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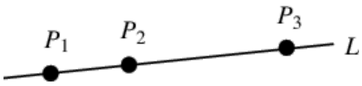
Collinear



Three or more points P_1, P_2, P_3, ..., are said to be collinear if they lie on a single straight line L. A line on which points lie, especially if it is related to a geometric figure such as a triangle, is sometimes called an axis.

Two points are trivially collinear since two points determine a line.

Three points $\mathbf{x}_i = (x_i, y_i, z_i)$ for $i = 1$, 2, 3 are collinear iff the ratios of distances satisfy

$$x_2 - x_1 : y_2 - y_1 : z_2 - z_1 = x_3 - x_1 : y_3 - y_1 : z_3 - z_1. \quad (1)$$

A slightly more tractable condition is obtained by noting that the area of a triangle determined by three points will be zero iff they are collinear (including the degenerate cases of two or all three points being concurrent), i.e.,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad (2)$$

or, in expanded form,

$$x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0. \quad (3)$$

This can also be written in vector form as

$$\mathrm{Tr}\,(\mathbf{x} \times \mathbf{y}) = 0, \quad (4)$$

where $\mathrm{Tr}\,(\mathbf{A})$ is the sum of components, $\mathbf{x} = (x_1, x_2, x_3)$, and $\mathbf{y} = (y_1, y_2, y_3)$.

The condition for three points $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ to be collinear can also be expressed as the statement that the distance between any one point and the line determined by the other two is zero. In three dimensions, this means setting $d = 0$ in the point-line distance

$$d = \frac{|(\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1)|}{|\mathbf{x}_2 - \mathbf{x}_1|}, \quad (5)$$

giving simply

$$|(\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_1 - \mathbf{x}_3)| = 0, \quad (6)$$

where × denotes the cross product.

Since three points are collinear if $\mathbf{x}_3 = \mathbf{x}_1 + c\,(\mathbf{x}_2 - \mathbf{x}_1)$ for some constant c, it follows that collinear points in three dimensions satisfy

$$\det\,(\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{x}_3) = \begin{vmatrix} x_1 & x_2 & x_1 + c\,(x_2 - x_1) \\ y_1 & y_2 & y_1 + c\,(y_2 - y_1) \\ z_1 & z_2 & z_1 + c\,(z_2 - z_1) \end{vmatrix} \quad (7)$$

$$= 0 \quad (8)$$

by the rules of determinant arithmetic. While this is a necessary condition for collinearity, it is not sufficient. (If any single point is taken as the origin, the determinant will clearly be zero. Another counterexample is provided by the noncollinear points $\mathbf{x}_1 = (16, 20, 20)$, $\mathbf{x}_2 = (5, 6, 6)$, $\mathbf{x}_3 = (15, 9, 9)$, for which $\det\,(\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{x}_3) = 0$ but $d = 22\,898 \neq 0$.)

Three points $\alpha_1 : \beta_1 : \gamma_1$, $\alpha_2 : \beta_2 : \gamma_2$, and $\alpha_3 : \beta_3 : \gamma_3$ in trilinear coordinates are collinear if the determinant

$$\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} = 0 \quad (9)$$

(Kimberling 1998, p. 29).

Let points P_1, P_2, and P_3 lie, one each, on the sides of a triangle $\Delta A_1 A_2 A_3$ or their extensions, and reflect these points about the midpoints of the triangle sides to obtain P_1', P_2', and P_3'. Then P_1', P_2', and P_3' are collinear iff P_1, P_2, and P_3 are (Honsberger 1995).

SEE ALSO:
Axis, Concyclic, Configuration, Directed Angle, Droz-Farny Theorem, General Position, Line, N-Cluster, Point-Line Distance--3-Dimensional, Sylvester's Line Problem

REFERENCES:

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Honsberger, R. *Episodes in Nineteenth and Twentieth Century Euclidean Geometry.* Washington, DC: Math. Assoc. Amer., pp. 153-154, 1995.

Kimberling, C. "Triangle Centers and Central Triangles." *Congr. Numer.* **129**, 1-295, 1998.

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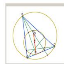
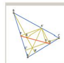
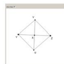


A Collinearity between the N Point Center, Foot of an Alt and a Midpoir



Four Collinea Points Relate the Altitudes



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