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TA Name: Alvaro Ruiz Martinez

Section: D07

Time: 8:00 PM

MATLAB Assignment 2

1. **Exercise 2.1**
   a. Sketch (by hand, without using MATLAB) the direction field of the differential equation dy⁄dx = y⁄5 for x and y values between -5 and 5. (You do not need to include this sketch in your write-up.)
      1.Sketched.
   b. On your direction field, add a curve (by hand) that approximates the solution passing through the point x = 0, y = 1.
      1.Drawn.
   c. Now solve the differential equation given in part (a), either working it out by hand or using the dsolve command that we saw in Assignment 1. Compare your answers to parts (a) and (b).

Matlab Code and Answer:

General Solution:

```
>> dsolve('Dy=y/5')

ans =

C1*exp(t/5)
```

Initial Value Problem(y(0) = 1):

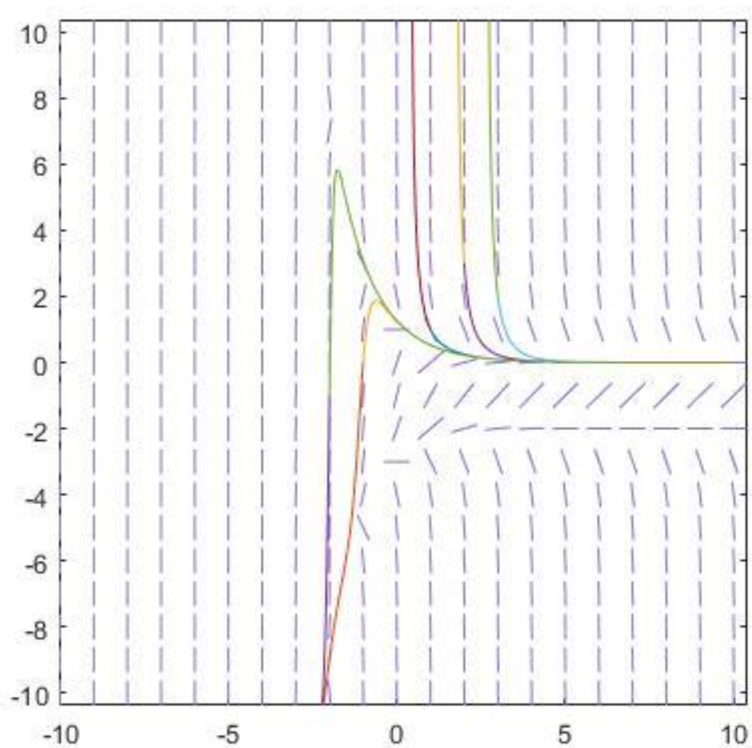
```
>> dsolve('Dy=y/5', ‘y(0) = 1’)

ans =

exp(t/5)
```

2. Exercise 2.2

Consider the differential equation

$$dy/dx = (e\text{-}x - y)(e\text{-}x + 2 + y).$$

a. Plot a direction field for (5) for x and y between -10 and 10. Use the hold commands and drawode to plot at least two solution curves on this direction field, one of which passes through the point (2,3) (that is, x = 2 and y = 3). Paste your plot into your Word document.



b. Considering how complicated differential equation (5) appears to be, why do you think we might want to plot a direction field?

Answer: Plotting a direction field is important because it allows you to see a functions behavior around equilibrium points. Visualizing a problem often allows you to understand a solution.

