① $\forall a, b \in \mathbb{R}^3, \exists c = a \hat{} b$

$\forall R \in SO(3)$ $\quad Rc = R(a \hat{} b) = (Ra) \hat{} (Rb)$

$$c = R^{-1}(Ra) \hat{} Rb$$

$$a \hat{} b = R^{-1}(Ra) \hat{} Rb$$

$$a \hat{} = R^{-1}(Ra) \hat{} R$$

$$R a \hat{} = (Ra) \hat{} R$$

$$R a \hat{} R^T = (Ra) \hat{}$$

② Let $\vec{v} \in \mathbb{R}^3$

$$(Rp) \times \vec{v} = (Rp) \times (RR^T)\vec{v}$$

$$= R(p \times (R^T \vec{v}))$$

$$(Rp) \hat{} \vec{v} = R(p \hat{} (R^T \vec{v}))$$

$$(Rp) \hat{} \vec{v} = Rp \hat{} R^T \vec{v}$$

$$(Rp) \hat{} = Rp \hat{} R^T$$