1. 左扰动：$\frac{\partial Rp}{\partial R} = \frac{\partial Rp}{\partial \varphi}$

$$= \lim_{\varphi \to 0} \frac{\exp(\varphi^\wedge)\exp(\phi^\wedge)p - \exp(\phi^\wedge)p}{\varphi}$$

$$\approx \lim_{\varphi \to 0} \frac{(I+\varphi^\wedge)\exp(\phi^\wedge)p - \exp(\phi^\wedge)p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\varphi^\wedge \exp(\phi^\wedge)p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{-(\exp(\phi^\wedge)p)^\wedge \varphi}{\varphi}$$

$$= \lim_{\varphi \to 0} -(\exp(\phi^\wedge)p)^\wedge$$

$$= -(Rp)^\wedge$$

右扰动：$\frac{\partial Rp}{\partial R} = \frac{\partial Rp}{\partial \varphi}$

$$= \lim_{\varphi \to 0} \frac{\exp(\phi^\wedge)\exp(\varphi^\wedge)p - \exp(\phi^\wedge)p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\exp(\phi^\wedge)(I+\varphi^\wedge)p - \exp(\phi^\wedge)p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\exp(\phi^\wedge)\varphi^\wedge p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{R\varphi^\wedge p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{-Rp^\wedge \varphi}{\varphi}$$

$$= -Rp^\wedge$$

2. 对 R_1 求导

左扰动：$\frac{\partial \ln(R_1R_2)^\vee}{\partial R_1} = \frac{\partial \ln(\exp(\varphi_1^\wedge)R_1R_2)^\vee}{\partial \varphi_1}$

$$\approx \frac{\partial (\ln(R_1R_2)^\vee + J_l^{-1}\varphi_1)}{\partial \varphi_1}$$

$$= J_l^{-1}$$

右扰动：$\frac{\partial \ln(R_1R_2)^\vee}{\partial R_1} = \frac{\partial \ln(R_1\exp(\varphi_1^\wedge)R_2)^\vee}{\partial \varphi_1}$

$$\approx \frac{\partial \ln(R_1R_2\exp((R_2^T\varphi_1)^\wedge))^\vee}{\partial \varphi_1}$$

$$= \frac{\partial ((\ln(R_1R_2)^\vee + J_r^{-1}R_2^T\varphi_1)}{\partial \varphi_1}$$

$$= J_r^{-1}R_2^T$$

对 R_2 求导

左扰动：$\frac{\partial \ln(R_1 R_2)^\vee}{\partial R_2} = \frac{\partial \ln(R_1 \exp(\varphi_2) R_2)^\vee}{\partial R_2}$

$= \frac{\partial \ln(R_1 R_2 \exp(R_2^T \varphi_2))^\vee}{\partial \varphi_2}$

$\approx \frac{\partial (\ln(R_1 R_2)^\vee + J_r^{-1} R_2^T \varphi_2)}{\partial \varphi_2}$

$= J_r^{-1} R_2^T$

右扰动：$\frac{\partial \ln(R_1 R_2)^\vee}{\partial R_2} = \frac{\partial \ln(R_1 R_2 \exp(\varphi_2))}{\partial \varphi_2}$

$\approx \frac{\partial (\ln(R_1 R_2)^\vee + J_r^{-1} \varphi_2)}{\partial \varphi_2}$

$= J_r^{-1}$