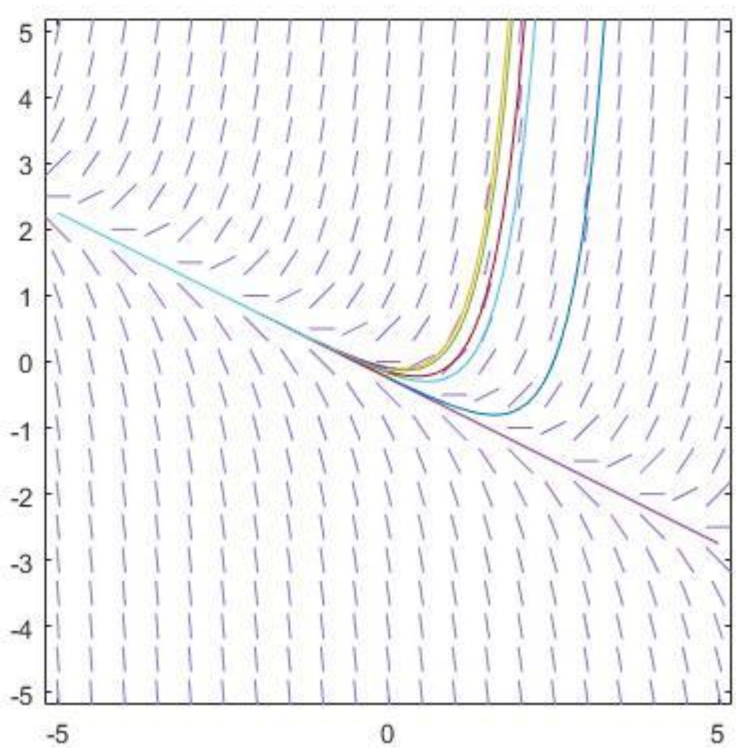
3. **Exercise 2.3**

Suppose the differential equation

(6) $dy/dx = x + 2y$

arises as a model in a physics experiment.

a. Plot a direction field for (6) with x and y between -5 and 5. On this direction field, plot the solution curve passing through (0,-1⁄4). Then draw three more solution curves passing through points very close to (0,-1⁄4) on the same figure. Paste your plot into your Word document.



b. Using this plot, think about what would happen if the initial value in the problem were not exactly (0,-1⁄4). Would this greatly affect the solution of the differential equation?

<u>Answer:</u> If the initial value of the problem was not (0, -1/4), the solution would be very different. It would not be a straight line solution any more. Instead the solution will look like a curve that grows or decays rapidly (appearing to approach positive or negative infinity) for all values of x and y.

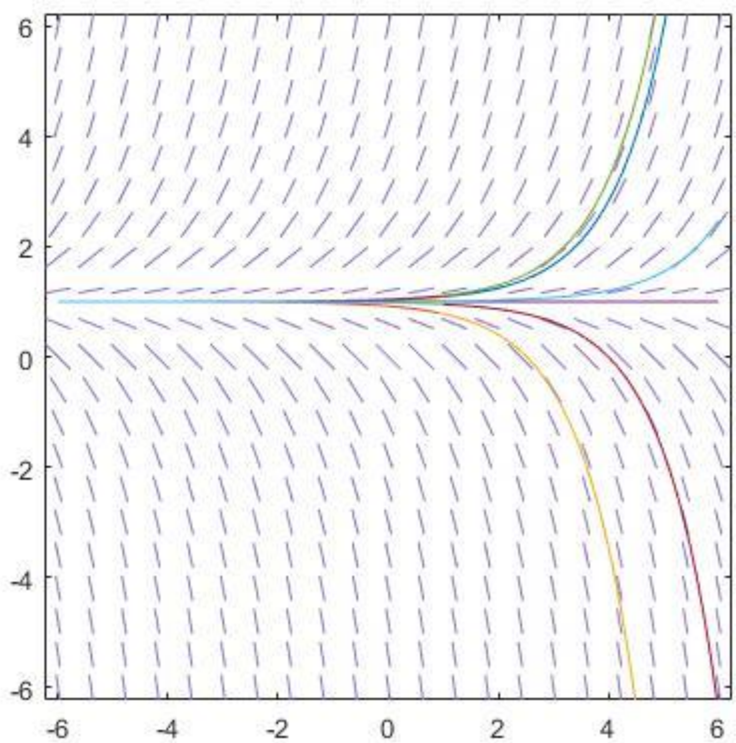
4. Exercise 2.4

Now suppose that

(7) $dy/dx = y - 1$

arises as a model.

a. Plot a direction field for (7) with x and y between -6 and 6. Suppose our experiment reveals that the initial value is about (1,1). On your direction field, plot the solution curve passing through (1,1), and also plot several solution curves going through other points near (1,1). Paste your plot into your Word document.

