Experimental particle physics

3. particle interactions in particle detectors

European School of Instrumentation in Particle & Astroparticle Physics

lots of material borrowed from the excellent course of H.-C. Schultz-Coulon
http://www.kip.uni-heidelberg.de/~coulon/Lectures/Detectors/

Today lecture and tutorial format for today…

Wednesday Jan 20
Experimental Particle Physics Lecture 3 Marco Delmastro
BREAK
Experimental Particle Physics Tutorial 1 Marco Delmastro

- *In normal times, if we were all sitting in the same room, I would hold a lecture in the first half of the morning, and we'd take turns at solving problems at the whiteboard after the coffee break*
- *This year, in order to avoid that you fall asleep while I speak to my screen:*
  - ✓ *I'll distribute the lecture content over the whole morning*
  - ✓ *… and at times I'll ask you to solve small problems in small groups (e.g. Zoom breakout rooms) and we'll discuss / solve them together afterwards*

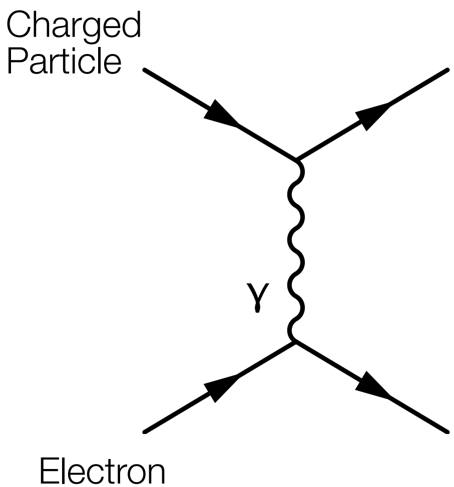
How do we detect particles?

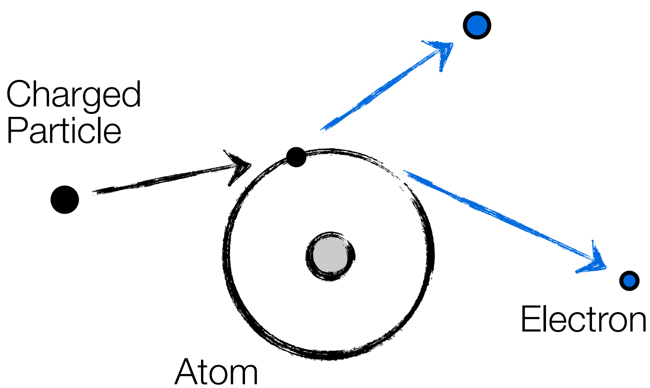
- In order to detect a particle, it must:
  - ✓ interact with the material of the detector
  - ✓ transfer energy in some recognizable fashion (signal)
- Detection of particles happens via their energy loss in the material they traverses

Charged particles	Ionization, Bremsstrahlung, Cherenkov, …	multiple interactions
Photons	Photo/Compton effect, pair production	single interactions…
Hadrons	Nuclear interactions	multiple interactions
Neutrinos	Weak interactions	

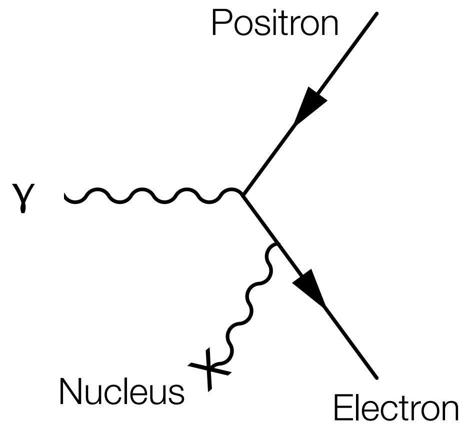
Example of particle interactions

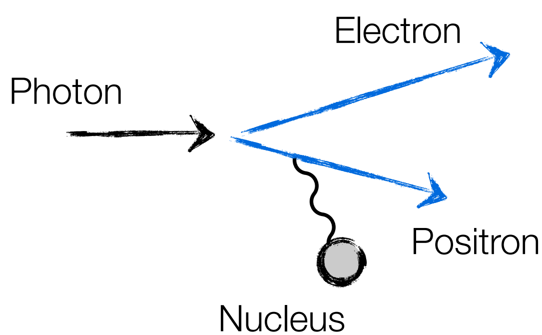
Ionization:



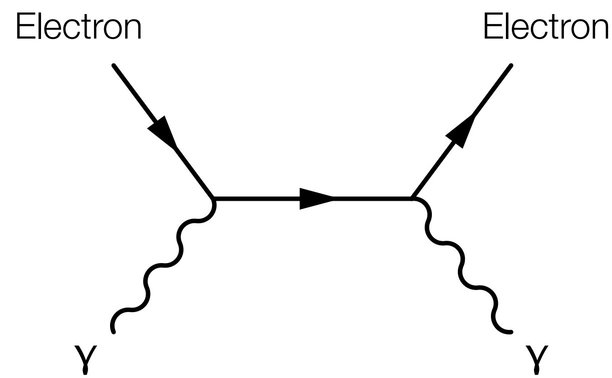


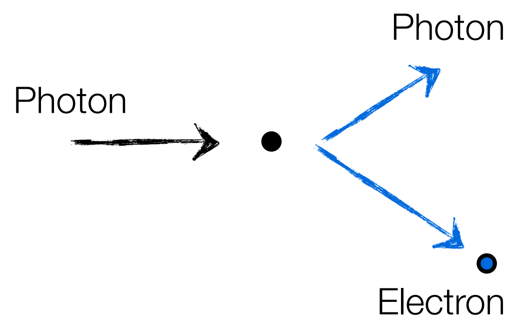
Pair production:





Compton scattering:

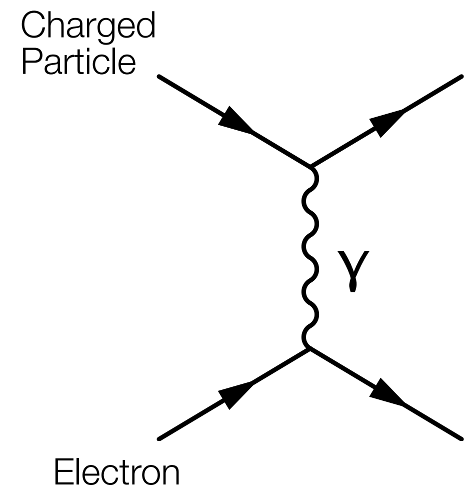




Energy loss by ionization: Bethe-Bloch formula

For now assume: $Mc^2 \gg m_e c^2$

i.e. energy loss for heavy charged particles
[dE/dx for electrons more difficult ...]



Interaction dominated
by elastic collisions with electrons ...

Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$\propto 1/\beta^2 \cdot \ln(\mathrm{const} \cdot \beta^2\gamma^2)$

Bethe-Bloch formula for heavy particles

[see e.g. PDG 2010]

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV g}^{-1} \text{ cm}^2$

$T_{max} = 2 m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma\, m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

z : Charge of incident particle
M : Mass of incident particle

Z : Charge number of medium
A : Atomic mass of medium

I : Mean excitation energy of medium
δ : Density correction [transv. extension of electric field]

$N_A = 6.022 \cdot 10^{23}$
[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8 \text{ fm}$
[Classical electron radius]

$m_e = 511 \text{ keV}$
[Electron mass]

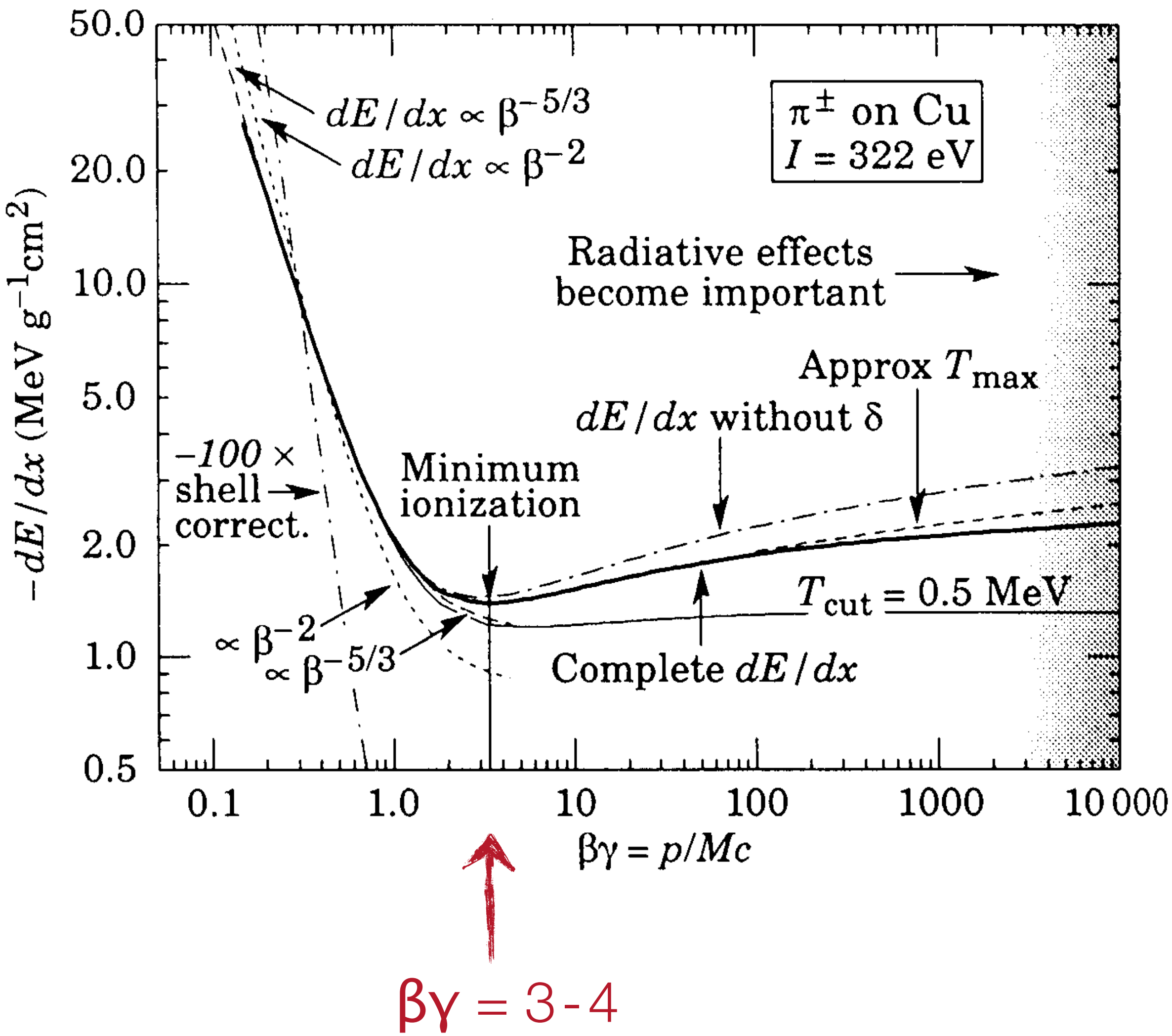
$\beta = v/c$
[Velocity]

$\gamma = (1-\beta^2)^{-2}$
[Lorentz factor]

Validity:

$.05 < \beta\gamma < 500$
$M > m_\mu$

Energy loss of pions in Cu



Minimum
ionizing particles (MIP): $\beta\gamma$ = 3-4

dE/dx falls ~ β^{-2}; kinematic factor
[precise dependence: ~ $\beta^{-5/3}$]

dE/dx rises ~ $\ln(\beta\gamma)^2$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large ($\beta\gamma$) due to
density effect (correction δ)
[polarization of medium]

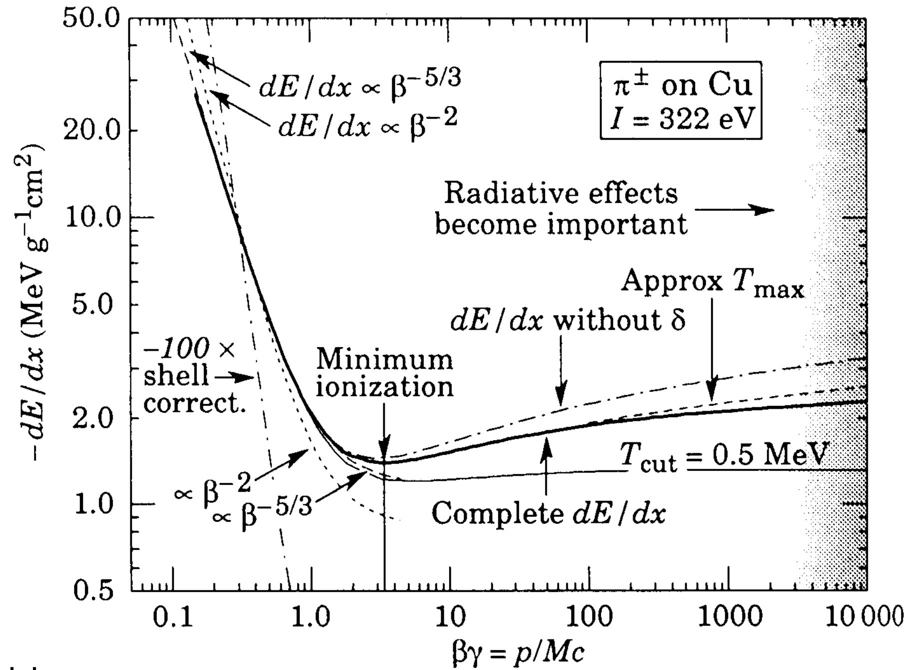
Units: MeV g^{-1} cm^2

MIP looses ~ 13 MeV/cm
[density of copper: 8.94 g/cm^3]

Understanding Bethe-Bloch

$1/\beta^2$-dependence:

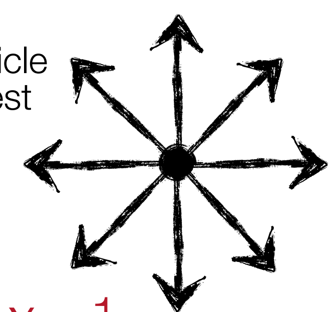
Slower particles fell electric force of atomic electrons for longer time …



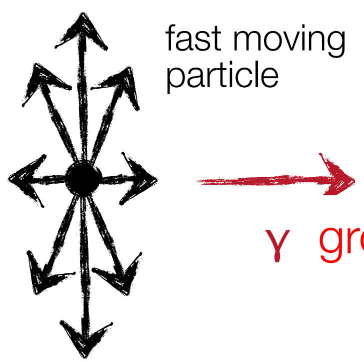
Relativistic rise for $\beta\gamma > 4$:

High energy particle: transversal electric field increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$. Thus interaction cross section increases ...

particle at rest



$\gamma = 1$



fast moving particle

γ grows

Corrections:

low energy : shell corrections

high energy : density corrections

Understanding Bethe-Bloch

Density correction:

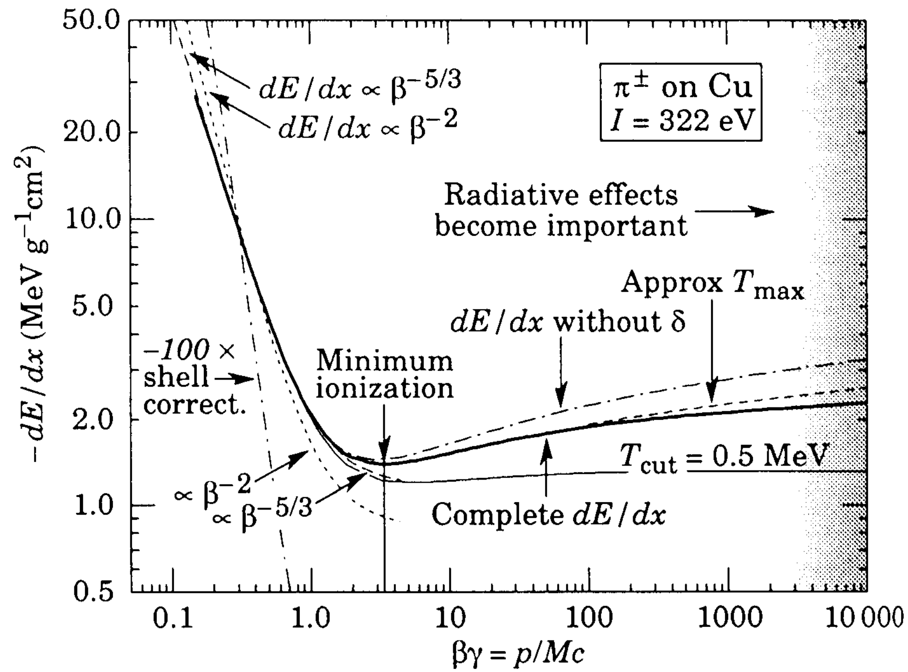
Polarization effect ...
[density dependent]

→ Shielding of electrical field far from particle path; effectively cuts of the long range contribution ...

