VARIANCE RISK PREMIUM IN EQUITY MARKETS

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1 THESIS INDEX

2 Introduction

The paper is organized into 3 parts:

- First, I aim to quantify the variance risk premium on the S&P 500 over a recent historical period. The approach used to do so is following that developed by Carr and Wu in their paper *Variance Risk Premiums, 2009*.
- Second, I test whether the results on variance risk premium can be accounted for by risk factor models. *The Capital Asset Pricing Model*, and the *Fama French Model* and are used for testing.
- Third, I implement a systematic trading strategy that is designed to monetize this premium. The strategy, best described as a rolling short variance strategy, is back tested on historical data with the help of an index. Results from different variants of the strategy are then discussed.

3 Quantifying Variance Risk Premium

A risk premium in layman's terms is an equilibrium concept that speaks to the minimum compensation (in expectation) required to take on a certain risk. In the context of a financial asset, one could argue that there are principally 2 sources of risk. Namely, the risk of fluctuations in the asset's value, and then the risk that the speed of these fluctuations may change. In other words, there is the risk of variation in asset returns ("return risk") and then there is the risk of variation in asset return variability itself. ("return variance risk")

This return variance risk (or simply "variance risk") and the equilibrium compensation required by market participants for taking on this risk (defined as the "variance risk premium") is the focus of this paper.

The following section discusses different methods of quantifying the variance risk premium, highlighting the strengths and weakness of each approach. It then uses one of these methods to quantify the same for the S&P 500 index.

3.1 Pure Options based measurement

This approach would use a sample average of the difference between the realized volatility of a given underlying and the implied volatility of its options for a certain maturity, over a long historical period. If indeed a variance risk premium exists, one should expect to find a sample average that is significantly different from 0.

The problem with using this approach is that it is hard to isolate the pure effect of the expected underlying return volatility on option implied volatilities from that of directional risk in the underlying. Hence our results using this approach would be biased.

3.2 Variance Swap based measurement

An alternative approach presented by *Carr and Wu* in their paper *Variance Risk Premiums* is to use a variance swap contract.

The big advantage of the variance swap contract is that its value is independent of directional risk in the underlying. In the next section, I undertake a brief literature review to review this result.

3.2.1 Literature Review on Variance Swaps

If one combines say a long position in a call option with a certain number of underlying shares, equal to the delta of the call option, and the money market account in such a way that this portfolio is costless to set up, and delta neutral always, this portfolio is described as the long volatility trade. With these assumptions, it could be argued that the portfolio value V, at any point in time, is purely exposed to the volatility of the underlying, since the portfolio is assumed to be delta neutral in continuous time. The assumption of continuous delta neutrality is however far from realistic as it assumes continuous rebalancing. In reality, rebalancing is discrete.

It can be shown that with discrete rebalancing (i.e. assuming the same position is kept over an interval of time $\Delta t > dt$) the approximate gain on the portfolio

$$PnL_{\delta t} = \frac{\partial V}{\partial t}\delta t + (1/2))\frac{\partial^2 V}{\partial S^2}(\delta S_t)^2$$

Now, by Black Scholes, it follows that

$$\frac{\partial V}{\partial t} = -(1/2))\frac{\partial^2 V}{\partial S^2}S_t^2\sigma_t^2$$

Therefore,

$$PnL_{\delta t} = (1/2))\frac{\partial^2 V}{\partial S^2}\left[(\delta S_t)^2 - S_t^2\sigma_t^2\delta t)\right]$$

Or Total PnL a given time period T can be written as:

$$PnL[0, T\ ] = \sum_{i=0}^{T}\frac{\partial^2 V}{\partial S^2}\left[(\delta S_t)^2 - S_t^2\sigma_t^2\delta t)\right]$$

The above expression states that the PnL on a discretely delta hedged long call option position is the product of the dollar gamma and the difference between the realised and implied variance, summed over all time periods (Δt) over time [0, T].

This is an interesting result because it shows that PnL of the delta hedged option is path dependent through the interaction of volatility and dollar gamma. That is, even if one's view on the realization of volatility turns out to be correct on average, over a given time period, one could still lose money on the above position because the dollar gamma of the option turned out to be low whenever one's view was right and turned out to be high whenever one's view was wrong. Thus on average, over a given time period, one could lose money even though one's view on volatility was correct.

Anthony Neuberger in his paper *"Volatility Trading, June 1990"* showed that one can get rid of the above path dependency and attain pure exposure to realized volatility, by buying and delta hedging a log contract

(where he defined a log contract as a claim whose terminal payoff is the logarithm of the terminal price of the asset)

Demeterfi, Derman, Kamal, Zou in their paper, *More Than You Ever Wanted To Know*About Volatility Swap, March 1999*, then showed that a variance swap can be replicated by a delta hedged short position in the log contract.

They also showed that this variance swap exposure via shorting a log contract can alternatively be replicated by a static portfolio of options weighted by the inverse of their square of their strike. In particular, they were able to show that this static portfolio of options has a constant exposure to volatility that is independent of the level of the underlying. And this is precisely the feature of a variance swap that makes it attractive.

3.2.2 Description of Variance Swap Contract

The contract's payoff is structured as a forward transaction, in that there is only 1 single payoff at maturity. This payoff is calculated as the difference between a pre-agreed estimate of realized variance over the life of the contract and a pre agreed variance swap rate (referred to as variance swap "strike"), both of which are annualized figures.

$$Payoff\ at\ maturity = VarianceNotional * [Realized\ Variance - Strike^2\ ]$$

Alternatively, in terms of vega notional[1]

$$Payoff\ at\ maturity = \frac{Vega\ Notional*[Realized\ Variance - Strike^2\ ]}{2*Strike}$$

[1] Vega Notional I the context of a variance swap is defined as the average profit or loss for a 1 vega (or 1 point difference) between realised volatility and the variance swap strike volatility.

3.2.3 Carr and Wu approach for estimating variance risk premium

Carr and Wu's approach combines a variance swap contract, with no-arbitrage principles to produce a variance risk premium estimator.

By no-arbitrage and since swaps are costless to enter, they argue that it must be the case that the variance swap rate or "strike" agreed at contract inception is equal to the risk neutral expected value of the future realized variance. That is,

$$\text{Variance Swap Strike}_{t,T} = E^{Q}_{t}\,[\text{Realized Variance}_{t,T}]$$

They then show that $\mathbf{E^{Q}_{t}}$ **[Realized Variance** $_{\mathbf{t,T}}$**]** can be approximated by a static portfolio of options (assuming zero interest rates and dividends) per the following expression

$$E_t^Q\,[RV_{t,T}] \;=\; K_{VAR} = \frac{2}{T-t}\left[\int_0^F \frac{P(K)dK}{K^2} + \int_F^\infty \frac{C(K)dK)}{K^2}\right]$$

Where

i) $P(K)$ denotes the time-t value of an out-of-the-money put option with strike price K > 0 and maturity T ≥ t

ii) $C(K)$ denotes time-t value of a call option with strike K > 0 and maturity T ≥ t

iii) K_{VAR} is the variance swap strike

Carr and Wu use the above equation as the basis for inferring variance swap rates from the prices of vanilla options. Finally they prove that a sample average of the difference between the realized variance and the variance swap rate (inferred per above equation) is a direct estimate of the average variance risk premium. That is :

Average Variance Risk Premium = Realized Variance – $\mathbf{E^{Q}_{t}}$ **[Realized Variance** $_{\mathbf{t,T}}$**]**

This expression is also exactly equivalent to the terminal payoff on a variance swap.

3.3 Quantifying S&P 500 Variance Risk Premium

In this section, I describe the data and methodology used to calculate the variance risk premium estimate for the S&P 500 index.

Data

The raw data used spans the period 31st December 2004 to 13th March 2015 and includes:

i) daily close of business S&P 500 index values

ii) daily close of business VIX index values

These datasets are used to create an overlapping and non-overlapping monthly return series. The overlapping return series calculates returns as if new 1 month swap positions were opened daily. The non-overlapping return series calculates returns as if new 1 month swap positions were opened monthly.

Methodology

Both overlapping and non-overlapping return series are constructed as follows:

i. Monthly variance swap returns used in the estimation of the variance risk premium are calculated per the following equation :

   Monthly Return $=\frac{252}{21}\sum_{k=1}^{21} Daily\ Realized\ Variance - (VIX_0\%)^2$

   Assumptions in calculation

   1. Unit variance notional,
   2. 1 month = 21 trading days
   3. Daily realized variance = square of daily log returns,

ii. The use of the VIX index in the above calculation follows closely the theoretical approach developed by Carr and Wu. Since it is well known that the VIX index is calculated as the risk neutral expected value of future realized variance over the next 21 business days (30 calendar days), it is equivalent to using the theoretical variance swap strike K_{VAR} derived by Carr and Wu per above equation.

iii. The sample average of the overlapping and non-overlapping return series taken separately, is then defined as the variance risk premium estimator.