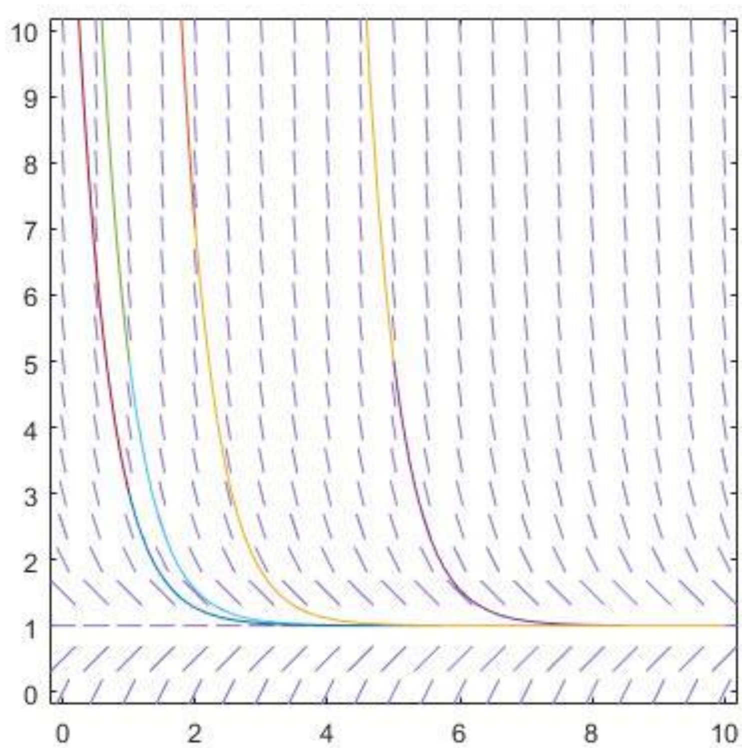


b. If the initial value were not exactly (1,1), how would this affect the solution?

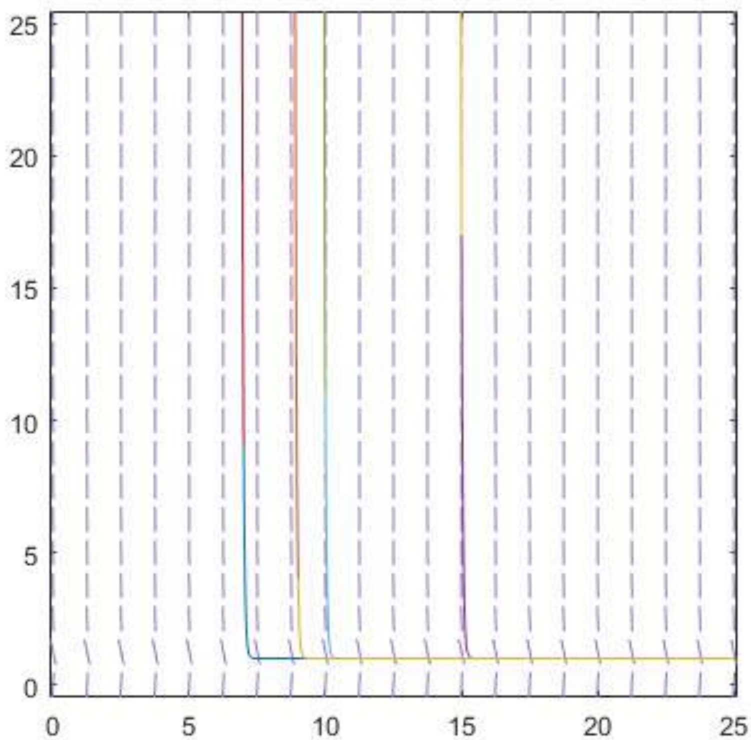
If the initial value weren't (1,1), the solution would behave far more dramatically (ie. would display increasing or decreasing behavior much earlier). We can see this from the direction fields.