More relevant at high γ ...
[Increased range of electric field; larger b_{max}; ...]

For high energies:

$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln\beta\gamma - 1/2$$



Density effect leads to saturation at high energy ...

Shell correction:

Arises if particle velocity is close to orbital velocity of electrons, i.e. $\beta c \sim v_e$.

Assumption that electron is at rest breaks down ...
Capture process is possible ...

Shell correction are in general small ...

Energy loss of (heavy) charged particles

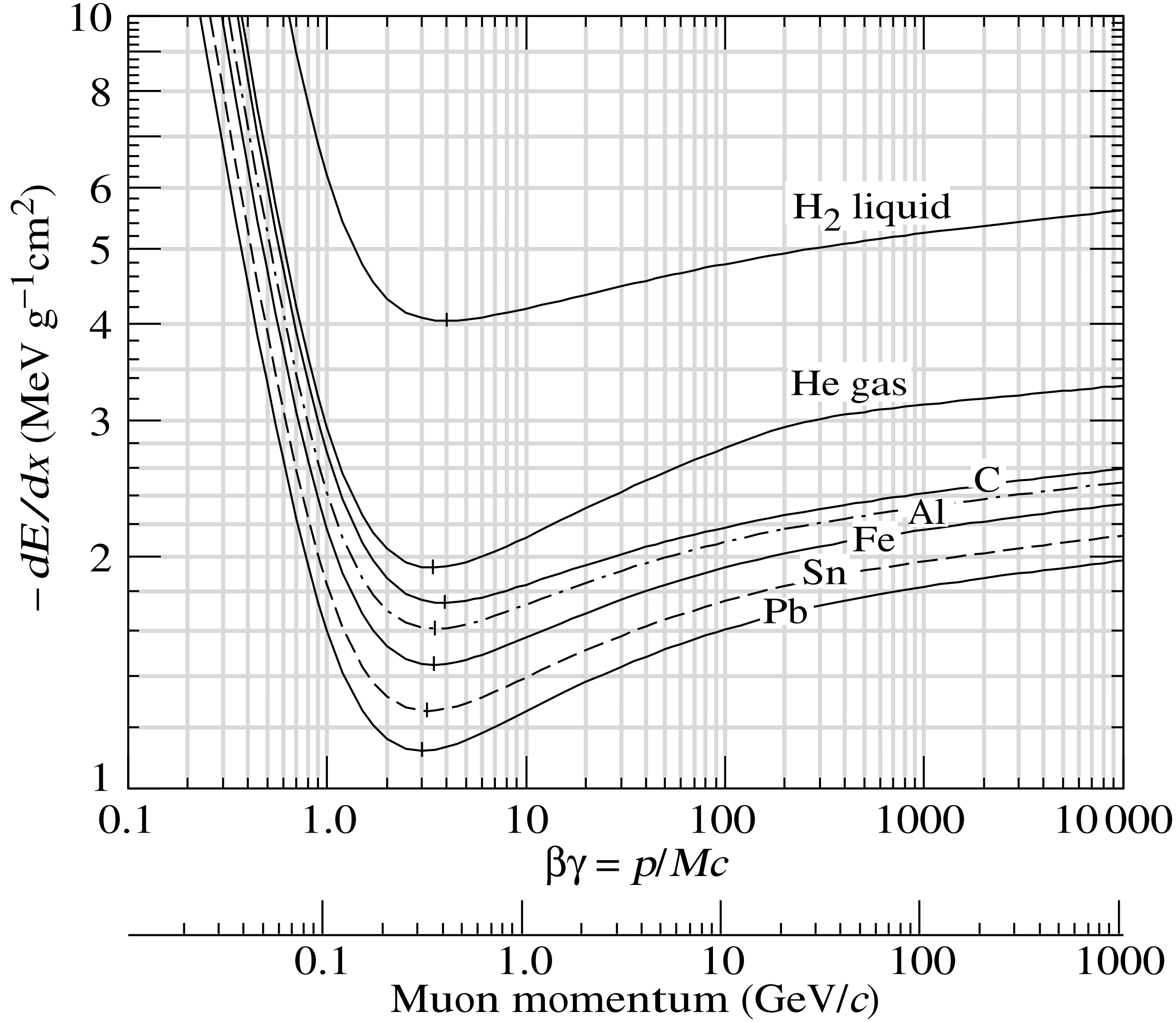
Dependence on

Mass A
Charge Z
of target nucleus

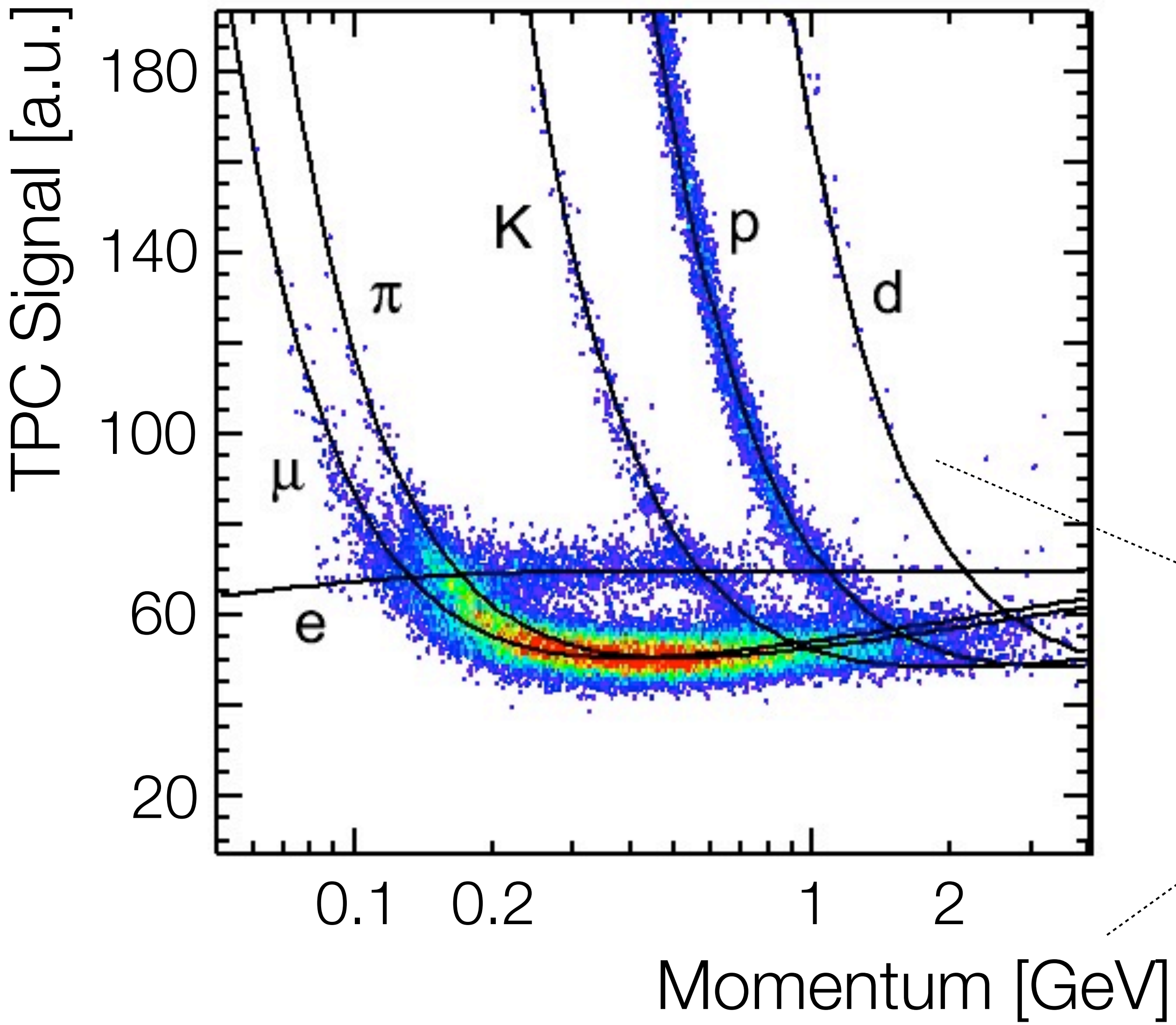
Minimum ionization:

ca. 1 - 2 MeV/g cm^{-2}
[H_2: 4 MeV/g cm^{-2}]

$$-\left\langle \frac{dE}{dx} \right\rangle \sim \frac{Z}{A}$$



Identifying particles by dE/dx



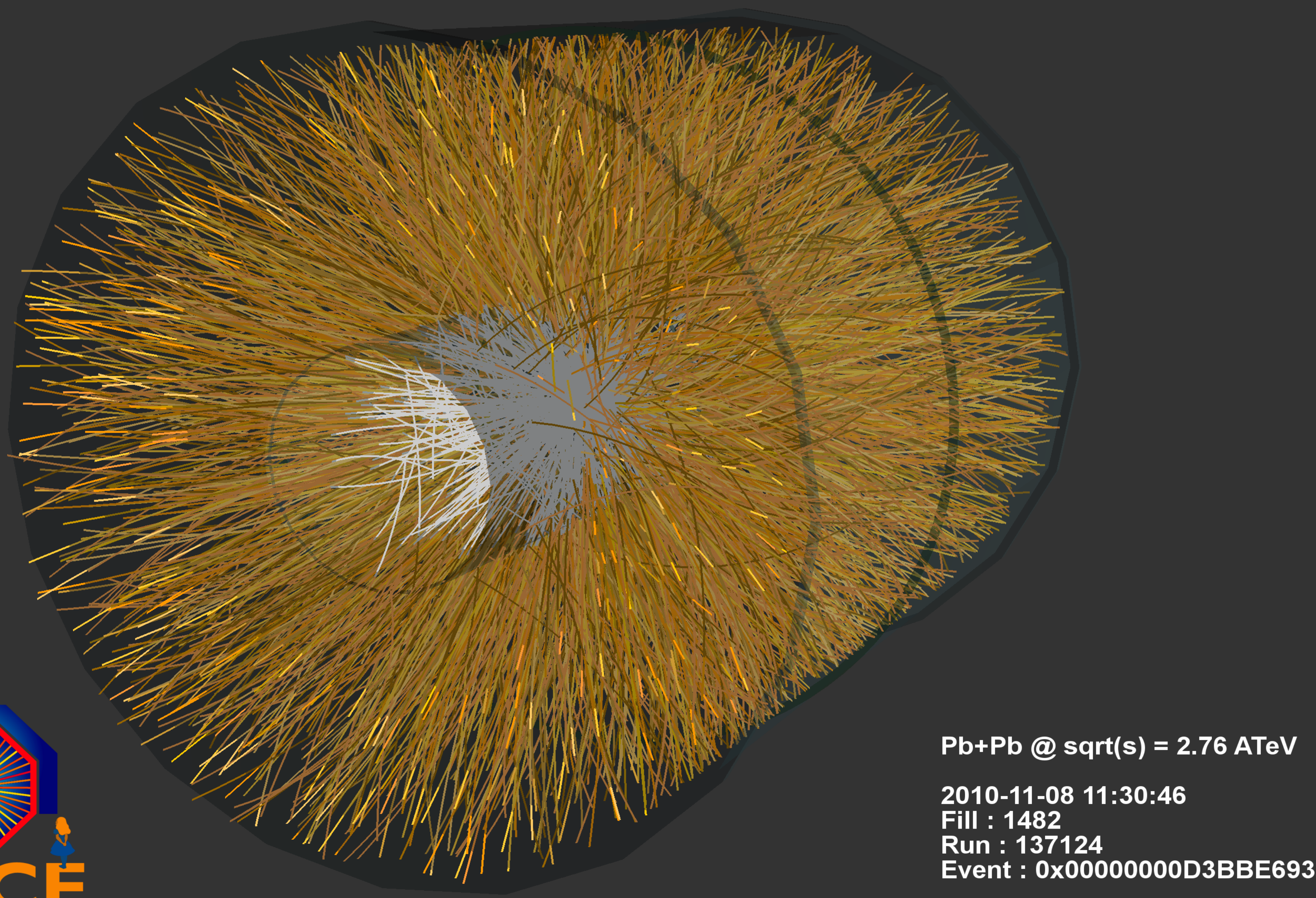
Measured
energy loss

[ALICE TPC, 2009]

Bethe-Bloch

Remember:
dE/dx depends on β!

ALICE TPC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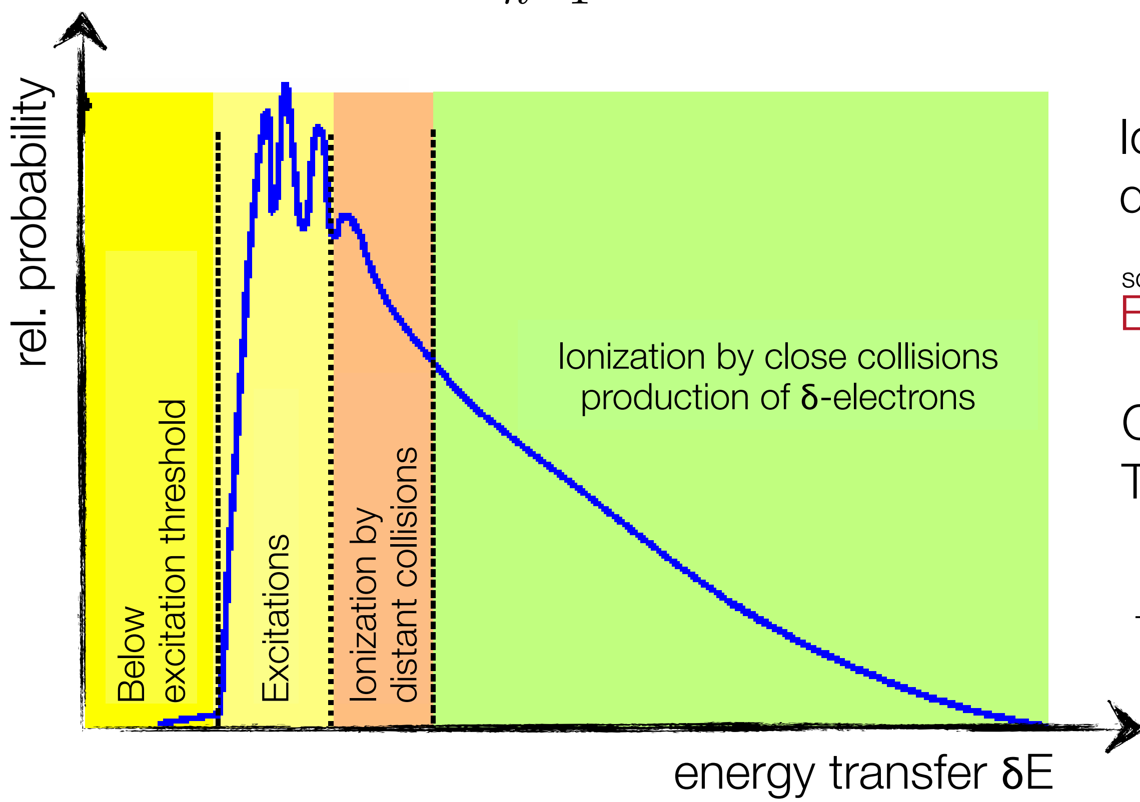
Event : 0x00000000D3BBE693

dE/dx fluctuations

Bethe-Bloch describes mean energy loss; measurement via energy loss ΔE in a material of thickness Δx with

$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N : number of collisions
δE : energy loss in a single collision



Ionization loss δE
distributed statistically ...

so-called
Energy loss 'straggling'

Complicated problem ...
Thin absorbers: Landau distribution

Standard Gauss with mean energy loss E_0
\+ tail towards high energies due to δ-electrons

see also Allison & Cobb
[Ann. Rev. Nucl. Part. Sci. 30 (1980) 253.]

