Applied Mathematical Modelling 52 (2017) 747–769

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Coordinating a three-echelon supply chain under price and quality dependent demand with sub-supply chain and RFM strategies

B.C. Giri[a], B. Roy[a,*], T. Maiti[a,b]

[a] Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India
[b] Department of Mathematics, Gurudas College, Kolkata- 700054, West Bengal, India

ARTICLE INFO

Article history:
Received 4 February 2016
Revised 23 March 2017
Accepted 23 May 2017
Available online 31 May 2017

Keywords:
Supply chain
Price and quality dependent demand
Sub-supply chain
Retail fixed markup

ABSTRACT

The paper considers a three-echelon supply chain which consists of one supplier, one manufacturer and one retailer for trading a single product. The market demand at the retailer is influenced by the retail price and the quality of the product. The quality of the finished product at the manufacturer depends on the supplier's raw material quality. We analyze the model for both deterministic and stochastic demand patterns. We first study the centralized and decentralized systems, and then the decentralized system with a sub-supply chain coordination strategy (where the manufacturer chooses to merge with either the supplier or the retailer and then acts as a single entity) and the two-level retail fixed mark-up (RFM) strategy. In the case of the two-level RFM strategy, the manufacturer and the retailer use fixed mark ups over the supplier's wholesale price. The proposed models are demonstrated through numerical examples. It is observed from the numerical study that the two-level RFM strategy is superior to the sub-supply chain coordination strategy. Further, the two-level RFM strategy in the stochastic demand scenario is not as effective as in the deterministic demand scenario.

1. Introduction

In today's competitive market, a business enterprise has to focus on proper demand analysis and coordination among supply chain entities in order to run the business efficiently. To analyze demand properly, we need to know the influential factors that can encourage customers to purchase a product. From real life situations, it can be observed that there are many factors such as the price of the commodity, after sales service, advertisement, quality of the product, etc. which can influence the market demand. In the case of buying a mobile phone or a laptop, price as well as durability or quality plays an important role in a potent customer's mind. Apple is well known for its superior product quality and high price; it only attracts high end customers. Customers who look for better value for money sometimes compromise with quality and buy those products which deem suitable for their needs.

Supply chain is a complex logistics system in which the raw materials are converted into finished goods and then distributed to the final users (Ghiani et al. [1]). As raw materials are needed to manufacture sellable products, the quality of the finished goods depends on both the manufacturer's production quality and the quality of the raw materials supplied. For

* Corresponding author.
E-mail addresses: bcgiri.jumath@gmail.com (B.C. Giri), roy.balaji86@gmail.com (B. Roy), tarun.ju@gmail.com (T. Maiti).

http://dx.doi.org/10.1016/j.apm.2017.05.039
0307-904X/

instance, consider computer manufacturing companies such as Dell, Lenovo and HP. They use processors, RAM, hard disks, etc. from different companies and assemble them together to produce a laptop or desktop computer. Ultimately, these laptop or desktop computers are sold to the end customers. Clearly, to produce high quality products, one needs good quality raw materials. Otherwise, one has to put in more effort to maintain quality standards which incurs higher production cost. Most of the existing literatures omit the importance of the raw material quality in the finished goods. In this paper, we study the effect of raw material quality on the pricing and other decision making strategies.

There are several entities involved in a supply chain, starting from the raw material supplier and ending with the consumer. Normally these entities act separately and try to optimize their own profits. It is true that the performance of an uncoordinated supply chain is worst when considered individual players or the whole supply chain system. To improve efficiency, the supply chain has to be coordinated. It can be coordinated by using a centralized approach or by implementing strategies such as buyback contract, revenue sharing contract, retail fixed markup (RFM) pricing strategy, sub-supply chain coordination strategy, etc. A centralized system is the most efficient one but in real life situations it is hardly achievable. In our model, we use the centralized system as the benchmark case and, to make the decentralized system more efficient, we use the sub-supply chain coordination strategy and two-level RFM strategy. The sub-supply chain coordination strategy is useful when the supply chain is comprised of more than two members. In the sub-supply chain coordination strategy, one supply chain member merges with another and is represented by a single entity. Our aim is to study these two strategies and determine which one is more efficient than the other to coordinate the system.

In the literature, a lot of work has been done on the price dependency of demand. We focus only on those works which have considered the effects of price and/or product quality on demand. Among the early works on supply chain, Reyniers and Tapiero [2] modeled the effect of price rebate and after-sales warranty cost on the choice of a supplier from the quality perspective, inspection policy of a manufacturer, and the resultant end product quality. Forker [3] linked quality management with process optimization to address both effectiveness and efficiency concerns. Banker et al. [4] considered quality competition in Nash environments. Baiman et al. [5] developed a model considering contracting and quality cost. Lin et al. [6] formulated a structural equation model of supply chain quality management and organizational performance. Robinson and Malhotra [7] defined the concept of supply chain quality management and its relevance to academic and industrial practice. Bernstein and Federgruen [8] considered coordination mechanisms for supply chains under price and service competition. Foster [9] developed a supply chain model focusing on quality management. Franca et al. [10] tried to evaluate trade-off between profit and quality. Xie et al. [11] discussed about quality investment and pricing decision in a risk-averse supply chain. Tse and Tan [12] proposed a supply chain product quality risk management framework, integrating both incremental calculus and marginal analysis. Wang and Li [13] discussed the impact of the accuracy of the quality or shelf-life indicator, and timing and frequency of the discount in a selling period on retailing performance. Yu and Ma [14] considered the impact of the decision sequence of pricing and quality investment in decentralized assembly system. Maiti and Giri [15] studied a closed-loop supply chain considering retail price and quality dependent demand. Darwish et al. [16] developed a vendor managed inventory model for single-vendor multi-retailer with quality consideration. Liu et al. [17] analyzed the joint dynamic pricing and investment strategy for perishable foods with price and quality dependent demand.

The above mentioned works are mostly based on a two-echelon supply chain system and consider only the finished product quality of the manufacturer. The importance of the raw material's quality in the finished good has not been studied extensively in the literature. We analyze, in this paper, a three-echelon supply chain system in which the finished product quality depends on raw material quality, and the end customer's demand depends on both price and quality.

It is true that coordinating a multi-echelon supply chain is more difficult than coordinating a two-echelon supply chain. Munson and Rosenblatt [18] tried to coordinate a three-echelon supply chain model with quantity discounts, assuming deterministic demand. Giannoccaro and Pontrandolfo [19] studied a three-echelon supply chain model with revenue sharing contract which is used by both the up-stream and down-stream supply chain members. Jaber et al. [20] studied a three-echelon supply chain under price discounts, price dependent demand, and profit sharing. Jaber and Goyal [21] investigated the coordination of order quantities among the players in a three-echelon supply chain with a centralized decision process, allowing more than one player at each echelon. Ding and Chen [22] proposed a flexible buyback contract which can fully coordinate a three-echelon supply chain. Zijm and Timmer [23] coordinated a three-echelon serial and distribution system. Huang and Huang [24] developed a multi-echelon model with price competition and deterministic demand. Georgiadis et al. [25] considered a four-echelon supply chain network with uncertainties in time and demand. Mirzapour et al. [26] developed a three-echelon multi-product multi-period supply chain model with uncertainties in cost parameters and demands. He and Zhao [27] considered a three-echelon supply chain with both demand and supply uncertainties. Seifert et al. [28] developed a three-echelon supply chain model considering price-only contract and sub-supply chain coordination. Omar et al. [29] proposed a JIT (just-in-time) three-level integrated manufacturing system for linearly time-varying demand. Zhang and Liu [30] employed a coordination mechanism in three-level green supply chain under non-cooperative game. Liu et al. [31] analyzed the impact of partial information sharing in a three-echelon supply chain. Saha [32] studied channel characteristics and coordination in a three-echelon dual channel supply chain. Cárdenas-Barrón and Treviño-Garza [33] proposed a general integer programing model for a three-echelon supply chain network.

In our model, we use the RFM strategy which was first studied by Monroe [34]. He claimed that fixed markup pricing is the most common form of pricing used by retailers. Later, Liu et al. [35] introduced the RFM strategy to coordinate a two-echelon supply chain with price-dependent demand. Liu et al. [36] then extended their work on RFM incorporating multiplicative stochastic demand. Liu et al. [37] developed a multi-period model with price dependent demand and two-way price commitment. Wang et al. [38] used markup pricing strategies between a dominant retailer and competitive manufacturers. Sharma and Banerjee [39] determined the optimal price markup policy for an inventory model with random price fluctuation and option for additional purchase. Liu et al. [40] developed a manufacturer's uniform pricing model considering channel choice with a retail price markup commitment strategy.

Dependency of the finished good quality on the raw material's quality is an undisputed fact. However, in supply chain literature, it is somewhat unexplored territory. In this paper, we study the effect of the raw material's quality on pricing and other decision making strategies in the supply chain. We also coordinate the supply chain system through two strategies, viz. sub-supply chain coordination strategy and two-level RFM strategy, and provide a comparative study on their performances. The rest of the paper is organized as follows: In Section 2, notations and problem description is provided. Section 3 deals with model development and procedure for finding the optimal decisions. Section 4 is devoted to numerical analysis. Managerial insights and conclusions are presented in Sections 5 and 6, respectively.

2. Notations and problem description

The notations used in this paper are as follows:

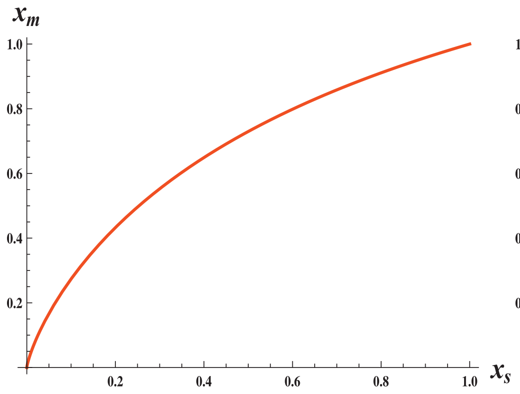
i	:	Supply chain entity, $i = s$ (supplier), m (manufacturer), r (retailer).
$c(>0)$	:	Unit raw material production cost of the supplier.
w_i	:	Unit wholesale price of the entity i, where $i = s, m$ and $w_m > w_s > c$.
$p(>w_m)$	:	Unit selling price of the retailer.
x_i	:	Quality of raw materials, where $i = s, m$.
$D(>0)$	:	Market demand.
$a(>0)$	:	Fixed part of the market demand.
$b(>0)$	:	Price sensitivity parameter of the market demand.
$\beta(>0)$	:	Quality sensitivity parameter of the market demand.
ε	:	Random variable representing the stochastic part of the demand with positive support, density function $f(.)$, cumulative distribution function $F(.)$ and mean μ.
q	:	Order size of the retailer.
z	:	Stocking factor when demand is stochastic.
h	:	Overage cost per unit for leftover inventory.
s	:	Lost sale cost per unit shortage at the retailer's end.
η_i	:	Per unit cost for quality improvement, where $i = s, m$.
g_i	:	Goodwill loss cost due to quality reason, where $i = s, m$.
α_i	:	Retail price markup of the entity i, where $i = m, r$.
$\lambda(>0)$	:	Quality dependency parameter of the manufacturer.

We develop a three-level supply chain model with a raw material supplier, a manufacturer and a retailer for trading a single product. The manufacturer needs one unit of raw material to produce one unit of end product. The demand D for the product is dependent on the retail price and quality of the finished product. Therefore, $D = D(p, x_m) = a - bp + \beta x_m$ [11,15], where a is a positive constant, b is the price sensitivity parameter and β is the quality sensitivity parameter.[1] We also address the uncertainty of market demand by assuming $D = D(p, x_m) + \varepsilon$ [11,42,43], where ε is a random variable representing the randomness of the market demand [42] due to changes in economic and business conditions [11]. Here, we do not consider the randomness of the price and product quality; we consider the randomness of the basic market demand $D(p, x_m) = a - bp + \beta x_m$, where the price and product quality have particular values [11,44]. To meet the customers demand, the retailer places an order of quantity q units to the manufacturer who buys q units of raw material from the supplier. The supplier and the manufacturer both can improve quality with added cost. It is also beneficial for them, as it can help them to lower the goodwill loss cost. However, the opportunity to improve quality for the manufacturer is rather limited as the raw material quality plays an important role in the final product quality.