3.3.1 Results for overlapping monthly return series

The summary statistics for the overlapping monthly series are as follows:

Mean (variance risk premium estimate)	-0.83%
t-statistic[2]	-7.04
Standard Error	0.12%
Median	-1.02%
Standard Deviation	5.95%
Sample Variance	0.35%
Kurtosis	43.05
Skewness	5.10
Range	89.18%
Minimum	-32.69%
Maximum	56.49%
Count	2546

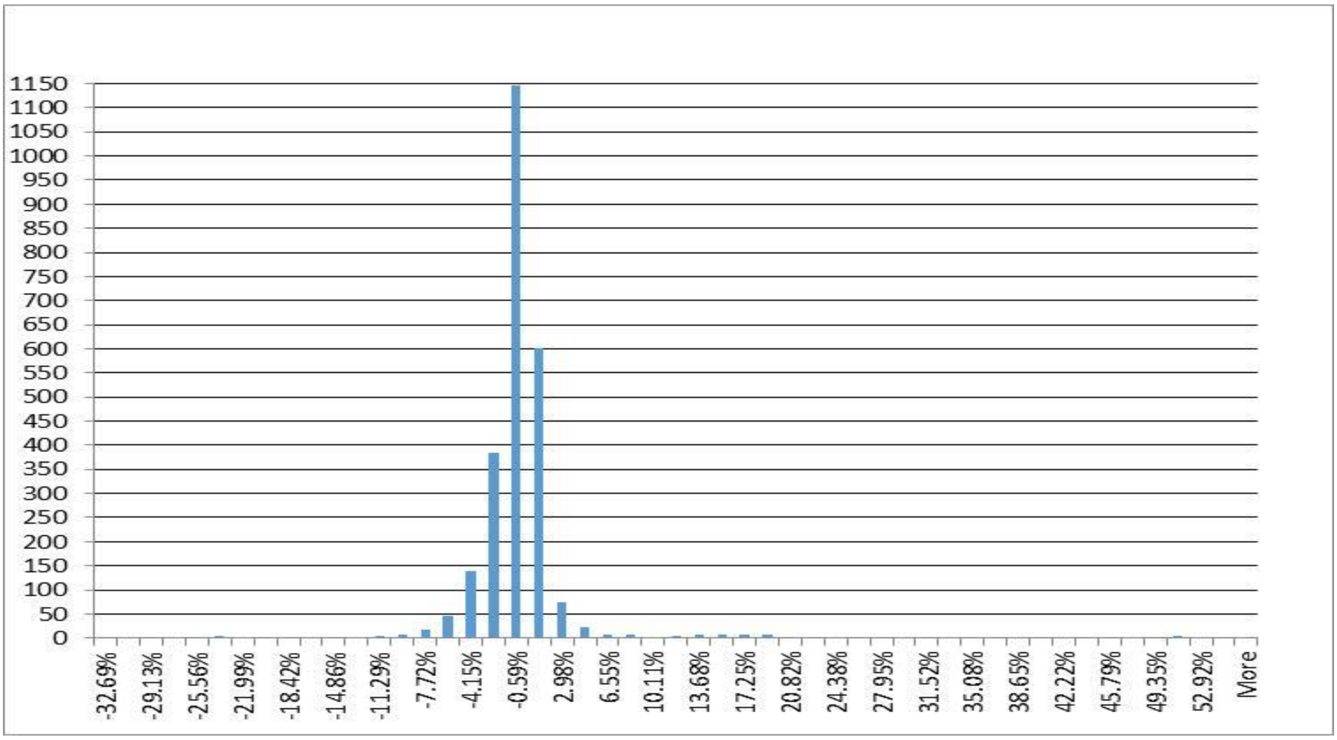

Figure: Variance Swap Returns for the overlapping monthly return series

[2] T-statistics results included in Appendix

Observations

1. The variance risk premium (VRP) estimate is negative (=-0.83%), with a significant t statistic. This is an important result as it as it confirms that variance risk on the S&P 500 over the chosen historical period is priced.
2. The negative VRP estimate also provides an important interpretation on the price of variance risk. Namely, that investors are happy to accept negative compensation in equilibrium (i.e. on average) to hedge against upward movements in variance. That is, investors are willing to pay a premium to hedge against variance risk.

   Conversely, investors that are short variance demand positive compensation in equilibrium (i.e. on average) in the form of this variance risk premium.

3. The VRP distribution is positively skewed with a fat right tail. This reveals a defining feature of variance risk: that variance realizes below the level implied by the variance swap strike most of the time, but exceeds it a small number of times (during volatility spikes)

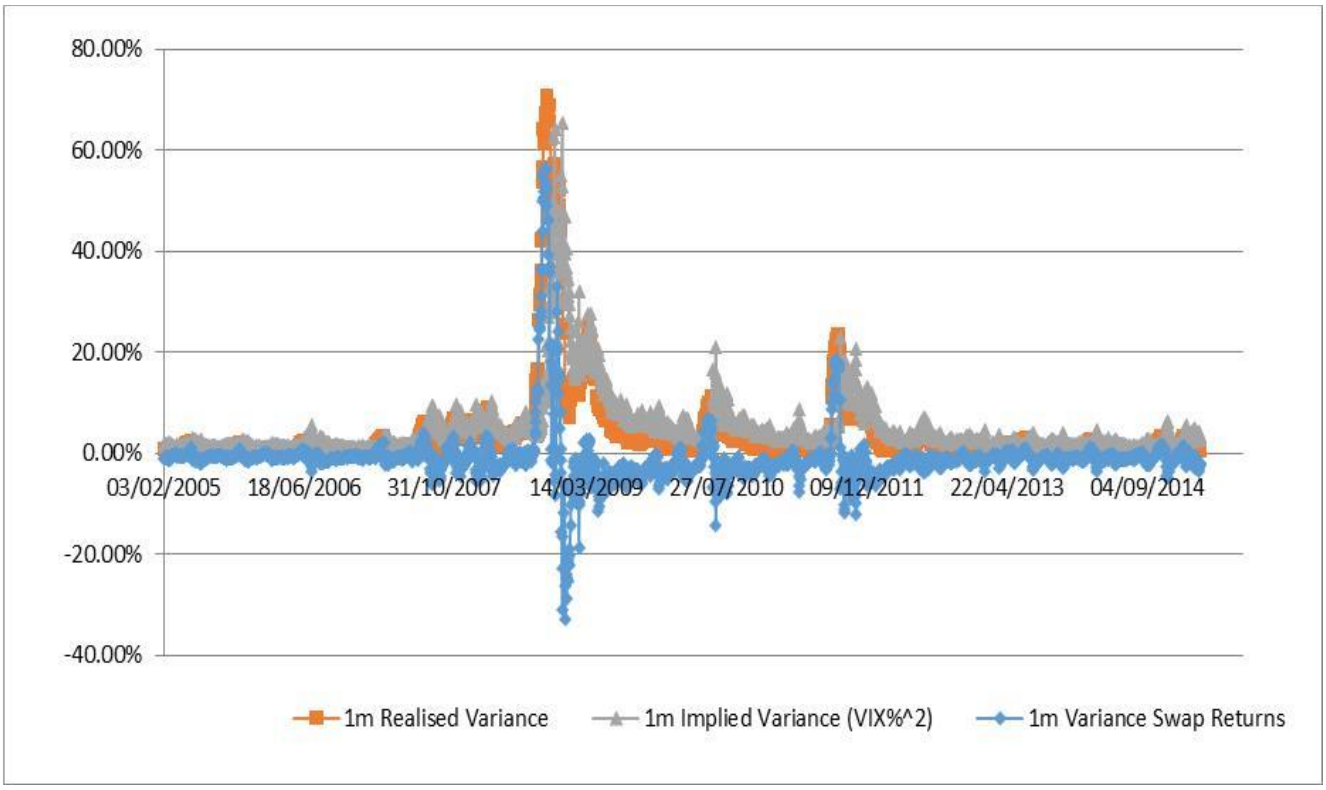

Figure: Variance Swap returns through time

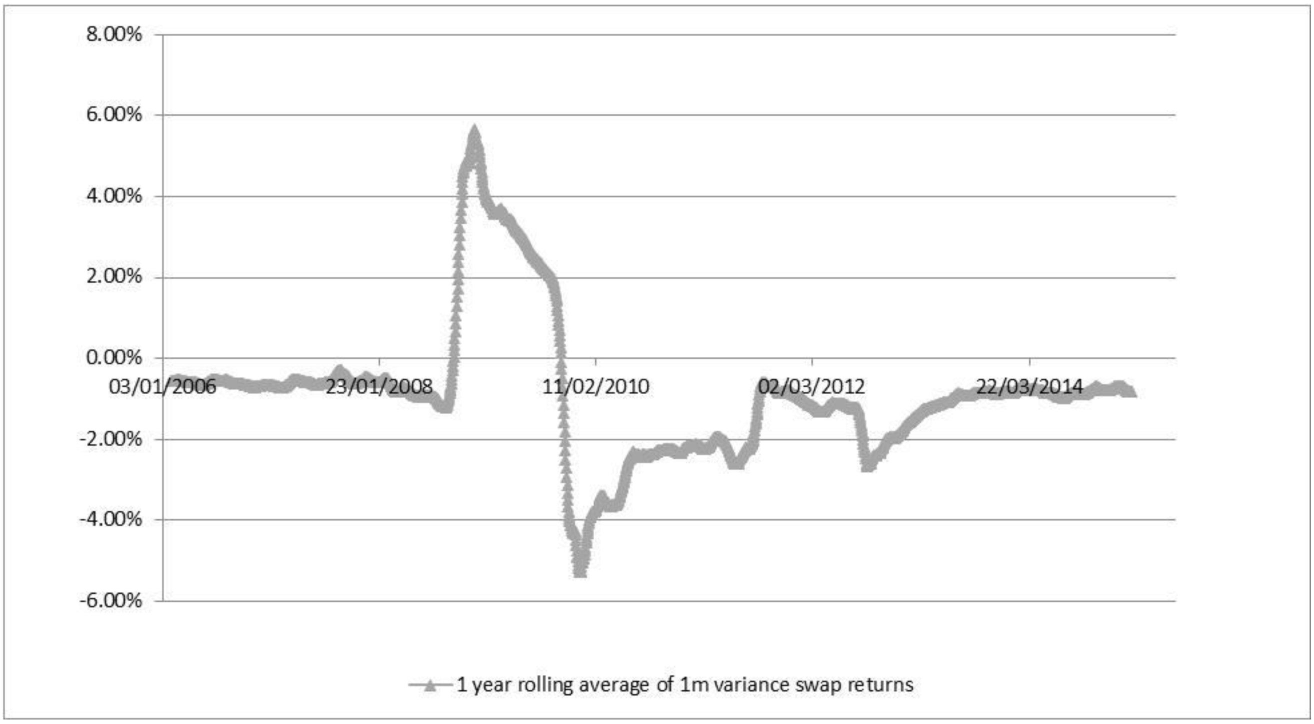

Figure: 1 year rolling average variance swap returns through time

3.3.2 Results for non-overlapping monthly return series

Mean (variance risk premium estimate)	-0.86%
t-statistic[3]	-1.31
Standard Error	0.66%
Median	-1%
Standard Deviation	7.24%
Sample Variance	0.52%
Kurtosis	29.94
Skewness	3.65
Range	85.30%
Minimum	-32.69%
Maximum	52.61%
Count	122