Mean particle range

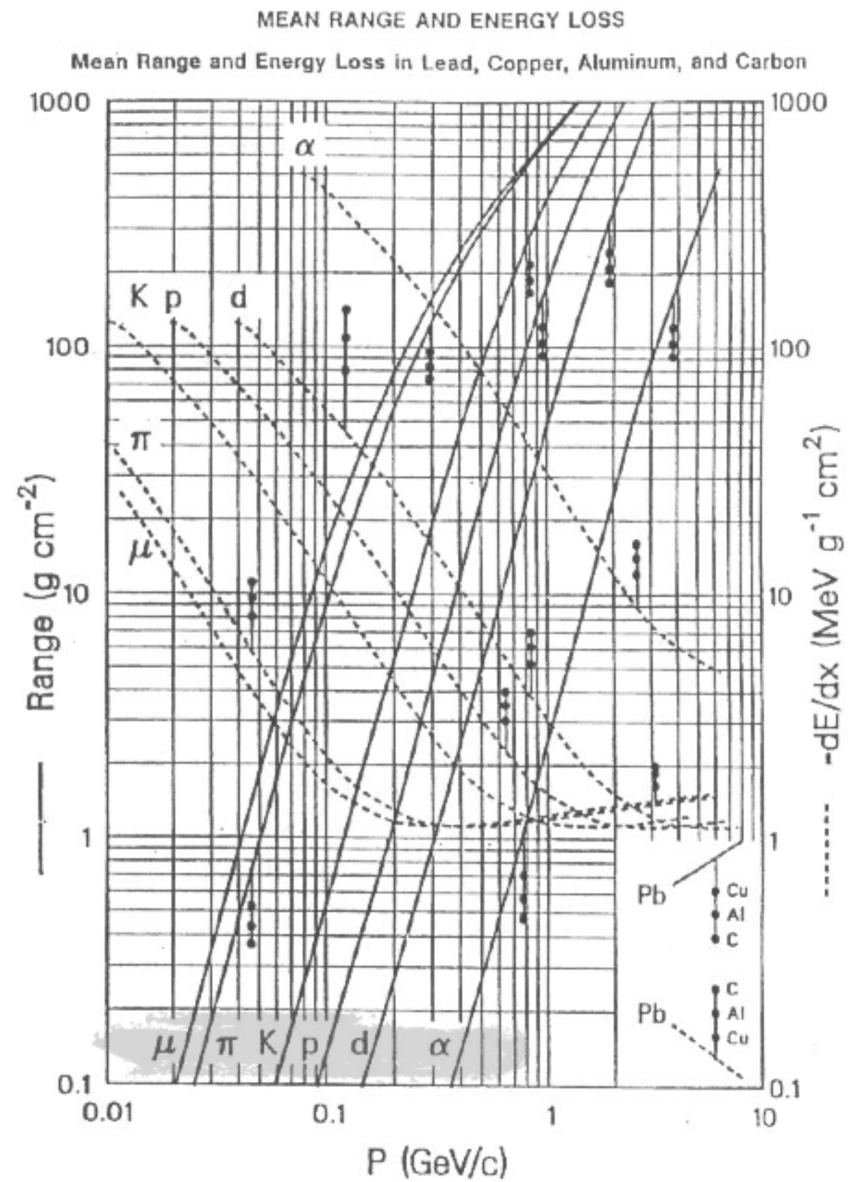
Integrate over energy loss
from E down to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

Example:

Proton with p = 1 GeV
Target: lead with ρ = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}
→ R = 200/11.34/1 cm ~ 20 cm



Energy loss of electrons

Bethe-Bloch formula needs modification

Incident and target electron have same mass m_e
Scattering of identical, undistinguishable particles

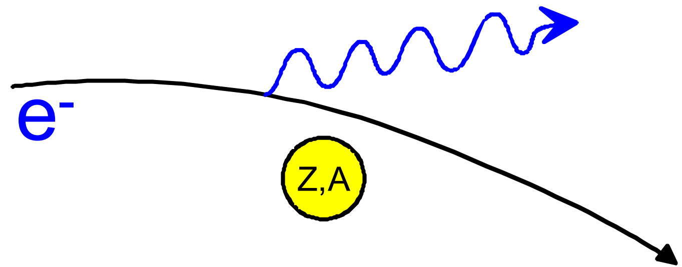
$$-\left\langle \frac{dE}{dx} \right\rangle_{\mathrm{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2 I^2} + F(\gamma) \right]$$

[T: kinetic energy of electron]

Remark: different energy loss for electrons and positrons at low energy as positrons are not identical with electrons; different treatment ...

Bremsstrahlung and Radiation Length

Bremsstrahlung arises if particles
are accelerated in Coulomb field of nucleus



$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to $1/m^2$ ➙ main relevance for electrons ...

... or ultra-relativistic muons

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \qquad \text{with} \quad X_0 = \frac{A}{4\alpha N_A \ Z^2 r_e^2 \ \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm^2]

➙ $$E = E_0 e^{-x/X_0}$$

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy

[i.e. 63%]

Critical Energy

Critical energy:

$$\frac{dE}{dx}(E_c)\bigg|_{\text{Brems}} = \frac{dE}{dx}(E_c)\bigg|_{\text{Ion}}$$

Approximation:

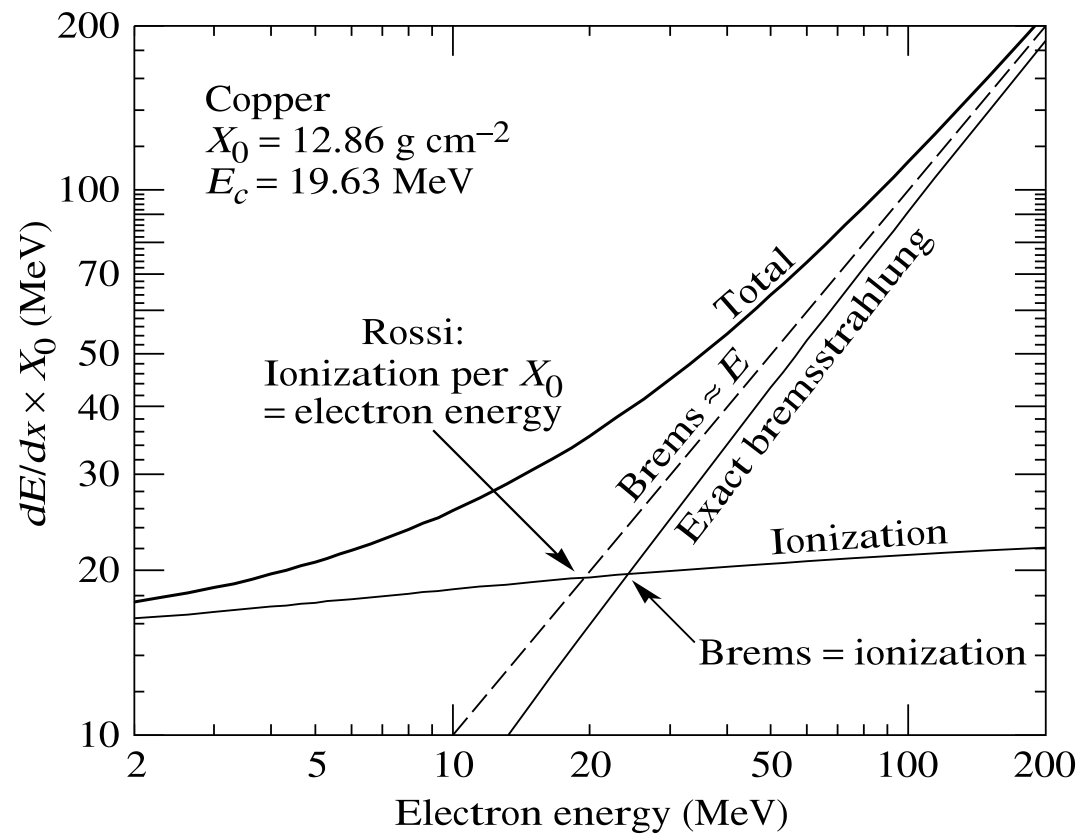
$$E_c^{\text{Gas}} = \frac{710\ \text{MeV}}{Z+0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610\ \text{MeV}}{Z+1.24}$$

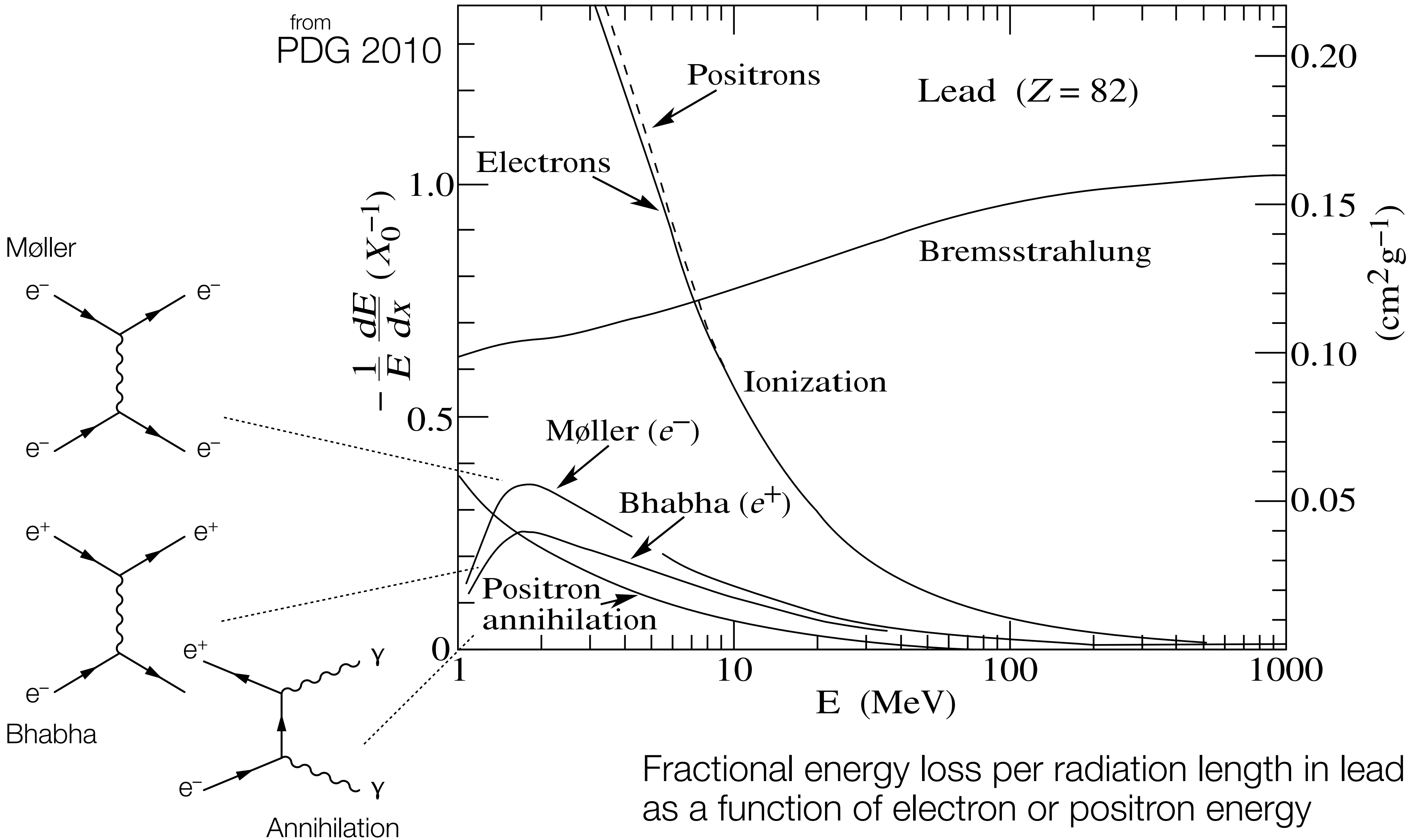
Example Copper:

$E_C \approx 610/30$ MeV ≈ 20 MeV

$$\left(\frac{dE}{dx}\right)_{\text{Tot}} = \left(\frac{dE}{dx}\right)_{\text{Ion}} + \left(\frac{dE}{dx}\right)_{\text{Brems}}$$

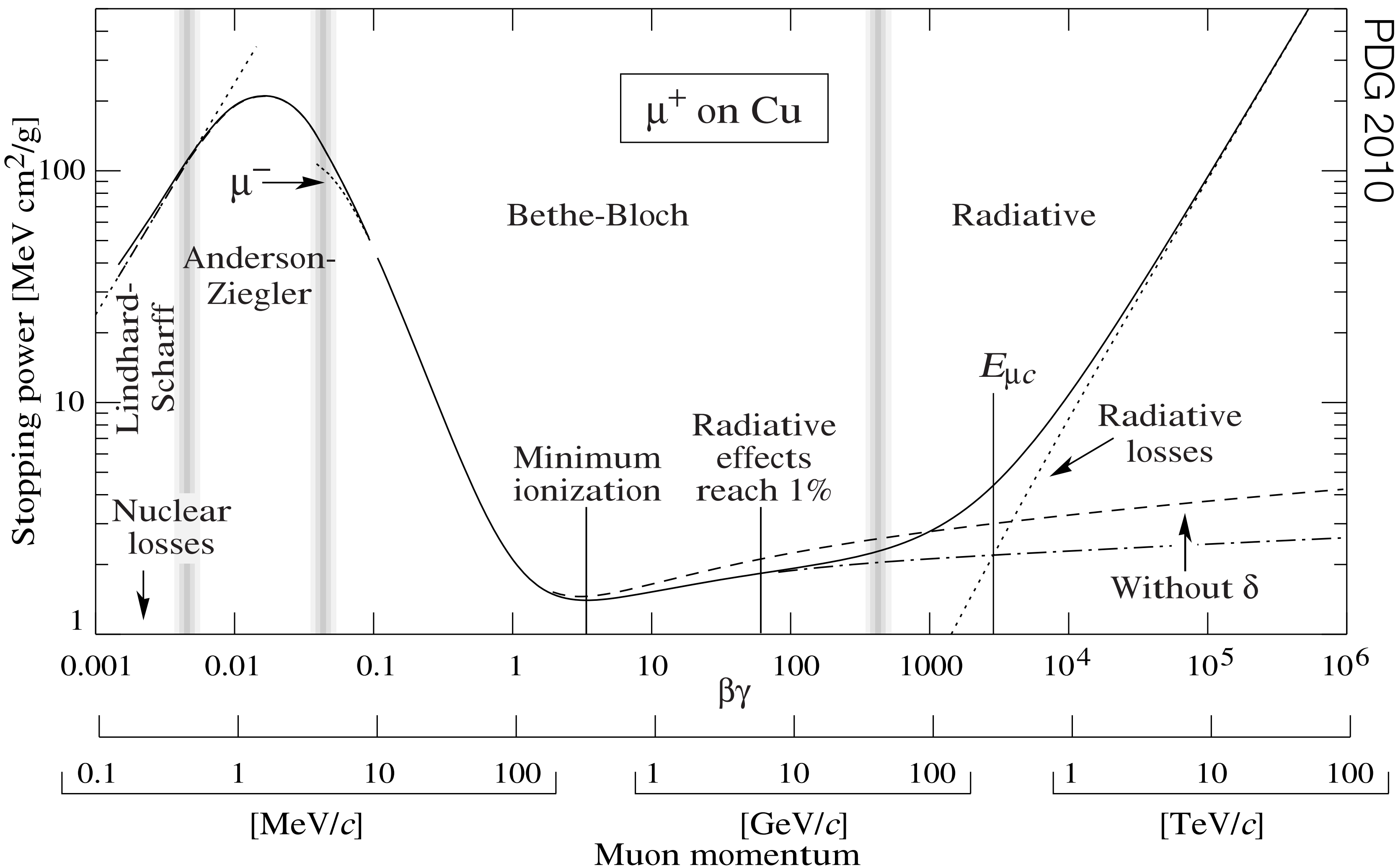


Total Energy Loss of Electrons



Fractional energy loss per radiation length in lead as a function of electron or positron energy

Energy loss for muons

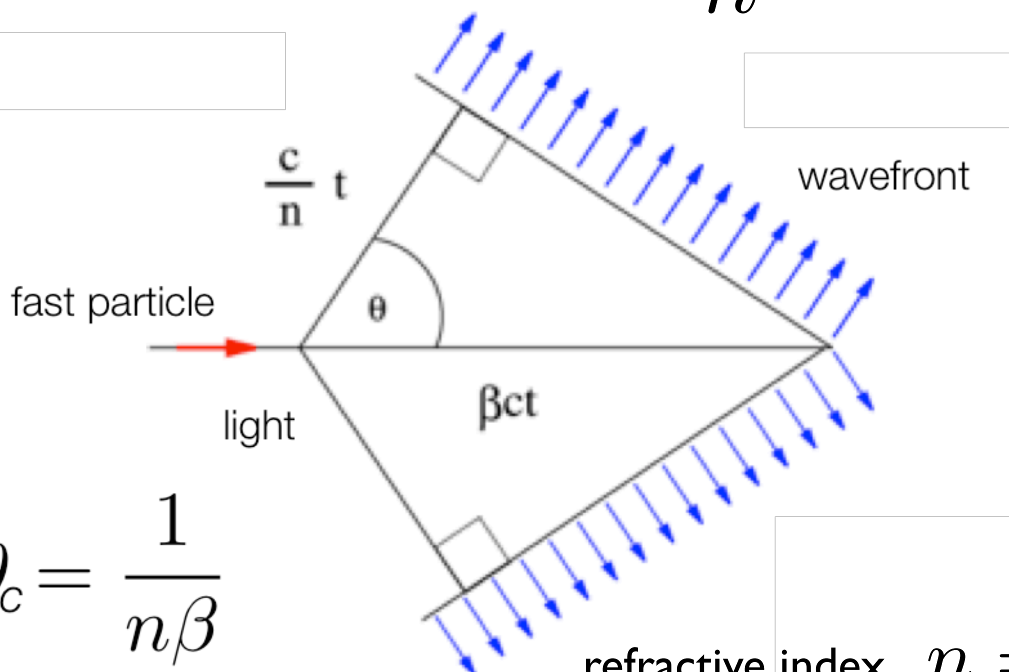


Cherenkov radiation

Particles moving in a medium with speed larger than speed of light in that medium will loose energy by emitting electromagnetic radiation

- ✓ Charged particle polarize medium generating an electrical dipole varying in time
- ✓ Every point in trajectory emits a spherical EM wave, waves constructively interfere…

$$\beta > \frac{1}{n}$$



$$\cos\theta_c = \frac{1}{n\beta}$$

refractive index $n = \sqrt{\varepsilon}$ dielectric constant

Cherenkov radiation: radiators

Parameters of Typical Radiator

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Air	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Water	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

Note: Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%).