5. **Exercise 2.5**
   a. Plot a direction field for (8) with A = 1, k = 2, and where the minimum value of t is zero (since we are not interested in negative times here). You can choose an appropriate maximum value for t and minimum and maximum values for y. Include the direction field in your Word document. Now plot some direction fields for other values of A, and graph a few solution curves on those direction fields using drawode by choosing some initial values. What property do you think A represents in real life? [Hint: Think about the temperature at which the solutions stabilize.]

      K = 2



      K = 20



The function and A in particular represent how hot and cold temperatures tend to stabilize to room temperature (or another cooling temperature like the inside of a refrigerator).

6. **Exercise 2.6**
    a. Let us try to figure out how long it will take to defrost a frozen chicken breast in the fridge, which keeps a constant temperature of 41°F. The chicken breast has been in the freezer for a while, so its temperature is uniform at -6°F. We'll suppose k = 0.4, based on the properties of the chicken.
        1. Recall that an initial value problem consists of a differential equation along with an initial condition. Write out the initial value problem that we must solve here. (We already have the differential equation, so this means you need to find the appropriate initial condition.)
            1. We know the initial temperature of the chicken is -6 degrees (F). The general solution is: dy/dt=k*(41 - y) and the initial condition is: y(0) = -6.
        2. To simulate the conditions in the fridge, we must pick the parameter A. What do you think the value of A should be?
            1. A is the resting temperature of the refrigerator. A = 41 degrees Fahrenheit.
        3. Let us consider the chicken breast fully defrosted when the temperature reaches 39°F. How long does it take to defrost a chicken breast under the above conditions? A rough estimate from a direction field plot is sufficient. [Hint: You may need to adjust the axes on your plots.]
            1. To find the time, plug into the equation. 41 – 47 * exp(- (2*t) / 5). The chicken reaches 39 degF in about 8 units of time.
        4. How much time would be saved if the chicken breast were thawed on the kitchen counter instead, given that room temperature is around 69°F?
            1. Just plug in the updated numbers. 69 – 75 * exp(-(2*t) / 5). It takes about 2.3 units of time to reach 39 degrees (F).

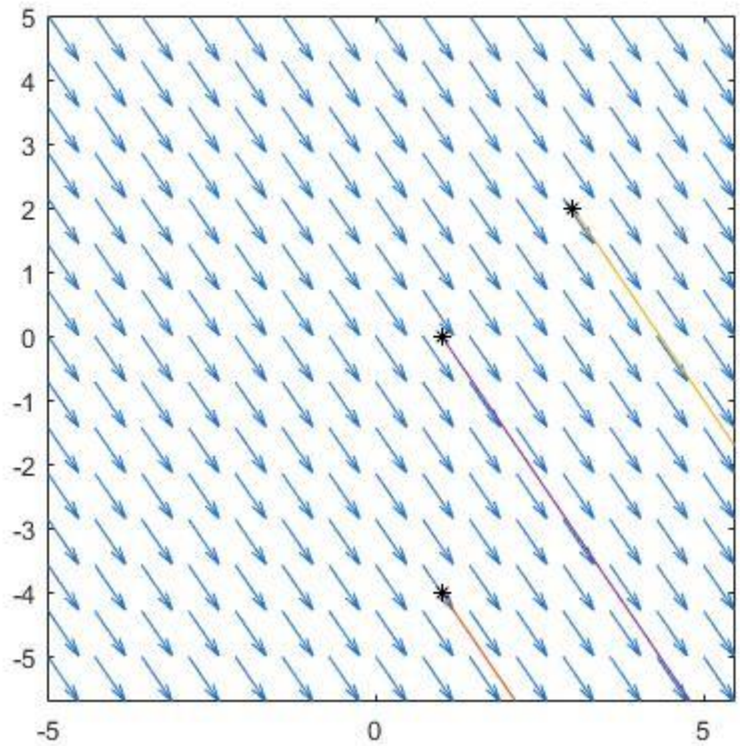
7. **Exercise 2.7**
    a. Use the techniques we've learned involving phaseplane to plot a phase portrait of (11), where the x and y values are between -5 and 5. Then, on the same plot, use drawphase to draw at least three different solution curves. Include the resulting plot in your Word document.