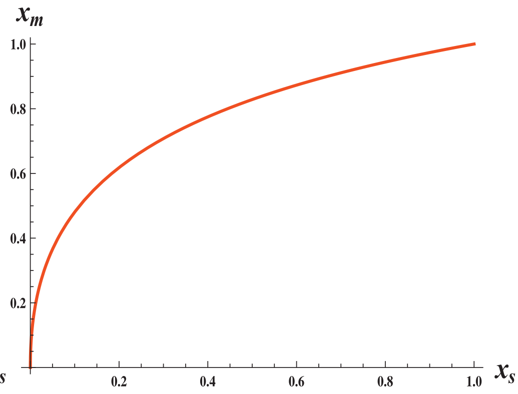
Suppose that the supplier's raw material quality is x_s. As it is almost impossible to achieve 100% pure raw materials, so $0 < x_s < 1$. The supplier can improve the raw material quality subject to additional cost. The manufacturer's product quality x_m depends on the raw material quality x_s. We assume, $x_m = 1 - \frac{1-x_s^\lambda}{1+x_s^\lambda}$, where $0 < x_s < 1$ and $\lambda(>0)$ is a quality dependency parameter. This indicates that when $\lambda > 1$, the function x_m is convex in x_s, and when $0 < \lambda \leq 1$, the function x_m is concave in x_s (proof is provided in Lemma 1). This suggests that the product quality is monotonically increasing with decreasing rate when $0 < \lambda \leq 1$, and with increasing rate when $\lambda > 1$ [45,46].[2] Fig. 1 depicts the behavior of the finished product quality over

[1] This form of demand indicates that higher pricing puts off customers, but higher product quality attracts them. In the same market segment, higher product quality generates higher demand when the retail price is fixed. Generally speaking, in the market for high quality products, high price is set for high-end customers who constitute the most price-insensitive segment of the market [41]. This policy is suitable to the consumers who are not conscious about the product quality but accept lower prices.

[2] As we know, in the food industry, high quality raw materials result in high quality food. Therefore, careful attention must be paid into cooperation between the producer of raw materials and the manufacturer who produces an end food product. The food quality is a rather complex term. It includes the perspective of quality from nutritional, hygienic, sensory and even technological viewpoints. All mentioned forms of quality attributes influence the quality of the end product. So, the behavior of the end product's quality cannot be captured by assuming linear dependency of raw material quality. Thats why we take a nonlinear form for the manufacturer's end product quality which increases with raw material quality (Ref. Devidek, J. Quality control of raw materials. Food Quality and Standards, Vol II). We measure quality in the scale of 0 to 1. In real life situations, the supplier's raw material quality or



(a) x_m vs. x_s when $\lambda = 0.8$



(b) x_m vs. x_s when $\lambda = 0.5$

Fig. 1. Manufacturer product quality with respect to raw material quality.

raw material quality for different values of λ. It is almost impossible to manufacture 100% pure raw material or finished goods. For this impurity, the supplier and the manufacturer may lose some goodwill, which incurs a cost (goodwill loss cost) at a rate g_i, where $i = s, m$[3]. We assume that there is no capacity constraint or lead time for the manufacturer or the supplier.

3. Model development and analysis

In this section, we first consider the deterministic demand scenario ($\varepsilon = 0$) and then the stochastic demand scenario ($\varepsilon \neq 0$). In both the cases, we develop centralized, decentralized and coordinated models which are also represented in Fig. 2. We denote optimal strategies in centralized, decentralized, MR-Nash, MS-Nash and RFM strategies by superscripts c, d, mr, ms and f, respectively.

3.1. Modeling with deterministic demand

As the demand at the retailer is deterministic, the order quantity of the retailer would be the same as the market demand in this scenario. At first, we consider the centralized supply chain and then the decentralized one. We also use the sub-supply chain strategy and retail fixed markup pricing strategy to coordinate the decentralized system.

3.1.1. Centralized supply chain

In the centralized supply chain scenario, a single decision maker is present and he sets the price, order quantity and quality of the product to maximize the whole system's profit. The revenue generated by selling D units is Dp units, cost of production is Dc units, quality improvement cost and goodwill loss cost (due to lack of quality) of the manufacturer and the supplier are $(\eta_m x_m^2,\ g_m(1 - x_m))$ and $(\eta_s x_s^2,\ g_s(1 - x_s))$, respectively [11].[4] Therefore, the profit of the system is given by

$$\Pi_c = (p - c)D - \eta_s x_s^2 - \eta_m x_m^2 - g_s(1 - x_s) - g_m(1 - x_m). \tag{1}$$

In the right-hand side of the above equation, the first term indicates the profit margin of the system, second and third terms indicate the quality improvement costs, and fourth and fifth term represent goodwill loss cost of the supplier and the manufacturer, respectively. It is easy to see that, for fixed quality of the product, the profit function Π_c is concave in p as $\frac{\partial^2 \Pi_c}{\partial p^2} = -2b < 0$.

Lemma 1. *When the retail price is fixed, the finished product quality is concave with respect to x_s if $0 < \lambda \leq 1$ and convex with respect to x_s if $\lambda > 1$. Proof of this lemma is provided in Appendix A.*

the manufacturer's finished product quality cannot be 100% pure. Thus, we take the manufacturer's end product quality as $x_m = 1 - \frac{1-x_s^{\lambda}}{1+x_s^{\lambda}}$, where $0 < x_s < 1$, which results in $0 < x_m < 1$.

[3] The purpose of measuring such costs is to provide broad guidelines for management decision-making and action. It is difficult to evaluate the very nature of the cost of quality. It can be measured periodically with a rough estimation of costs as lost customer goodwill, cost of damage to the companys reputation, etc. These evaluations should be possible utilizing special audits, statistical sampling and other market studies. These exercises can be directed together by marketing, accounting and quality personnel. Since these costs are often huge, we must estimate these costs. However, these estimations need not be done very frequently. Yearly reviews are generally adequate to show inclines in these measures [47].

[4] The investment for quality has a decreasing return to scale, viz., the next dollar invested by the manufacturer returns less quality than the last dollar invested, i.e. it is harder (and it also costs more) to provide the next unit of quality than the last one. This can be reflected by the quadratic form of the cost of providing quality. The quadratic functional form is used in Tsay and Agrawal [48] for service level and Chao et al. [49]; Li et al. [50] and Maiti and Giri [15] for quality level.

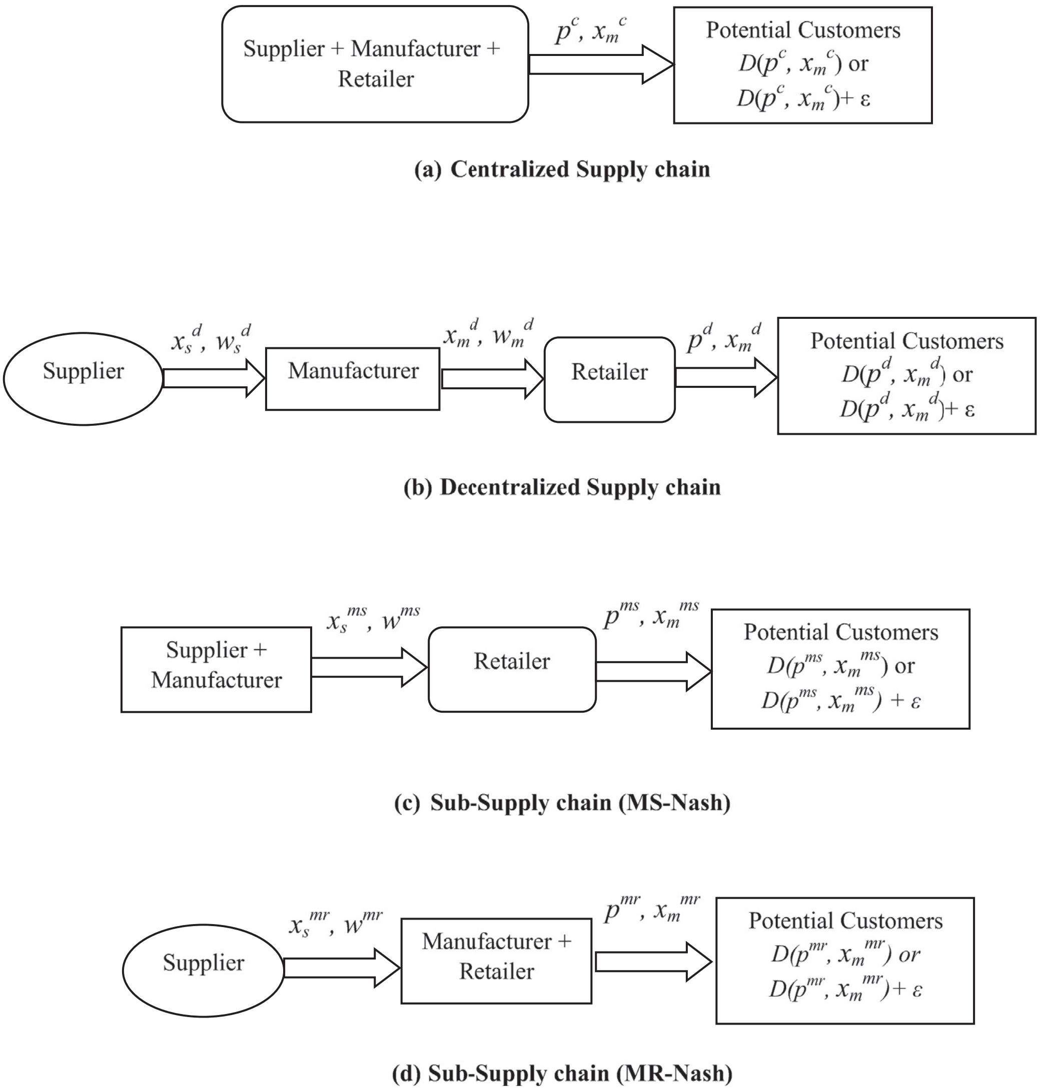


Fig. 2. Various supply chain scenarios.

Proposition 1. *When the retail price is fixed, the profit function of the system is concave with respect to raw material quality* x_s *if* $0 < \lambda \leq 1$ *and* $p > c + \frac{2\eta_m - g_m}{\beta}$.

Proposition 2. *When the quality of the product is fixed, the optimal price and order quantity are given by*

$$p^c(x_s) = \frac{a + bc + \left(a + bc + 2\beta\right)x_s^\lambda}{2b(1 + x_s^\lambda)}, \tag{2}$$

$$q^c(x_s) = \frac{a - bc + \left(a - bc + 2\beta\right)x_s^\lambda}{2b\left(1 + x_s^\lambda\right)}. \tag{3}$$

Proofs of Propositions 1 and 2 are given in Appendix B and Appendix C, respectively.
Now, solving the equation

$$\frac{\partial \Pi_c}{\partial x_s} = \frac{\lambda\beta(a + x_s^\lambda + 2\beta x_s^\lambda)x_s^{\lambda-1} + b\left[(g_s - 2\eta_s x_s)(1 + x_s^\lambda)^3 + \lambda(2g_m - c\beta)(1 + x_s^\lambda){x_s}^{\lambda-1} - 8\eta_m\lambda x_s^{2\lambda-1}\right]}{b(1 + x_s^\lambda)^3}$$
$$= 0,$$

we get the optimal raw material quality of the product. Subsequently, we use this value into the expressions for retail price, finished product quality and ordering quantity to determine the optimal decisions. Using these values in Eq. (1), we get the optimal profit of the system.

Proposition 3. *Optimal price and order quantity have the following properties:*

$$(i)\quad \frac{\partial p^c(x_s)}{\partial x_s} > 0, \quad \frac{\partial q^c(x_s)}{\partial x_s} > 0,$$

$$(ii)\quad \frac{\partial p^c(x_s)}{\partial \lambda} < 0, \quad \frac{\partial q^c(x_s)}{\partial \lambda} < 0.$$

Proof of this proposition is given in Appendix D. From the above we see that, as the raw material quality increases, the retail price and order quantity both increase. To produce better quality product, the cost of production increases and as a result, the retail price increases. The quality of the product has a positive effect on demand. So, with the increase in raw material quality, the order quantity increases. On the other hand, with the increase in the quality dependency parameter of the manufacturer, the retail price and order quantity decrease. This is due to a decrease in finished product quality.

3.1.2. Decentralized supply chain

In the decentralized scenario, there is a vertical Nash game where each entity in the supply chain tries to maximize it's own profit non-cooperatively. As a result, the supplier optimizes it's wholesale price (w_s) and raw material quality (x_s); the manufacturer optimizes its wholesale price (w_m), and the retailer optimizes its retail price (p) and order quantity (q). In this case, as the demand is deterministic, the order quantity is equal to the market demand, i.e. $q = D$.

The profit of the supplier is given by

$$\Pi_s = (w_s - c)D - \eta_s x_s^2 - g_s(1 - x_s), \tag{4}$$

where, in the right-hand side of the above equation, the first term indicates the profit margin, and the second and third terms indicate the quality improvement cost and goodwill loss cost of the supplier, respectively.

The profit of the manufacturer is given by

$$\Pi_m = (w_m - w_s)D - \eta_m x_m^2 - g_m(1 - x_m), \tag{5}$$

where, in the right-hand side of the above equation, the first term indicates the profit margin, and the second and third terms indicate the quality improvement cost and goodwill loss cost of the manufacturer, respectively.