Example:
[Proton with E_{kin} =1 GeV passing through 1 cm water]

$\beta = p/E \approx 0.875$; $\cos\theta_C = 1/n\beta = 0.859$ ➛ $\theta_C = 30.8°$
$d^2N/(dEdx) = 370 \sin^2\theta_C\ eV^{-1}\ cm^{-1} \approx 100\ eV^{-1}\ cm^{-1}$

➛ $\Delta E_{loss} = \langle E\rangle\, d^2N/(dEdx)\, \Delta E\, \Delta x$
$= 2.5\ eV \cdot 100\ eV^{-1}\ cm^{-1} \cdot 5\ eV \cdot 1\ cm = 1.25\ keV$

Visible light only!
[E = 1 - 5 eV; λ = 300 - 600 nm]

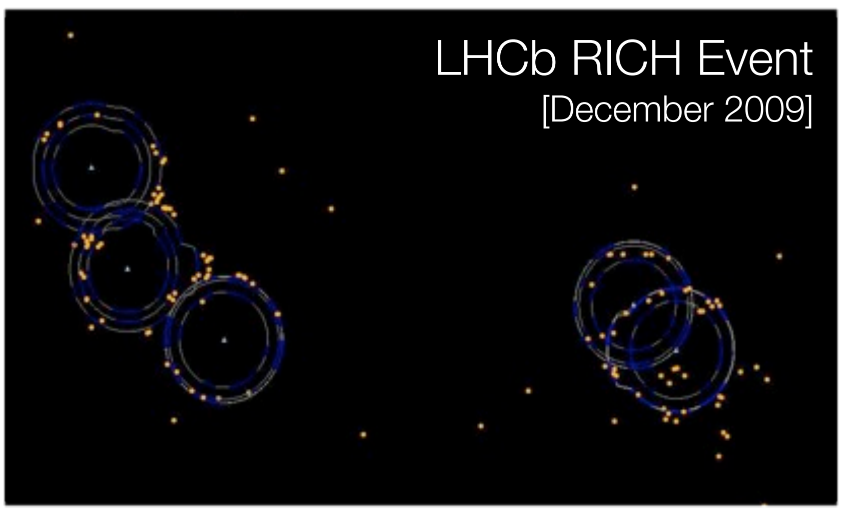
$\Delta E_{loss} < 1.25$ keV

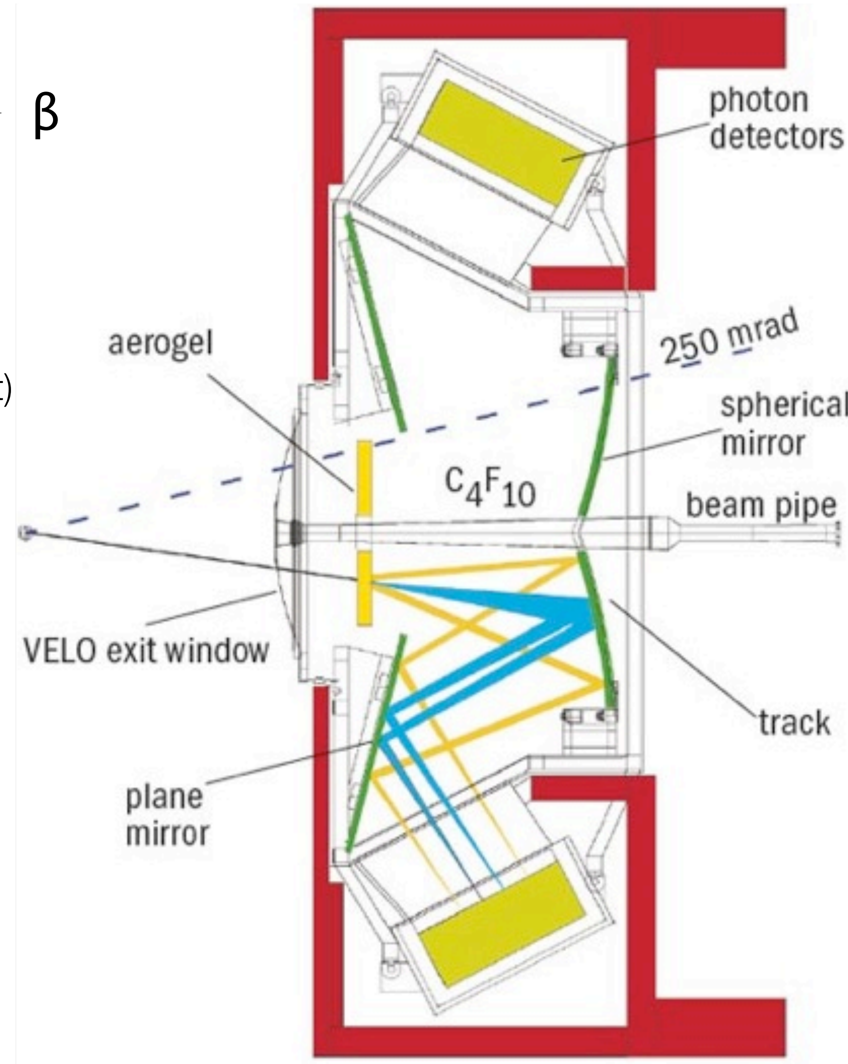
LHCb RICH

Measurement of Cherenkov angle:
Use medium with known refractive index n ➝ β

Principle of:

- RICH (Ring Imaging Cherenkov Counter)
- DIRC (Detection of Internally Reflected Cherenkov Light)
- DISC (special DIRC; e.g. Panda)





LHCb RICH

Transition radiation

Transition radiation occurs if a relativistic particle (large γ) passes the boundaries between two media with different refraction indices…

- ✓ Intensity of radiation is logarithmically proportional to γ

Angular distribution strongly forward peaked [Interference; coherence condition]

$$\theta \leq 1/\gamma$$

Coherent radiation is generated only over a very small formation length

$$D = \gamma c/\omega_p$$

Plasma frequency [from Drude model]

Volume element from which coherent radiation is emitted ...

$$V = \pi D \rho^2_{max}$$

$\rho_{max} = \gamma v/\omega$ [transversal range ... ... with large polarization]

Maximum energy of radiated photons limited by plasma frequency ... [results from requiring $V \neq 0 \rightarrow \omega = \gamma\omega_p$]

$$E_{max} = \gamma\hbar\omega_p$$

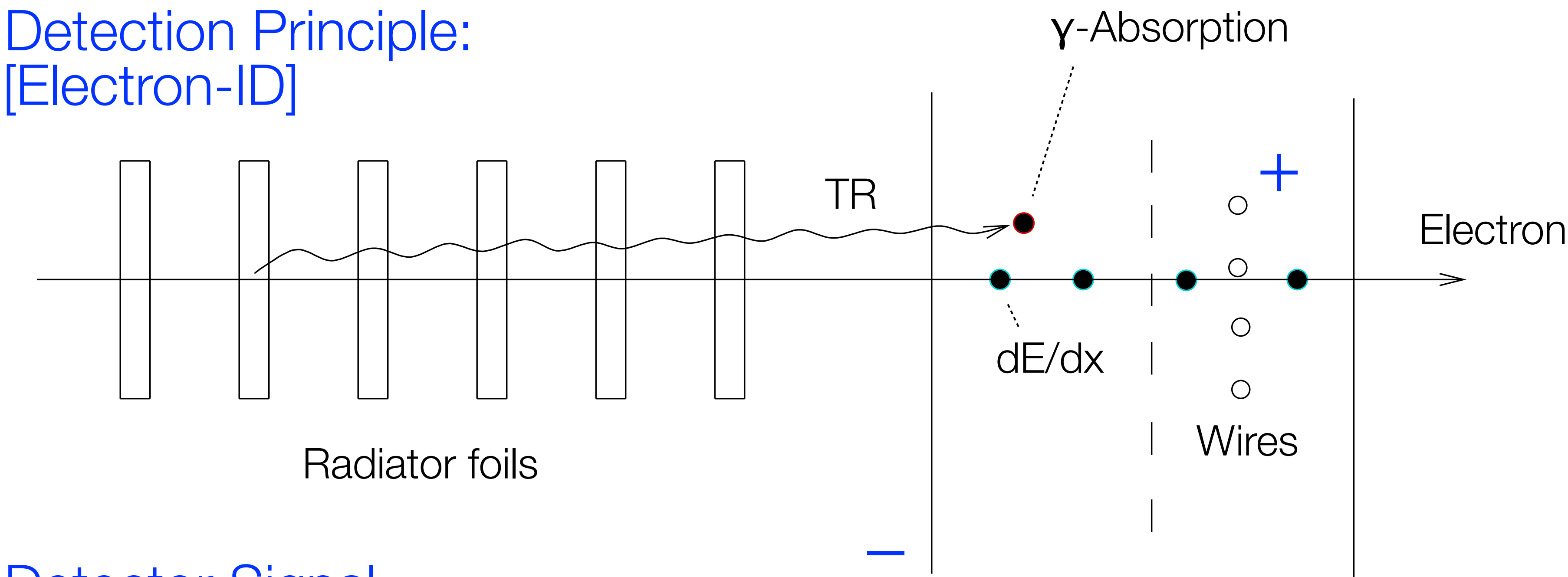
[X-Rays ➛ large γ!!]

Typical values: CH_2: $\hbar\omega_p = 20$ eV; $\gamma = 10^3$ [Air: $\hbar\omega_p = 0.7$ eV]

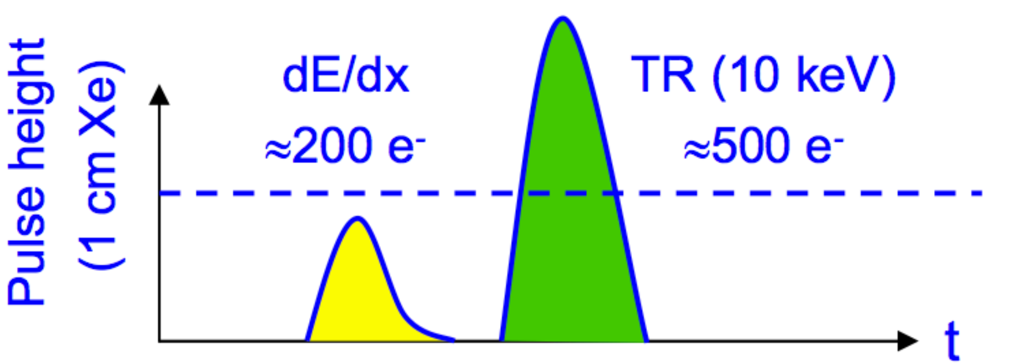
$$D = 10\ \mu m$$

[d > D: absorption dominates]

Identifying particles with transition radiation



Detector Signal:



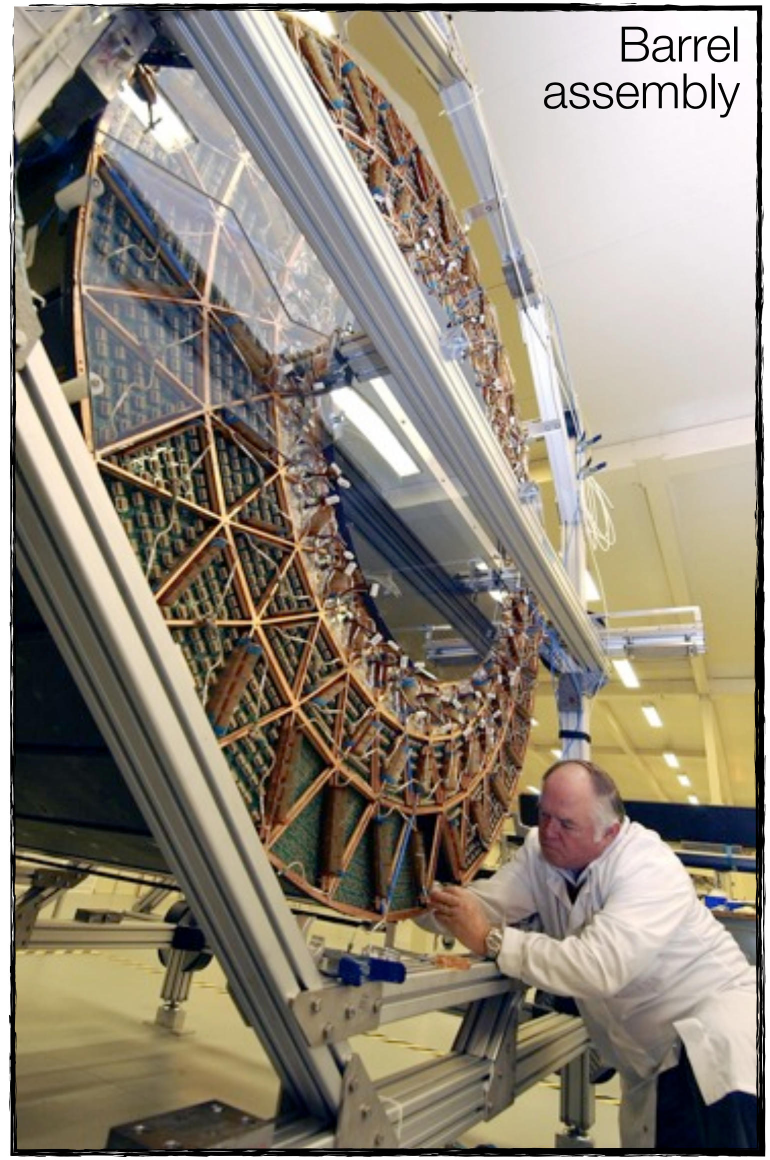
- Detector should be sensitive to $3 \lesssim E_\gamma \lesssim 30$ keV.
  - ✓ Gaseous detectors
- In gas $\sigma_{\text{photo effect}} \propto Z^5$
- Gases with high Z are required
  - ✓ e.g. Xenon (Z=54)