    Now, try changing the values of x′ and y′ from 2 and -3 to other constant values. Describe in your Word document how this changes the phase portrait.

    <u>Matlab Code and Plot:</u>

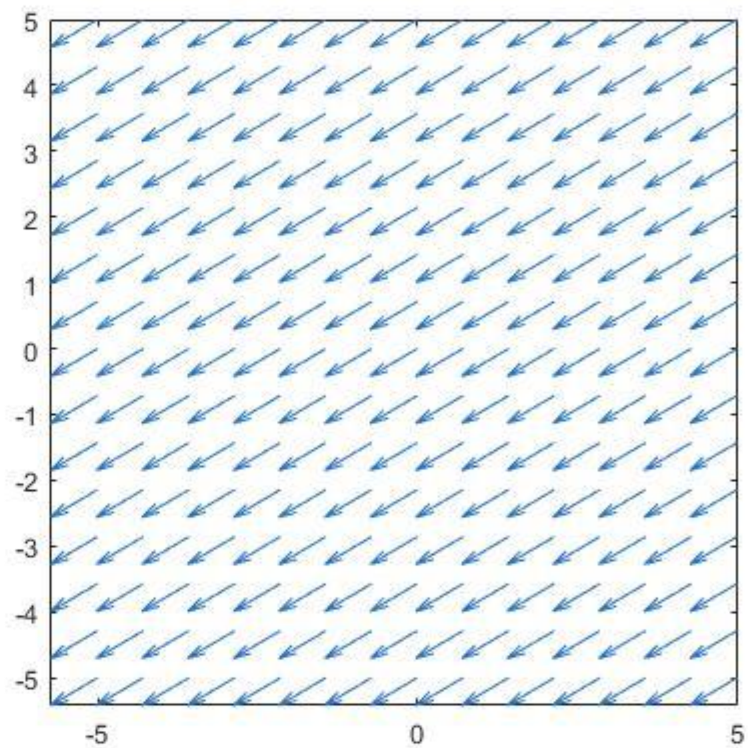
```
>> g= @(t, Y) [2; -3]

phaseplane (g, [-5, 5], [-5, 5], 15)

hold on

drawphase (g, 50, 1, -4)

drawphase (g, 50, 3, 2)

drawphase (g, 50, 1, 0)

hold off

g =


  function_handle with value:
```

@(t,Y)[2;-3]



Changing values

```
>> g = @(t,Y) [-6; -5]
g = @(t,Y)[-6;-5]
>> phaseplane(g, [-5,5], [-5,5], 15)
```



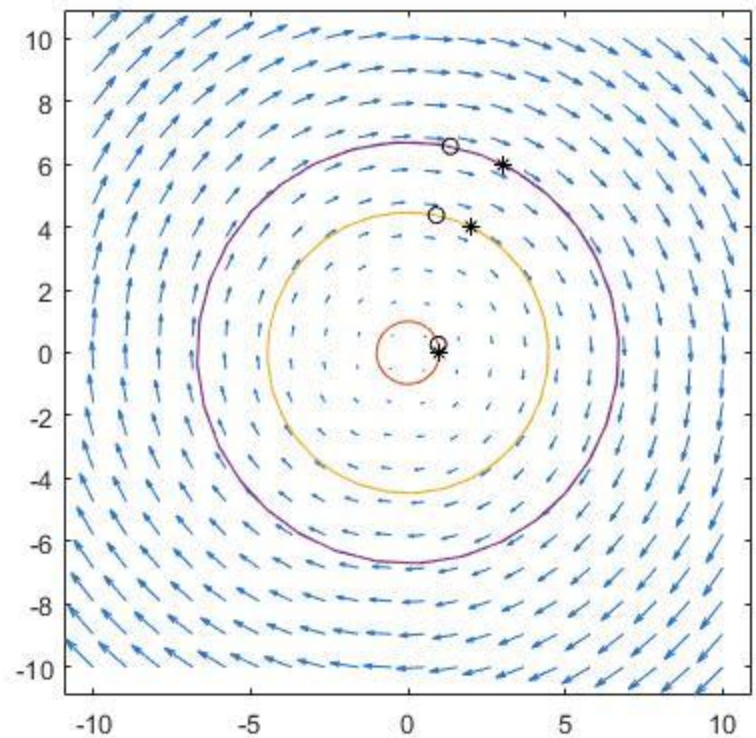
Comment: For other constant values of the function, the phase portrait changes the direction and the slope depending on the sign of the constant numbers, but it remains the same straight line, so the solutions to all constants look similar, other than the directions.