Finally, the profit of the retailer is given by

$$\Pi_r = (p - w_m)D. \tag{6}$$

It is easy to see that the profit function Π_s is concave in w_s and x_s. Also, the profit functions Π_m and Π_r are concave with respect to w_m and p, respectively. The profit functions of the supplier and the manufacturer are increasing with respect to w_s and w_m, respectively. Since $w_s < w_m < p$, the supplier's wholesale price can not be equal to the manufacturer's wholesale price or the manufacturer's wholesale price can not be equal to the retailer's retail price. We assume that the profit margins of the supplier, manufacturer and the retailer are equal. We take the unique wholesale price of the supplier and the manufacturer as $w_m = (2p + c)/3$ and $w_s = (2c + p)/3$, respectively [51–53].

Proposition 4. *When the quality of the product is fixed, the optimal reactions of the supplier, manufacturer and the retailer are given by*

$$w_s^d(x_s) = \frac{a + 3bc + (a + 3bc + 2\beta)x_s^\lambda}{4b(1 + x_s^\lambda)}, \tag{7}$$

$$w_m^d(x_s) = \frac{a + bc + (a + bc + 2\beta)x_s^\lambda}{2b(1 + x_s^\lambda)}, \tag{8}$$

$$p^d(x_s) = \frac{3a + bc + (3a + bc + 6\beta)x_s^\lambda}{4b(1 + x_s^\lambda)}, \tag{9}$$

$$q^d(x_s) = \frac{a - bc + (a - bc + 2\beta)x_s^\lambda}{4(1 + x_s^\lambda)}. \tag{10}$$

Proof of the proposition is provided in Appendix E.

Now, solving the equation

$$\frac{\partial \Pi_s}{\partial x_s} = \frac{\lambda\beta(a + x_s^\lambda + 2\beta x_s^\lambda)x_s^{\lambda-1} - b(1 + x_s^\lambda)\left[4(2\eta_s x_s - g_s)(1 + x_s^\lambda)^2 + c\beta\lambda x_s^{\lambda-1}\right]}{4b(1 + x_s^\lambda)^2} = 0,$$

we get the optimal raw material quality of the product. Subsequently, we can find out the optimal retail price, finished product quality and ordering quantity. Using these results in Eqs. (4)–(6), we can get the optimal profit of the system.

Proposition 5. *Optimal reactions ((7)–(10)) of the supplier, manufacturer and retailer have the following properties:*

$$(i)\quad \frac{\partial w_s^d(x_s)}{\partial x_s} > 0,\quad \frac{\partial w_m^d(x_s)}{\partial x_s} > 0,\quad \frac{\partial p^d(x_s)}{\partial x_s} > 0,\quad \frac{\partial q^d(x_s)}{\partial x_s} > 0,$$

$$(ii)\quad \frac{\partial w_s^d(x_s)}{\partial \lambda} < 0,\quad \frac{\partial w_m^d(x_s)}{\partial \lambda} < 0,\quad \frac{\partial p^d(x_s)}{\partial \lambda} < 0,\quad \frac{\partial q^d(x_s)}{\partial \lambda} < 0.$$

The proof of the proposition is omitted as it is similar to that of Proposition 4.

3.1.3. Sub-supply chain coordination

Sub-supply chain coordination is a very useful concept and it is widely used in supply chain management. Here, the manufacturer chooses to merge with either the supplier or the retailer and then acts as a single entity. In other words, the manufacturer chooses to open its own raw material manufacturing facility or retailing facility. If Lenovo decides to manufacture the motherboard of a laptop by itself then it is an example of a manufacturer-supplier merger. Similarly, when mobile manufacturers use their exclusive showrooms to sell their products, it can be termed as manufacturer-retailer merger. When the manufacturer combines with the supplier, we call it MS-Nash and when the manufacturer combines with the retailer, we call it MR-Nash.

A. MS-Nash

Here, the supplier and the manufacturer act as a single entity and try to maximize their joint profit. On the other hand, the retailer tries to maximize his/her own profit. Both the parties follow the Nash bargaining game and try to optimize their profits independently. In this case, the joint profit margin of the supplier and the manufacturer should be equal to that of the retailer [51–53]. Therefore, we take the joint wholesale price of the supplier and the manufacturer as $w=(c+p)/2$, similar to the case of decentralized scenario. The joint profit of the supplier and the manufacturer, and the profit of the retailer is given by

$$\Pi_{ms} = (w-c)D - \eta_s x_s^2 - g_s(1-x_s) - \eta_m x_m^2 - g_m(1-x_m), \tag{11}$$

$$\Pi_r = (p-w)D. \tag{12}$$

Proceeding similarly as in the previous section, we can easily prove the concavity of the profit functions with respect to retail price and raw material quality. We can also find the optimal decisions of the players from the first order optimality conditions.

Proposition 6. *When the quality of the product is fixed, the optimal responses are given by*

$$w^{ms}(x_s) = \frac{a+2bc+(a+2bc+2\beta)x_s^\lambda}{3b(1+x_s^\lambda)}, \tag{13}$$

$$p^{ms}(x_s) = \frac{2a+bc+(2a+bc+4\beta)x_s^\lambda}{3b(1+x_s^\lambda)}, \tag{14}$$

$$q^{ms}(x_s) = \frac{a-bc+(a-bc+2\beta)x_s^\lambda}{3(1+x_s^\lambda)}. \tag{15}$$

From the first order optimality condition, we can determine the optimal value of raw material quality i.e. x_s^{ms} and using it in Eqs. (13)–(15), we can find the optimal reactions of the supply chain entities. Finally, using these optimal values in (11) and (12), we can get the optimal profit for the whole supply chain.

B. MR-Nash

In this case, the manufacturer and the retailer act as a single entity and try to maximize their joint profit using the Nash bargaining game. On the contrary, the supplier tries to maximize his own profit. By the same argument as given in the previous section, the joint profit margin of the retailer and the manufacturer should be equal to the profit margin of the supplier. We take $w=(p+c)/2$ [51–53] and the profits are given by

$$\Pi_s = (w-c)D - \eta_s x_s^2 - g_s(1-x_s), \tag{16}$$

$$\Pi_{mr} = (p-w)D - \eta_m x_m^2 - g_m(1-x_m). \tag{17}$$

The optimal reactions of the supplier and the combined unit of the manufacturer and the retailer are given in the following proposition which can be easily derived from first order optimality conditions.

Proposition 7. *When the quality of the product is fixed, the optimal responses are*

$$w^{mr}(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^{\lambda}}{3b(1 + x_s^{\lambda})}, \tag{18}$$

$$p^{mr}(x_s) = \frac{2a + bc + (2a + bc + 4\beta)x_s^{\lambda}}{3b(1 + x_s^{\lambda})}, \tag{19}$$

$$q^{mr}(x_s) = \frac{a - bc + (a - bc + 2\beta)x_s^{\lambda}}{3(1 + x_s^{\lambda})}. \tag{20}$$

From the first order optimality condition, we can get the optimal quality of the raw material and the rest is the same like the previous sections.

3.1.4. Retail fixed markup (RFM) strategy

Under RFM strategy, the supplier decides his wholesale price (w_s) and the manufacturer uses a markup α_m over the wholesale price of the supplier. The retailer also enacts a markup α_r over the manufacturer's wholesale price. The problem then reduces to find out the optimal wholesale price of the supplier. The wholesale price of the manufacturer and the retail price of the retailer are decided by the RFM strategy such that $p = (1 + \alpha_r)w_m$ and $w_m = (1 + \alpha_m)w_s$ [35,36]. In this scenario, the profits of the supplier, manufacturer and the retailer are given by

$$\Pi_s^f = (w_s - c)\big(a - b(1 + \alpha_r)(1 + \alpha_m)w_s + \beta x_m\big) - \eta_s x_s^2 - g_s(1 - x_s), \tag{21}$$

$$\Pi_m^f = \alpha_m w_s\big(a - b(1 + \alpha_r)(1 + \alpha_m)w_s + \beta x_m\big) - \eta_m x_m^2 - g_m(1 - x_m), \tag{22}$$

$$\Pi_r^f = \alpha_r(1 + \alpha_m)w_s\big(a - b(1 + \alpha_r)(1 + \alpha_m)w_s + \beta x_m\big). \tag{23}$$

Proposition 8. *When the quality of the product is fixed, the optimal responses of the supplier, manufacturer and the retailer are given by*

$$w_s^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^{\lambda} + bc(1 + x_s^{\lambda})(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1 + x_s^{\lambda})(1 + \alpha_r)(1 + \alpha_m)}, \tag{24}$$

$$w_m^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^{\lambda} + bc(1 + x_s^{\lambda})(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1 + x_s^{\lambda})(1 + \alpha_r)}, \tag{25}$$

$$p^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^{\lambda} + bc(1 + x_s^{\lambda})(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1 + x_s^{\lambda})}, \tag{26}$$

$$q^f(x_s) = \frac{a - bc + \big[a - bc(1 + \alpha_r)(1 + \alpha_m) + 2\beta\big]x_s^{\lambda}}{2\big(1 + x_s^{\lambda}\big)}. \tag{27}$$

The proof is provided in the Appendix F. From the first order optimality condition, we can get the optimal quality of the raw material and the rest is the same like the previous sections.

Proposition 9. *The following relationships hold for $p^i(x_s)$ and $q^i(x_s)$:*

$$(i)\quad p^d(x_s) > p^{mr}(x_s) = p^{ms}(x_s) > p^f(x_s) > p^c(x_s),$$

$$(ii)\quad q^c(x_s) > q^f(x_s) > q^{mr}(x_s) = q^{ms}(x_s) > q^d(x_s).$$

Comparing Eqs. (2), (9), (14), (19) and (26), we can easily get the results provided in Proposition 9(i). Similarly, comparing (3), (10), (15), (20) and (27), we get the results given in Proposition 9(ii). From above, we see that both strategies fall short of the centralized system but can achieve better results than the decentralized system. This indicates that sub-supply chain and RFM strategies can be very useful to improve the results of the decentralized supply chain.

3.2. Modeling with stochastic demand

Now, we develop the model with stochastic demand. We assume that unfulfilled demand at the retailer's end is lost and there is an overage cost for the leftover inventory.

3.2.1. Centralized supply chain

As the market demand is stochastic, the order quantity may not fulfill the demand accurately. We assume that the retailer places an order of q units to the manufacturer. The manufacturer buys q units of raw material from the supplier to produce the end product, which incurs a cost of cq units. When the market demand exceeds the order quantity, shortage occurs. It generates a revenue of qp units and incurs a shortage cost of $s(D-q)$ units at the retailer's end. On the other hand, if the market demand does not exceed the order quantity, it generates a revenue of Dp units with overage cost $h(q-D)$ units. The quality improvement cost and the goodwill loss cost for the manufacturer and the supplier are same as those of the deterministic demand scenario. Thus, the expected profit for the system is given by

$$\Pi_c = E\{p(q \wedge D) - s(D-q)^+ - h(q-D)^+ - cq\} - \eta_s x_s^2 - \eta_m x_m^2 - g_s(1-x_s) - g_m(1-x_m),$$

where $X \wedge Y = min\{X, Y\}$ and $X^+ = max\{X, 0\}$. We define the stocking factor z as $z = q - D(p, x_m)$ to cover the randomness of demand. This stocking factor act as order-up-to level in the newsvendor model, to separate the random component of the demand from deterministic trend. Then the expected profit becomes

$$\begin{aligned}\Pi_c &= E\{p[D(p,x_m) + (\varepsilon \wedge z)] - s(\varepsilon - z)^+ - h(z-\varepsilon)^+\} - c[z + D(p,x_m)] - \eta_s x_s^2,\\ &\quad -\eta_m x_m^2 - g_s(1-x_s) - g_m(1-x_m)\\ &= (p-c)[D(p,x_m) + \mu] - (h+c)\Lambda(z) - (p+s-c)\Theta(z) - \eta_s x_s^2 - \eta_m x_m^2\\ &\quad -g_s(1-x_s) - g_m(1-x_m),\end{aligned} \tag{28}$$

where $\Lambda(z) = \int_0^z (z-\xi)dF(\xi)$ and $\Theta(z) = \int_z^\infty (\xi - z)dF(\xi)$ are the expected overage and stock out quantities, respectively. Our objective is to maximize the expected profit of the system with respect to the stocking factor, the retail price and the quality. We use sequential optimization technique to obtain the optimal results.