ATLAS Transition Radiation Tracker

Straw Tube Tracker
with interspace filled with foam

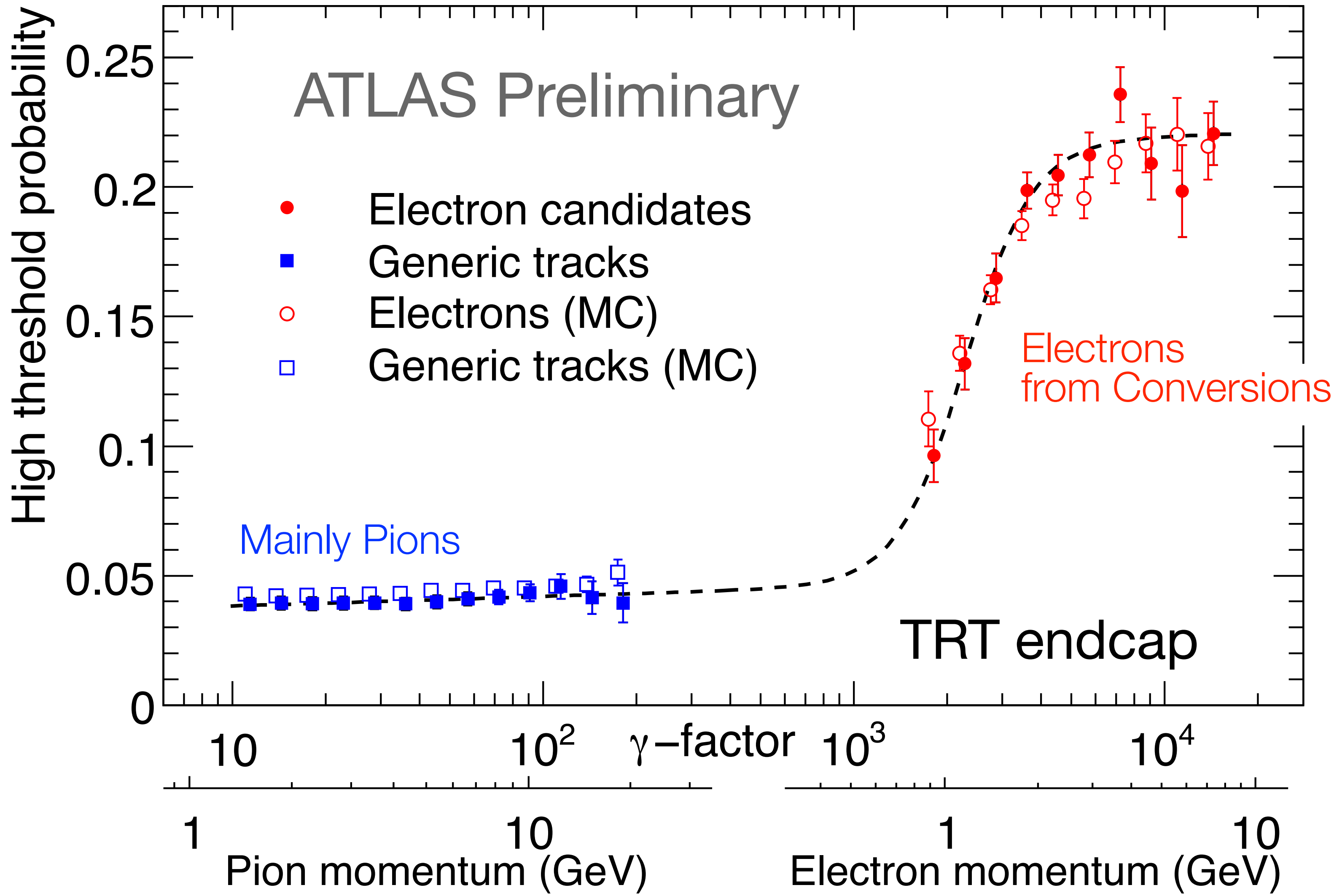
➛ Tracking & transition radiation

End-cap assembly



Barrel assembly

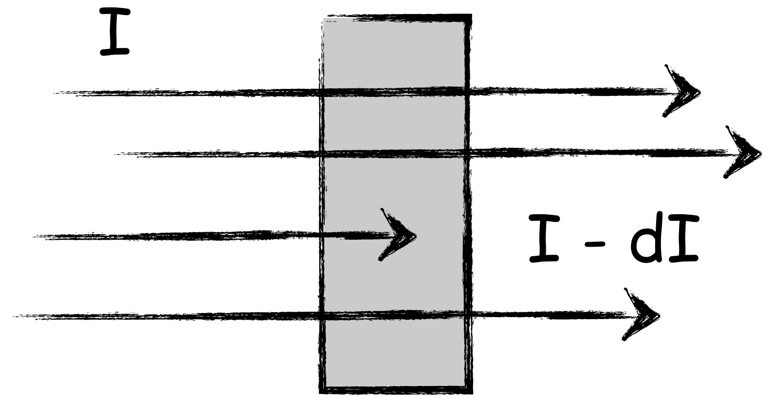
Identifying particles with transition radiation



Interaction of photons with matter

Characteristic for interactions of photons with matter:

A single interaction removes photon from beam !



Possible Interactions

Photoelectric Effect
Compton Scattering
Pair Production

$$dI = -\mu\, I\, dx$$

[μ : absorption coefficient]

depends on
E, Z, ρ

Rayleigh Scattering (γA ➛ γA; A = atom; coherent)
Thomson Scattering (γe ➛ γe; elastic scattering)
Photo Nuclear Absorption (γK ➛ pK/nK)
Nuclear Resonance Scattering (γK ➛ K* ➛ γK)
Delbruck Scattering (γK ➛ γK)
Hadron Pair production (γK ➛ h^+h^- K)

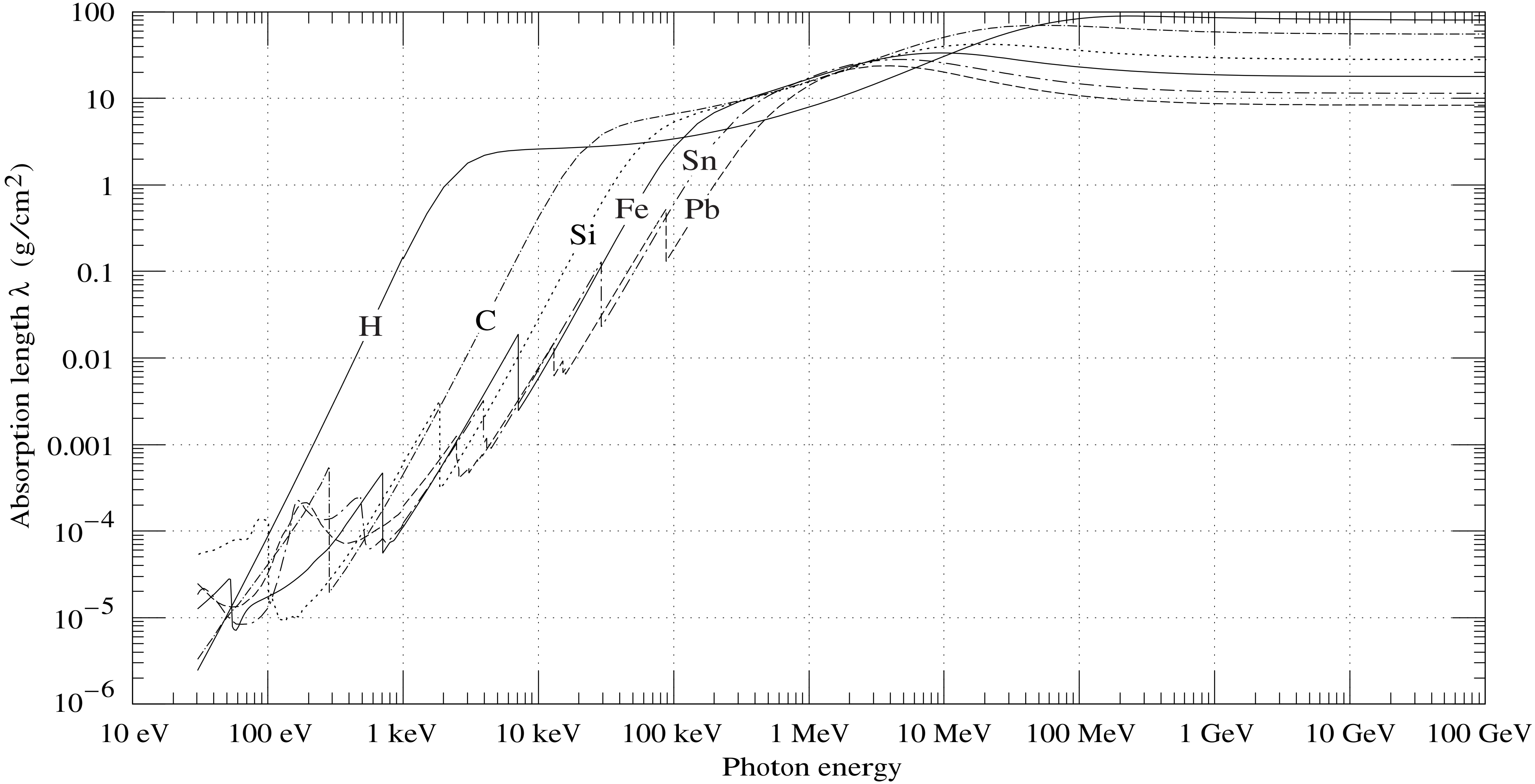
➛ Beer-Lambert law:

$$I(x) = I_0 e^{-\mu x}$$

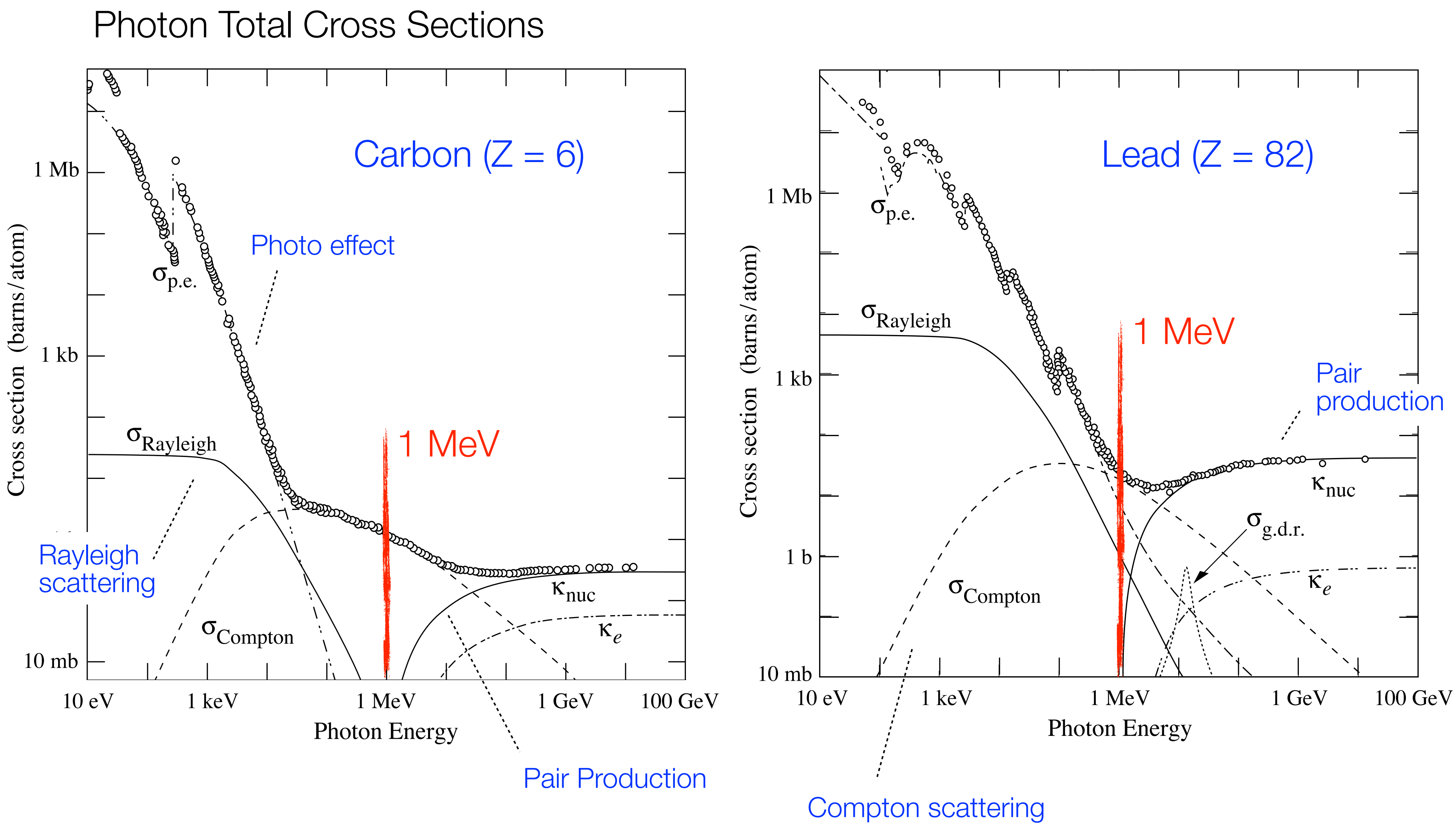
with $\lambda = 1/\mu = 1/n\sigma$
[mean free path]

Interaction of photons with matter

from PDG



Interaction of photons with matter



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Pair production

Cross Section:
[for $E_\gamma \gg m_e c^2$]

$$\sigma_{\text{pair}} \approx \underbrace{\frac{7}{9}\left(4\,\alpha r_e^2 Z^2 \ln\frac{183}{Z^{\frac{1}{3}}}\right)}_{A/N_A X_0}$$

[X_0: radiation length]
[in cm or g/cm^2]

Absorption coefficient:

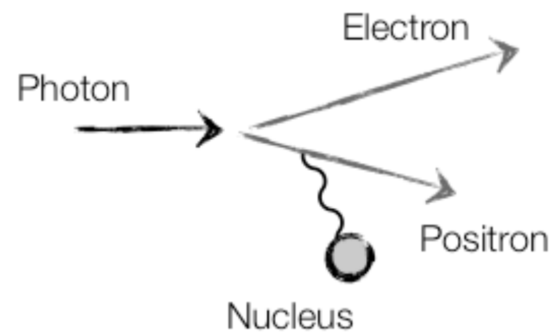
$\mu = n\sigma$ [with n: particle density]

$\mu = \rho \cdot N_A/A\ \sigma_{\text{pair}}$

$= 7/9\ \dfrac{1}{X_0}$

[where now X_0 is in cm]

$$I(x) = I_0 e^{-\mu x}$$



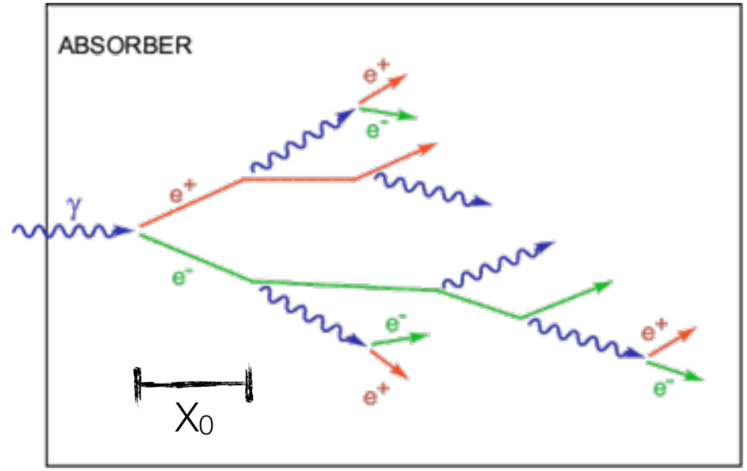
	ρ [g/cm^3]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Air	$1.2 \cdot 10^{-3}$	$30 \cdot 10^3$

Electromagnetic showers

Reminder:

Dominant processes
at high energies ...