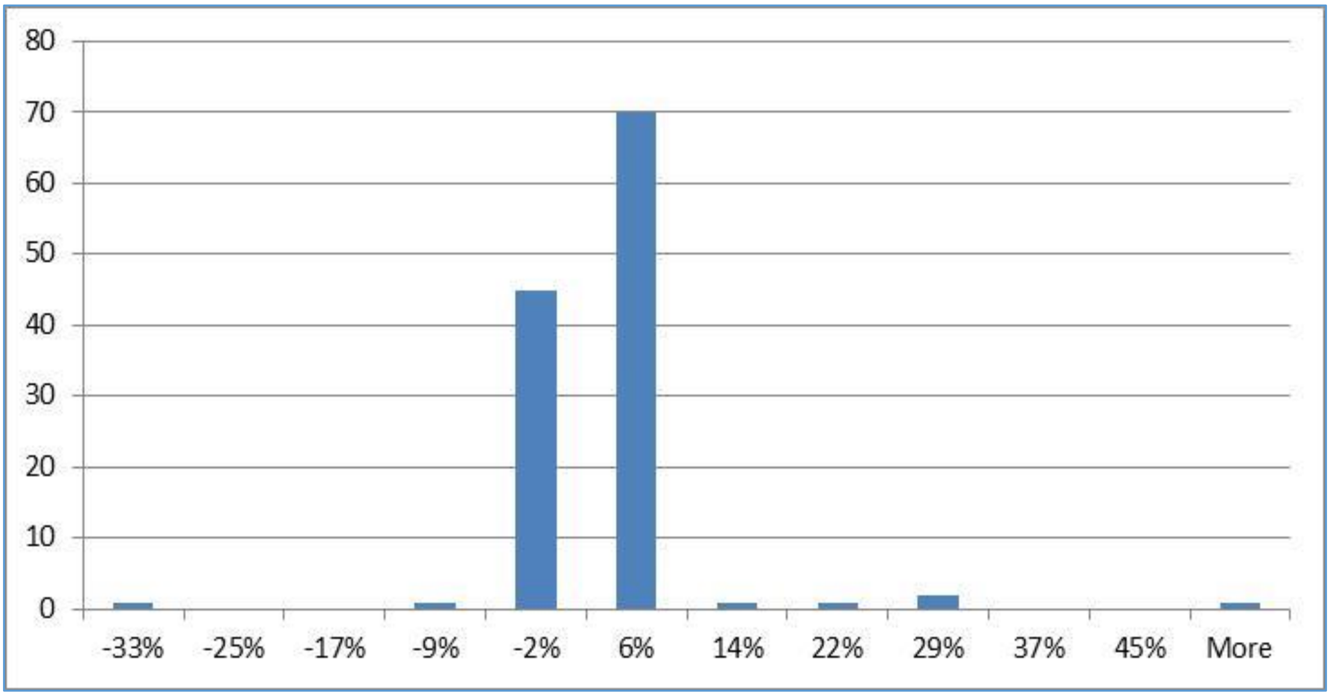

Figure: Variance Swap Returns for non-overlapping monthly return series

[3] T-statistics results included in Appendix

Observations

1. The variance risk premium (VRP) estimate is still negative (=-0.86%) but no longer significant even at the 10% confidence level. Bearing in mind the smaller sample size (122 observations for this return series), the estimate is still large enough to suggest that variance risk on the S&P 500 is priced.
2. The negative VRP estimate has the same interpretation as in the overlapping series above : that investors are willing to pay a premium to hedge against upward movements in variance.
3. The observed positive skew also has the same interpretation as in the overlapping series.

4 Can risk factor models account for the Variance Risk Premium?

The debate on whether return variance is an independent source of risk or a derived source of risk is a long standing one. The literature on the subject presents contrasting views.

In stochastic volatility models, academics have modelled return variance as an independent source of risk (eg: Heston model). There have also been models that have modelled return variance as a function of the underlying returns.

In this section, the objective is to test which of the above theories holds true for variance risk on the S&P 500. Namely, whether S&P 500 return variance is indeed an independent source of risk, or whether it is induced entirely by the return risk of the S&P 500 index.

The way I test this is by regressing the non-overlapping monthly variance swap return series (computed from the previous section) on different factor models. Description of the different factors is included in the Appendix.

4.1 Capital Asset Pricing Model (CAPM) Regression: Results

CAPM regression				
Dependent Variable: 1m Variance Swap Returns				
Method: Least Squares				
Date: 06/10/15 Time: 12:59				
Sample: 1 122				
Included observations: 122				
HAC standard errors & covariance (Bartlett kernel, Newey-West automatic				
bandwidth = 10.3440, NW automatic lag length = 6)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
intercept	-0.249146	0.575853	-0.432656	0.666
MRP	-0.943818	0.360303	-2.619509	0.0099
R-squared	0.324007	Mean dependent var		-0.856207
Adjusted R-squared	0.318374	S.D. dependent var		7.243701

Observations:

1. The coefficient on MRP (or Market Risk Premium) is negative and significant. In particular, the coefficient is very close to -1. This is an important result as it suggests that variance risk on the S&P 500 can be accounted for by the strong negative correlation of variance swap returns with index returns.
2. The value of the intercept or the "alpha" is negative but not significantly different from 0. This confirms that in a CAPM world, there is no residual variance swap return that is left unexplained by the negative correlation with market returns.

The above 2 observations thus confirm that variance risk premium on the S&P 500 in a CAPM world is induced entirely by the market risk premium.

4.2 Fama French Model (FF) Regression: Results

In this section, I test whether adding the Fama French Factors (SMB or Small minus Big, and HML or High minus Low) to the MRP factor alters the results from the previous CAPM regression.

Fama French regression				
Dependent Variable: 1m Variance Swap Returns				
Method: Least Squares				
Date: 06/10/15 Time: 13:02				
Sample: 1 122				
Included observations: 122				
HAC standard errors & covariance (Bartlett kernel, Newey-West automatic				
bandwidth = 8.6976, NW automatic lag length = 6)				
Variable	Coefficier	Std. Error	t-Statistic	Prob.
C	-0.22048	0.636457	-0.346423	0.7296
MRP	-0.96803	0.384235	-2.519365	0.0131
SMB	-0.13519	0.294048	-0.459749	0.6465
HML	0.313861	0.291753	1.075777	0.2842
R-squared	0.334811	Mean dependent var		-0.85621
Adjusted R-squared	0.317899	S.D. dependent var		7.243701

Observations:

1. The coefficient on MRP is still negative and significant, confirming that the previous results for CAPM continue to hold.
2. The coefficient on SMB is negative but not significantly different from 0.
3. The coefficient on HML is positive but again not significantly different from 0

The above observations thus confirm that variance risk premium on the S&P 500, even in a Fama French world, can be largely explained by its negative correlation with S&P 500 returns.

5 TRADING STRATEGIES TO MONETIZE THE VARIANCE RISK PREMIUM

So far we have established that a statistically significant negative variance risk premium exists when looking at historical data on the S&P 500. That is investors are willing to pay a premium to hedge away upwards movements in volatility. We have also established that this negative premium can almost entirely be accounted for by the market risk premium. That is, a long position in variance swaps effectively acts as an insurance asset, paying out large amounts during market crashes and drawing down on a small premium during low volatility periods.

Given these results, the obvious question that comes to mind is how to monetize this premium.

In this next section, I will discuss a particular type of systematic trading strategy that aims to monetize this premium. I will then implement this strategy on the S&P 500 and track its performance with the help of an index.

5.1 ROLLING SHORT VARIANCE STRATEGY

This is a strategy that aims to capture the differential between implied and realized risk in the S&P 500 market by systematically selling volatility (or variance). The strategy is implemented using a rolling short position in a variance swap.

A fairly popular investment strategy in the marketplace, these strategies are typically sold by packaging the strategy's performance in the form of an index, and then selling the index returns in OTC swap format or via a fully funded note.

The strategy and index construction[4] can be explained in the following steps:

1. The strategy assumes a certain start date.
2. The index assumes a starting value of 100 on the start date.
3. The initial Vega Notional is calculated as a constant "alpha" (typically set to 1%) times the starting index value. Thus, the Vega Notional starts at 1 but is updated on every monthly roll based on the closing value of the index on the roll date. Likewise, the Variance Notional starts at a number equal to the Vega Notional divided by 2 times the Initial Variance Swap Strike, but is updated per monthly roll based on the new Vega Notional and the new Variance Swap Strike for the next month.
4. On the start date, the strategy enters into a 1 month short variance swap position at the prevailing 1 month variance swap strike. (mid-market level)
    a. For the strategy implementation in this paper, I proxy the variance swap strike mid-level with the 1 month VIX index[5].

[4] The description applies equally to a 1 month and 3 month swap strategy

[5] As discussed in an earlier section, the VIX is the risk neutral expected value of future 1 month realized variance. Assuming away any market frictions and costs, the VIX is therefore the closest available approximation to market variance swap levels.

5. On any day‘t’ between the start date (t=0) and the expiry date of the 1 month swap, the strategy’s index tracks the Mark to Market (MTM) value of the swap.

   The MTM value of a variance swap is essentially the present discounted value of the expected PnL at expiry, and is calculated as follows:

   MTM= Not*[% time elapsed*$RVaR_t$ + % time remaining*$IVaR_{t,T}$ - $IVaR_{0,T}$]*$DF_{t,T}$

Where:

$IVaR_{0,T}$ = initial variance swap strike struck at time ‘0’ for a swap maturing at time ‘T’

$IVaR_{t,T}$ = new variance swap strike as of time‘t’ for a variance swap maturing at time ‘T’

$RVaR_t$= cumulative realised variance (annualized) from time ‘0’ to time‘t’

$DF_{t,T}$ = discount factor at time ‘t’ for a cash flow occurring at time ‘T’

Not = -Variance Notional = -Vega Notional/(2*Initial Variance Swap Strike)[6]

6. On the swap expiry date or “roll date”,
   a. The MTM of the strategy is simply equal to the swap PnL at expiry. This PnL then accrues to the previous month’s starting index value such that the index value for the next month starts at :

      New Month Index Value = Previous Month’s Index Value + PnL

   b. Since the vega notional and variance notional are defined as functions of the index value, these too get updated for the next month’s swap in the following manner :

      New month Vega Notional = alpha * New month Index Value

      New Variance Notional = New Vega Notional/(2*New month Variance Swap Strike)

   c. Also, on the “roll” date, the strategy enters into a new short 1 month variance swap position based on an updated variance notional, and based on the current mid-level variance swap strike (This is proxied by the VIX index in my implementation)

7. Thereafter, though successive days and months, the index tracks the value of the strategy through a repetition of steps 4-6

[6] Since the swap MTM is calculated for a long position, the notional is assigned a negative sign to reflect the short nature of the strategy

5.2 ROLLING SHORT VARIANCE ON THE S&P 500 : DATA AND METHODOLOGY

In this section, I describe the data and methodology used to construct the index.

