Math 184A Final Exam

Fall 2016

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. You are free to make use of any result in the textbook or proved in class. You may use up to 12 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please sit in the seat indicated below.

Name:

ID Number:

Seat:

Problem	1	2	3	4	5	6	Total
Score							

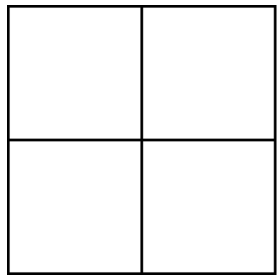
Question 1 (Fibonacci Identity, 15 points)**.** *Recall that the Fibonacci numbers are defined by* $F_1 = F_2 = 1$ *and* $F_{n+2} = F_{n+1} + F_n$ *for all* $n \geq 0$*. Prove that*

$$\sum_{k=1}^{n} F_{2k-1} = F_{2n}$$

for all positive integers n*.*

Question 2 (Equal Length Cycles, 15 points)**.** *How many permutations of* [10] *have all cycles the same length? Any closed form solution is acceptable, you do not need to compute the actual integer.*

Question 3 (Square Coloring, 15 points)**.** *I have a pallet of n colors. In how many ways can I color the squares in the diagram below with these colors so that no two adjacent regions are given the same color? Note: the diagonally opposite regions are not adjacent.*



Question 4 (Generating Function Construction, 15 points)**.** *Let a_n be the number of compositions of n into an even number of parts each of which is at most 5. Give an explicit formula for the generating function*

$$\sum_{n=0}^{\infty} a_n x^n.$$

Question 5 (Identical Parts, 20 points). *Suppose that you are given more than $n/4$ compositions of n into three parts. Show that at least two parts of these compositions are the same (these could be two parts from the same composition or parts from different compositions). Proving that this can be done with a greater number of compositions (i.e. replacing $n/4$ in the problem statement with something larger) will be worth partial credit.*

Question 6 (Ramsey Lower Bound, 20 points). *(a) Prove that for any integers n, m, k that*

$$R(n, m+k-1) \geq R(n, m) + R(n, k) - 1. \qquad \textit{[15 points]}$$

(b) Use this to show that $R(3, 3n+1) \geq 8n+1$ for all $n \geq 1$. You may use the result from part (a), even if you did not complete that part. [5 points]