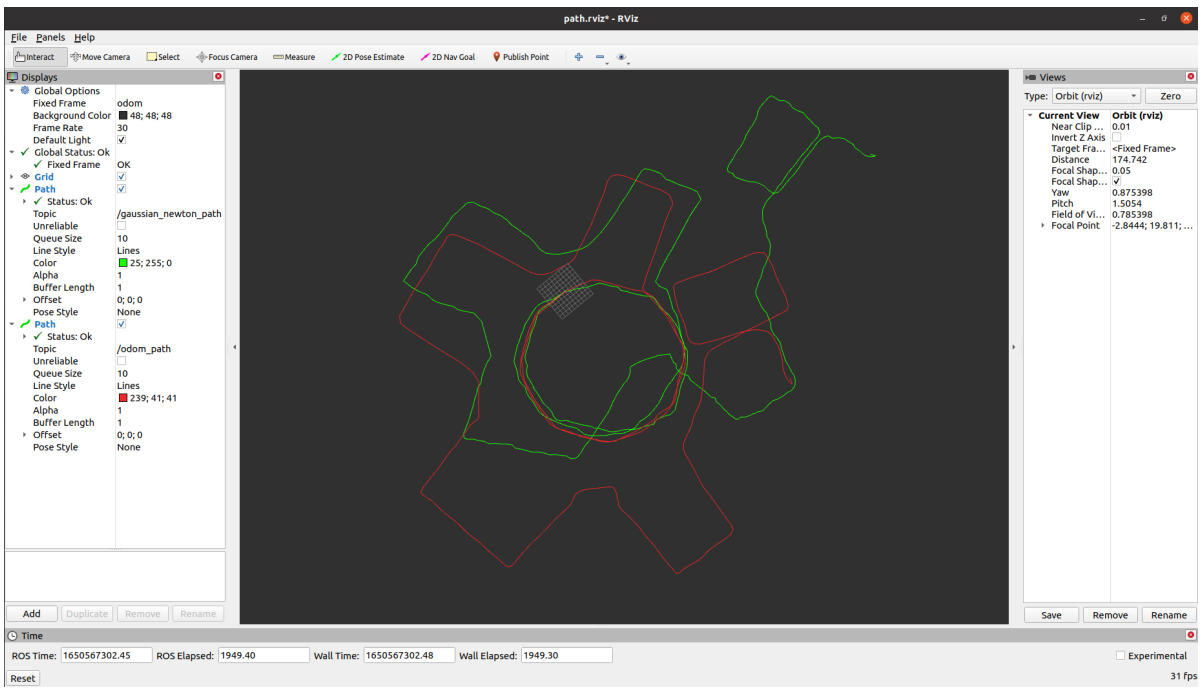
1.See gauss_newton_ws

运行结果：



2. 用LM法来代替GN法求解非线性最小二乘可以让结果更加准确。由于我们GN的H矩阵存在可能为奇异矩阵或者病态的情况，增量的稳定性较差，导致算法不收敛。假设H非奇异也非病态时，如果我们求出来的步长 Δ x太大，也会导致我们采用的局部近似（一阶泰勒展开）不够准确，这样一来我们甚至都无法保证它的迭代收敛。

3. (1) $p = (x, y, \theta)$ is the current pose

x_i are laser points of the current frame

x_i' are laser points projected to the last frame

Σ_i covariance q_i mean of the distribution to x_i'

$$score(p) = \sum_i exp\left(\frac{-(x_i' - q_i)^T \Sigma_i^{-1} (x_i' - q_i)}{2}\right)$$

(2) $f = -score = -\sum_i \exp\left(-\frac{q^t \Sigma_i^{-1} q}{2}\right)$ where $q = x_i' - q_i$

$g = \frac{\partial f}{\partial p}$ where $g_i = \frac{\partial f}{\partial p}$

$H = \frac{\partial g}{\partial p}$ where $H_{ij} = \frac{\partial f}{\partial p_i \partial p_j}$

$g(x) = 0 \Rightarrow g(x+\Delta x) = g(x) + g'(x)\Delta x = 0$

$g'(x)\Delta x = -g(x)$

$H\Delta x = -g$

$g_i = \frac{\partial f}{\partial p} = -\frac{\partial s}{\partial q}\frac{\partial q}{\partial p_i} = q^t \Sigma^{-1} \frac{\partial q}{\partial p_i} \exp\left(-\frac{q^t \Sigma^{-1} q}{2}\right)$

where $J = \frac{\partial q}{\partial p_i} = \begin{pmatrix} 1 & 0 & -x\sin\theta - y\cos\theta \\ 0 & 1 & x\cos\theta - y\sin\theta \end{pmatrix}$

$H_{ij} = -\frac{\partial s}{\partial p_i \partial p_j}$

$= -\exp\left(\frac{-q^t \Sigma^{-1} q}{2}\right)\left(\left(-q^t \Sigma^{-1}\frac{\partial q}{\partial p_i}\right)\left(-q^t \Sigma^{-1}\frac{\partial q}{\partial p_j}\right) + \left(-q^t \Sigma^{-1}\frac{\partial^2 q}{\partial p_i \partial p_j}\right) + \left(-\frac{\partial q^t}{\partial q_j}\Sigma^{-1}\frac{\partial q}{\partial p_i}\right)\right)$

where $\frac{\partial^2 q}{\partial p_i \partial p_j} = \begin{cases} \begin{pmatrix} -x\cos\theta + y\sin\theta \\ -x\sin\theta - y\cos\theta \end{pmatrix} & i = j = 3 \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \text{otherwise} \end{cases}$

4. 1. 遍历粗分辨率下4个位置，先使best _ score = – ∞，第一次选取最大的99分粗分辨率节点

2. 选取的99分粗分辨率节点为根节点，进行分枝，进入细分辨率节点，遍历后选取最大的87分的细分辨率节点，更新best _ score = 87

3. 返回粗分辨率节点，第一个节点是85分，小于best_score best，剪枝

4. 在粗分辨率节点继续查找，到第三个节点，98分大于best_score，进入细分辨率节点，遍历后得到95分节点大于best_score，更新best_score = 95

5. 返回粗分辨率节点继续查找，目前循环到第四节点，96分大于best_score，进入细分辨率节点，遍历后所有叶子节点都小于best_score ，返回粗分辨率节点，粗分辨率节点为空，结束