CSCE 222 [Section 501] Discrete Structures for Computing
Spring 2019 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Friday, 3/8/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Kim Nguyen**

Total 100 (+ 20 extra) points.

Problem 1. (4 pts $\times$ 5 = 20 points) Section 2.4, Exercise 10, page 177

Solution. .
a) $a_n = 6a_{n-1}$, $a_0 = 2$
$a_0 = 2, a_1 = 12, a_2 = 72, a_3 = 432, a_4 = 2592, a_5 = 15552, a_6 = 93312$
b) $a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$
$a_0 = 2, a_1 = -1, a_2 = -3, a_3 = -2, a_4 = 1, a_5 = 3, a_6 = 2$
c) $a_n = 3a_{n-1}^2$, $a_0 = 1$
$a_0 = 1$, $a_1 = 3$, $a_2 = 27$, $a_3 = 2187$, $a_4 = 14348907$, $a_5 = 6.1767E14$, $a_6 = 1.1446E30$
d) $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$
$a_0 = -1$, $a_1 = 0$, $a_2 = 1$, $a_3 = 3$, $a_4 = 13$, $a_5 = 74$, $a_6 = 613$
e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$
$a_0 = 1$, $a_1 = 1$, $a_2 = 2$, $a_3 = 2$, $a_4 = 1$, $a_5 = 1$, $a_6 = 1$

Problem 2. (5 pts $\times$ 4 = 20 points) Section 2.4, Exercise 16 a)–d), page 178

Solution. .
a) $a_n = 5(-1)^n$
b) $a_n = 3n + 1$
c) $a_n = 4 - \frac{n(n+1)}{2}$
d) $a_n = -1 - n^2$

Problem 3. (5 pts $\times$ 2 = 10 points) Section 2.4, Exercise 34 a) and c), page 179

Solution. .
a) -3
c) 9

Problem 4. (12 points) Section 5.1, Exercise 8, page 350

Solution. .

$\underline{InductionBasis:}$ Since $P(1) : 2(-7)^1 = \frac{1-(-7)^2}{4} = -14$ is true, then the assertion $P(n)$ is true.

$\underline{InductionStep:}$ Suppose $P(n)$ holds for all integers $n > 0$, then $P(n+1)$: $\frac{1-(-7)^{n+1}}{4} + 2(-7)^{n+1} = \frac{1-(-7)^{n+1}+8(-7)^{n+1}}{4} = \frac{1-(1-8)(-7)^{n+1}}{4} = \frac{1-(-7)^{n+2}}{4}$. Hence $P(n+1)$ holds, thus we can conclude that $P(n)$ implies $P(n+1)$.

Problem 5. (12 points) Section 5.1, Exercise 10, page 350

Solution. .
a) $\frac{n}{n+1}$
b)
$\underline{InductionBasis:}$ Since $P(1) : \frac{1}{1+1} = \frac{1}{2}$ is true then the assertion $P(n)$ is true.

$\underline{InductionStep:}$ Suppose $P(n)$ holds for all integers $n > 0$, then
$P(n+1) : \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+1)(n+2)+(n+1)}{(n+2)(n+1)^2} = \frac{n^2+2n+1}{(n+2)(n+1)} = \frac{n+1}{n+2}$.
Hence $P(n+1)$ holds and thus we can conclude that $P(n)$ implies $P(n+1)$

Problem 6. (12 points) Section 5.1, Exercise 12, page 351

Solution. .
$\underline{InductionBasis:}$ Since $P(1) : 1 - \frac{1}{2} = \frac{3}{(3)(2)} = \frac{1}{2}$ is true, then the assertion $P(n)$ is true.

$\underline{InductionStep:}$ Suppose $P(n)$ holds for all integers $n > 0$, then
$P(n+1) : \frac{2^{n+1}+(-1)^n}{3\cdot 2^n} + (\frac{1}{2})^{n+1} = \frac{2^{n+1}+(-1)^n}{3\cdot 2^n} + \frac{1}{2^{n+1}} = \frac{2^{2n+2}+}{}$

Problem 7. (12 points) Section 5.1, Exercise 14, page 351

Solution. .
$\underline{InductionBasis:}$ Since $P(1) : (1-1)2^{n+1} + 2 = 1 \cdot 2^1 = 2$ is true, then the assertion $P(n)$ is true.

$\underline{InductionStep:}$ Suppose $P(n)$ holds for all integers $n > 0$, then
$P(n+1) : (n-1)2^{n+1} + 2 + (n+1)2^{n+1} = (2n)2^{n+1} + 2 = n2^{n+2} + 2$. Hence $P(n+1)$ holds and thus we can conclude that $P(n)$ implies $P(n+1)$

Problem 8. (12 points) Section 5.1, Exercise 34, page 351

Solution. .
<u>*InductionBasis* :</u> Since $P(0) : 0^3 - 0 = 0$ which is divisible by 6. Thus the assertion P(n) is true.

<u>*InductionStep*</u> : Suppose $P(n)$ holds for all integers $n > 0$, then $P(n+1) : (n+1)^3 - (n+1) = (n^3 - n) + 3n(n+1)$. One of n or $n+1$ is even so $n(n+1)$ must be even and thus two divides $3n(n+1)$ and 3 also divides that factor. Thus we can see that six divides $n^3 - n$ for all non-negative integers. Hence $P(n)$ holds and thus we can conclude that $P(n)$ implies $P(n+1)$

Problem 9. (10 points) Section 5.1, Exercise 50, page 352

Solution. The base case is incorrect and thus the proof by induction is incorrect.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you electronically sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?