8. Exercise 2.8

a. Use phaseplane to plot a phase portrait of (12), where the x and y values are between -10 and 10. [Remember that when you're defining your function g(t,Y), you need to use Y(1) to represent your first dependent variable and Y(2) for the second one.] Then, on the same plot, use drawphase to draw at least three different solution curves. Include the resulting plot in your Word document.

Use phaseplane to plot a phase

```
>> g = @(t, Y) [Y(2); -Y(1)]
phaseplane(g, [-10, 10], [-10, 10], 20)
hold on
drawphase (g, 50, 1, 0)
drawphase (g, 50, 2, 4)
drawphase (g, 50, 3, 6)
hold off


g =


  function_handle with value:


    @(t,Y)[Y(2);-Y(1)]
```



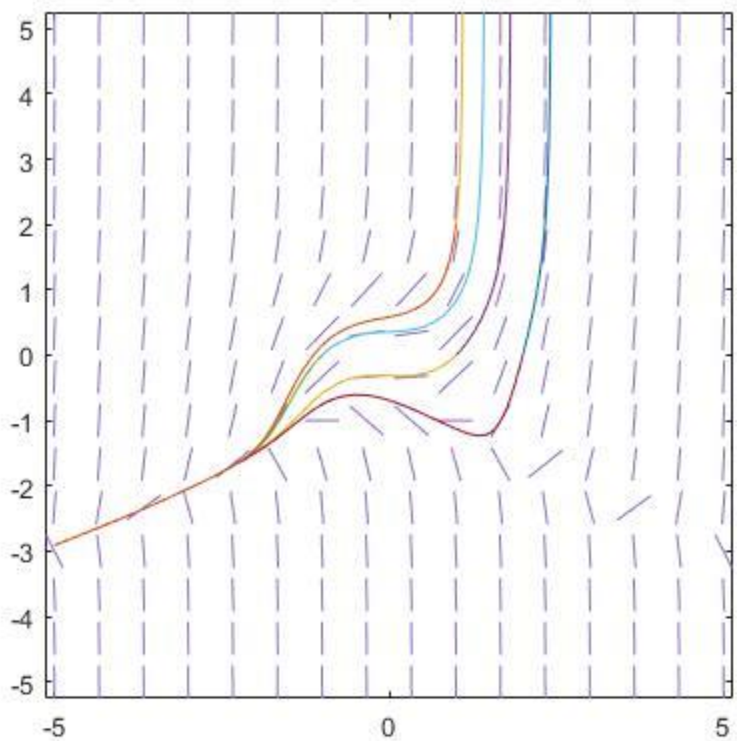
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-5
0
5
10

9. **Exercise 2.9**

   a. Use slopefield to draw a direction field for the differential equation

   y′ = x^2 + y^3,

   and then draw some solution curves on the resulting direction field using drawode. Include your figure in your Word document.