The dataset used spans from 31st July 2006 to 13th March 2015. It includes

i) Close of Business S&P 500 index values,

ii) Close of business VIX & VXV index values (1month and 3 month indices)

The daily index calculation for all rolling short variance swap strategies discussed in this paper can be summarized as follows:

$$\textbf{Index}_t = \textbf{Index}_{\text{Previous Roll}} + \textbf{MTM value of current swap}_{t,T}$$

Where:

i) Starting value of Index $_{\text{Previous Roll}}$ is set to 100, and where every subsequent value of Index $_{\text{Previous Roll}}$ is set to the index value on the roll date

ii) MTM value of current swap is calculated as detailed in the previous section, with the following additional details

   a. The CBOE VIX Index is used as the fair mid market variance swap strike for a monthly strategy
   b. The CBOE VXV index (3 month volatility index) is used as the fair mid market variance swap strike for a 3 month strategy
   c. The fair variance swap strike for time to maturity less than 21 trading days is calculated using an interpolation of the following form :

Fair Swap Strike = sqrt [(Days to maturity) * (VIX^2-0)/(21)]

   d. The fair variance swap strike for time to maturity greater than 21 trading days and less than 63 trading days (for a 3 month strategy) is calculated using an interpolation between the VIX and the VXV indices per the following:

Fair Swap Strike= sqrt [(VIX^2) + ((VXV^2 – VIX^2)/21)*(Days to maturity-21)]

5.2.1 Results for 1 month strategy

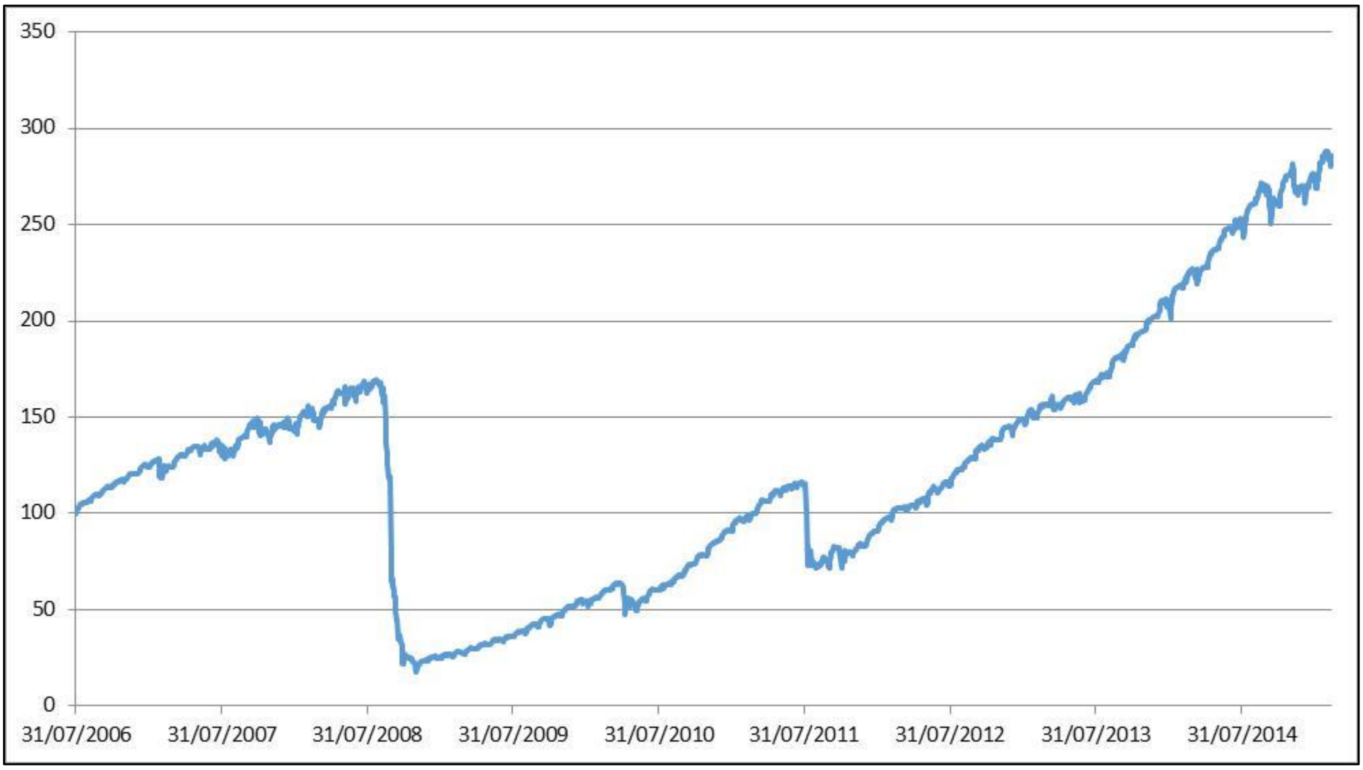

Figure: Index performance for rolling short 1 month strategy, with vega notional reset =1% of roll date index value

Observations

i. The strategy shows a steady upward performance throughout the above historical period barring 2 short periods of sharp downturns. These downturns are observed between
    i. 12th September 2008 - 5th December 2008, and
    ii. 3rdAugust 2011 - 26th September 2012

ii. The cumulative net return on the strategy over the above period is 186%

iii. The annualized net return on the strategy is equal to 13%

iv. The annualized volatility on the strategy is equal to 50.9%

v. The strategy's Sharpe ratio is thus equal to 0.25.

vi. The maximum drawdown[7] over the 10 year period is -89.0% observed between July and December 2008.

vii. Despite the two sharp downturns, the average yearly return is 28.6% over the last 10 years (per below)

[7] Maximum drawdown is defined as the single largest difference between the highest entry point and the lowest exit point on an index. It is a measure of the worst potential loss incurred over a given historical period.

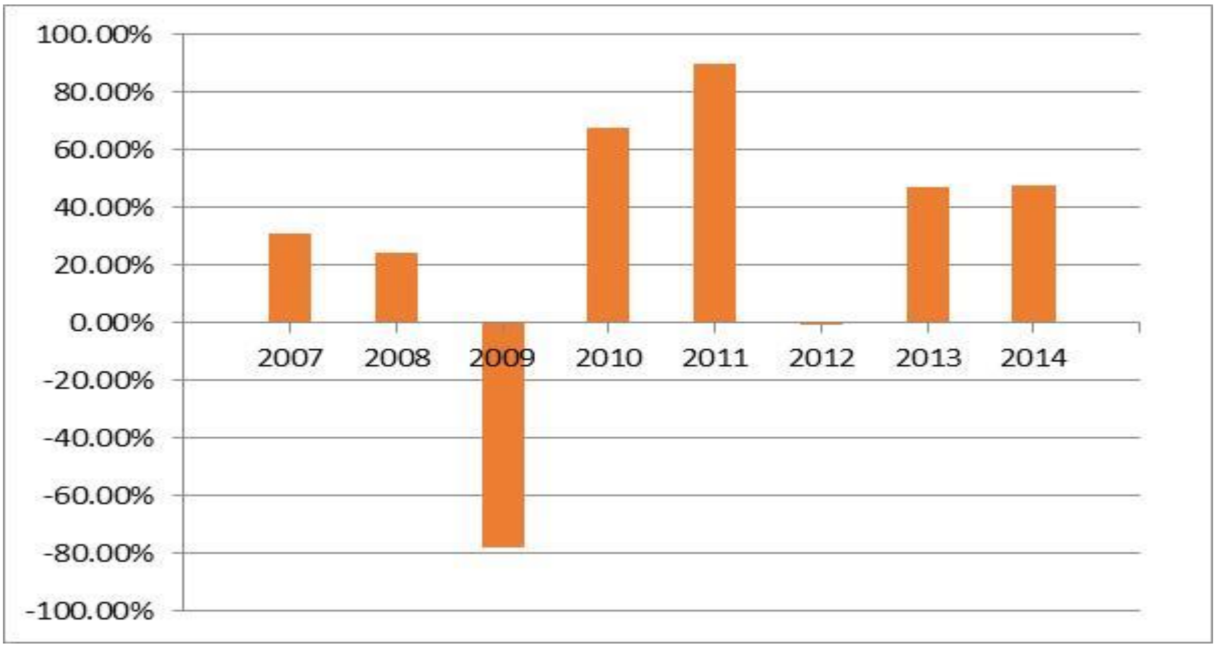

Figure: Year on year index returns

viii. The monthly return distribution (below) shows large negative skewness and large kurtosis

a. The mean monthly return is 2%, with a standard deviation of 10.5%.
b. The distribution has a fat left tail, a property symptomatic of short variance ("short insurance") strategies. They incur large losses a small number of times but make relatively small profits most of the time[8].

