$\xi = (\rho, \phi)^T \quad \xi^\wedge = \begin{pmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{pmatrix}$

$(\xi^\wedge)^n = \begin{pmatrix} (\phi^\wedge)^n & (\phi^\wedge)^{n-1}\rho \\ 0^T & 0 \end{pmatrix}$ for $n \geq 1$

$$\exp(\xi^\wedge) = \sum_{n=0}^{\infty} \frac{1}{n!}(\xi^\wedge)^n = \sum_{n=0}^{\infty} \frac{1}{n!}\begin{pmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{pmatrix}^n$$

$$= I + \sum_{n=1}^{\infty} \frac{1}{n!}\begin{pmatrix} (\phi^\wedge)^n & (\phi^\wedge)^{n-1}\rho \\ 0^T & 0 \end{pmatrix}$$

$$= I + \begin{pmatrix} \sum_{n=1}^{\infty} \frac{1}{n!}(\phi^\wedge)^n & \sum_{n=1}^{\infty} \frac{1}{n!}(\phi^\wedge)^{n-1}\rho \\ 0^T & 0 \end{pmatrix}$$

$$= I + \begin{pmatrix} \sum_{n=1}^{\infty} \frac{1}{n!}(\phi^\wedge)^n & \sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\phi^\wedge)^n\rho \\ 0^T & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sum_{n=0}^{\infty} \frac{1}{n!}(\phi^\wedge)^n & \sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\phi^\wedge)^n\rho \\ 0^T & I \end{pmatrix}$$

Let $\phi = \theta a$,

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\phi^\wedge)^n = \sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\theta a^\wedge)^n$$

$$= I + \frac{1}{2!}(\theta a^\wedge) + \frac{1}{3!}(\theta a^\wedge) + \frac{2}{4!}(\theta a^\wedge)^3 + \dots$$

$$= I + \left(\frac{1}{2!}\theta - \frac{1}{4!}\theta^3 + \dots\right)a^\wedge + \left(\frac{1}{3!}\theta^2 - \frac{1}{5!}\theta^4 + \dots\right)a^\wedge a^\wedge$$

$$= I + \frac{1}{\theta}\left(\frac{1}{2!}\theta^2 - \frac{1}{4!}\theta^4 + \dots\right)a^\wedge + \frac{1}{\theta}\left(\frac{1}{3!}\theta^3 - \frac{1}{5!}\theta^5 + \dots\right)(aa^T - I)$$

$$= I + \frac{1}{\theta}(1-\cos\theta)a^\wedge + \frac{1}{\theta}\left(\frac{1}{3!}\theta^3 - \frac{1}{5!}\theta^5 + \dots\right)aa^T -$$

$$\frac{1}{\theta}\left(\frac{1}{3!}\theta^3 - \frac{1}{5!}\theta^5 + \dots\right)I$$

$$= \frac{1}{\theta}(1-\cos\theta)a^\wedge + \frac{1}{\theta}\left(\frac{1}{3!}\theta^3 - \frac{1}{5!}\theta^5 + \dots\right)aa^T +$$

$$\frac{1}{\theta}\left(\theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \dots\right)I$$

$$= \frac{1}{\theta}(1-\cos\theta)a^\wedge + \frac{1}{\theta}\left(\frac{1}{3!}\theta^3 - \frac{1}{5!}\theta^5 + \dots\right)aa^T + \frac{1}{\theta}\sin\theta I$$

$\triangleq J$