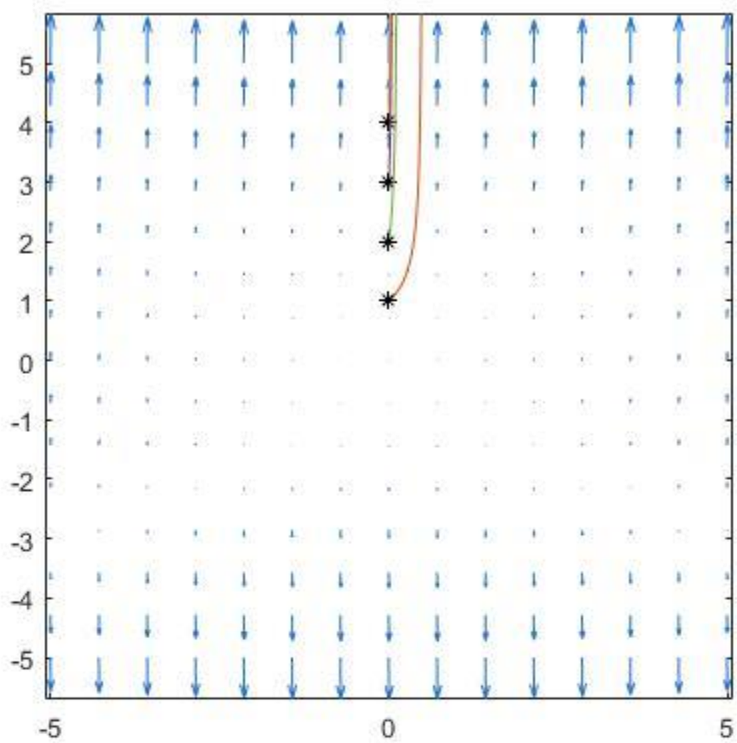
Now consider the system

(13)   dx⁄dt = 1

dy⁄dt = x2 + y3

where x(0) = 0.

Tell MATLAB to create a new figure using the figure command. Then use phaseplane to draw a phase portrait for the system (13), and plot a phase path on your diagram using drawphase; the y1start value represents x(0) and should therefore be zero, while the y2start value can be any initial value for y(0) of your choosing. Finally, try adding a few more phase paths using other values for y(0). Paste the resulting figure into your Word document.



What's the relationship between your phase portrait figures and your direction field figure? [Hint: When you drew the solutions on your direction field, you should have picked several different starting values for x and y. Try drawing some phase paths on your phase portrait with the same initial values. Do the resulting curves match up?]

Observing both graphs, the phase portrait plot and the direction field plot are both very similar to each other as they follow similar behaviors. For example, they have slope of 0 close to t = 0. They also both increase in slope as y increases and likewise a decrease in slop as y decreases. They also look similar and both solution curves are affected by different initial values.

