0.0000i
  -0.0165 + 0.6668i  -0.0165 - 0.6668i  -0.4895 + 0.0000i  -0.0105 + 0.0000i
   0.6930 + 0.0000i   0.6930 + 0.0000i   0.8717 + 0.0000i   0.9991 + 0.0000i

eigval =

  -0.0329 + 0.9467i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0329 - 0.9467i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.5627 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0073 + 0.0000i
```

Answer to B

All of the real parts of the eigenvalues are negative which means that the system will tend to move back to a steady state. This means that it is stable.

Answer to C

The eigenvector that corresponds with the eigenvalue -0.0073 has the highest pitch and therefore is most closely associated with pitch.

Exercise 4.6

Recall that we specified A as: $\begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.4650 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$

and B as $\begin{bmatrix} 0.01 \\ -0.175 \\ 0.153 \\ 0 \end{bmatrix}$

a. If F = [0 7 0 -1], what is the matrix BF?
b. If F = [0 5 0 -0.1], what is the matrix BF?
c. If F = [0 5 0 -0.1], what is the matrix A + BF? Are its eigenvalues the same as or different from those of A?
d. Find F = [F1 F2 F3 F4] such that the damping of yaw oscillation is increased and the plane still responds decently to pilot commands. More specifically: we want the complex eigenvalues to have real part less than -0.2, and we want there to be a real eigenvalue within 0.02 of zero. [Hint: There is a solution with F1 = 0 = F3 and F4 = -0.09, so you only need to fiddle with F2 to find an appropriate value.]

Answer to A

```
>> F = [0, 7, 0, -1]

F =

    0     7     0    -1

>> B*F

ans =

         0    0.0700         0   -0.0100

         0   -1.2250         0    0.1750

         0    1.0710         0   -0.1530

         0         0         0         0
```

Answer to B

```
>> F = [0, 5, 0, -0.1]

F =
```

```
     0    5.0000         0   -0.1000
```

>> B*F

ans =

```
     0    0.0500         0   -0.0010
     0   -0.8750         0    0.0175
     0    0.7650         0   -0.0153
     0         0         0         0
```

Answer to C

>> A + B*F

ans =

```
   -0.0558   -0.9468    0.0802    0.0405
    0.5980   -0.9900   -0.0318    0.0175
   -3.0500    1.1530   -0.4650   -0.0153
         0    0.0805    1.0000         0
```

These eigenvalues are different from the eigenvalues of A.

Answer to D

>> eig(A+B*[0 2.5 0 -.09])

ans =

```
  -0.2057 + 0.9227i
  -0.2057 - 0.9227i
  -0.0160 + 0.0000i
  -0.6458 + 0.0000i
```

When F2 is approximately 2.5; it increases dampening in yaw oscillation and gives the pilot decent control over descent.