The solution is quite easy given as a Python function (Python 3.3+):

```
@lru_cache(maxsize=None) # memoize
def num_handshakes(n):
    if n % 2 == 1: return 0
    elif n == 0: return 1
    res = 0
    for i in range(0, n, 2):
        res += num_handshakes(i) * num_handshakes(n-2-i)
    return res
```

Example:

```
>>> num_handshakes(8)
14
```

This basically implements @Buhb's divide-and-conquer approach. Another solution, once we know the answer is related to the Catalan numbers:

```
from math import factorial as fac
def catalan(n):
    return fac(2*n) // fac(n+1) // fac(n)

def num_handshakes(n):
    if n % 2 == 1: return 0
    return catalan(n//2)
```