

# Math 184A Exam 2

Spring 2018

**Instructions:** Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please be sure to sit in the seat indicated below for the exam.

**Name:**

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**ID Number:**

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**Seat:**

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Problem	1	2	3	Total
Score				

**Question 1** (Recurrence Relation Generating Function, 30 points). *Consider the sequence  $a_n$  defined by:*

$$a_n = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n = 1 \\ 3a_{n-1} - a_{n-2} & \text{otherwise.} \end{cases}$$

*Give a closed form formula for the generating function  $A(x) := \sum_{n=0}^{\infty} a_n x^n$ .*

**Question 2** (Committee Forming, 35 points). *Let  $C_n$  be the number of ways of taking a group of  $n$  people, and breaking them up into an odd number of committees so that each person is in exactly one committee and so that each committee has a designated leader. Give a closed form expression for the exponential generating function  $F(x) = \sum_{n=0}^{\infty} C_n x^n / n!$  [Note:  $C_n$  here is as defined above and NOT the Catalan number]*

**Question 3** (Words Without Multiplicity Two Letters, 35 points). *How many six letter words (by which I mean strings of 6 English letters) are there so that no letter occurs exactly twice in this word? Your answer should be a simple expression involving standard operations and standard combinatorial quantities like binomial coefficients and Stirling numbers.*