Proposition 10. *When the retail price is fixed, the profit of the system is concave with respect to the raw material quality x_s for $0 < \lambda \le 1$ and $p > c + \frac{2\eta_m - g_m}{\beta}$.*

As the stochastic terms do not contain raw material quality, the proof of the above proposition is exactly the same as that of Proposition 1. Keeping the raw material quality fixed, we find the first and second order partial derivatives of Π_c with respect to the stocking factor z and the retail price p as

$$\begin{aligned}\frac{\partial \Pi_c}{\partial z} &= -(c+h) + (p+s+h)[1-F(z)],\\ \frac{\partial^2 \Pi_c}{\partial z^2} &= -(p+s+h)f(z),\\ \frac{\partial \Pi_c}{\partial p} &= a - bp + \beta x_m - b(p-c) + \mu - \Theta(z),\\ \frac{\partial^2 \Pi_c}{\partial p^2} &= -2b.\end{aligned}$$

Clearly, Π_c is concave in z for a given p. Now, solving the first order optimality condition $\frac{\partial \Pi_c}{\partial z} = 0$, we get the optimal value of the stocking factor (z) for any given p as

$$z^c(p) = F^{-1}\left(1 - \frac{c+h}{p+s+h}\right). \tag{29}$$

Again, since Π_c is concave in p, solving the first order optimality condition $\frac{\partial \Pi_c}{\partial p} = 0$, we get the optimal value of the retail price for fixed z and x_s as

$$p^c(z, x_s) = \frac{a + bc + \mu + \left(a + bc + \mu + 2\beta\right)x_s^\lambda - (1 + x_s^\lambda)\Theta(z)}{2b(1 + x_s^\lambda)}. \tag{30}$$

Proposition 11. *For stochastic demand, (i) the optimal retail price is greater than that of the deterministic demand and (ii) the stocking factor increases with the retail price.*

The proof of the proposition is provided in Appendix G. When demand is stochastic, there is a probability of stock out or unsold product. To compensate the loss due to stock out or unsold quantity, the retail price becomes higher than that in case of deterministic demand. We see that when price increases, the optimal stocking factor increases to cover the uncertainty of demand.

Now, using the first order optimality condition, we find the optimal raw material quality (x_s^c) from Eq. (28).

Theorem 1. *Under the linear additive demand function, the single-period stochastic centralized supply chain's solution is to set quality to x_s^c, price to p^c, and order to $a - bp^c + \beta x_s^c + z^c$, subject to:*

(i) *if $F(\cdot)$ is an arbitrary distribution, then the entire support must be searched to find z^c;*
(ii) *if $F(\cdot)$ satisfies $2r(z)^2 + (dr(z)dz) > 0$ where $r(z) = f(z)/(1 - F(z))$ is the hazard rate, then z^c is the largest z satisfying the first-order condition.*

The theorem is proved in Appendix H following Petruzzi and Dada [42].

3.2.2. Decentralized supply chain

Similar to that of the deterministic case, we use vertical Nash game where each entity in the supply chain tries to maximize its own profit non-cooperatively. In this scenario, the profits of the supplier, manufacturer and the retailer can be derived in a manner similar to those of the deterministic scenario. The profits of the supplier, manufacturer and the retailer are given by

$$\Pi_s = (w_s - c)q - \eta_s x_s^2 - g_s(1 - x_s), \tag{31}$$

$$\Pi_m = (w_m - w_s)q - \eta_m x_m^2 - g_m(1 - x_m), \tag{32}$$
$$\Pi_r = E\left\{p(q \wedge D) - s(D-q)^+ - h(q-D)^+ - w_m q\right\}.$$

Substituting $z = q - D(p, x_m)$ in the retailer's profit function and simplifying, we get

$$\Pi_r = (p - w_m)[D(p, x_m) + \mu] - (h + w_m)\Lambda(z) - (p + s - w_m)\Theta(z). \tag{33}$$

Note that, in Π_r, the quality aspect is omitted and c is replaced by w_m. The optimal stocking factor of the retailer for given raw material quality and price is given by

$$z^d(p) = F^{-1}\left(1 - \frac{w_m + h}{p + s + h}\right). \tag{34}$$

Similar to the deterministic case, we assume that the profit margins of the supplier, manufacturer and the retailer are equal, and we take the unique wholesale prices of the manufacturer and the supplier as $w_m = (2p + c)/3$ and $w_s = (2c + p)/3$, respectively [51–53].

Proposition 12. *For given stocking factor and raw material quality, the optimal reactions of the supplier, manufacturer and the retailer are given by*

$$w_s^d(z, x_s) = \frac{a + 3bc + \mu + (a + 3bc + \mu + 2\beta)x_s^\lambda - (1 + x_s^\lambda)\Theta(z)}{4b(1 + x_s^\lambda)}, \tag{35}$$

$$w_m^d(z, x_s) = \frac{a + bc + \mu + (a + bc + \mu + 2\beta)x_s^\lambda - (1 + x_s^\lambda)\Theta(z)}{2b(1 + x_s^\lambda)}, \tag{36}$$

$$p^d(z, x_s) = \frac{3a + bc + 3\mu + (3a + bc + 3\mu + 6\beta)x_s^\lambda - 3(1 + x_s^\lambda)\Theta(z)}{4b(1 + x_s^\lambda)}. \tag{37}$$

Eqs. (35)–(37) can be easily obtained from the first order optimality conditions (see Appendix I). However, we could not find explicit expressions for stocking factor and raw material quality of the supplier.

Proposition 13. *For $w_s^d(z, x_s)$, $w_m^d(z, x_s)$ and $p^d(z, x_s)$, we have the following relations:*

$$p^d(z, x_s) > p^d(x_s),\ \ w_m^d(z, x_s) > w_m^d(x_s),\ \ w_s^d(z, x_s) > w_s^d(x_s).$$

The proof is omitted as it is similar to that of Proposition 11.

3.2.3. Sub-supply chain coordination

As in the deterministic scenario, we use sub-supply chain coordination by merging the manufacturer with the supplier or the retailer. We first analyze the MS-Nash model and then the MR-Nash model.

A. MS-Nash

In this case, the joint profit of the manufacturer and the supplier, and the profit of the retailer are given by

$$\Pi_{ms} = (w - c)q - \eta_s x_s^2 - g_s(1 - x_s) - \eta_m x_m^2 - g_m(1 - x_m), \tag{38}$$

$$\Pi_r = (p - w)[D(p, x_m) + \mu] - (h + w)\Lambda(z) - (p + s - w)\Theta(z). \tag{39}$$

It is to be noted here that the profit function of the retailer is the same as that in the decentralized case. So, the optimal stocking factor of the retailer for given raw material quality is given by

$$z^{ms}(p) = F^{-1}\left(1 - \frac{w + h}{p + s + h}\right). \tag{40}$$

Likewise the deterministic scenario, we assume $w = (p + c)/2$ [51–53] and the rest of the solution procedure follows the similar treatment as done previously.

Proposition 14. *When the quality of the raw material and stocking factor are known, the optimal pricing strategies are given by*

$$w^{ms}(z, x_s) = \frac{a + 2bc + \mu + (a + 2bc + \mu + 2\beta)x_s^{\lambda} - (1 + x_s^{\lambda})\Theta(z)}{3b(1 + x_s^{\lambda})}, \tag{41}$$

$$p^{ms}(z, x_s) = \frac{2a + bc + 2\mu + (2a + bc + 2\mu + 4\beta)x_s^{\lambda} - 2(1 + x_s^{\lambda})\Theta(z)}{3b(1 + x_s^{\lambda})}. \tag{42}$$

Comparing the above results with those of the deterministic scenario, one can see that the pricing is higher in comparison to the deterministic scenario.

B. MR-Nash

In this case, the profit of the supplier, and the joint profit of the manufacturer and the retailer are given by

$$\Pi_s = (w - c)q - \eta_s x_s^2 - g_s(1 - x_s), \tag{43}$$

$$\Pi_{mr} = (p - w)[D(p, x_m) + \mu] - (h + w)\Lambda(z) - (p + s - w)\Theta(z) - \eta_m x_m^2 - g_m(1 - x_m). \tag{44}$$

Following a similar procedure as outlined previously, we can easily obtain the solution of the system. The optimal stocking factor of the retailer for given raw material quality is given by

$$z^{mr}(p) = F^{-1}\left(1 - \frac{w + h}{p + s + h}\right). \tag{45}$$

Similar to the deterministic scenario, we assume $w = (p + c)/2$ [51–53]. The rest of the solution procedure is identical to the one already done before.

Proposition 15. *When the quality of the raw material and stocking factor are known, the optimal pricing strategies are given by*

$$w^{mr}(z, x_s) = \frac{a + 2bc + \mu + (a + 2bc + \mu + 2\beta)x_s^{\lambda} - (1 + x_s^{\lambda})\Theta(z)}{3b(1 + x_s^{\lambda})}, \tag{46}$$

$$p^{mr}(z, x_s) = \frac{2a + bc + 2\mu + (2a + bc + 2\mu + 4\beta)x_s^{\lambda} - 2(1 + x_s^{\lambda})\Theta(z)}{3b(1 + x_s^{\lambda})}. \tag{47}$$

Here again, the above results indicate that the pricing is higher in the stochastic scenario than that of the deterministic scenario.

3.2.4. Retail fixed markup (RFM) strategy

Similar to the deterministic scenario, under RFM strategy, we assume $p = (1 + \alpha_r)w_m$ and $w_m = (1 + \alpha_m)w_s$ [35,36]. The profits of the supplier, manufacturer and the retailer are given by

$$\Pi_s^f = (w_s - c)q - \eta_s x_s^2 - g_s(1 - x_s), \tag{48}$$

$$\Pi_m^f = \alpha_m w_s q - \eta_m x_m^2 - g_m(1 - x_m), \tag{49}$$

$$\Pi_r^f = \alpha_r(1 + \alpha_m)w_s[D(p, x_m) + \mu] - (h + (1 + \alpha_m)w_s)\Lambda(z) - ((1 + \alpha_r)(1 + \alpha_m)w_s + s - (1 + \alpha_m)w_s)\Theta(z). \tag{50}$$

For given wholesale price and fixed quality, the retailer's profit function is concave in stocking factor as

$$\frac{\partial^2 \Pi_r^f}{\partial z^2} = -((1 + \alpha_r)(1 + \alpha_m)w_s + s + h)f(z) < 0.$$

The optimal stocking factor is determined as

$$z^f(w_s) = F^{-1}\left(1 - \frac{(1 + \alpha_m)w_s + h}{(1 + \alpha_r)(1 + \alpha_m)w_s + s + h}\right). \tag{51}$$

Proposition 16. *When the quality of the product is fixed, the optimal responses of the supplier, manufacturer and retailer are given by*

$$w_s^f(z, x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^{\lambda} + bc(1 + x_s^{\lambda})(\alpha_r + \alpha_r\alpha_m + \alpha_m) + z(1 + x_s^{\lambda})}{2b(1 + x_s^{\lambda})(1 + \alpha_r)(1 + \alpha_m)}, \tag{52}$$

Table 1
Optimal results when demand is deterministic and RFM $\alpha_m = 0.6$ and $\alpha_r = 0.4$.

Optimal decisions	Centralized (c)	Decentralized (d)	Sub-supply chain MS-Nash(ms)	MR-Nash(mr)	RFM (f)
x_s	0.625	0.655	0.435	0.764	0.575
x_m	0.769	0.791	0.606	0.866	0.730
w_s	–	187.90	–	233.91	159.26
w_m	–	325.79	155.00	–	254.81
p	325.77	463.69	417.48	417.82	356.73
q	13.79	6.89	9.20	9.19	12.24
Π_m	–	944.40	1682.36	–	1163.76
Π_s	–	947.19	–	1686.40	1334.08
Π_r	–	950.76	1687.97	1683.59	1247.18
Π_i	3793.13	2842.35	3370.33	3369.99	3745.03
Π_i/Π_c	1	0.749	0.889	0.888	0.987

Table 2
Optimal results when demand is stochastic and RFM $\alpha_m = 0.6$ and $\alpha_r = 0.4$.

Optimal decisions	Centralized (c)	Decentralized (d)	Sub-supply chain MS-Nash(ms)	MR-Nash(mr)	RFM (f)
x_s	0.781	0.692	0.499	0.825	0.618
x_m	0.877	0.818	0.665	0.904	0.764
w_s	–	202.77	–	263.36	174.05
w_m	–	355.54	263.20	–	278.47
p	412.10	508.31	476.39	476.72	389.86
q	29.31	8.19	12.07	12.08	13.89
Π_s	–	1247.8	–	2571.10	1344.61
Π_m	–	1244.97	2566.08	–	1720.16
Π_r	–	958.15	1885.11	1880.03	1444.86
Π_i	5892.16	3450.91	4451.20	4451.12	4509.63
Π_i/Π_c	1	0.586	0.755	0.755	0.765

$$w_m^f(z, x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^\lambda + bc(1 + x_s^\lambda)(\alpha_r + \alpha_r\alpha_m + \alpha_m) + z(1 + x_s^\lambda)}{2b(1 + x_s^\lambda)(1 + \alpha_r)}, \tag{53}$$

$$p^f(z, x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^\lambda + bc(1 + x_s^\lambda)(\alpha_r + \alpha_r\alpha_m + \alpha_m) + z(1 + x_s^\lambda)}{2b(1 + x_s^\lambda)}. \tag{54}$$

The derivation of Eqs. (52)–(54) are given in Appendix J. The above expressions are similar to those of the deterministic scenario; the only difference is the numerator which contains an extra term $z(1 + x_s^\lambda)$. Thus, the price here is greater than that of the deterministic scenario. From the first order optimality condition, we can easily obtain $q^f(z, x_s)$, the optimal quality of the raw material.

