

3COLOR is in NP because a coloring can be verified in polynomial time. We show $3\text{SAT} \leq_P 3\text{COLOR}$. Let $\phi = c_1 \wedge c_2 \wedge \cdots \wedge c_l$ be a 3cnf formula over variables x_1, x_2, \dots, x_m , where the c_i 's are the clauses. We build a graph G_ϕ containing $2m + 6l + 3$ nodes: 2 nodes for each variable; 6 nodes for each clause; and 3 extra nodes. We describe G_ϕ in terms of the subgraph gadgets given in the Hint.

G_ϕ contains a variable gadget for each variable x_i , two OR-gadgets for each clause, and one palette gadget. The four bottom nodes of the OR-gadgets will be merged with other nodes in the graph, as follows. Label the nodes of the palette gadget T, F, and R. Label the nodes in each variable gadget $+$ and $-$ and connect each to the R node in the palette gadget. In each clause, connect the top of one of the OR-gadgets to the F node in the palette. Merge the bottom node of that OR-gadget with the top node of the other OR-gadget. Merge the three remaining bottom nodes of the two OR-gadgets with corresponding nodes in the variable gadgets so that if a clause contains the literal x_i , one of its bottom nodes is merged with the $+$ node of x_i whereas if the clause contains the literal $\overline{x_i}$, one of its bottom nodes is merged with the $-$ node of x_i .