What solution is obtained? Does this solution vary as a function of the initial guess and if so why?

How to run:

Open the terminal under the directory which save c1749059.py.

Input the command: python c1749059.py

How to test:

Change the initial guess number in c1749059.py file. The variable is x0.

When initial guess is equal to 1.0

Optimization terminated successfully. (Exit mode 0)

Current function value: -0.9999999999999649

Iterations: 4

Function evaluations: 4

Gradient evaluations: 4

[ 1.57079606]

When initial guess is equal to 2.0

Optimization terminated successfully. (Exit mode 0)

Current function value: -0.9999999107300288

Iterations: 3

Function evaluations: 3

Gradient evaluations: 3

[ 1.57037379]

When initial guess is equal to 5.0

Optimization terminated successfully. (Exit mode 0)

Current function value: -0.9999996868852562

Iterations: 4

Function evaluations: 5

Gradient evaluations: 4

[ 7.85319029]

When initial guess is equal to 10.0

Optimization terminated successfully. (Exit mode 0)

Current function value: -0.9999999999535361

Iterations: 6

Function evaluations: 7

Gradient evaluations: 6

[ 7.85399127]

Yes, the solution varies as a function of the initial guess.

It is because the function is harmonic functions, which means the function have infinite maximum. The solution is harmonic. In other words, the solution will change with the change of initial guess.

1. Is a randomly selected student more likely to be an undergraduate or graduate student? In your answer state all steps taken to obtain your solution.

Suppose there are two kinds of student, student = Graduate and student = undergraduate

P(student=graduate) = 0.3, P(student=undergraduate) = 0.7

A random selected student is more likely to be an undergraduate.

Since P(student= undergraduate) = 0.7 > P(student=graduate) = 0.3

It is more likely to be an undergraduate. Because the percentage of students who are graduate students is 30%, and the percentage of students who are undergraduate is 70%.

1. What is the probability that a female student is a graduate student? In your answer state all steps taken to obtain this probability.

Since the gender and undergraduate and graduate is not independent.

Suppose student = graduate as G and student= undergraduate as U and gender= Female as F, we can get:

Female in graduate = F|G: P (F|G) = 0.33

Female in undergraduate = F|U: P (F|U) = 0.25

Female student is graduate student: G|F, it is probability is: P (G|F)

P(G|F)

= P(F|G) \* P(G)/ P(F)

= P (F|G) \* P (G)/ (P (G)\*P (F|G) +P (F|U) \*P (U))

=0.33 \* 0.3/ (0.33\*0.3+0.25\*0.7)

=0.361

The probability that a female student is a graduate student is 0.569.

Suppose that 30% of graduate students attend all their lectures and 10% of undergraduate students attend all their lectures. If a student is female and attends all her lectures, is she more likely to be a graduate or undergraduate student. You may assume conditional independence between the probability that a student attends all their lectures and the probability that a student is male or female. In your answer state all steps taken to obtain your solution. [2 marks].

Suppose the event “attend” as “A”

Since the gender and attendance is independent

The 30% of graduate students attend all their lectures: P (A|G) =0.3

The 10% of undergraduate students attend all their lectures: P (A|U) =0.1

Then,

P (G|FA) =P (FA|G) \*P (G)/P (FA|G) \*P(G) + P(FA|U) \*P(U)

=P (F|G) \*P (A|G) \*P (G)/P(F|G) \*P(A|G) \*P(G)+P(F|U) \*P(A|U) \*P(U)

= 0.33\*0.3\*0.3/ (0.33\*0.3\* 0.3+0.25\*0.1\*0.7)

=0.629

P (U|FA) =P (FA|U) \*P (U)/ P (FA|U) \*P(U) + P(FA|G) \*P(G)

=P (F|U) \*P (A|U) \*P (U)/P(F|U) \*P(A|U) \*P(U) + P(F|G)\*P(A|G)\*P(G)

=0.25\*0.1\*0.7/ 0.25\*0.1\*0.7+0.33\*0.3\*0.3

=0.371

So P (G|FA) =0.629>P (U|FA) =0.371

She more likely to be a graduate.

Question 3

1. Estimate the conditional probabilities P(X=1|Class=T) and P(Z=1|Class=F). In your answer state all steps taken to obtain your solution.

P(X=1|Class=T) = P (X=1 ∩ Class=T)/ P (Class=T) = (3/10)/ (5/10) = 3/5

P (Z=1|Class=F) = P (Z=1 ∩ Class=F)/ P (Class=F) = (1/10)/ (5/10) = 1/5

1. Compare P(X=1), P(Y=1) and P (X=1, Y=1). State the relationship between X and Y. In your answer state all steps taken to obtain your solution. [2 marks]

P(X=1) = 5/10

P(Y=1) = 4/10

P (X=1, Y=1) = 2/10

If and only if A and B are independent:

Then we have:

P(A) \* P(B) = P(A∩B)

In this case, A and B are conditionally independent.

1. Compare P (X=1, Y=1|Class=T) with P(X=1|Class=T) and P(Y=1|Class=T). Are the random variables X and Y conditionally independent given the class? In your answer state all steps taken to obtain your solution. [2 marks]

In the standard notation of probability theory, X and Y are conditionally independent given Z if and only if:

P (X, Y| Z) =P (X | Z) \* P(Y|Z)

P (X, Y| Z) = P (X=1, Y=1|Class=T) = P (X=1 ∩ Y=1 ∩ Class=T)/ P(Class=T) = (1/10)/ (5/10) =1/5

P (X | Z) = P(X=1|Class=T) = P (X=1 ∩ Class=T)/ P(Class=T) = (3/10)/ (5/10) =3/5

P(Y|Z) = P(Y=1|Class=T) = P (Y=1 ∩ Class=T)/ P(Class=T) = (2/10)/ (5/10) =2/5

1/5 ≠ 3/5 \* 2/5

So, X and Y are not conditionally independent given the class.