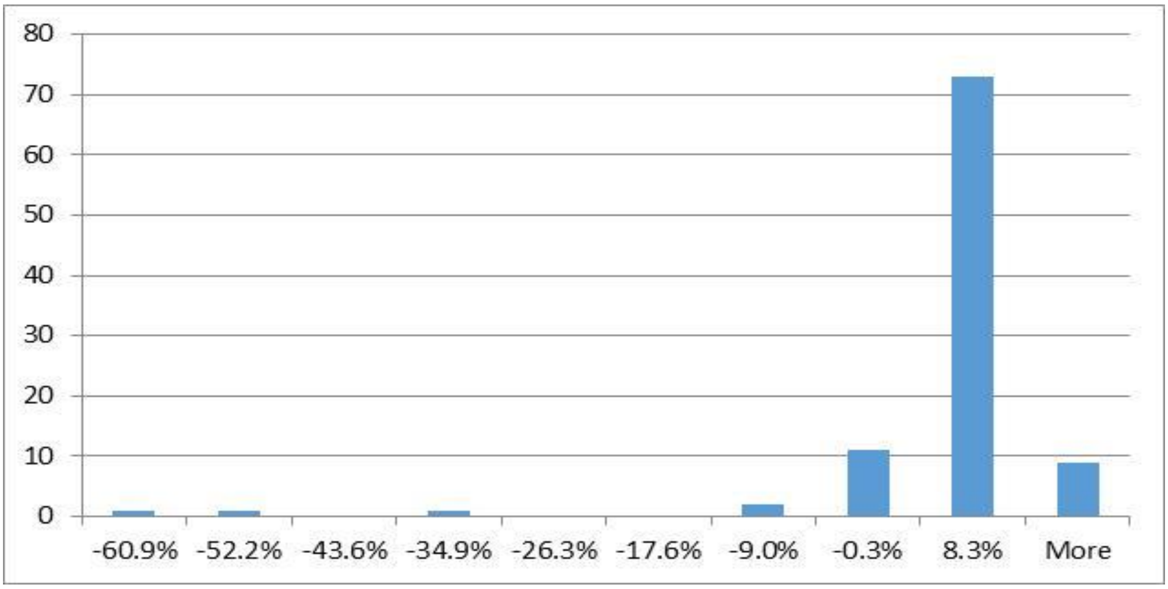

Figure: Monthly index return distribution

[8] Part of this asymmetric distribution can also be attributed to the fact that short positions in variance swaps are short convexity with respect to realized volatility. That is, for the same move in realized volatility away from the strike, they lose more than they gain.

ix. The daily return distribution shows a similar structure to that of the monthly distribution. The mean daily return is 0.07%, with standard deviation of 1.97%

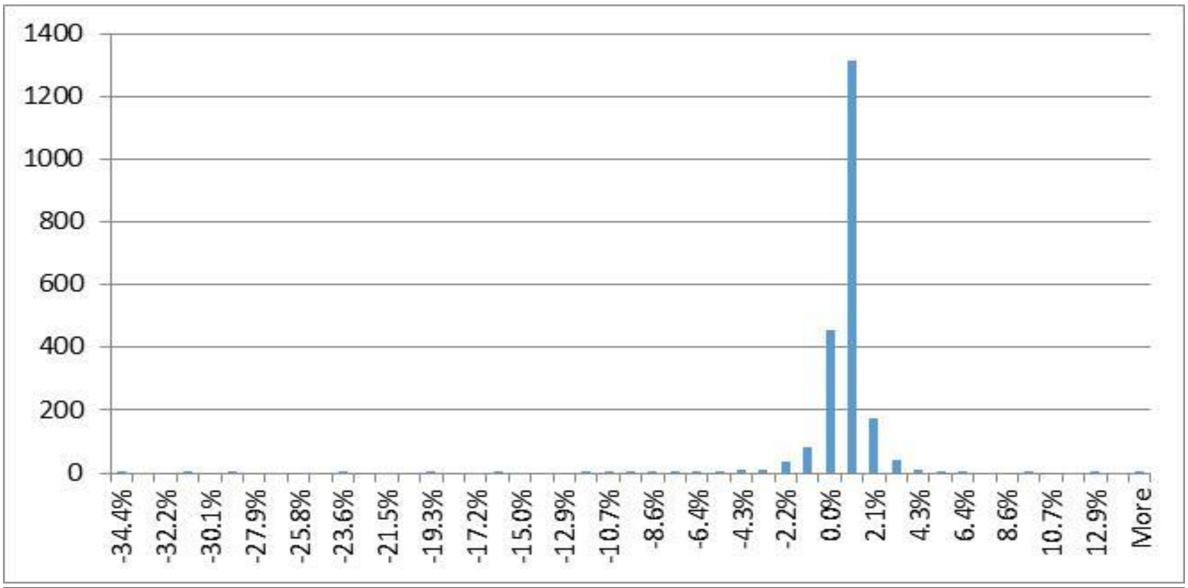

Figure: Daily Index Return Distribution

5.2.2 Description of strategy performance through the 2 downturns

i) 12th September 2008 - 5th December 2008

The aforementioned period was the most turbulent period in financial markets since September 11th 2001. The turbulence was sparked by the bankruptcy declaration of Lehman Brothers Inc on the 14th of September.

Equity markets around the world experienced a near free fall on the back of this news. The S&P 500 dropped 54 points or 4.5%, marking its biggest 1 day fall since September 11th, 2001. This was triggered by a steep fall in major financials making up the S&P 500 index.

In the context of our strategy, this 1 day move alone translated to an annualized realized volatility of approximately 77%, bumping up the cumulative realized volatility on the existing variance swap by 25%. The VIX or the fair variance swap rate also experienced a major 1 day re-adjustment. It closed 23% higher at 31.70 from 25.66 as of 12th September close.

Together, the rise in realized volatility and the rise in the VIX caused the MTM on the existing swap to drop by 8% in 1 day (this can measured by the 1 day 8% drop in the index) The maximum drawdown on the strategy also occurred during this period as detailed above.

ii) 3rdAugust 2011 - 26th S-September 2011 (August 2011 stock markets fall)

This was another extremely volatile period in global equity markets. Heightened volatility was triggered by fears about the spread of the European sovereign debt crisis, and by the downgrade of the US's AAA credit rating.

The sovereign debt crisis was a manifestation of the growing budget deficits of certain peripheral Euro zone countries since 2009. Due to inaction to contain these deficits, and in some cases, due to underreporting, the reaction to the news that the debt to GDP ratio had grown to 120% of GDP for Greece, for example, spooked markets. Concerns about sovereign credit and the resulting possibility of contagion created panic.

The other major event that sparked a volatility spike was the downgrade of America's credit rating from AAA to AA+ (Standard and Poor's) on 6 August 2011. This was the first time that the US had lost its AAA rating, since being conferred the same in 1941.

In the context of our strategy, these events eroded about 34% off value over this 2 month period, and about 29% over a 1 day period. The 1 day 7% drop in the S&P 500 following the U.S. credit downgrade alone translated to an annualized realized volatility of approximately 110%., bumping up the cumulative realized volatility on the existing variance swap by 43%. The VIX also experienced a major 1 day re-adjustment, closing 50% higher at 48.

5.2.3 Takeaways from 1 month strategy performance

The above evidence confirms certain defining features of volatility. Namely, that

a. Volatility can suddenly increase in spikes
b. Volatility can be observed to experience different regimes (crisis and post crisis)

What this evidence implies for our short variance swap strategy is that it would be expected to experience sharp losses over short periods, and make small gains over long periods.

The only caveat to the above statement is that the strategy performance is clearly sensitive to the back test period. The fact that our backtest period included the biggest financial crisis in history (since the Great Depression of the 1930s) and a historically unprecedented sovereign default crisis, is obviously a deterrent to its performance.

In the next section, I compare the performance of a 1 month short variance swap strategy with a 3 month short variance swap strategy.

5.3 Results for rolling 1 month and 3 month short variance strategies on S&P 500

Figure: Index performance for 1 month, 3 month strategies (Vega notional reset =1%)

Observations

i. On a pure net return basis, the rolling 1 month strategy clearly outperforms the rolling 3 month strategy, over the above historical period. However, it also exhibits higher volatility.

ii. Below summarizes the performance characteristics of the two strategies

	1 month strategy	3 month strategy
Cumulative Net Return	186%	19%
Annualized Net Return	13%	2%
Annualized Volatility	51%	27%
Sharpe Ratio	0.25	0.08
Maximum Drawdown	-89.0% (Aug-Dec 2008)	-69.7% (Aug-Nov 2008)

iii. The 1 month strategy is found to have a higher Sharpe Ratio and so is also the superior strategy on a risk adjusted basis.

Why does the 3 month underperform relative to the 1 month?

There could be a few possible explanations for this.

1. First, by construction the 1 month strategy accommodates for changes in market volatility faster than the 3 month strategy, because it resets on a monthly basis as opposed to a quarterly basis. That is, the monthly roll on the 1 month strategy means that it rolls into a new swap after updating the current fair market variance swap strike. This update means that the new variance swap is sold at a strike level that incorporates recent changes in volatility conditions and risk aversion. The quarterly roll on the 3 month means the update takes longer.

2. Second, it could be that the 1 month variance swap strike incorporates a higher degree of risk aversion than the 3 month variance swap strike. That is, if the implied volatility skew in the S&P 500 options market is higher for shorter maturity options than for longer maturity options, it would be reflected in the different variance swap strikes.

3. <u>Path Dependency of volatility</u>: Another argument can be made about how singling out these particular paths for these 2 strategies is biasing the true unknown path that these indices may take in the future. That is, if one were evaluating the relative investment potential of these 2 strategies, it would not make sense to use just 1 back test.

   And herein lies the point about dependency of these strategies on the path of volatility itself. That is, the payoff on any of these strategies depends crucially on the start and end dates of the back test and therefore by implication on the “roll dates” or dates on which these strategies roll.

   In the section after next, I try to estimate the average performance of these strategies over the same historical period but by running several parallel strategies of the same maturity, but each of which rolls on a different date.

