

Discussion Session 2.7

1. Find the limit.

$$\lim_{t \rightarrow 0} \left(e^{-3t} \mathbf{i} + \frac{t^2}{\sin^2 t} \mathbf{j} + \cos 2t \mathbf{k} \right).$$

2. Find a vector equation and parametric equations for the line segment that joins $P(-2, 1, 0)$ to $Q(5, 2, -3)$.

3. At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

4. Suppose \mathbf{u} and \mathbf{v} are vector functions that possess limits as $t \rightarrow a$ and let c be a constant. Prove the following properties of limits.

(a) $\lim_{t \rightarrow a} [\mathbf{u}(t) + \mathbf{v}(t)] = \lim_{t \rightarrow a} \mathbf{u}(t) + \lim_{t \rightarrow a} \mathbf{v}(t)$

(b) $\lim_{t \rightarrow a} [\mathbf{u}(t) \cdot \mathbf{v}(t)] = \lim_{t \rightarrow a} \mathbf{u}(t) \cdot \lim_{t \rightarrow a} \mathbf{v}(t)$

5. If $\mathbf{r}(t) = \langle t^4, t, t^2 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$.

6. If $\mathbf{r}(t) \neq 0$, show that $\frac{d}{dt} |\mathbf{r}(t)| = \frac{1}{|\mathbf{r}(t)|} \mathbf{r}(t) \cdot \mathbf{r}'(t)$