Proposition 17. *For $p^i(z, x_s)$ and $q^i(z, x_s)$, we have the following relationships:*

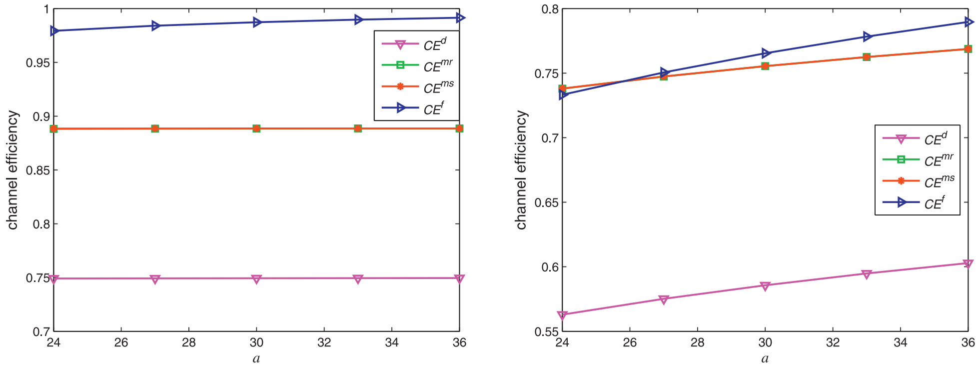
(i) $p^d(z, x_s) > p^{mr}(z, x_s) = p^{ms}(z, x_s) > p^f(z, x_s) > p^c(z, x_s)$,
(ii) $q^c(z, x_s) > q^f(z, x_s) > q^{mr}(z, x_s) = q^{ms}(z, x_s) > q^d(z, x_s)$.

Comparing Eqs. (30), (37), (42), (47) and (54), we can easily get the results provided in Proposition 17(i). Similarly, comparing $D(p, x_m)$ and Eqs. (29), (34), (40), (45) and (51), we get the results given in Proposition 17 (ii).

4. Numerical analysis

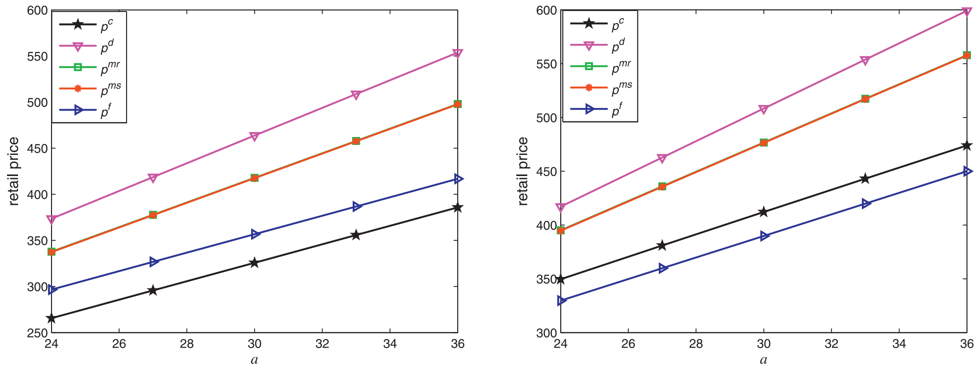
To demonstrate the model with deterministic demand, we take the following numerical example: $a = 30$, $b = .05$, $c = 50$, $\beta = 0.1$, $g_s = 0.4$, $\eta_s = 8$, $\eta_m = 10$, $g_m = .5$, $\lambda = 1$, $h = 8$, $s = 1$ in appropriate units. For the model with stochastic demand, we assume that the randomness of the demand follows an exponential distribution i.e. $f(x, \alpha) = \alpha e^{-\alpha x}, x > 0$ with mean $\mu = 10$ and $\alpha = 0.1$. The results obtained are summarized in Tables 1 and 2.

From Tables 1 and 2, we can see that the centralized system generates the maximum profit, as expected. The RFM strategy successfully coordinates the decentralized system and makes a win-win situation for each party involved in the supply chain. Further, the RFM strategy performs better for the deterministic demand rather than the stochastic demand, and nearly matches with the centralized system profit. It is to be observed that sub-supply chain coordination is quite fruitful in case of stochastic demand, and it improves profit of each party involved. In case of deterministic demand, sub-supply



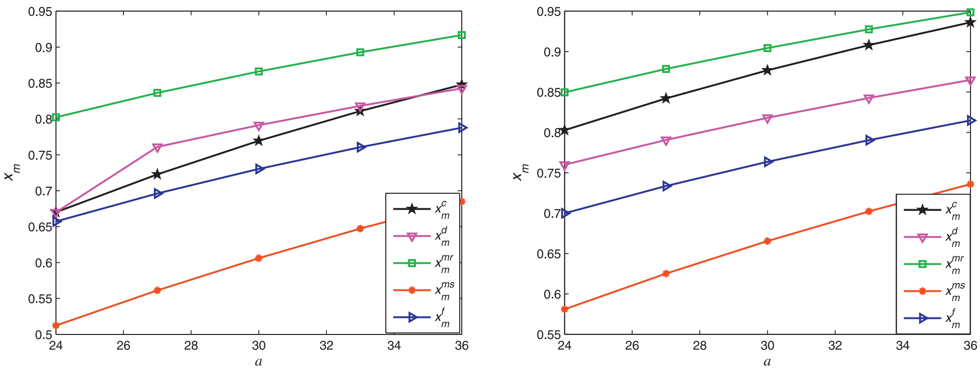
(a) a vs. CE in deterministic scenario (b) a vs. CE in stochastic scenario

Fig. 3. Sensitivity of a on channel efficiency(CE).



(a) a vs. price in deterministic scenario (b) a vs. price in stochastic scenario

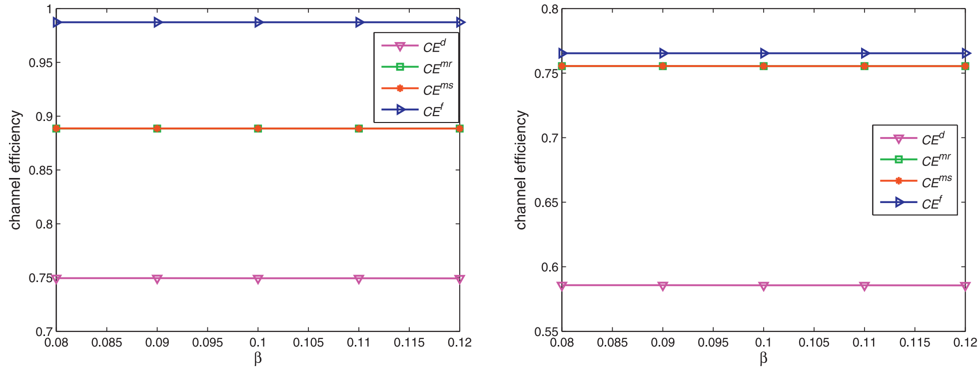
Fig. 4. Sensitivity of a on retail price.



(a) a vs. quality in deterministic scenario (b) a vs. quality in stochastic scenario

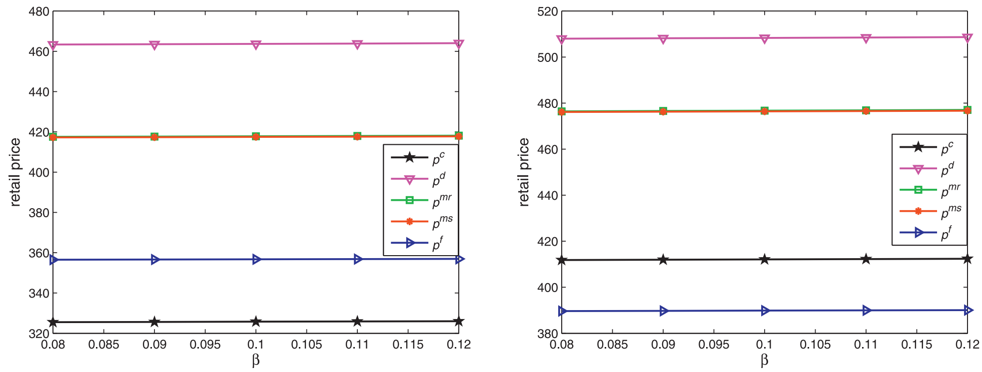
Fig. 5. Sensitivity of a on product quality.

chain coordination is not very effective in improving each party's profit although the total profit of the chain is enhanced compared to that of the decentralized system. In sub-supply chain coordination, when the manufacturer is combined with the supplier, the quality of the product falls drastically. On the other hand, when the manufacturer is combined with the retailer, the product quality is much higher. In spite of quality differences in two cases of sub-supply chain coordination, the total profit, the retail price and profits of the supply chain entities are almost same. We now investigate how the key findings of our model vary with various parameter-values. We reflect them in Figs. 3–13.



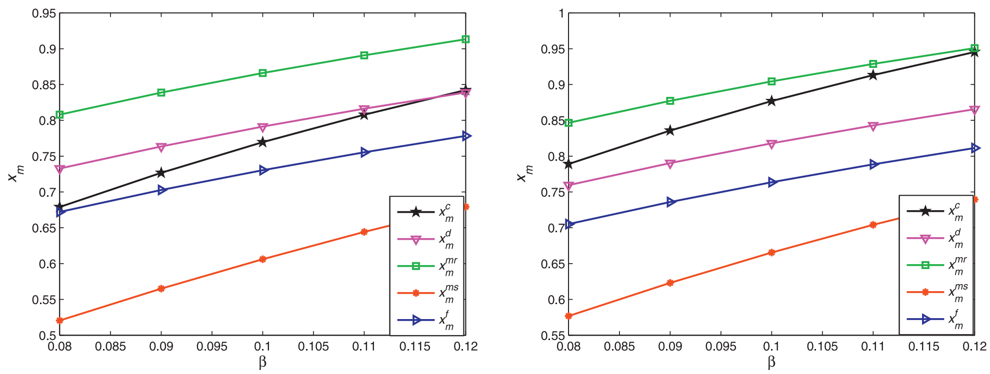
(a) β vs. CE in deterministic scenario (b) β vs. CE in stochastic scenario

Fig. 6. Sensitivity of β on channel efficiency(CE).



(a) β vs. price in deterministic scenario (b) β vs. price in stochastic scenario

Fig. 7. Sensitivity of β on retail price.

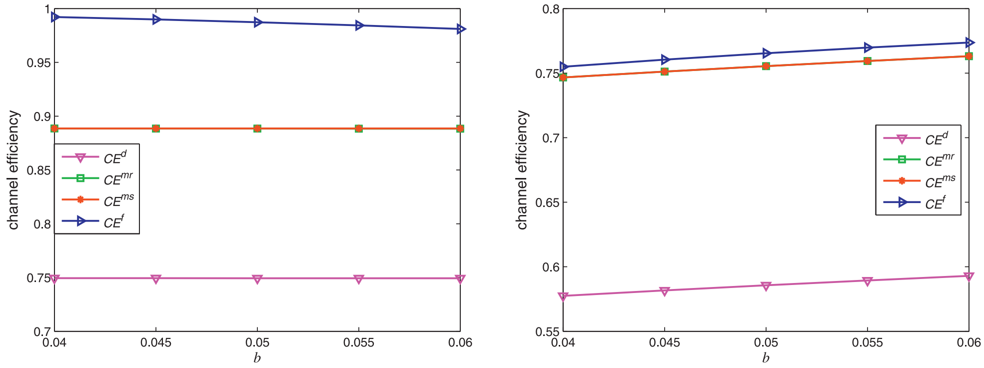


(a) β vs. quality in deterministic scenario (b) β vs. quality in stochastic scenario

Fig. 8. Sensitivity of β on product quality.

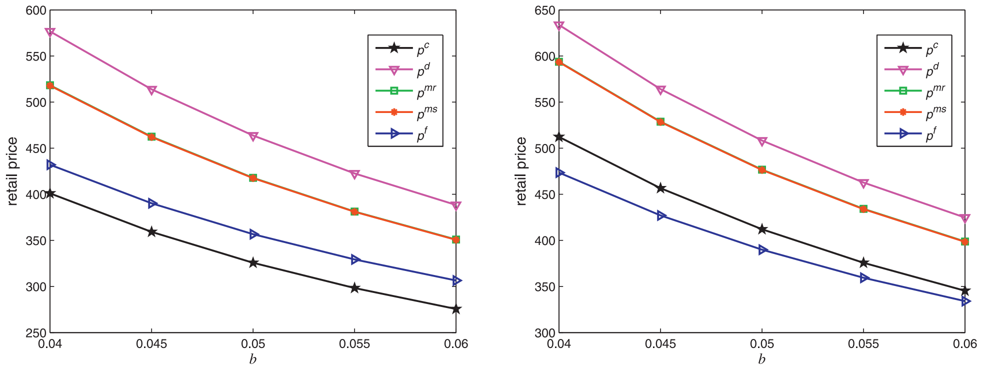
Figs. 3–11 show the changes in channel efficiency, retail price and product quality with respect to changes in deterministic part of the market demand (a), the retail price sensitivity parameter (b) and the quality sensitivity parameter (β), respectively. Figs. 12 and 13 compare the behavior of the profit functions of the supplier, manufacturer and the retailer in the decentralized system and the system with RFM strategy. Figs. 3a–13a are drawn for the deterministic demand whereas Figs. 3b–13b are drawn for the stochastic demand.

