3.21

16.2 Line Integrals

1. Evaluate the line integral, where C is the given plane curve.

$$\int_C xy^4 ds, \quad C \text{ is the right half of the circle } x^2 + y^2 = 16$$

2. Evaluate the line integral, where C is the given space curve.

$$\int_C x^2 y ds, \quad C : x = \cos t, y = \sin t, z = t, \ 0 \le t \le \pi/2$$

3. Evaluate the line integral, where C is the given space curve.

$$\int_C y dx + z dy + x dz, \quad C : x = \sqrt{t}, y = t, z = t^2, \ 1 \le t \le 4$$

4. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t)$.

$$\mathbf{F}(x, y) = xy^2\mathbf{i} - x^2\mathbf{j},$$

$$\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le 1$$

16.3 The Fundamental Theorem for Line Integrals

5. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y) = (y^2 - 2x)\mathbf{i} + 2xy\mathbf{j}$$

6. Evaluate the integrals along the given curve C.

$$\mathbf{F}(x, y) = (1 + xy)e^{xy}\mathbf{i} + x^2 e^{xy}\mathbf{j},$$

$$C : \mathbf{r}(t) = \cos t\mathbf{i} + 2\sin t\mathbf{j}, \quad 0 \le t \le \pi/2$$