A Drinfeld presentation of twisted Yangians

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Based on joint work with Weiqiang Wang and Weinan Zhang

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2. Twisted Yangians
3. Main result

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- Yangian $\mathrm{Y}(\mathfrak{gl}_N)$ is a unital associative $\mathbb{C}$-algebra with generators $t_{ij}^{(r)}$ for $1 \leqslant i,j \leqslant N$ and $r > 0$ subject to the relations

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$$T(u) = (t_{ij}(u))_{1\leqslant i,j\leqslant N}, \quad t_{ij}(u) = \delta_{ij} + \sum_{r>0} t_{ij}^{(r)} u^{-r}.$$

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Drinfeld presentation

- Cartan matrix $C = (c_{ij})$ (ADE for simplicity) and Chevalley generators $e_i, f_i, h_i,\ i \in \mathbb{I}$ for $\mathfrak{g}$

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[Drinfeld'88] Drinfeld new/current presentation

$\mathrm{Y}(\mathfrak{g})$ is generated by $\xi_{i,r}, x_{i,r}^{\pm}$, $i \in \mathbb{I}$ and $r \in \mathbb{N}$ subject to

$$[\xi_{i,r}, \xi_{j,s}] = 0 \quad [x_{i,r}^{+}, x_{j,s}^{-}] = \delta_{ij}\xi_{i,r+s} \quad [\xi_{i,0}, x_{j,s}^{\pm}] = \pm c_{ij} x_{j,s}^{\pm}$$

$$[\xi_{i,r+1}, x_{j,s}^{\pm}] - [\xi_{i,r}, x_{j,s+1}^{\pm}] = \pm\frac{c_{ij}}{2}\{\xi_{i,r}, x_{j,s}^{\pm}\}$$

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$$\mathrm{Sym}_{r_1,\cdots,r_n}\big[x_{i,r_1}^{\pm}, [x_{i,r_2}^{\pm}, \cdots [x_{i,r_n}^{\pm}, x_{j,s}^{\pm}] \cdots]\big] = 0, \quad i \neq j, n = 1 - c_{ij}$$

- $\xi_{i,r} \rightsquigarrow h_i z^r, \quad x_{i,r}^{+} \rightsquigarrow e_i z^r, \quad x_{i,r}^{-} \rightsquigarrow f_i z^r$

Isomorphism between RTT and Drinfeld presentations

- Gauss decomposition (special case $N = 2$)

$$T(u) = \begin{bmatrix} 1 & 0 \\ f_{21}(u) & 1 \end{bmatrix} \begin{bmatrix} d_1(u) & 0 \\ 0 & d_2(u) \end{bmatrix} \begin{bmatrix} 1 & e_{12}(u) \\ 0 & 1 \end{bmatrix}$$

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- [Drinfeld'88, Brundan-Kleshchev'05] type A
- [Jing-Liu-Molev'18, Guay-Regelskis-Wendlandt'18] types BCD
  GRW'18 uses different argument – minimalistic presentations

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Symmetric pairs

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- AI (split type A)

$$\theta : e_i \to -f_i, \quad f_i \to -e_i, \quad h_i \to -h_i$$

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- Quasi-split AIII

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τ is a Dynkin diagram automorphism

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$$S^t(-u) = S(u) \pm \frac{S(u) - S(-u)}{2u}$$

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- Evaluation maps $\mathcal{Y}^{tw} \twoheadrightarrow \mathfrak{o}_N$ or $\mathfrak{sp}_{2n}$
- $\mathcal{Y}^{tw}$ is a deformation of twisted current algebra $\mathfrak{g}[z]^{\check{\theta}}$, where

$$\check{\theta} : \mathfrak{g}[z] \to \mathfrak{g}[z], \quad gz^r \mapsto \theta(g)(-z)^r$$

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Goal

Goal: Find a Drinfeld (new) presentation for twisted Yangians.

Drinfeld presentation is useful for Yangians and quantum affine algebras

- finite-dimensional representations [Drinfeld,Chari-Pressley,Kashiwara groups]
- q-characters [Frenkel-Reshetikhin]
- monoidal categorifications [Hernandez-Leclerc,Kashiwara groups]
- vertex representations [Frenkel-Jing'88]
- quantizations of slices of Grassmannians [Kamnitzer-Webster-Weekes-Yacobi]
- $\cdots$ and more

Drinfeld presentation for twisted Yangians

Definition

Let $C = (c_{ij})_{i,j\in\mathbb{I}}$ be Cartan matrix of type A. Define $\mathbb{C}$-algebra $\mathbf{Y}^\imath$ with generators $h_{i,r}, b_{i,s}$ for $i \in \mathbb{I}, r, s \in \mathbb{N}$ and relations

$$[h_{i,r}, h_{j,s}] = 0, \qquad h_{i,2r} = 0$$

$$[h_{i,r+1}, b_{j,s}] - [h_{i,r-1}, b_{j,s+2}] = c_{ij}\{h_{i,r-1}, b_{j,s+1}\} + \frac{1}{4}c_{ij}^2[h_{i,r-1}, b_{j,s}]$$

$$[b_{i,r+1}, b_{j,s}] - [b_{i,r}, b_{j,s+1}] = \frac{c_{ij}}{2}\{b_{i,r}, b_{j,s}\} + 2\delta_{ij}(-1)^r h_{i,r+s+1}$$

$$[b_{i,r}, b_{j,s}] = 0 \qquad (c_{ij} = 0)$$

$$\mathrm{Sym}_{k_1,k_2}\big[b_{i,k_1}, [b_{i,k_2}, b_{j,r}]\big] = (-1)^{k_1+1}[h_{i,k_1+k_2+1}, b_{j,r-1}] \quad (c_{ij} = -1)$$

where $h_{i,-1} = 1$.

Main result

Theorem [L-Wang-Zhang'23]

The twisted Yangian $\mathcal{Y}^{tw}$ (quotient center) is isomorphic to $\mathbf{Y}^{\imath}$.

Explicit isomorphism is given as follows:

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- A suitable solution of reflection equation G
- nontrivial Serre relations

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  - all even nilpotent elements for type BC (type D is in progress) [L-Peng-Tappeiner-Topley-Wang]

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- Finite $\mathcal{W}$-algebra of classical type
  - rectangular case [Brown'07]
  - all even nilpotent elements for type BC (type D is in progress) [L-Peng-Tappeiner-Topley-Wang]
- Quantization of fixed-point subvarieties of slices of affine Grassmannians [L-Wang-Weekes]

Thanks!