5.4 Results for Dynamic Vega versus Constant Vega Notional Strategies

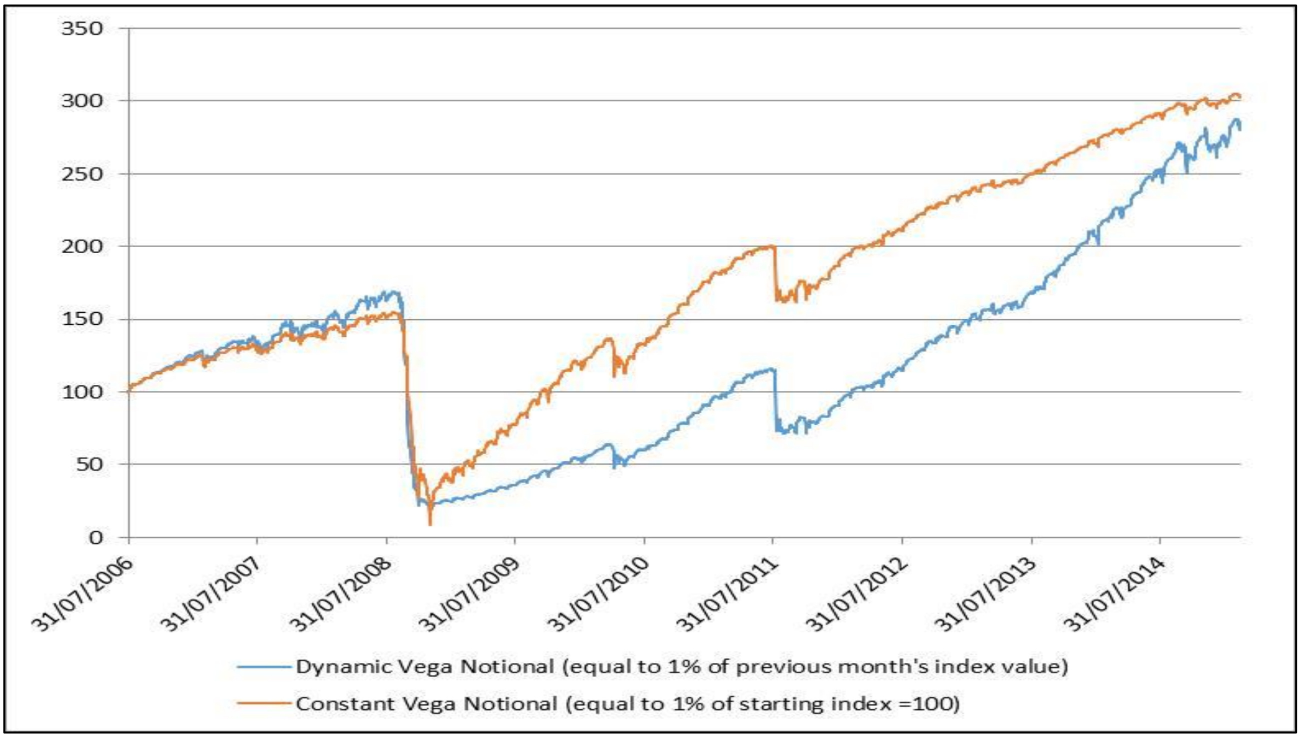

Figure: Index performance for 1 month strategy with different notional re-sets

Observations

1. The dynamic vega strategy displays two important features
   a. It takes longer to recover from a trough compared to a constant vega strategy (Jul 2008-Mar-2015) This can be explained by the fact that its vega notional reduces dynamically as the index loses value. When it starts to recover from its lowest point, it thus starts with a lower vega notional (lower base) compared to the constant vega strategy, and thus takes longer to recover its former value.
   b. It outperforms the constant vega strategy on uptrends. The only outperformance that is easily visible is the one between July 2006-July-2008. This can be explained by the fact that the vega notional of the dynamic strategy increases upwards following index gains, allowing the strategy to feed off this increased vega notional in the successive uptrend months.

5.5 Results for staggered strategies with different roll dates, Dynamic notional : 1 month strategy

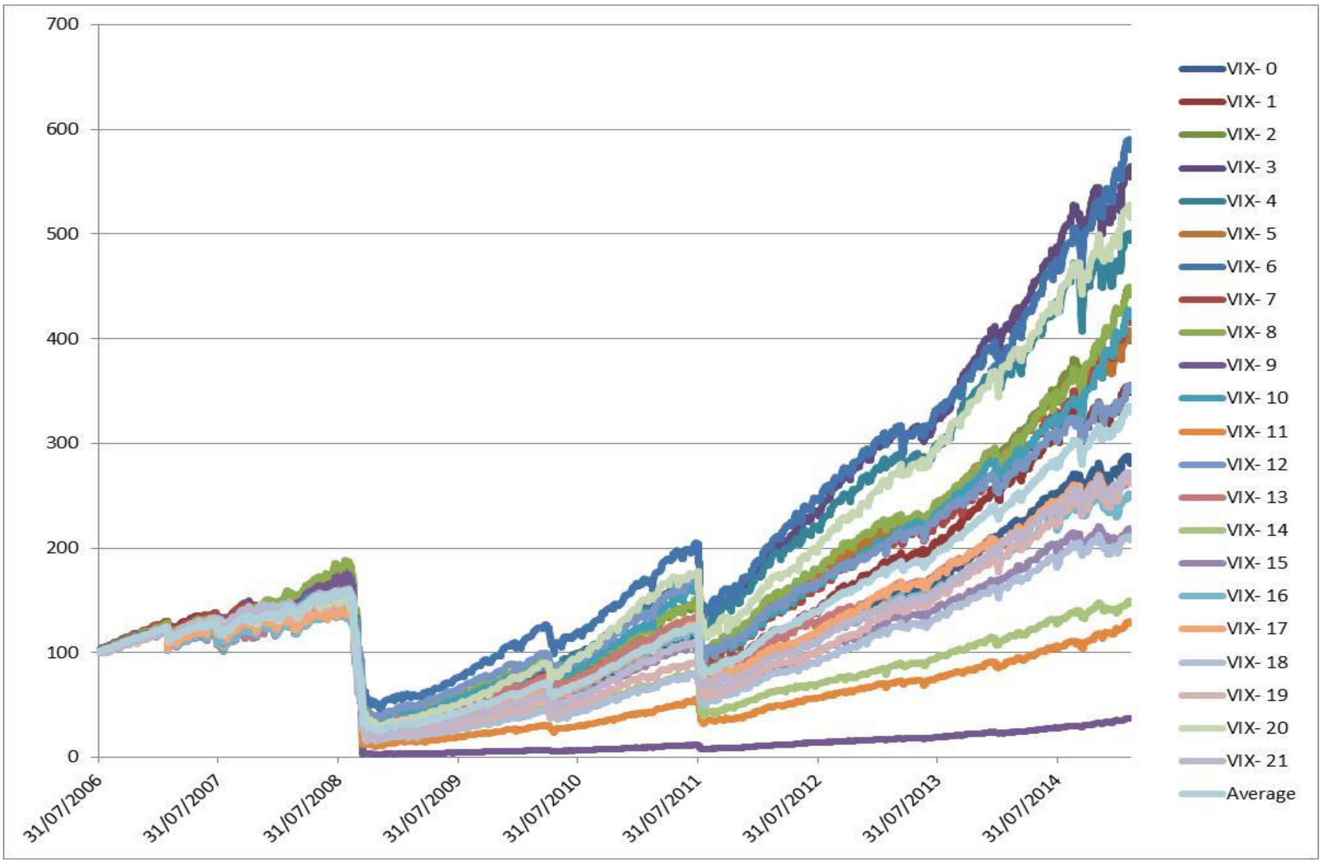

Figure: 22 one month rolling strategies, each with different roll dates (Vega notional=1% of index on roll date)

DESCRIPTION AND NOTATION

Here, all 22 strategies are identical, only distinguished by their start dates (which are staggered by 1 day) and consequently their roll dates (which by definition are 1 day apart). So for example, the first strategy VIX-0 starts on the 31st July and rolls every 21 trading days, the second strategy VIX-1 starts on the 1st of August and rolls every 21 trading days, and so and so forth.

That is,

VIX-0: rolling 1 month short variance strategy starting on day 0 (31st July 2007)

VIX-1: rolling 1 month short variance strategy starting on day 1 (1st Aug 2007)

VIX-2: rolling 1 month short variance strategy starting on day 14 (2nd Aug 2007)

…………

OBSERVATIONS

1. All strategies display a broadly similar pattern, witnessing a sharp correction in 2008 and 2011 (as discussed earlier)
2. The interesting result however is that some perform above average and some perform well below average. The 6 day staggered strategy for example displays the best performance ending up with a terminal index value of 585 or a cumulative net return of 484%, while the 9 day staggered strategy displays the worst performance ending up with a terminal index value of 37 or cumulative net return of -63%. The average of all these strategies ends up with a cumulative net return of 233%.

3. The divergence between the best and the worst performance is exaggerated due to the dynamic notional re-setting property of these strategies. All these strategies re-adjust their vega notional to 1% of the index level on the roll date. As such, per discussed in the previous section, these strategies get a kick to their performance on an uptrend but also take longer to recover from a trough.

5.6 Results for staggered strategies with different roll dates, Constant notional : 1 month strategy

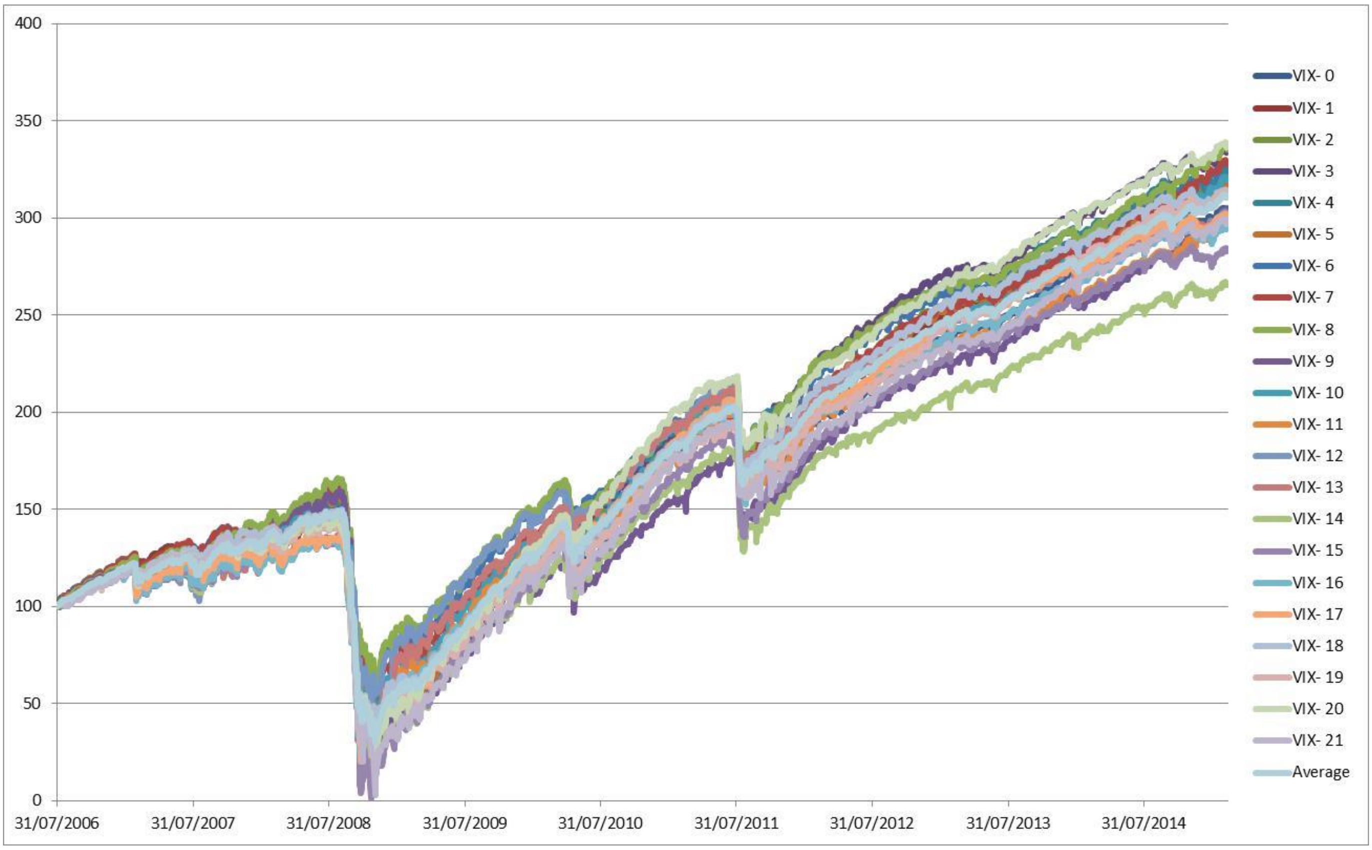

Figure: 22 one month rolling strategies, each with different roll dates, but constant vega notional=1% of 100