(i) For deterministic demand, with an increase in a, channel efficiencies of decentralized, MS-Nash and MR-Nash scenarios remain unchanged, but the channel efficiency of the system with RFM strategy increases slightly (see Fig. 3a). For



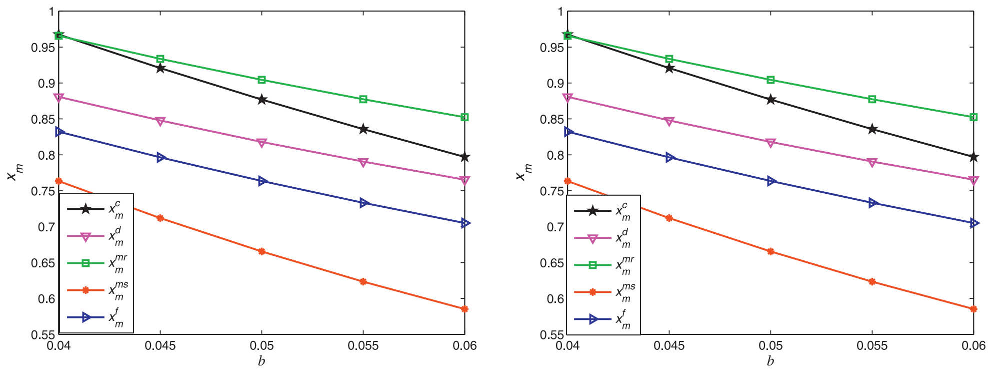
(a) b vs. CE in deterministic scenario (b) b vs. CE in stochastic scenario

Fig. 9. Sensitivity of b on channel efficiency(CE).



(a) b vs. price in deterministic scenario (b) b vs. price in stochastic scenario

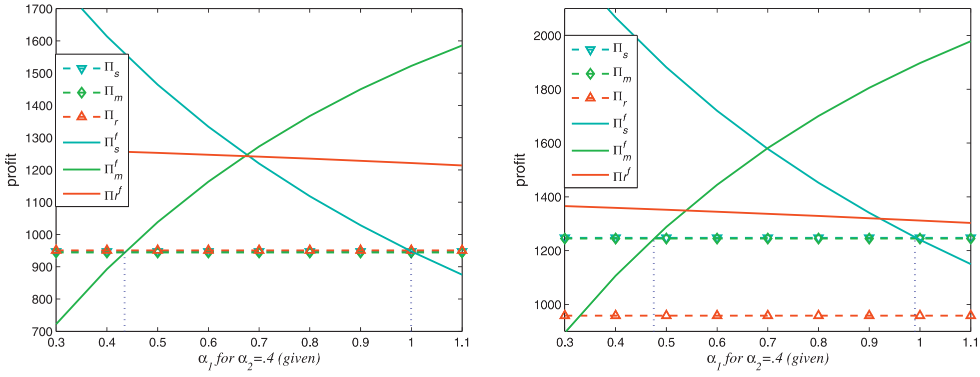
Fig. 10. Sensitivity of b on retail price.



(a) b vs. quality in deterministic scenario (b) b vs. quality in stochastic scenario

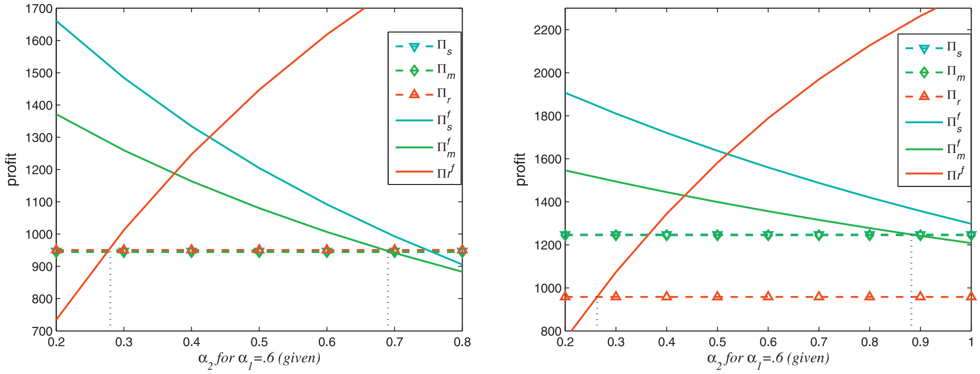
Fig. 11. Sensitivity of b on product quality.

stochastic demand, the channel efficiency of each system increases with an increase in a (see Fig. 3b). In stochastic scenario, the higher value of a means better coverage of uncertainty of the market in each scenario, which improves channel efficiency. The retail price and the product quality of each system also increases with an increase in a (see Figs. 4, 5) and the rate of increase in deterministic case is also similar to that in stochastic case. Further, a higher value of a encourages production of high quality end product to attract more customers. To produce high quality product, the production cost goes up, which results in higher retail price.



(a) α_m vs. profit in deterministic scenario (b) α_m vs. profit in stochastic scenario

Fig. 12. Win-win situation for all players when the manufacturer's markup is varying.



(a) α_r vs. profit in deterministic scenario (b) α_r vs. profit in stochastic scenario

Fig. 13. Win-win situation for all players when the retailer's markup is varying.

(ii) With an increase in the parameter β, the channel efficiency and the retail price remain almost same (see Figs. 6 and 7) but the product quality increases steadily (see Fig. 8) for both the deterministic and stochastic demand patterns. The product quality sensitivity parameter β affects all channels in a similar way. On the other hand, an increase in β directly increases the dependency of the product quality on the market demand, but it does not affect the retail price of the commodity directly. This is the reason behind rather ineffectiveness of channel efficiency and the retail price, and a significant improvement in the product quality with increasing β.

(iii) An increase in the value of the parameter b results in a decrease in the retail price and the product quality for each system in both deterministic and stochastic demand scenarios (see Figs. 10 and 11). The decreasing pattern is also similar to both the situations. An increase in price sensitivity parameter b directly decreases the market demand. To cope with the increasing b and maintain steady market demand, the retail price has to be reduced. A reduction in the retail price causes a decrease in production budget which affects the quality of the product negatively. In the case of channel efficiency, the change is very little in comparison with the change in the retail price or product quality (see Fig. 9). In the deterministic situation, the channel efficiency of the system with RFM strategy decreases slightly (see Fig. 9a) but for the decentralized, MS-Nash and MR-Nash scenarios, the channel efficiency remains unchanged. It is due to the fact that the pricing decision is mainly taken by the supplier, and the manufacturer and the retailer use fixed markup over the supplier's pricing decision. On the other hand, in the stochastic case, there is a slight increase in the channel efficiency of the system (see Fig. 9b). The added randomness of demand in the stochastic scenario copes better with the increasing b. That's why the channel efficiency is better in this scenario.

(iv) In Fig. 12, we keep α_r fixed and vary α_m. We observe that the profits of the supplier and the retailer decrease and that of the manufacturer increases with increase in α_m. However, the rate of decrease in profit for the retailer is much lower in comparison to that of the supplier. If the manufacturer increases α_m, the supplier has to reduce his/her price to maintain market demand. As a result, the profit of the supplier decreases. It also affects the retailer and diminishes his/her profit. On the other hand, the manufacturer increases his/her margin, which generates higher profit. In the RFM strategy, the pricing decision is made by the supplier. So, changing α_m affects directly the pricing of the supplier but indirectly the pricing of the retailer. This is the reason for a lower rate of profit decrease of the retailer in comparison to that of the supplier. Comparing the results of the RFM policy with the decentralized policy in the deterministic scenario, we see that RFM policy provides a win-win situation for each supply chain entity when α_m lies between 0.43 and 1 (see Fig. 12a). In the stochastic scenario, the win-win situation arises (see Fig. 12b) when the value of α_m lies between 0.47 and 0.99.

(v) In Fig. 13, we keep α_m fixed and vary α_r. We see that the profit of the retailer increases with an increase in α_r, and those of the supplier and the manufacturer decrease with increase in α_r. The rate of decrease is slightly lower for the manufacturer. Here, an increase in α_r causes an increase in the retailer's margin as well as profit, and a decrease in pricing of the supplier ultimately reduces profit. The manufacturer is also affected negatively by the increase in α_r, but as the supplier makes the pricing decision, the effect on the supplier is more than the manufacturer. In the deterministic scenario (see Fig. 13a), when α_r takes values between 0.28 and 0.69, the RFM strategy provides a win-win situation for each supply chain entity. The win-win situation arises in stochastic scenario (see Fig. 13b) when the value of α_r lies between 0.26 and 0.88.

5. Managerial implications

Efficient supply chain management is necessary to successfully run a business enterprise. To make supply chain efficient, a supply chain manager has to make several important decisions. Here, we consider a three-echelon supply chain where the end customer demand is driven by the retail price and the finished product quality. We emphasize the dependency of the finished product quality on the raw material quality. We also address the issue of uncertainty of the market demand. We answer several relevant questions: How much to order? What retail price to set? What quality products to produce and how to coordinate? These answers not only enrich supply chain managers, but also help them to take optimal decisions. Each supply chain member has also to make several important decisions. The supplier has to decide his/her raw material quality and wholesale price, the manufacturer has to find out how much raw material to purchase, what wholesale price to set and how to maintain a certain finished product quality. The retailer has to optimize his/her order quantity and set retail price. We obtain these decisions and finally establish two strategies which enhance the efficiency of the supply chain.

It is the job of the manager to come up with strategies that can improve the profit of the company. Coordinating with other supply chain members and establishing contracts among them is one such strategy. In our model, we study the system with the sub-supply chain coordination strategy which is a very effective strategy for the manufacturing firm. The manufacturing firm, in this set up, merges either with the raw material provider or with the retailer and jointly take decisions. In another attempt, we apply the RFM strategy among the supply chain members. We show that such ventures improve the efficiency of the supply chain.

6. Conclusions

The main objective of our study is to find out the optimal ordering, pricing and quality management strategy in a three-echelon supply chain where the market demand is price and product quality dependent. First, we find out the optimal decisions in the centralized scenario which is the benchmark case. We then consider the decentralized scenario and establish two strategies, viz. sub-supply chain coordination and RFM strategies, to improve the profitability of the decentralize system. From our study, we see that the RFM strategy performs better than the sub-supply chain strategy in coordinating the decentralized system. We also observe that the performance of the RFM strategy is better when the market demand does not involve any uncertainty, i.e. the market demand is deterministic. In this model, we analyze the effect of the supplier's raw material quality on the manufacturer's finished product quality. We find that the supplier's raw material quality has a positive effect on the pricing and order quantity of the product. On the other hand, the quality decision parameter of the manufacturer has a negative effect on the pricing and order quantity of the product. We also observe relatively higher order quantity, pricing, quality and profit in the stochastic demand scenario than those of the deterministic demand scenario. In the sub-supply chain strategy, when the manufacturer and the retailer work together, the quality is higher. On the contrary, when the manufacturer is combined with the supplier, the quality drops drastically.

The present model can be extended in several ways. It can be extended by incorporating multiple manufactures or retailers. One may consider a multiple products scenario or a product which requires multiple types of raw material to produce. Our model considers a single channel scenario; it can be extended to a dual channel scenario where the manufacturer can use his/her own direct channel to sale the product to the end customers, besides traditional retail channel.

Acknowledgments

The authors are thankful to the anonymous reviewers for their helpful comments and constructive suggestions on the earlier versions of the manuscript. This work is supported by Council of Scientific & Industrial Research (CSIR), Government of India under Award no.:09/096(0706)/2011-EMR-I.

