Math 184A Homework 1

Fall 2015

This homework is due Monday October 5th in discussion section. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeXis recommend though not required.

Question 1 (Summation Polynomials, 40 points). *Use induction to prove the following:*

(a) Show for all positive integers n that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

[10 points]

(b) Show that if p and q are polynomials so that

(i) $q(0) = 0$

(ii) $p(n) = q(n) - q(n-1)$ *for all n*

that

$$\sum_{i=1}^{n} p(i) = q(n)$$

for all positive integers n. [10 points]

(c) Show that for any polynomial p, there exists a polynomial q so that

$$\sum_{i=1}^{n} p(i) = q(n)$$

for all positive integers n. [Hint: use induction on the degree of p and note that $n^d - (n-1)^d = dn^{d-1}$ + lower order terms.] [20 points]

Question 2 (Polygonal Triangulations, 20 points). *Let $n = 2m$ for $m \geq 2$. Show by induction on m that a convex n-gon can be triangulated (that is divided into triangles whose vertices are vertices of the original polygon) in at least 2^{m-1} different ways. [Hint: First cut your polygon into a 4-gon and an $(n-2)$-gon.]*

Question 3 (Proving Pigeonhole by Induction, 20 points). *Prove the pigeon hole principle by induction on the number of holes.*

Question 4 (Repeating Decimals, 20 points). *Show that any rational number (that is a number of the form $\frac{n}{m}$ for integers n, m) has a decimal expansion that repeats after some point. [Hint: Consider computing the decimal expansion using the standard long division algorithm. Show that eventually, the number you are trying to divide by m on the next step repeats.]*

Question 5 (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*