Notation and Description per above

Observations

1. The divergence in strategy performance using a constant vega notional is drastically reduced compared to the divergence with a dynamic vega notional. In fact we see a concentration or clustering of performance across these 21 strategies.
2. The average cumulative net return across strategies is 211%, with the range across strategies lying between 166% and 238%.

5.7 Results for staggered strategies with different roll dates, Dynamic notional : 3 month strategy

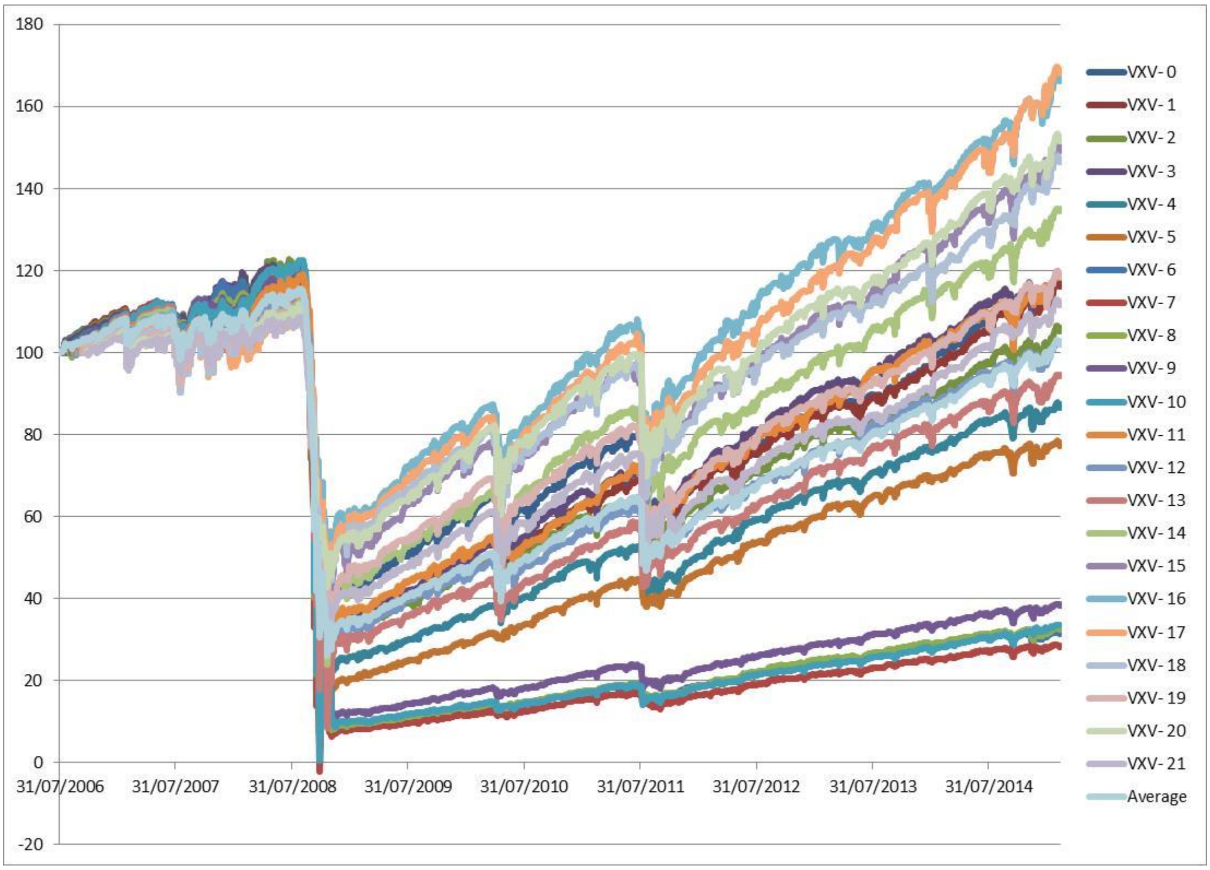

Figure: 22 three month rolling strategies, each with different roll dates (vega notional=1% of index on roll date)

Notation and Description per above, except now VXV refers to the 3 month strategy

Observations

1. As for the 1 month strategy with dynamic vega notional, the divergence between the best and worst performance is large but much smaller in relative terms. For example, the terminal index values range between 28 and 169 in this case,while in the 1 month case, they ranged between 36 and 584.
2. The average terminal index value is just above 100, yielding a net cumulative return of 2%

5.8 Results for staggered strategies with different roll dates, Constant notional: 3 month strategy

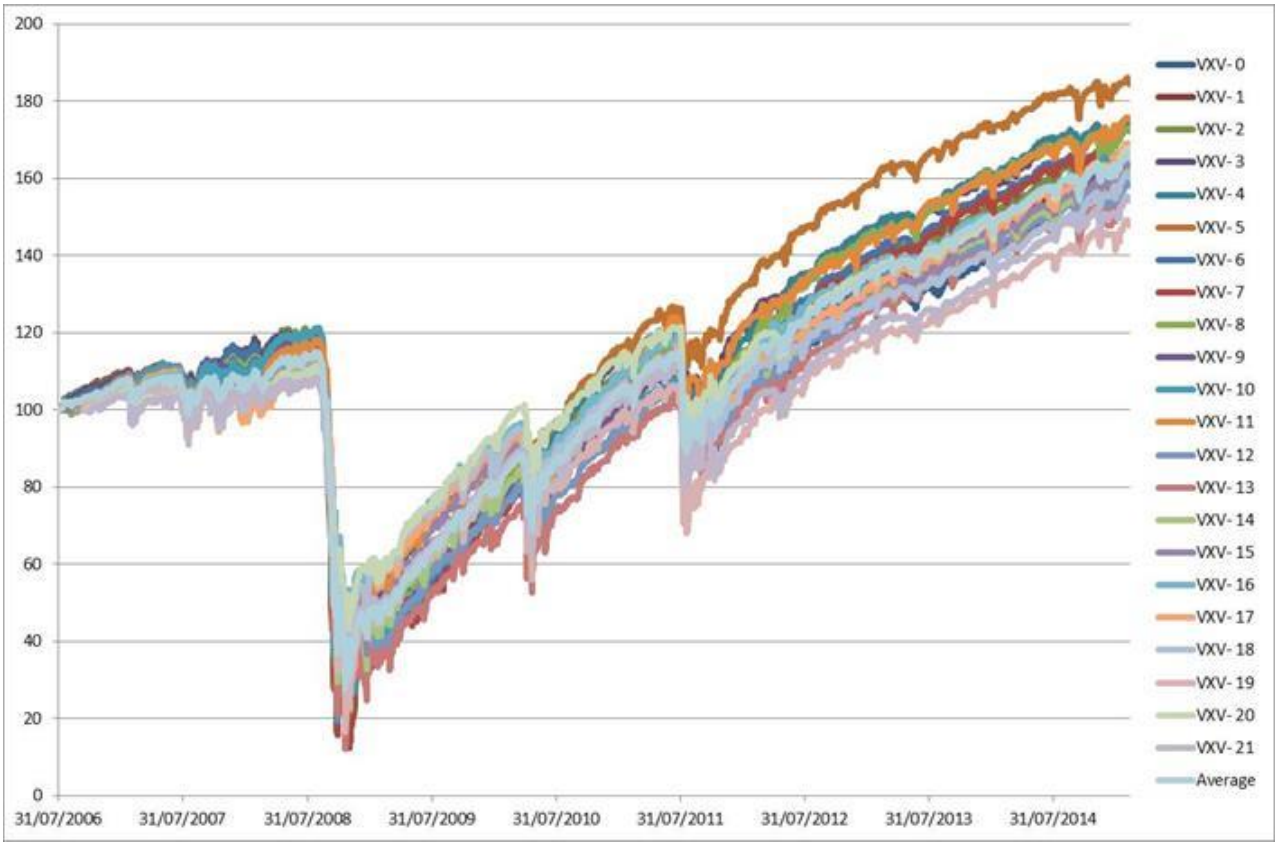

Figure : 22 three month rolling strategies, each with different roll dates (constant vega notional=1% of 100)

Notation and Description per above

RESULTS:

1. The divergence with a constant vega notional is again drastically reduced compared to the divergence with a dynamic vega notional, similar to our results for the 1 month strategy.
2. The average cumulative net return across strategies is 65%, lower than similar results for the 1 month strategy.

5.8.1 Takeaways from Staggered Strategies with different roll dates

Performance

The average 1 month performance is found to dominate the average 3 month performance across different notional readjustment scenarios, while the variability in terminal index values is found to be higher for the 1 month strategy relative to the 3 month.

Path Dependency

Above results highlight a very important property of variance. That returns on these short variance strategies are path dependent. While earlier, we talked about the path dependency of a delta hedged call option strategy on the path of the underlying, here we are no longer talking about the path dependency of variance strategies on the path of the underlying. Instead, we are talking about the path dependency of these variance strategies on the path of the underlying return variance.