Photons : Pair production
Electrons : Bremsstrahlung



Pair production:

$$\sigma_{\text{pair}} \approx \frac{7}{9}\left(4\,\alpha r_e^2 Z^2 \ln\frac{183}{Z^{\frac{1}{3}}}\right)$$

$$= \frac{7}{9}\frac{A}{N_A X_0}$$

[X_0: radiation length]
[in cm or g/cm^2]

Absorption coefficient:

$$\mu = n\sigma = \rho\frac{N_A}{A}\cdot\sigma_{\text{pair}} = \frac{7}{9}\frac{\rho}{X_0}$$

Bremsstrahlung:

$$\frac{dE}{dx} = 4\alpha N_A\frac{Z^2}{A}r_e^2\cdot E\,\ln\frac{183}{Z^{\frac{1}{3}}} = \frac{E}{X_0}$$

$$\to E = E_0 e^{-x/X_0}$$

After passage of one X_0 electron has only $(1/e)^{\text{th}}$ of its primary energy ...
[i.e. 37%]

Electromagnetic showers

Further basics:

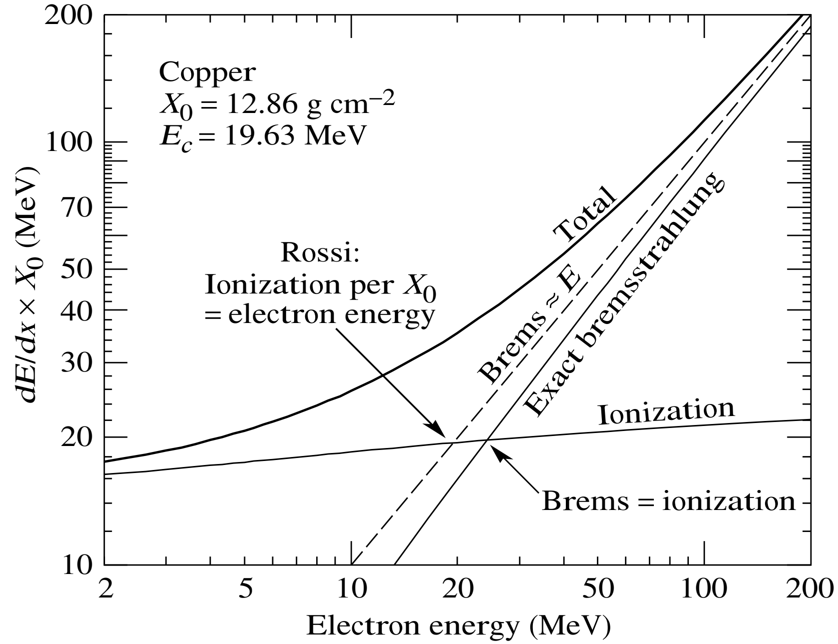
Critical Energy [see above]:

$$\frac{dE}{dx}(E_c)\Big|_{\text{Brems}} = \frac{dE}{dx}(E_c)\Big|_{\text{Ion}}$$

Approximations:

$$E_c^{\text{Gas}} = \frac{710\text{ MeV}}{Z+0.92} \qquad \left[E_c^{\text{Sol/Liq}} = \frac{610\text{ MeV}}{Z+1.24}\right]$$

$$\left(\frac{dE}{dx}\right)_{\text{Brems}} \Big/ \left(\frac{dE}{dx}\right)_{\text{Ion}} \approx \frac{Z\cdot E}{800\text{ MeV}}$$



with:

$$\frac{dE}{dx}\Big|_{\text{Brems}} = \frac{E}{X_0} \quad \& \quad \frac{dE}{dx}\Big|_{\text{Ion}} \approx \frac{E_c}{X_0} = \text{const.}$$

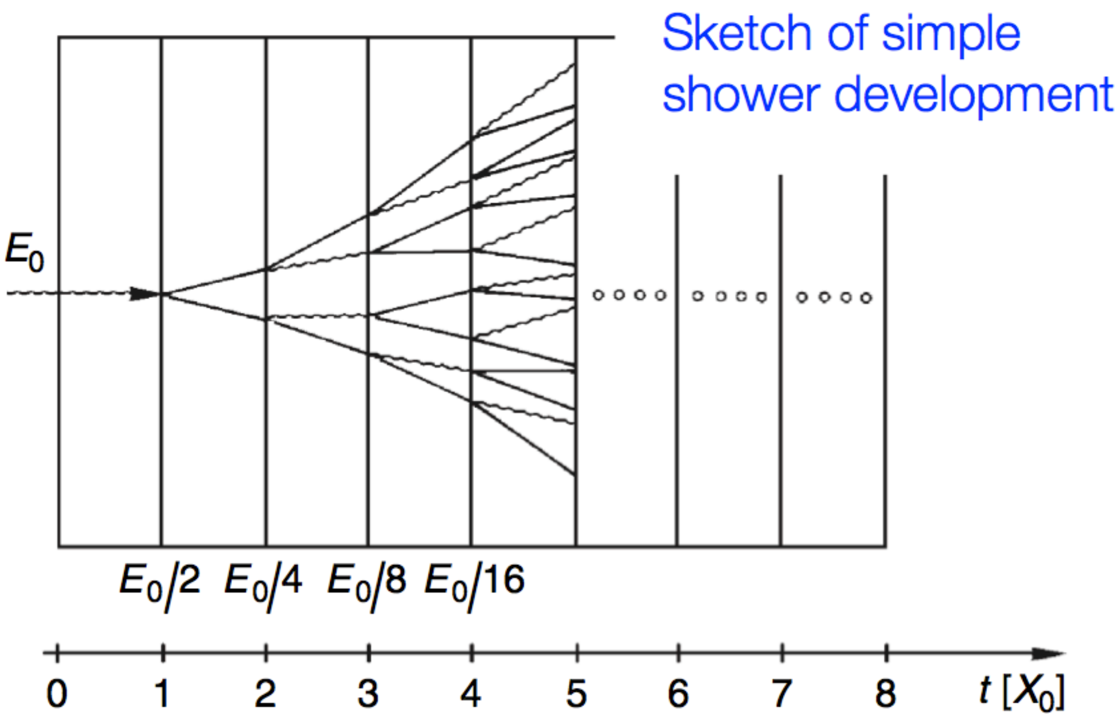
Transverse size of EM shower given by radiation length via Molière radius

$$R_M = \frac{21\text{ MeV}}{E_c} X_0$$

[see also later]

R_M : Moliere radius
E_c : Critical Energy [Rossi]
X_0 : Radiation length

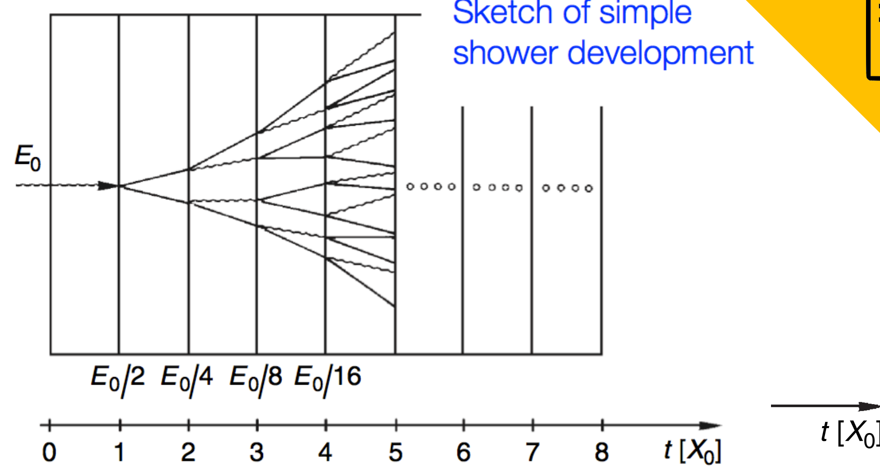
A simple shower model



- Assumptions
  - ✓ Only 2 interactions happenings: e→eγ (bremsstrahlung) and γ→ee (pair production)
  - ✓ Interactions happen exactly after 1 X_0
  - ✓ Secondaries bring exactly half of their originating particle energy
  - ✓ Shower stops when each particle have $E_i = E_c$ (critical energy)
- Compute shower longitudinal development as a function of E_0 and E_c

A simple shower model

Sketch of simple shower development



Simple shower model:
[continued]

Shower characterized by:

Number of particles in shower
Location of shower maximum
Longitudinal shower distribution
Transverse shower distribution

Longitudinal components;
measured in radiation length ...

... use: $t = \dfrac{x}{X_0}$

Number of shower particles after depth t:

$$N(t) = 2^t$$

Energy per particle after depth t:

$$E = \frac{E_0}{N(t)} = E_0 \cdot 2^{-t}$$

$$\rightarrow \quad t = \log_2(E_0/E)$$

Total number of shower particles with energy E_1:

$$N(E_0, E_1) = 2^{t_1} = 2^{\log_2(E_0/E_1)} = \frac{E_0}{E_1}$$

Number of shower particles at shower maximum:

$$N(E_0, E_c) = N_{\max} = 2^{t_{\max}} = \frac{E_0}{E_c} \propto E_0$$

Shower maximum at:

$$t_{\max} \propto \ln(E_0/E_c)$$

A simple shower model

Simple shower model:
[continued]

Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle ...

Some numbers: $E_c \approx 10$ MeV, $E_0 = 1$ GeV ➛ $t_{max} = \ln 100 \approx 4.5$; $N_{max} = 100$
$E_0 = 100$ GeV ➛ $t_{max} = \ln 10000 \approx 9.2$; $N_{max} = 10000$

$$t_{\max}[X_0] \sim \ln \frac{E_0}{E_c}$$

Electromagnetic showers

Typical values for X_0, E_c and R_M of materials used in calorimeter

	X_0 [cm]	E_c [MeV]	R_M [cm]
Pb	0.56	7.2	1.6
Scintillator (Sz)	34.7	80	9.1
Fe	1.76	21	1.8
Ar (liquid)	14	31	9.5
BGO	1.12	10.1	2.3
Sz/Pb	3.1	12.6	5.2
PB glass (SF5)	2.4	11.8	4.3

EM shower longitudinal development

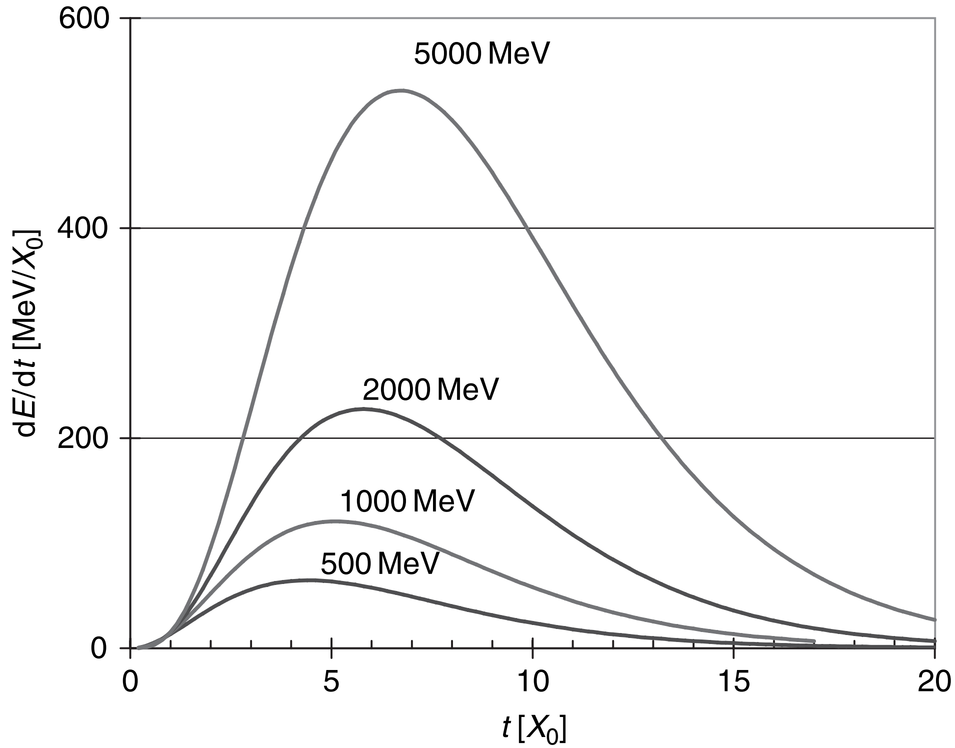
Longitudinal profile

Parametrization:
[Longo 1975]

$$\frac{dE}{dt} = E_0 \, t^{\alpha} e^{-\beta t}$$

α,β : free parameters
t^{α} : at small depth number of secondaries increases ...
$e^{-\beta t}$: at larger depth absorption dominates ...

Numbers for E = 2 GeV (approximate):
α = 2, **β** = 0.5, t_{max} = **α/β**



More exact
[Longo 1985]

$$\frac{dE}{dt} = E_0 \cdot \beta \cdot \frac{(\beta t)^{\alpha-1} e^{-\beta t}}{\Gamma(\alpha)}$$

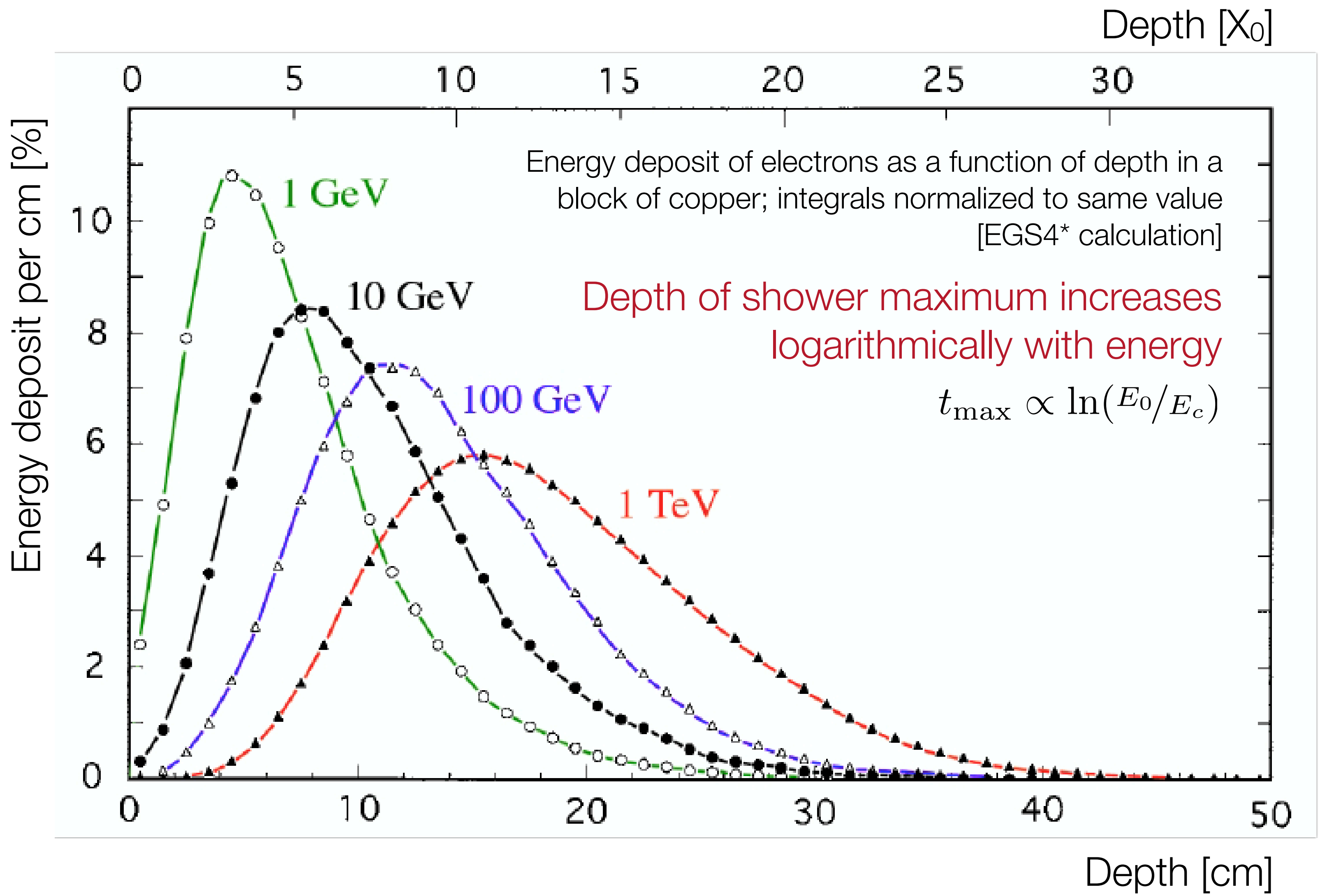
[Γ: Gamma function]

$$\rightarrow \quad t_{\max} = \frac{\alpha - 1}{\beta} = \ln\left(\frac{E_0}{E_c}\right) + C_{e\gamma}$$

with:

$C_{e\gamma} = -0.5$ [γ-induced]
$C_{e\gamma} = -1.0$ [e-induced]

EM shower longitudinal development



*EGS = Electron Gamma Shower

EM shower transverse development

Transverse shower development ...

Opening angle
for bremsstrahlung and pair production

$$\langle\theta^2\rangle \approx \left({}^{m}/_{E}\right)^2 = {}^{1}/_{\gamma^2}$$

Small contribution as $m_e/E_c = 0.05$

Multiple scattering
deflection angle in 2-dimensional plane ...

