CSCE 222 [Section 501] Discrete Structures for Computing
Spring 2019 – Hyunyoung Lee

Problem Set 3

Due dates: Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Friday, 2/15/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

Name: Kim Nguyen **Section: 501**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: Kim Nguyen

***** Please make sure that you are solving the correct problems from the 8th Edition of the Rosen book, not the 7th Edition! *****

Problem 1. (2 points × 6 subproblems = 12 points) Section 2.1, Exercise 10, page 132.

Solution. .
a) 2 is not an element of that set.
b) 2 is not an element of that set.
c) 2 is an element of that set.
d) 2 is an element of that set.
e) 2 is an element of that set.
f) 2 is not an element of that set.

Problem 2. (2.5 points × 4 subproblems = 10 points) Section 2.1, Exercise 26, page 132.

Solution. a) This set is not a power set of any set.
b) This set is a power set of B =

$$\{ \text{ a } \}$$

c) This set is a power set of C =

$$\{ \text{ } \emptyset\text{,a } \}$$

d) This set is a power set of D =

$$\{ \text{ a,b } \}$$

Problem 3. (10 points) Section 2.1, Exercise 28, page 132. *Use definitions and justify each step of your argument.*

Solution. Let A×B = $\{x|$ x ∈ A ∧ x ∈ B $\}(by definition of cartesian products)$, C×D = $\{x|$ x ∈ C ∧ x ∈ D $\}$, $(by definition of cartesian products)$ A⊆C, and B⊆D.
Then
(A⊆C $\iff$ (x∈A ⇒ x∈C)) ∧ (B⊆D $\iff$ (y∈B ⇒ y∈D)).
Then (x,y)∈A×B ⇒ (x,y)∈C×D. Thus we can conclude that A×B ⊆ C×D.

Problem 4. (2 points × 4 subproblems = 8 points) Section 2.2, Exercise 4, page 144.

Solution. .

a) A ∪ B = $\{a, b, c, d, e, f, g, h\}$
b) A ∩ B = $\{a, b, c, d, e\}$
c) A - B = ∅
d) B - A = $\{f, g, h\}$

Problem 5. (5 points × 2 subproblems = 10 points) Section 2.2, Exercise 16 c) and d), page 144. *Use definitions, and explain each step using definitions and/or laws.*

Solution. .

c) A - B $\iff$ $\{x|$ x ∈ A ∧ x ∉ B$\}$(by definition of sets)
⊆ $\{x|$ x ∈ A $\}$= A (by definition of subsets)
d) A ⌒ (B - A) = ∅
$\iff$ $\{x|$ x ∈ A $\}$⌒ $\{x|$ x ∈ B ∧ x ∉ A$\}$=∅ (by definition of sets)
$\iff$ A ⌒ $\overline{\text{A}}$ ⌒ B = ∅ (from above)
$\iff$ (A ⌒ $\overline{\text{A}}$) ⌒ B = ∅ (by associative law)
$\iff$ ∅ ⌒ B = ∅ (since a and not a = ∅)
$\iff$ ∅ = ∅ (since x ⌒ ∅ = ∅ for all x)

Problem 6. (5 points × 2 subproblems = 10 points) Section 2.2, Exercise 56 a) and c), page 145.

Solution. .
a) $\bigcup_{i=1}^{\infty} A_i$ = all positive integers
$\bigcap_{i=1}^{\infty} A_i = \infty$
c) $\bigcup_{i=1}^{\infty} A_i$ = all real numbers
$\bigcap_{i=1}^{\infty} A_i$ = [-1,1]

Problem 7. (2.5 points × 4 subproblems = 10 points) Section 2.3, Exercise 12, page 162.

Solution. a) This function is one-to-one.
b) This function is not one-to-one.
c) This function is one-to-one.
d) This function is not one-to-one.

Problem 8. (2.5 points × 2 subproblems = 5 points) Section 2.3, Exercise 14 a) and b), page 162.

Solution. a) onto
b) not onto

Problem 9. (2.5 points × 4 subproblems = 10 points) Section 2.3, Exercise 60, page 164.

Solution. a) One byte
b) Two bytes
c) 63 bytes
d) 375 bytes

Problem 10. (15 points) Prove that

$$\left\lceil \left\lceil \frac{x}{2} \right\rceil / 2 \right\rceil = \left\lceil \frac{x}{4} \right\rceil$$

holds for all real numbers x. *Use the definition of the ceiling function as we discussed in class.*

Solution. .
Let $\lceil \lceil x/2 \rceil /2 \rceil = k \iff k - 1 < \lceil x/2 \rceil /2 \leq k$ and $\lceil x/2 \rceil = j$
$\iff j - 1 < x/2 \leq j \iff 2j - 2 < x \leq 2j$. Then
$k - 1 < \lceil x/2 \rceil /2 \leq k \iff k - 1 < j/2 \leq k \iff 2k - 2 < j \leq 2k$
$\iff 4k - 4 < 2j \leq 4k \iff 4k - 4 < x \leq 4k \iff k - 1 < x/4 \leq k$.
Thus we can conclude that $\lceil \lceil x/2 \rceil /2 \rceil = \lceil x/2 \rceil = k$.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you electronically sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?

LaTeX symbols for sets and functions

1. Set of integers that are less than or equal to n: $\{x \in \mathbf{Z} \mid x \leq n\}$
2. x is a real number: $x \in \mathbf{R}$
3. x is not an integer: $x \notin \mathbf{Z}$
4. Cardinality of set A: $|A|$
5. Union of set A and set B: $A \cup B$
6. Generalized union: $\bigcup_{i=1}^{\infty} A_i$
7. Intersection of set A and set B: $A \cap B$
8. Generalized intersection: $\bigcap_{i=1}^{\infty} A_i$
9. The empty set: $\emptyset$
10. Set A is a subset of set B: $A \subseteq B$
11. Set A is a proper subset of set B: $A \subset B$
12. Cartesian product of set A and set B: $A \times B$
13. Complement of set A: A^C or $\overline{A}$
14. Ellipsis: $\ldots$ or $\cdots$
15. Ceiling function: $\lceil 3.14 \rceil = 4$
16. Floor function: $\lfloor 3.14 \rfloor = 3$
17. Square root: $\sqrt{b^2 - 4ac}$