Appendix A

Proof of Lemma 1

Taking the first and second order partial derivatives of finished product quality x_m with respect to raw material quality x_s, we get

$$\frac{\partial x_m}{\partial x_s} = \frac{2\lambda {x_s}^{\lambda-1}}{(1+x_s^{\lambda})^2} > 0,$$
$$\frac{\partial^2 x_m}{\partial {x_s}^2} = -\frac{2\lambda {x_s}^{\lambda-2}(1-\lambda+x_s^{\lambda}(1+\lambda))}{(1+x_s^{\lambda})^3}.$$

From above, we see that, $\frac{\partial^2 x_m}{\partial {x_s}^2} < 0$, when $1-\lambda+x_s^{\lambda}(1+\lambda) > 0$.
From our assumption, $0 < x_s < 1$ and $\lambda > 0$. This implies, $0 < x_s^{\lambda} < 1$.
When $x_s > 0$, and $\lambda < 1$, we have $1-\lambda+x_s^{\lambda}(1+\lambda) > 0 \Rightarrow \frac{\partial^2 x_m}{\partial {x_s}^2} < 0$.
Again, for $\lambda = 1$, $x_s^{\lambda}(1+\lambda) > 0 \Rightarrow \frac{\partial^2 x_m}{\partial {x_s}^2} < 0$. Therefore, $\frac{\partial^2 x_m}{\partial {x_s}^2} < 0$ for $0 < \lambda \le 1$.
Now, for $\lambda > 1$, we get $1-\lambda+x_s^{\lambda}(1+\lambda) < 0$ as $x_s < 1 \Rightarrow x_s^{\lambda} << 1$. Therefore, $\frac{\partial^2 x_m}{\partial {x_s}^2} > 0$ for $\lambda > 1$.

Appendix B

Proof of Proposition 1

The total profit of the system is

$$\Pi_c = (p-c)D - \eta_s x_s^2 - \eta_m x_m^2 - g_s(1-x_s) - g_m(1-x_m).$$

Taking the first and second order partial derivatives of Π_c with respect to x_s, we get

$$\frac{\partial \Pi_c}{\partial x_s} = \left[(p-c)\beta - 2\eta_m x_m + g_m\right]\frac{\partial x_m}{\partial x_s} - 2\eta_s x_s - g_s,$$
$$\frac{\partial^2 \Pi_c}{\partial {x_s}^2} = \left[(p-c)\beta - 2\eta_m x_m + g_m\right]\frac{\partial^2 x_m}{\partial {x_s}^2} - 2\eta_s - 2\eta_m\left(\frac{\partial x_m}{\partial x_s}\right)^2.$$

From above, we see that

$$\frac{\partial^2 \Pi_c}{\partial {x_s}^2} < 0, \text{ when } \frac{\partial^2 x_m}{\partial {x_s}^2} < 0, \text{ and } (p-c)\beta + g_m > 2\eta_m, \text{ as } 0 < x_m < 1.$$

Now, from Lemma 1, we see that

$$\frac{\partial^2 x_m}{\partial {x_s}^2} < 0, \text{ for } 0 < \lambda \le 1.$$

Therefore,

$$\frac{\partial^2 \Pi_c}{\partial {x_s}^2} < 0, \text{ if } 0 < \lambda \le 1 \text{ and } p > c + \frac{2\eta_m - g_m}{\beta}.$$

Appendix C

Proof of Proposition 2

The total profit of the system is

$$\Pi_c = (p-c)D - \eta_s x_s^2 - \eta_m x_m^2 - g_s(1-x_s) - g_m(1-x_m).$$

Taking the first order partial derivative of Π_c with respect to p, for given value of raw material quality x_s, we get

$$\frac{\partial \Pi_c}{\partial p} = a - bp + \beta x_m - b(p-c).$$

We have $x_m = 1 - \frac{1-x_s^{\lambda}}{1+x_s^{\lambda}}$. The first order optimality condition $\frac{\partial \Pi_c}{\partial p} = 0$ gives the optimal value of p when the raw material quality x_s is fixed. Thus, solving $\frac{\partial \Pi_c}{\partial p} = 0$, for p, we get

$$p^c(x_s) = \frac{a + bc + \left(a + bc + 2\beta\right)x_s^{\lambda}}{2b(1+x_s^{\lambda})}.$$

In the deterministic scenario, the order quantity is equal to the market demand i.e. $q = D$. Now, using the value of the optimal retail price $p^c(x_s)$ and finished product quality x_m in the expression for the market demand i.e. $D(p, x_m) = a - bp + \beta x_m$, we obtain the optimal order quantity for fixed raw material quality as

$$q^c(x_s) = \frac{a - bc + \big(a - bc + 2\beta\big)(x_s^c)^\lambda}{2b\big(1 + (x_s^c)^\lambda\big)}.$$

Appendix D

Proof of Proposition 3

We have

$$p^c(x_s) = \frac{a + bc + \big(a + bc + 2\beta\big)x_s^\lambda}{2b(1 + x_s^\lambda)},$$

$$q^c(x_s) = \frac{a - bc + \big(a - bc + 2\beta\big)(x_s^c)^\lambda}{2b\big(1 + (x_s^c)^\lambda\big)}.$$

Taking the first order partial derivatives of $p^c(x_s)$ and $q^c(x_s)$ with respect to x_s, we get

$$\frac{\partial p^c(x_s)}{\partial x_s} = \frac{\alpha\beta {x_s}^{\lambda-1}}{b(1 + x_s^\lambda)^2} > 0, \ \text{as } b, \ \alpha, \ \beta, \ \lambda > 0,$$

$$\frac{\partial q^c(x_s)}{\partial x_s} = \frac{\alpha\beta {x_s}^{\lambda-1}}{(1 + x_s^\lambda)^2} > 0, \ \text{as } b, \ \alpha, \ \beta, \ \lambda > 0.$$

Again, taking the first order partial derivatives of $p^c(x_s)$ and $q^c(x_s)$ with respect to λ, we get

$$\frac{\partial p^c(x_s)}{\partial \lambda} = \frac{\beta {x_s}^{\lambda} \log x_s}{b(1 + x_s^\lambda)^2} < 0, \ \text{as } b, \ \beta, \ \lambda > 0 \text{ and } \log x_s < 0 \ (\text{as } 0 < x_s < 1),$$

$$\frac{\partial q^c(x_s)}{\partial \lambda} = \frac{\beta {x_s}^{\lambda} \log x_s}{(1 + x_s^\lambda)^2} < 0, \ \text{as } b, \ \beta, \ \lambda > 0 \text{ and } \log x_s < 0 \ (\text{as } 0 < x_s < 1).$$

Appendix E

Proof of Proposition 4

The profit of the retailer is given by

$$\Pi_r = (p - w_m)D.$$

Taking the first order partial derivative of Π_r with respect to p, for given value of x_s, we get

$$\frac{\partial \Pi_r}{\partial p} = \frac{(a - 2bp + bw_m)(1 + x_s^\lambda) + 2x_s^\lambda \beta}{(1 + x_s^\lambda)}.$$

The first order optimality condition $\frac{\partial \Pi_r}{\partial p} = 0$ gives the optimal value of p. We have $w_m = (2p + c)/3$ and $w_s = (2c + p)/3$. Now, solving simultaneously the equations

$$\frac{(a - 2bp + bw_m)(1 + x_s^\lambda) + 2x_s^\lambda \beta}{(1 + x_s^\lambda)} = 0, \quad w_m = (2p + c)/3 \text{ and } w_s = (2c + p)/3,$$

we get

$$w_s^d(x_s) = \frac{a + 3bc + (a + 3bc + 2\beta)x_s^\lambda}{4b(1 + x_s^\lambda)},$$

$$w_m^d(x_s) = \frac{a + bc + (a + bc + 2\beta)x_s^\lambda}{2b(1 + x_s^\lambda)},$$

$$p^d(x_s) = \frac{3a + bc + (3a + bc + 6\beta)x_s^\lambda}{4b(1 + x_s^\lambda)}.$$

As in Appendix B, we get the optimal order quantity q for fixed raw material quality x_s by putting the values of $p^d(x_s)$ and x_m in the expression $D(p, x_m) = a - bp + \beta x_m$, which is given by

$$q^d(x_s) = \frac{a - bc + (a - bc + 2\beta)x_s^\lambda}{4(1 + x_s^\lambda)}.$$

Appendix F

Proof of Proposition 8
The profit of the supplier is given by

$$\Pi_s^f = (w_s - c)\big(a - b(1+\alpha_r)(1+\alpha_m)w_s + \beta x_m\big) - \eta_s x_s^2 - g_s(1-x_s).$$

Taking the first order partial derivative of Π_s^f with respect to w_s, for given value of x_s, we get

$$\frac{\partial \Pi_s^f}{\partial w_s} = a + b(c - 2w_s)(1+\alpha_m)(1+\alpha_s) + \frac{2x_s^\lambda \beta}{1+x_s^\lambda}.$$

Solving the first order optimality condition of w_s, i.e. $\frac{\partial \Pi_s^f}{\partial w_s} = 0$, we get

$$w_s^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^\lambda + bc(1+x_s^\lambda)(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1+x_s^\lambda)(1+\alpha_r)(1+\alpha_m)}.$$

Using the optimal value of w_s in the RFM strategy parameters i.e. $p = (1+\alpha_r)w_m$ and $w_m = (1+\alpha_m)w_s$, we get the optimal value of wholesale price of the manufacture and the retail price of the retailer as

$$w_m^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^\lambda + bc(1+x_s^\lambda)(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1+x_s^\lambda)(1+\alpha_r)},$$

$$p^f(x_s) = \frac{a + 2bc + (a + 2bc + 2\beta)x_s^\lambda + bc(1+x_s^\lambda)(\alpha_r + \alpha_r\alpha_m + \alpha_m)}{2b(1+x_s^\lambda)}.$$

Similarly, from $D(p, x_m) = a - bp + \beta x_m$, we get the optimal order quantity as

$$q^f(x_s) = \frac{a - bc + \big[a - bc(1+\alpha_r)(1+\alpha_m) + 2\beta\big]x_s^\lambda}{2\big(1+x_s^\lambda\big)}.$$

Appendix G

Proof of Proposition 11
We have

$$\begin{aligned} p^c(z, x_s) &= \frac{a + bc + \mu + \big(a + bc + \mu + 2\beta\big)x_s^\lambda - (1+x_s^\lambda)\Theta(z)}{2b(1+x_s^\lambda)} \\ &= p^c(x_s) + \frac{\mu - \Theta(z)}{2b}, \end{aligned}$$

where $p^c(x_s) = \frac{a+bc+(a+bc+2\beta)x_s^\lambda}{2b(1+x_s^\lambda)}$, the retail price when the demand is certain. Thus, $p^c(z,\ x_s) > p^c(x_s)$ as expected demand $(\mu) >$ stock out $(\Theta(z))$.
Again, $\frac{dz^c}{dp} = \frac{(c+h)}{f(z^c)(p+g+h)^2} > 0$. Therefore, the stocking factor increases with increasing retail price.

Appendix H

Proof of Theorem 1
We have

$$\begin{aligned} \frac{\mathrm{d}\Pi_c}{\mathrm{d}z} &= -(c+h) + \big(p^c(z, x_s) + s + h\big)[1 - F(z)] \\ &= -(c+h) + \Big(p^c(x_s) + \frac{\mu - \Theta(z)}{2b} + s + h\Big)[1 - F(z)]. \end{aligned}$$

To identify that the values of z satisfy the first order optimality condition, let $R(z) \equiv \frac{\mathrm{d}\Pi_c}{\mathrm{d}z}$

$$\begin{aligned} \frac{\mathrm{d}R(z)}{\mathrm{d}z} &= \frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{\mathrm{d}\Pi_c}{\mathrm{d}z}\right] \\ &= -\frac{f(z)}{2b}\left[2b\big(p^c(x_s) + s + h\big) + \mu - \Theta(z) - \frac{1 - F(z)}{r(z)}\right], \end{aligned}$$

where $r(z)=f(z)/(1-F(z))$ is the hazard rate. Again,

$$\frac{\mathrm{d}^2R(z)}{\mathrm{d}z^2}=\left[\frac{\mathrm{d}\Pi_c/\mathrm{d}z}{f(z)}\right]\frac{\mathrm{d}f(z)}{\mathrm{d}z}-\frac{f(z)}{2b}\left[[1-F(z)]+\frac{f(z)}{r(z)}+\frac{[1-F(z)][\mathrm{d}\Pi_c/\mathrm{d}z]}{r(z)^2}\right],$$

$$\left.\frac{\mathrm{d}^2R(z)}{\mathrm{d}z^2}\right|_{\mathrm{d}\Pi_c/\mathrm{d}z=0}=-\frac{f(z)[1-F(z)]}{2br(z)^2}\left[2r(z)^2+\frac{\mathrm{d}r(z)}{\mathrm{d}z}\right].$$

Now, if $F(\cdot)$ satisfies $2r(z)^2+(dr(z)dz)>0$, then it follows that $R(z)$ is either monotone or unimodal, which implies that $R(z)\equiv\frac{\mathrm{d}\Pi_c}{\mathrm{d}z}$ has at most two roots. Also, $\lim_{z\to\infty}R(z)=-(c+h)<0$. Therefore, if $R(z)$ has only one root, it corresponds to the maximum of Π_c; if it has two roots, the larger of the two represents the maximum and the smaller represents the minimum of Π_c and we denote it by z^c.

