

$$\begin{aligned}
 1. \text{左扰动: } \frac{\partial R\rho}{\partial R} &= \frac{\partial R\rho}{\partial \varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\exp(\varphi^T) \exp(\varphi^T) \rho - \exp(\varphi^T) \rho}{\varphi} \\
 &\approx \lim_{\varphi \rightarrow 0} \frac{(I + \varphi^T) \exp(\varphi^T) \rho - \exp(\varphi^T) \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\varphi^T \exp(\varphi^T) \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{-(\exp(\varphi^T) \rho)^T \varphi}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} -(\exp(\varphi^T) \rho)^T \\
 &= -(R\rho)^T
 \end{aligned}$$

$$\begin{aligned}
 \text{右扰动: } \frac{\partial R\rho}{\partial R} &= \frac{\partial R\rho}{\partial \varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\exp(\varphi^T) \exp(\varphi^T) \rho - \exp(\varphi^T) \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\exp(\varphi^T) (I + \varphi^T) \rho - \exp(\varphi^T) \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\exp(\varphi^T) \varphi^T \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{R \varphi^T \rho}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} -R \rho^T \varphi \\
 &= -R \rho^T
 \end{aligned}$$

2. 对 R_1 求导

$$\begin{aligned}
 \text{左扰动: } \frac{\partial \ln(R_1 R_2)^T}{\partial R_1} &= \frac{\partial \ln(\exp(\varphi_1^T) R_1 R_2)^T}{\partial \varphi_1} \\
 &\approx \frac{\partial (\ln(R_1 R_2)^T + J_1^{-T} \varphi_1)}{\partial \varphi_1} \\
 &= J_1^{-T}
 \end{aligned}$$

$$\begin{aligned}
 \text{右扰动: } \frac{\partial \ln(R_1 R_2)^T}{\partial R_1} &= \frac{\partial \ln(R_1 \exp(\varphi_1^T) R_2)^T}{\partial \varphi_1} \\
 &\approx \frac{\partial \ln(R_1 R_2 \exp(R_2^T \varphi_1)^T)^T}{\partial \varphi_1} \\
 &= \frac{\partial (\ln(R_1 R_2)^T + J_2^{-T} R_2^T \varphi_1)}{\partial \varphi_1} \\
 &= J_2^{-T} R_2^T
 \end{aligned}$$

对 R_2 求导

$$\begin{aligned} \text{左扰动: } \frac{\partial \ln(R_1 R_2)^V}{\partial R_2} &= \frac{\partial \ln(R_1 \exp(\varphi_2) R_1)^V}{\partial R_2} \\ &= \frac{\partial \ln(R_1 R_2 \exp(R_2^T \varphi_2))^V}{\partial \varphi_2} \\ &\approx \frac{\partial (\ln(R_1 R_1)^V + J_r^{-1} R_2^T \varphi_2)}{\partial \varphi_2} \end{aligned}$$

$$\begin{aligned} \text{右扰动: } \frac{\partial \ln(R_1 R_2)^V}{\partial R_1} &= \frac{\partial \ln(R_1 R_2 \exp(\varphi_2))^V}{\partial \varphi_2} \\ &\approx \frac{\partial (\ln(R_1 R_2)^V + J_r^{-1} \varphi_2)}{\partial \varphi_2} \\ &= J_r^{-1} \end{aligned}$$