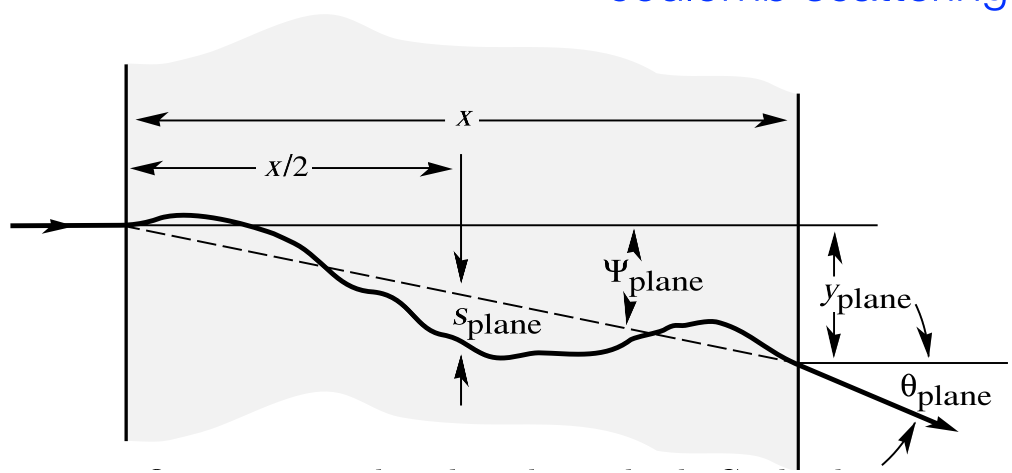
$$\langle\theta_k^2\rangle = \sum_{m=1}^{k} \theta_m^2 = k\langle\theta^2\rangle$$

$$\sqrt{\langle\theta^2\rangle} \approx \frac{13.6\ \mathrm{MeV}/c}{p}\sqrt{\frac{x}{X_0}} \qquad [\beta = 1]$$

In 3-dimensions extra factor √2:

$$\sqrt{\langle\theta^2\rangle_{3\mathrm{d}}} \approx \frac{19.2\ \mathrm{MeV}/c}{p}\sqrt{\frac{x}{X_0}} \qquad [\beta = 1]$$

Multiple
coulomb scattering



Assuming the approximate range of electrons to be X_0 yields lateral extension: $R = \langle\theta\rangle\cdot X_0$ ...

$$R_M = \langle\theta\rangle_{x=X_0}\cdot X_0 \approx \frac{21\mathrm{MeV}}{E_C}\,X_0$$

Molière Radius;
characterizes lateral shower spread ...

EM shower transverse profile

Transverse profile

Parametrization:

$$\frac{dE}{dr} = \alpha e^{-r/R_M} + \beta e^{-r/\lambda_{\min}}$$

α,β : free parameters

R_M : Molière radius

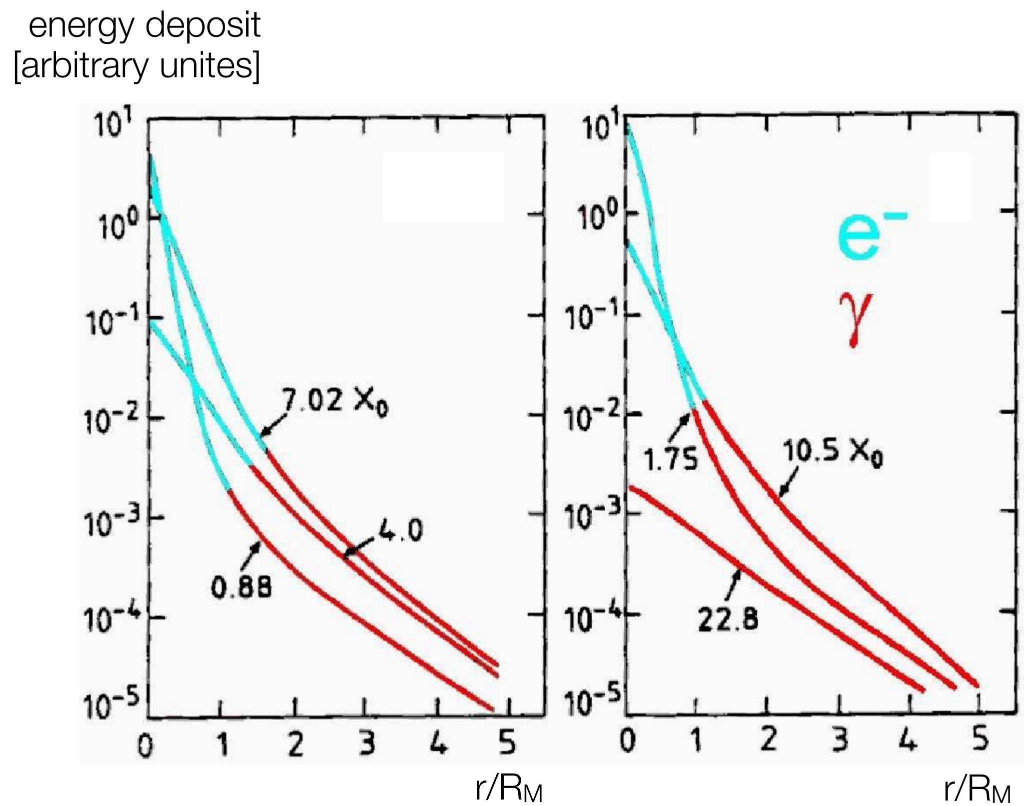
λ_{min} : range of low energetic photons ...

Inner part: coulomb scattering ...

Electrons and positrons move away from shower axis due to multiple scattering ...

Outer part: low energy photons ...

Photons (and electrons) produced in isotropic processes (Compton scattering, photo-electric effect) move away from shower axis; predominant beyond shower maximum, particularly in high-Z absorber media...

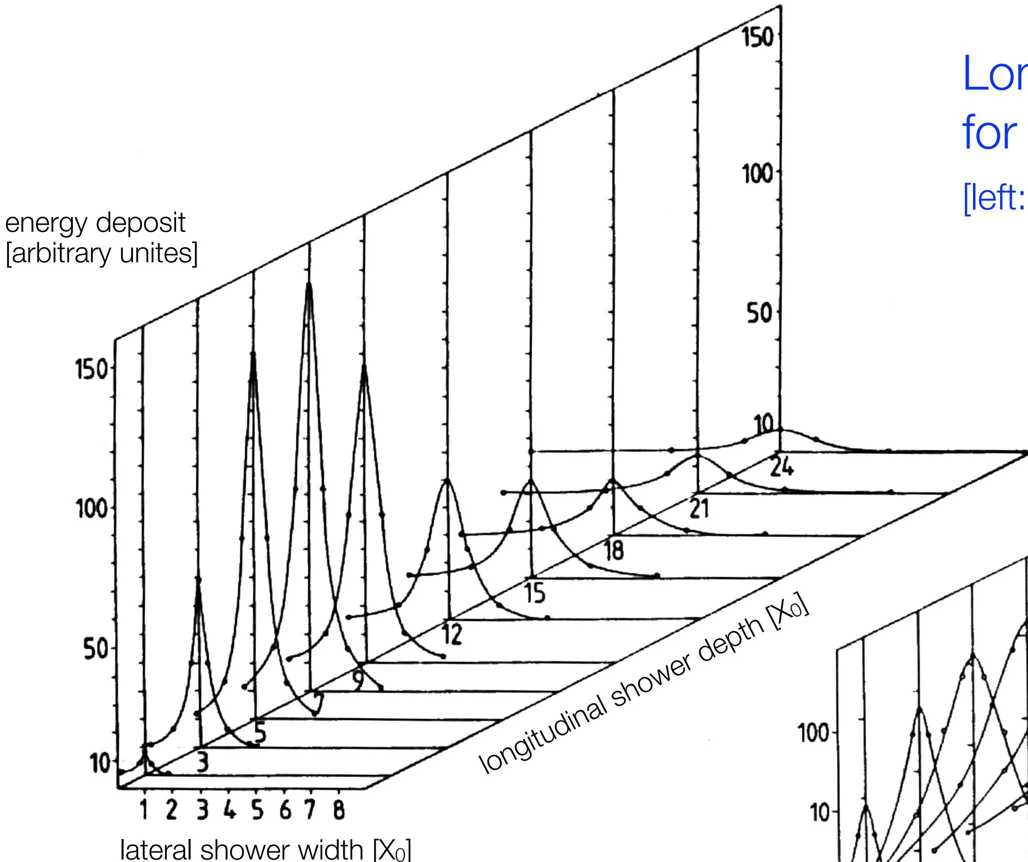


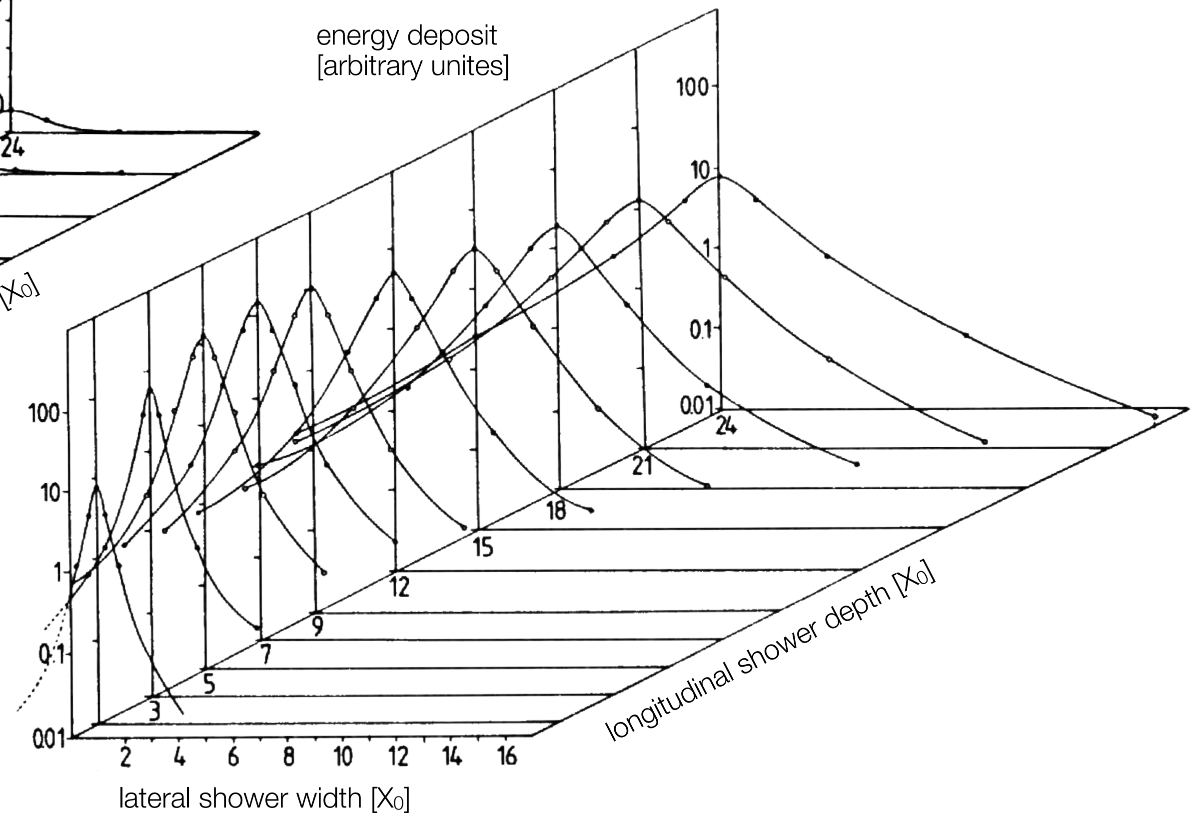
Shower gets wider at larger depth ...

EM shower profiles

Longitudinal and transversal shower profile for a 6 GeV electron in lead absorber ...

[left: linear scale; right: logarithmic scale]





EM showers in a nutshell

Radiation length: $$X_0 = 716.4\ \mathrm{g\,cm^{-2}} \frac{A}{Z(Z+1)\ln\frac{287}{\sqrt{Z}}}$$

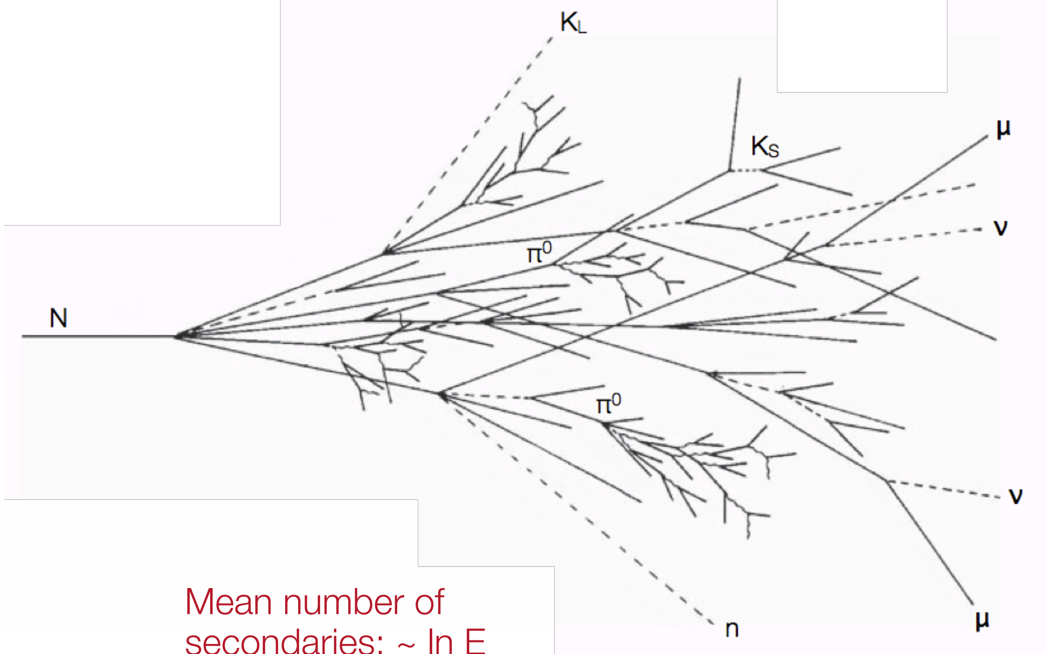
Critical energy: $$E_c^{\mathrm{Gas}} = \frac{710\ \mathrm{MeV}}{Z+0.92} \qquad E_c^{\mathrm{Sol/Liq}} = \frac{610\ \mathrm{MeV}}{Z+1.24}$$

Shower maximum: $$t_{\max} = \ln\frac{E}{E_c} - \begin{cases} 1.0 & \text{e}^- \text{ induced shower} \\ 0.5 & \gamma \text{ induced shower} \end{cases}$$

Longitudinal
energy containment: $$L(95\%) = t_{\max} + 0.08Z + 9.6\ [X_0]$$

Transverse
Energy containment: $$R(90\%) = R_M$$ $$R(95\%) = 2R_M$$ $$R_M = \frac{21\ \mathrm{MeV}}{E_c} X_0$$

Hadronic showers



Mean number of secondaries: ~ ln E

Typical transverse momentum: p_t ~ 350 MeV/c

Shower development:

1. p + Nucleus ➛ Pions + N* + ...
2. Secondary particles ...

   undergo further inelastic collisions until they fall below pion production threshold

3. Sequential decays ...

   π_0 ➛ γγ: yields electromagnetic shower
   Fission fragments ➛ β-decay, γ-decay
   Neutron capture ➛ fission
   Spallation ...