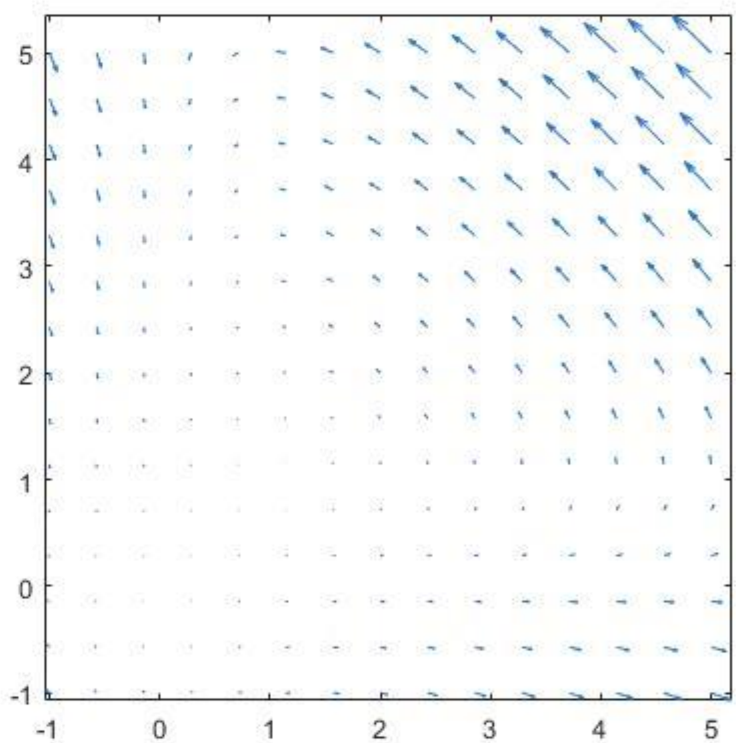
10. Exercise 2.10

a. Use MATLAB to produce a phase plane of the system (14) above, with the parameter values a = b = c = d = 1. Set the minimum values of x and y to -1 and the maximum values to 5. Where in the x-y plane are the physically possible solutions? (Remember, x and y represent populations.)
b. On your phase plane, use drawphase to plot three solution curves in the first quadrant from time zero to time 75. Note that because of rounding error, drawphase might fail to close a loop exactly. Include the graph in your Word document.
c. The predator–prey system has the following states:
   1. The populations of foxes and rabbits are both relatively small.
   2. The small number of foxes allows the rabbit population to increase.
   3. The increased number of rabbits allows the number of foxes to increase.
   4. The increase in the fox population causes the rabbit population to decrease.
   5. The decreased supply of rabbits causes the fox population to decrease, returning to state A.

   On your plot in your Word document, mark one of the solutions (that is, one loop) with the states A, B, C, D, and E at the locations where they occur on that solution.

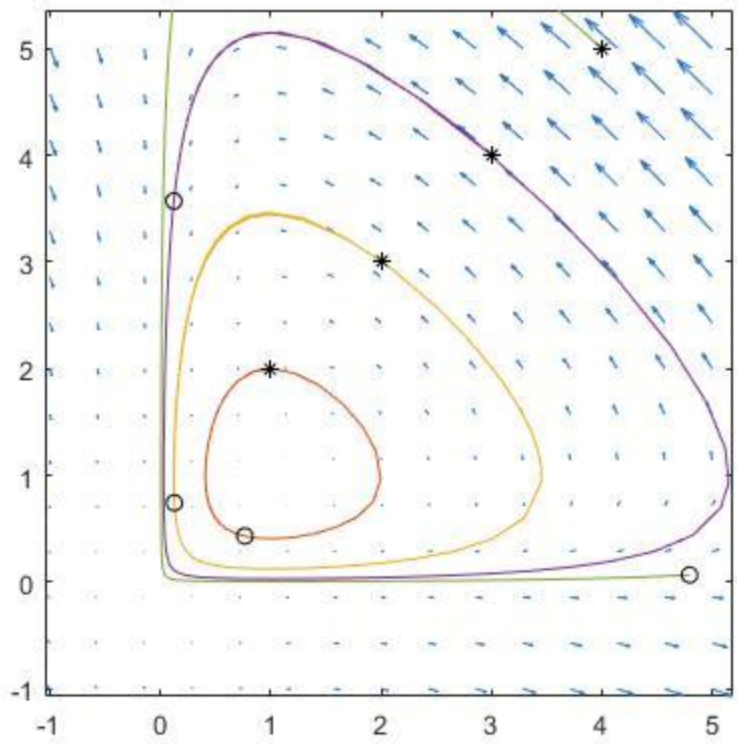
<u>Part A:</u>

```
>> g = @(x,Y)[Y(1)*(1-Y(2));Y(2)*(Y(1)-1)]
phaseplane(g, [-1,5], [-1,5], 15)


g =


  function_handle with value:


    @(x,Y)[Y(1)*(1-Y(2));Y(2)*(Y(1)-1)]
```



We know that x and y represent populations of rabbits and foxes, respectively. Therefore, the first quadrant of the xy-plane would have solutions because populations have to be all positive and cannot take up negative values.

Part B:



Code:

```
g = @(x,Y)[Y(1)*(1-Y(2));Y(2)*(Y(1)-1)]
phaseplane(g, [-1, 5], [-1, 5], 15)
hold on
drawphase(g, 10, 1, 2)
drawphase(g, 10, 2, 3)
drawphase(g, 10, 3, 4)
drawphase(g, 10, 4, 5)
hold offs
```