Staggered strategies with different roll dates are by definition always exposed to i) two different realized variance paths, ii) two different initial swap rates. For example, the outperformance of a given staggered strategy could be explained by the fact that it rolled immediately after a major volatility spike. Or that it experienced a big volatility move only on the last day before it's roll, as a result of which the impact of the move was relatively small. The opposite explanation applies for staggered strategies that underperform.

6 CONCLUSION

The paper finds that variance risk premium on the S&P 500 over the tested historical period is priced.

It also finds that the observed negative variance risk premium can almost entirely be accounted for by the market risk premium.

Finally, the paper finds that systematic trading strategies aimed at monetizing this premium typically generate negative skewed PnL distributions, exhibit path dependency on the underlying return volatility, and tend to perform better for shorter maturities relative to longer maturities, on average.

7 BIBLIOGRAPHY

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8 APPENDIX

8.1 Description of Risk Factors used in Regressions

All historical risk factor data on MRP, SMB, HML and UMD is sourced from the following website

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

The Fama/French benchmark factors, MRP or Rm-Rf, SMB, and HML, are constructed from six size/book-to-market benchmark portfolios, where these 6 portfolios are re-adjusted every year at the end of June.

1. Rm-Rf, is a factor that equals the the excess return on the market. It is calculated as the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates).

2. SMB (Small Minus Big) is the average return on three small portfolios minus the average return on three big portfolios,

SMB = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth).

3. HML (High minus Low) is the average return on two value portfolios minus the average return on two growth portfolios,

HML = 1/2 (Small Value + Big Value)- 1/2 (Small Growth + Big Growth).

8.2 T-statistic results for the variance swap return series

For the overlapping monthly return series

Dependent Variable: VAR				
Method: Least Squares				
Date: 06/13/15 Time: 12:51				
Sample: 1 2546				
Included observations: 2546				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.008307	0.001179	-7.04382	0
R-squared	0	Mean dependent var		-0.00831
Adjusted R-squared	0	S.D. dependent var		0.059506
S.E. of regression	0.059506	Akaike info criterion		-2.8051
Sum squared resid	9.011668	Schwarz criterion		-2.8028
Log likelihood	3571.887	Hannan-Quinn criter.		-2.80426
Durbin-Watson stat	0.089623			

For the non-overlapping monthly return series

Dependent Variable: VAR				
Method: Least Squares				
Date: 06/13/15 Time: 12:52				
Sample: 1 122				
Included observations: 122				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.856207	0.655814	-1.30556	0.1942
R-squared	0	Mean dependent var		-0.85621
Adjusted R-squared	0	S.D. dependent var		7.243701
S.E. of regression	7.243701	Akaike info criterion		6.806305
Sum squared resid	6349.016	Schwarz criterion		6.829288
Log likelihood	-414.1846	Hannan-Quinn criter.		6.81564
Durbin-Watson stat	1.275099			

8.3 Code for Index Simulation

```
Sub main()

    Dim ROW_START As Integer

    Dim ROW_END As Integer

    Dim COL_END As Integer


    Dim Volatility As String

    Dim LIBOR As String

    Dim Duration As Integer

    Dim UpdateNotional As Boolean

    ROW_START = 2

    ROW_END = 2171

    COL_END = 7


    Columns(COL_END + 1).ClearContents

    Columns(COL_END + 2).ClearContents

    Columns(COL_END + 3).ClearContents

    Columns(COL_END + 4).ClearContents

    Scenario = 1

    UpdateNotional = True

    If Scenario = 1 Then

        'Scenario 1

        Volatility = "VIX"

        LIBOR = "LBR-1m"

        Duration = 21

      x = CalculateSwapIndex(ROW_START + 0, ROW_END, COL_END + 1, "VIX-0", Volatility, LIBOR, Duration, UpdateNotional)

      y = CalculateSwapIndex(ROW_START + 7, ROW_END, COL_END + 2, "VIX-7", Volatility, LIBOR, Duration, UpdateNotional)

      z = CalculateSwapIndex(ROW_START + 14, ROW_END, COL_END + 3, "VIX-14", Volatility, LIBOR, Duration, UpdateNotional)

    Else

        'Scenario 2

        Volatility = "VXV"

        LIBOR = "LBR-3m"

        Duration = 63
```

```
x = CalculateSwapIndex(ROW_START + 0, ROW_END, COL_END + 1, "VXV-0", Volatility, LIBOR, Duration, UpdateNotional)

y = CalculateSwapIndex(ROW_START + 21, ROW_END, COL_END + 2, "VXV-21", Volatility, LIBOR, Duration, UpdateNotional)

z = CalculateSwapIndex(ROW_START + 42, ROW_END, COL_END + 3, "VXV-42", Volatility, LIBOR, Duration, UpdateNotional)

End If

'Calculate Average of above 3

        Cells(1, COL_END + 4) = "Average"

        For Row = ROW_START To ROW_END

            x = Cells(Row, COL_END + 1)

            y = Cells(Row, COL_END + 2)

            Z = Cells(Row, COL_END + 3)


            NumOfEmpties = 0

            If IsEmpty(x) Then

                NumOfEmpties = NumOfEmpties + 1

            End If

            If IsEmpty(y) Then

                NumOfEmpties = NumOfEmpties + 1

            End If

            If IsEmpty(Z) Then

                NumOfEmpties = NumOfEmpties + 1

            End If


            If IsEmpty(x) And IsEmpty(y) And IsEmpty(Z) Then

                Average = 0

            Else

                Average = (x + y + Z) / (3 - NumOfEmpties)

            End If


            Cells(Row, COL_END + 4) = Average

        Next


End Sub


'Row Start helps allow for a Staggered Start

'Col Output info is to indicate where the output should be printed
```

```
'Volatility Index indicates whether to use VIX or VXV

'Duration indicates how long until we reset strategy

Function CalculateSwapIndex (ROW_START As Integer, ROW_END As Integer, COL_OUTPUT_NUM As Integer, COL_OUTPUT_NAME As String, VolatilityIndex As String, _LIBOR As String, Duration As Integer, UpdateNotional As Boolean)

    Worksheets("1m VaR swap strategy").Select

    Application.ScreenUpdating = False

    'Constants

    alpha = 0.01

    indexLast = 100

    indexCurrent = 100


    'Internal Params

    daycount = 0

    CumVar = 0

    DailyVar = 0


    COL_IDX = COL_OUTPUT_NUM

    COL_DAY = 1

    COL_DTE = 2

    COL_SP5 = 3


    COL_VIX = 4

    COL_VXV = 5

    If VolatilityIndex = "VIX" Then

        COL_VOLATILITY = COL_VIX

    ElseIf VolatilityIndex = "VXV" Then

        COL_VOLATILITY = COL_VXV

    Else

        MsgBox ("Invalid VIX entered, select a valid VIX")

    End If

    VOLATILITY_FIRST_VALUE = Cells(ROW_START, COL_VOLATILITY)

      If LIBOR = "LBR-1m" Then

        COL_LBR = 6

    ElseIf LIBOR = "LBR-3m" Then

        COL_LBR = 7

    Else
```

```
  MsgBox ("Invalid LIBOR entered, select a valid LIBOR")
End If
    Cells(1, COL_OUTPUT_NUM) = COL_OUTPUT_NAME

'Calculated paramters from sheet
VolatilityLast = Cells(ROW_START, COL_VOLATILITY)
VarNotional = -((indexLast * alpha) / (2 * VolatilityLast))

For Row = ROW_START To ROW_END

    If (daycount = 0) Then
        daycount = daycount + 1
        Cells(Row, COL_IDX) = indexCurrent
        Cells(Row, COL_IDX).Font.Bold = True
        Cells(Row, COL_DAY) = daycount
    Else

        DaysLeft = Duration - daycount
        PercTimeRemain = DaysLeft / Duration
        PercTimeElapsed = daycount / Duration

        SP500Yest = Cells(Row - 1, COL_SP5)
        SP500Today = Cells(Row, COL_SP5)
        LBR = Cells(Row, COL_LBR)

        If Not SP500Today = "" Then
            Volatility = Cells(Row, COL_VOLATILITY)
            DailyVar = Math.Log(SP500Today / SP500Yest) ^ 2
            CumVar = CumVar + DailyVar

            RealVar = Math.Sqr(CumVar * 252 / daycount) * 100

            'Interpolating between VIX and VXV
            If (Duration = 21) Then
                ImplVar = Sqr(PercTimeRemain * Cells(Row, COL_VOLATILITY) ^ 2)
```

```
        ElseIf (Duration = 63) Then
            VIX = Cells(Row, COL_VIX)
            VXV = Cells(Row, COL_VXV)

            If (DaysLeft <= 21) Then
                ImplVar = Sqr(DaysLeft * (VIX ^ 2) / 21)
            Else
                ImplVar = Sqr(VIX ^ 2 + ((VXV ^ 2 - VIX ^ 2) / 42) * (DaysLeft - 21))
            End If
        End If

DiscountFactor = 1 / (1 + (DaysLeft / 252) * (LBR * 0.01))

        ChangeInVar = (RealVar ^ 2) * PercTimeElapsed + (ImplVar ^ 2) * PercTimeRemain - (VolatilityLast ^ 2)
        indexCurrent = indexLast + VarNotional * DiscountFactor * ChangeInVar

        If (daycount < Duration) Then
            daycount = daycount + 1
        ElseIf (daycount Mod Duration = 0) Then
            VolatilityLast = (Cells(Row, COL_VOLATILITY))
            If UpdateNotional Then
                VarNotional = (-(indexCurrent * alpha) / (2 * VolatilityLast))
            Else
                VarNotional = -((100 * alpha) / (2 * VolatilityLast))
            End If
            indexLast = indexCurrent
            daycount = 1
            CumVar = 0
        End If

        Cells(Row, COL_IDX) = indexCurrent
        Cells(Row, COL_IDX).Font.Bold = False
        Cells(Row, COL_DAY) = daycount

    End If
```

```
        End If
    Next

    Debug.Print (Round(Cells(ROW_START + Duration * 1, COL_IDX), 2))
    Debug.Print (Round(Cells(ROW_START + Duration * 2, COL_IDX), 2))
    Debug.Print (Round(Cells(ROW_START + Duration * 3, COL_IDX), 2))
    Debug.Print (Round(Cells(ROW_START + Duration * 4, COL_IDX), 2))

End Function
```