Appendix I

Proof of Proposition 12

The profit of the retailer is given by

$$\Pi_r=(p-w_m)[D(p,x_m)+\mu]-(h+w_m)\Lambda(z)-(p+s-w_m)\Theta(z).$$

Taking the first order partial derivative of Π_r with respect to p, for given value of stocking factor z and raw material quality x_s, we get

$$\frac{\partial\Pi_r}{\partial p}=\frac{(a-2bp+bw_m+\mu-\Theta(z))(1+x_s^\lambda)+2x_s^\lambda\beta}{(1+x_s^\lambda)}.$$

The first order optimality condition $\frac{\partial\Pi_r}{\partial p}=0$ gives the optimal value of p. We also know that $w_m=(2p+c)/3$ and $w_s=(2c+p)/3$. Now, simultaneously solving the following:

$$\frac{(a-2bp+bw_m+\mu-\Theta(z))(1+x_s^\lambda)+2x_s^\lambda\beta}{(1+x_s^\lambda)}=0,\quad w_m=(2p+c)/3 \text{ and } w_s=(2c+p)/3,$$

we get

$$w_s^d(z,x_s)=\frac{a+3bc+\mu+(a+3bc+\mu+2\beta)x_s^\lambda-(1+x_s^\lambda)\Theta(z)}{4b(1+x_s^\lambda)},$$

$$w_m^d(z,x_s)=\frac{a+bc+\mu+(a+bc+\mu+2\beta)x_s^\lambda-(1+x_s^\lambda)\Theta(z)}{2b(1+x_s^\lambda)},$$

$$p^d(z,x_s)=\frac{3a+bc+3\mu+(3a+bc+3\mu+6\beta)x_s^\lambda-3(1+x_s^\lambda)\Theta(z)}{4b(1+x_s^\lambda)}.$$

Appendix J

Proof of Proposition 16

The stocking factor of the retailer is $z=q-D(p,x_m)$. This implies that the order quantity is $q=z+D(p,x_m)$. The profit of the supplier is given by

$$\begin{aligned}\Pi_s^f&=(w_s-c)q-\eta_s x_s^2-g_s(1-x_s)\\&=(w_s-c)\big(a-b(1+\alpha_r)(1+\alpha_m)w_s+\beta x_m+z\big)-\eta_s x_s^2-g_s(1-x_s).\end{aligned}$$

Taking the first order partial derivative of Π_s^f with respect to w_s, for given value of x_s, we get

$$\frac{\partial\Pi_s^f}{\partial w_s}=a+z+b(c-2w_s)(1+\alpha_m)(1+\alpha_s)+\frac{2x_s^\lambda\beta}{1+x_s^\lambda}.$$

Solving the first order optimality condition $\frac{\partial\Pi_s^f}{\partial w_s}=0$, we get

$$w_s^f(z,x_s)=\frac{a+2bc+(a+2bc+2\beta)x_s^\lambda+bc(1+x_s^\lambda)(\alpha_r+\alpha_r\alpha_m+\alpha_m)+z(1+x_s^\lambda)}{2b(1+x_s^\lambda)(1+\alpha_r)(1+\alpha_m)}.$$

Using the optimal value of w_s in $p=(1+\alpha_r)w_m$ and $w_m=(1+\alpha_m)w_s$, we get the optimal value of wholesale price of the manufacture and the retail price of the retailer as

$$w_m^f(z,x_s)=\frac{a+2bc+(a+2bc+2\beta)x_s^\lambda+bc(1+x_s^\lambda)(\alpha_r+\alpha_r\alpha_m+\alpha_m)+z(1+x_s^\lambda)}{2b(1+x_s^\lambda)(1+\alpha_r)},$$

$$p^f(z,x_s)=\frac{a+2bc+(a+2bc+2\beta)x_s^\lambda+bc(1+x_s^\lambda)(\alpha_r+\alpha_r\alpha_m+\alpha_m)+z(1+x_s^\lambda)}{2b(1+x_s^\lambda)}.$$

References

[1] G. Ghiani, G. Laporte, R. Musmanno, Introduction To Logistics Systems Planning and Control, John Wiley and Sons, Inc., Hoboken, New Jersey, 2004.
[2] D.J. Reyniers, C.S. Tapiero, The delivery and control of quality in supplier producer contracts, Manag. Sci. 41 (1995) 1581–1589.
[3] L.B. Forker, Factors affecting supplier quality performance, J. Oper. Manag. 15 (1997) 243–269.
[4] R.D. Banker, I. Khosla, K.K. Sinha, Quality and competition, Manag. Sci. 44 (1998) 1179–1192.
[5] S. Baiman, P.E. Fischer, M.V. Rajan, Information, contracting, and quality costs, Manag. Sci. 46 (2000) 776–789.
[6] C. Lin, W.S. Chow, C.N. Madu, C. Kuei, P. Yu, A structural equation model of supply chain quality management and organizational performance, Int. J. Prod. Econ. 96 (2005) 355–365.
[7] C.J. Robinson, M.K. Malhotra, Defining the concept of supply chain quality management and its relevance to academic and industrial practice, Int. J. Prod. Econ. 96 (2005) 315–337.
[8] F. Bernstein, A. Federgruen, Coordination mechanisms for supply chains under price and service competition, Manuf. Ser. Oper. Manag. 9 (2007) 242–262.
[9] S.T. Foster, Jr. towards an understanding of supply chain quality management, J. Oper. Manag. 26 (2008) 461–467.
[10] R.B. Franca, E.C. Jones, C.N. Richards, J.P. Carlson, Multi-objective stochastic supply chain modeling to evaluate tradeoffs between profit and quality, Int. J. Prod. Econ. 127 (2010) 292–299.
[11] G. Xie, W. Yue, S. Wang, K.K. Lai, Quality investment and price decision in a risk-averse supply chain, Eur. J. Oper. Res. 214 (2011) 403–410.
[12] Y.K. Tse, K.H. Tan, Managing product quality risk and visibility in multi-layer supply chain, Int. J. Prod. Econ. 139 (2012) 49–57.
[13] X. Wang, D. Li, A dynamic product quality evaluation based pricing model for perishable food supply chains, Omega 40 (2012) 906–917.
[14] J. Yu, S. Ma, Impact of decision sequence of pricing and quality investment in decentralized assembly system, J. Manuf. Syst. 32 (2013) 664–679.
[15] T. Maiti, B.C. Giri, A closed loop supply chain under retail price and product quality dependent demand, J. Manuf. Syst. 37 (3) (2015) 624–637.
[16] M.A. Darwish, O.M. Odah, S.K. Goyal, Vendor managed inventory models for single-vendor multi-retailer supply chains with quality consideration, Int. J. Ind. Syst. Eng. 18 (4) (2015) 499–527.
[17] G. Liu, J. Zhang, W. Tang, Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand, Ann. Oper. Res. 226 (1) (2015) 397–416.
[18] C. Munson, M. Rosenblatt, Coordinating a three-level supply chain with quantity discounts, IIE Trans. 33 (2001) 371–384.
[19] I. Giannoccaro, P. Pontrandolfo, Supply chain coordination by revenue sharing contracts, Int. J. Prod. Econ. 89 (2004) 131–139.
[20] M. Jaber, I. Osman, A. Guiffrida, Coordinating a three-level supply chain with price discounts, price dependent demand, and profit sharing, Int. J. Integr. Sup. Manag. 2 (2006) 28–48.
[21] M. Jaber, S. Goyal, Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers, Int. J. Prod. Econ. 116 (2008) 95–103.
[22] D. Ding, J. Chen, Coordinating a three level supply chain with flexible return policies, Omega 36 (2008) 865–876.
[23] H. Zijm, J. Timmer, Coordination mechanisms for inventory control in three-echelon serial and distribution systems, Ann. Oper. Res. 158 (2008) 161–182.
[24] Y. Huang, G.Q. Huang, Price coordination in a three-level supply chain with different channel structures using game-theoretic approach, Int. J. Manag. Sci. Eng. Manag. 5 (2) (2010) 83–94.
[25] M.C. Georgiadis, P. Tsiakis, P. Longinidis, M. Sofioglou, Optimal design of supply chain networks under uncertain transient demand variations, Omega 39 (2011) 254–272.
[26] S.M.J.M. Al-e Hashem, H. Malekly, M.B. Aryanezhad, A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty, Int. J. Prod. Econ. 134 (2011) 28–42.
[27] Y. He, X. Zhao, Coordination in multi-echelon supply chain under supply and demand uncertainty, Int. J. Prod. Econ. 139 (2012) 106–115.
[28] R.W. Seifert, R.I. Zequeira, S. Liao, A three-echelon supply chain with price only contracts and sub-supply chain coordination, Int. J. Prod. Econ. 138 (2012) 345–353.
[29] M. Omar, R. Sarkar, W.A.M. Othman, A just-in-time three-level integrated manufacturing system for linearly time-varying demand process, Appl. Math. Model. 37 (2013) 1275–1281.
[30] T.C. Zhang, L.P. Liu, Research on coordination mechanism in three-level green supply chain under non-cooperative game, Appl. Math. Model. 37 (2013) 3369–3379.
[31] Z. Liu, Q. Zhao, S. Wang, J. Shi, Modeling the impact of partial information sharing in a three-echelon supply chain, Asia. Pac. J. Oper. Res. 30 (2013) 1350020. [23 pages].
[32] S. Saha, Channel characteristics and coordination in three-echelon dual-channel supply chain, Int. J. Syst. Sci. 47 (3) (2014) 740–754.
[33] L.E. Cárdenas-Barrón, G.T. no Garza, An optimal solution to a three echelon supply chain network with multi-product and multi-period, Appl. Math. Model. 38 (2014) 1911-1918.
[34] K.B. Monroe, Pricing: Making Profitable Decisions, McGraw-Hill, New York, 1990.
[35] Y. Liu, M.J. Fry, A.S. Raturi, Vertically restrictive pricing in supply chains with price-dependent demand, Nav. Res. Logis. 53 (2006) 485–501.
[36] Y. Liu, M.J. Fry, A.S. Raturi, Retail price markup commitment in decentralized supply chains, Eur. J. Oper. Res. 192 (2009) 277–292.
[37] Y. Liu, F. Qin, M.J. Fry, A.S. Raturi, Multi-period modeling of two-way price commitment under price-dependent demand, Eur. J. Oper. Res. 221 (2012) 546–556.
[38] J.C. Wang, A.M. Wang, Y.Y. Wang, Markup pricing strategies between a dominant retailer and competitive manufacturers, Comput. Ind. Eng. 64 (2013) 235–246.
[39] A. Sharma, S. Banerjee, Optimal price markup policy for an inventory model with random price fluctuations and option for additional purchase, Int. J. Prod. Econ. 146 (2013) 620–633.
[40] H. Liu, M. Lei, X. Liu, Manufacturer's uniform pricing and channel choice with a retail price markup commitment strategy, J. Syst. Sci. Sys. Eng. 23 (2014) 111–126.
[41] C. Chambers, P. Kouvelis, J. Semple, Quality-based competition, profit-ability, and variable costs, Manag. Sci. 52 (12) (2006) 1884–1895.
[42] N.C. Petruzzi, M. Dada, Pricing and the newsvendor problem: a review with extensions, Manag. Sci. 47 (2) (1999) 183–194.
[43] C.C. Lin, Y.C. Wu, Combined pricing and supply chain operations under price-dependent stochastic demand, App. Math. Model. 38 (56) (2014) 1823–1837.
[44] Y. He, X. Zhao, L. Zhao, J. He, Coordinating a supply chain with effort and price dependent stochastic demand, Appl. Math. Model. 33 (2009) 2777–2790.
[45] M.A. Ahmed, E. Saadany, M.Y. Jaber, A production/remanufacturing inventory model with price and quality dependant return rate, Comput. Ind. Eng. 58 (2010) 352–362.
[46] J. Vörös, Product balancing under conditions of quality inflation, cost pressures and growth strategies, Eur. J. Oper. Res. 141 (1) (2002) 153–166.
[47] P. Keller, T. Pyzdek, The Handbook for Quality Management, McGraw-Hill, 2013.
[48] A. Tsay, N. Agrawal, Channel conflict and coordination in the e-commerce age, Prod. Oper. Manag. 13 (1) (2004) 93–110.
[49] G.H. Chao, S.M.R. Iravani, R.C. Savaskan, Quality improvement incentives and product recall cost sharing contracts, Manag. Sci 55 (7) (2009) 1122–1138.
[50] Y. Li, L. Xu, D. Li, Examining relationships between return policy, product quality and pricing strategy in online direct selling, Int. J. Prod. Econ. 144 (2013) 451–460.

[51] S. Jørgensen, G. Zaccour, Equilibrium pricing and advertising strategies in a marketing channel, J. Optim. Theory. Appl. 102 (1) (1999) 111–125.
[52] J. Xie, A. Neyret, Co-op advertising and pricing models in manufacturer retailer supply chains, Comput. Ind. Eng. 56 (4) (2009) 1375–1385.
[53] B.C. Giri, T. Maiti, Profit improvement through retailer-stackelberg in a multi-echelon supply chain of deteriorating product with price-sensitive demand, J. Ind. Prod. Eng. 31 (4) (2014) 187–198.