Solutions to HW5 of Math 103A, Fall 2018

P72. Q1.

$$\langle 2, 3\rangle = \langle \gcd(2, 3)\rangle = \langle 1\rangle = \mathbb{Z}_{12}.$$

Q2.

$$\langle 4, 6\rangle = \langle \gcd(4, 6)\rangle = \langle 2\rangle.$$

Q4.

$$\langle 12, 30\rangle = \langle \gcd(12, 30)\rangle = \langle 6\rangle.$$

Q7.

a. $(a^2b)a^3 = a^3b$,

b. $(ab)(a^3b) = ((ab)a^3)b = (a^2b)b = a^2$,

c. $b(a^2b) = (ba^2)b = (a^2b)b = a^2$.

Q8.

Since e, a are connected by an unarrowed solid line, we know $a^2 = e$ and the solid line connecting, say x, y, represents $xa = y$. Similarly, the dash line represents b and $b^2 = e$. Using this, we can write

out the group table:

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Q9.

Similarly, the solid line represents a and the dash line represents b. Thus we have

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	c	b	f	d
b	b	d	e	f	a	c
c	c	f	a	d	e	b
d	d	b	f	e	c	a
f	f	c	d	a	b	e