Cascade energy distribution:
[Example: 5 GeV proton in lead-scintillator calorimeter]

Ionization energy of charged particles (p,π,μ)	1980 MeV [40%]
Electromagnetic shower (π^0,η^0,e)	760 MeV [15%]
Neutrons	520 MeV [10%]
Photons from nuclear de-excitation	310 MeV [6%]
Non-detectable energy (nuclear binding, neutrinos)	1430 MeV [29%]
	5000 MeV [29%]

Substantial electromagnetic fraction

f_{em} ~ ln E
[variations significant]

Hadronic showers

Hadronic interaction:

Cross Section:

at high energies
also diffractive contribution

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$$

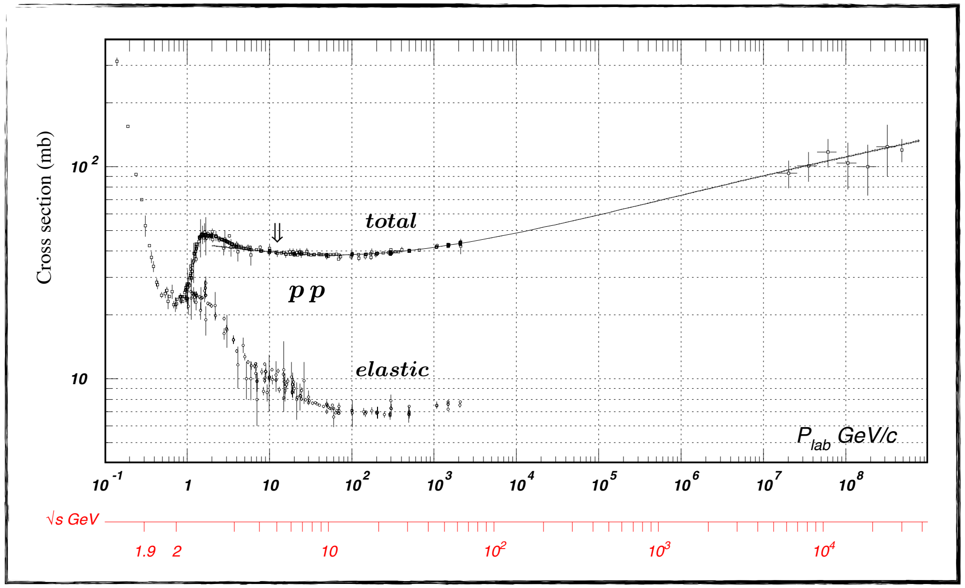
For substantial energies
σ_{inel} dominates:

$\sigma_{\text{el}} \approx 10 \text{ mb}$

$\sigma_{\text{inel}} \propto A^{2/3}$ [geometrical cross section]

$$\therefore \; \sigma_{\text{tot}} = \sigma_{\text{tot}}(pA) \approx \sigma_{\text{tot}}(pp) \cdot A^{2/3}$$

[σ_{tot} slightly grows with $\sqrt{s}$]



Total proton-proton cross section
[similar for p+n in 1-100 GeV range]

Hadronic interaction length:

$$\lambda_{\text{int}} = \frac{1}{\sigma_{\text{tot}} \cdot n} = \frac{A}{\sigma_{pp}\, A^{2/3} \cdot N_A \rho} \sim A^{1/3} \quad \text{[for } \sqrt{s} \approx 1 - 100 \text{ GeV]}$$

$$\approx 35 \; {}^{\text{g}}/_{\text{cm}^2} \cdot A^{1/3}$$

which yields:

$$N(x) = N_0 \exp(-x/\lambda_{\text{int}})$$

Remark: In principle one should distinguish between collision length $\lambda_W \sim 1/\sigma_{\text{tot}}$ and interaction length $\lambda_{\text{int}} \sim 1/\sigma_{\text{inel}}$ where the latter considers inelastic processes only (absorption) ...

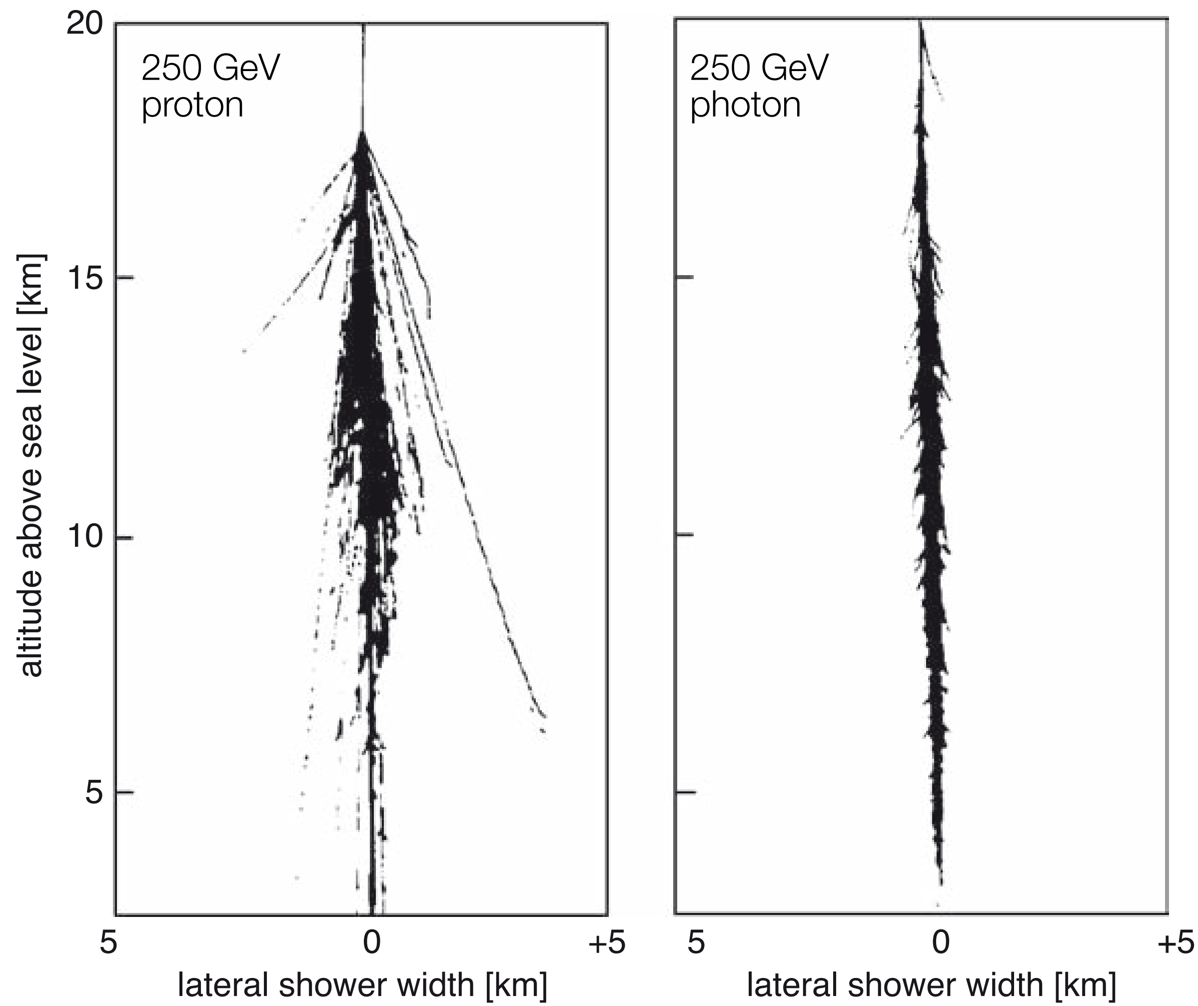
Interaction length characterizes both, longitudinal and transverse profile of hadronic showers ...

Hadronic vs. EM showers

Comparison

hadronic vs. electromagnetic shower ...

[Simulated air showers]



Hadronic vs. EM showers

Hadronic vs. electromagnetic interaction length:

$$\left.\begin{array}{l} X_0 \sim \dfrac{A}{Z^2} \\ \lambda_{\rm int} \sim A^{1/3} \end{array}\right] \rightarrow \frac{\lambda_{\rm int}}{X_0} \sim A^{4/3}$$

$$\lambda_{\rm int} \gg X_0$$

[λ_{int}/X_0 > 30 possible; see below]

Typical
Longitudinal size: 6 ... 9 λ_{int} [95% containment] [EM: 15-20 X_0]

Typical
Transverse size: one λ_{int} [95% containment] [EM: 2 R_M; compact]

Hadronic calorimeter need more depth than electromagnetic calorimeter ...

Some numerical values for materials typical used in hadron calorimeters

	λ_{int} [cm]	X_0 [cm]
Scint	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
C	38.1	18.8

Hadronic shower development

Hadronic shower development:

[estimate similar to e.m. case]

Depth (in units of λ_{int}):

$$t = \frac{x}{\lambda_{\text{int}}}$$

Energy in depth t:

$$E(t) = \frac{E}{\langle n \rangle^t} \quad \& \quad E(t_{\max}) = E_{\text{thr}}$$

[with $E_{thr} \approx$ 290 MeV]

$$E_{\text{thr}} = \frac{E}{\langle n \rangle^{t_{\max}}}$$

Shower maximum:

$$\langle n \rangle^{t_{\max}} = \frac{E}{E_{\text{thr}}}$$

Number of particles lower by factor E_{thr}/E_c compared to e.m. shower ...

Intrinsic resolution: worse by factor $\sqrt{E_{thr}/E_c}$

$$t_{\max} = \frac{\ln\left(E/E_{\text{thr}}\right)}{\ln\langle n \rangle}$$

But:

Only rough estimate as ...

energy sharing between shower particles fluctuates strongly ...

part of the energy is not detectable (neutrinos, binding energy); partial compensation possible (n-capture & fission)

spatial distribution varies strongly; different range of e.g. $\pi^\pm$ and π^0 ...

electromagnetic fraction, i.e. fraction of energy deposited by $\pi^0 \to \gamma\gamma$ increases with energy ...

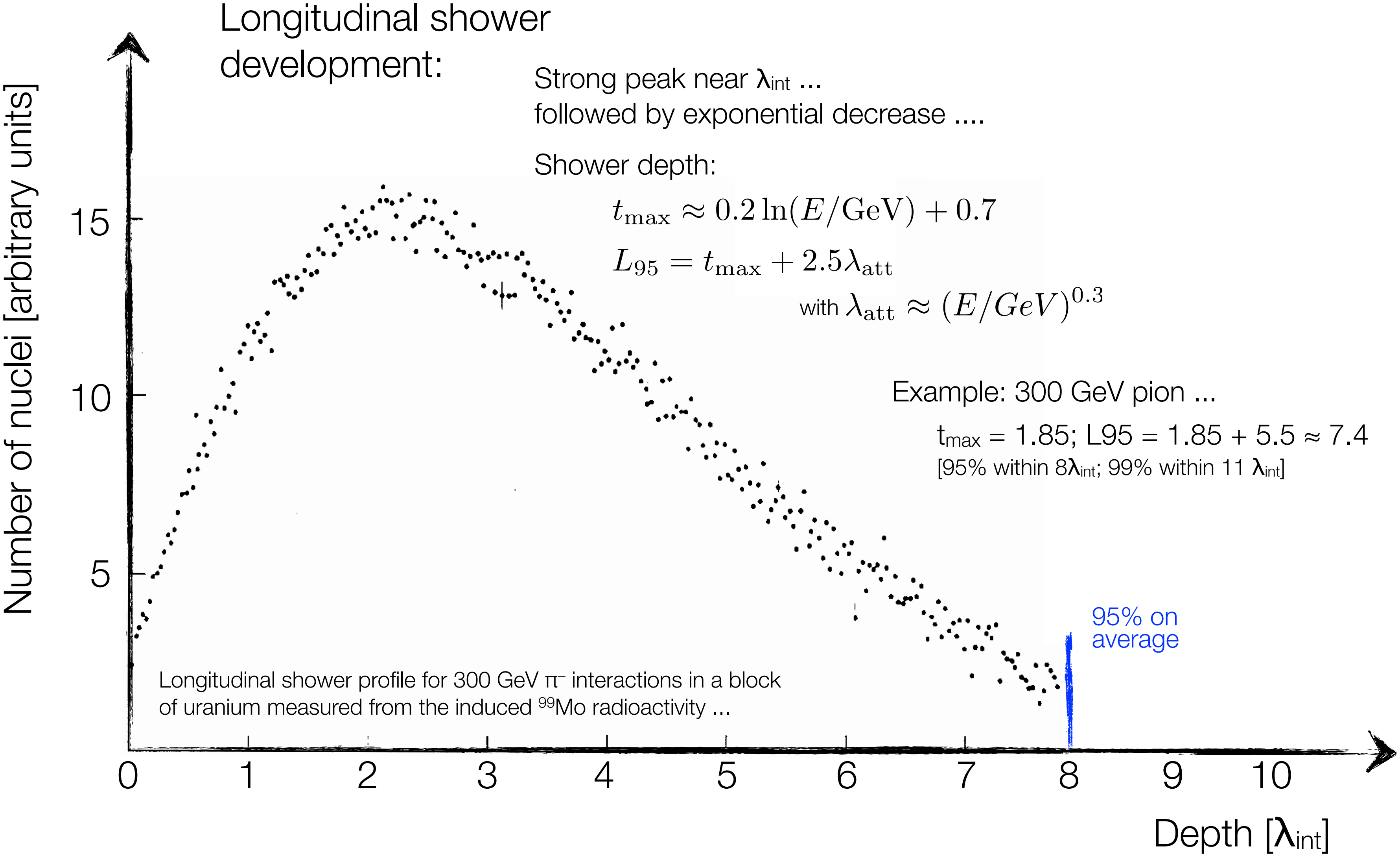
$$f_{\text{em}} \approx f_{\pi^0} \sim \ln E/(1\ \text{GeV})$$

Explanation: charged hadron contribute to electromagnetic fraction via $\pi^- p \to \pi^0 n$; the opposite happens only rarely as π^0 travel only 0.2 μm before its decay ('one-way street') ...

At energies below 1 GeV hadrons loose their energy via ionization only ...

Thus: need Monte Carlo (GEISHA, CALOR, ...) to describe shower development correctly ...

Hadronic shower longitudinal development



$$t_{\mathrm{max}} \approx 0.2 \ln(E/\mathrm{GeV}) + 0.7$$

$$L_{95} = t_{\mathrm{max}} + 2.5\lambda_{\mathrm{att}}$$

with $\lambda_{\mathrm{att}} \approx (E/GeV)^{0.3}$

Hadronic shower transverse development

Transverse shower profile

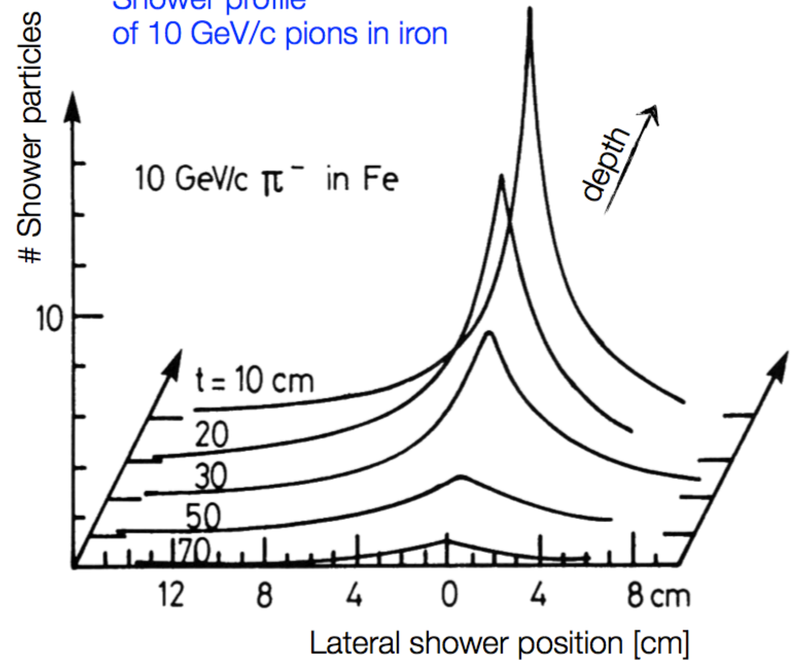
Typical transverse momenta of secondaries: $\langle p_t \rangle \simeq 350 \mathrm{MeV}/c$ ...

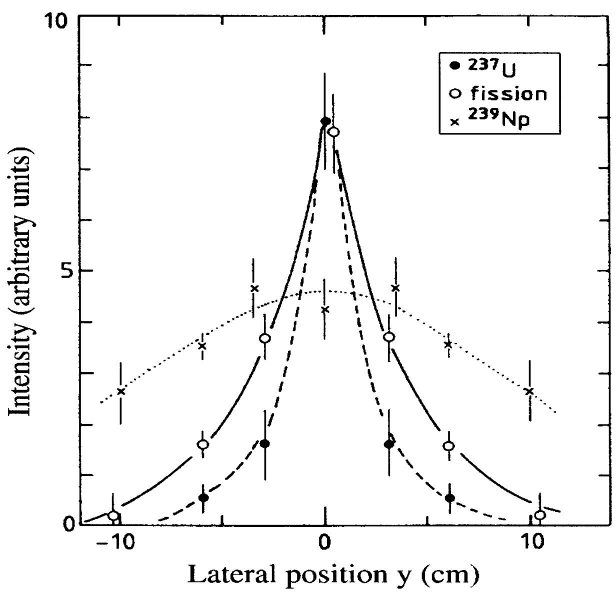
Lateral extend at shower maximum: $R_{95\%} \simeq \lambda_{int}$ ...

Electromagnetic component leads to relatively well-defined core: $R \simeq R_M$ ...

Exponential decay after shower maximum ...

Shower profile
of 10 GeV/c pions in iron





Lateral profile for
300 GeV π^-

[target material ^{238}U]
[measured at depth 4 λ_{int}]

More π^0's and γ in core

Energetic neutrons and charged pions form a wider core

Thermal neutrons generate broad tail

Measurement from induced radioactivity:

- ^{99}Mo (fission): neutron induced ...
  [energetic neutron component]
- ^{237}U: mainly produced via ^{238}U(γ,n)^{237}U ...
  [electromagnetic component]
- ^{239}Np: from ^{239}U decay ...
  [thermal neutrons]

Ordinate indicates decay rate of different radioactive nuclides ...