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Section 002

1.

a. P = “The Good Place”
Q = Have a rampant gambling addiction
R = Likes bacon
N = Enjoys rolling in the grass

$\forall x\ (P(x) \lor Q(x))$ // premise i
$\forall x\ (\neg Q(x) \lor R(x))$ // premise ii
$\forall x\ (N(x) \to \neg R(x))$ // premise iii
$\exists x\ (\neg P(x))$ // premise iv

$\therefore \exists x\ (\neg N(x))$ // conclusion

b.

1.	$\forall x\ (P(x) \lor Q(x))$	Premise
2.	$\forall x\ (\neg Q(x) \lor R(x))$	Premise
3.	$\forall x\ (N(x) \to \neg R(x))$	Premise
4.	$\exists x\ (\neg P(x))$	Premise
5.	$P(a) \lor Q(a)$	Universal instantiation, for (a) being arbitrary
6.	$\neg Q(a) \lor R(a)$	Universal instantiation
7.	$N(a) \to \neg R(a)$	Universal instantiation
8.	$\neg(P(c))$	Existential generalization, for (c) being some element
9.	$Q(a)$	Disjunctive syllogism (using 5 and 8)
10.	$Q(a) \to R(a)$	RBI (using 6)

11.	R(a)	Modus ponens (using 9 and 10)
12.	R(a) → ¬N(a)	Contraposition (using 7)
13.	¬N(a)	Modus ponens (using 11 and 12)
∴	∃x (¬N(c))	Existential generalization

2.

a. Direct proof
ax +by = e
cx +dy = f
ad - bc = bc - ad

To prove bc - ad ≠ 0, multiply each equation by c and d

Since ax + by = e doesn't have c so you multiply by c
Since cx + dy = f doesn't have a so you multiply by a
c(ax +by = e) = acx + bcy = ce
a(cx + dy = f) = acx + ady = af
Therefore, x and y are real number

Subtraction:

acx + bcy = ce
\- acx + ady = af
\----------------------------------
bcy - ady = ce - af
(bc - ad)y = ce - af

$$y = \frac{ce - af}{bc - ad}$$, bc - ad ≠ 0

Plug in y to the equation ax + by = e

$$ax + b\left(\frac{ce - af}{bc - ad}\right) = e$$

$$ax + \frac{bce - abf}{bc - ad} = e$$

$$ax = e - \frac{bce - abf}{bc - ad}$$
$$ax = \frac{e(bc - ad)}{bc - ad} - \frac{bce - abf}{bc - ad}$$
$$ax = \frac{bce - ade}{bc - ad} - \frac{bce - abf}{bc - ad}$$
$$ax = \frac{bce - ade - bce + abf}{bc - ad}$$
$$ax = \frac{abf - ade}{bc - ad}$$
$$ax = a\,(\frac{bf - de}{bc - ad})$$
$$x = \frac{bf - de}{bc - ad}$$

Since x is $\frac{bf - de}{bc - ad}$ and y is $\frac{ce - af}{bc - ad}$, bc - ad cannot be 0 and x and y will never be 0 in the denominator. Therefore, the two equations can be solved with real numbers of x and y.

b. Every positive integer can be expressed as the sum of two perfect squares.

If x and y are both positive integers
$x^2 + y^2 = \forall z$

$\forall$z is 3 is when $x^2 + y^2$ cannot be true because it cannot prove the integers sum of two perfect squares. $x^2 + y^2 = 3$. So x^2 and y^2 have to be less than 3 to prove the statement.
Since x^2 and y^2 cannot be 3, x or y has to be 0 and 1 in order to become true. Not when $x^2 = 2$

When $x^2 = 0$, y^2 must be 3, $0 + y^2 = 3$ but from above that cannot happen.
Vice versa,
When $y^2 = 0$, x^2 must be 3, $x^2 + 0 = 3$ but from above again that cannot happen.

Therefore, every positive integer can be expressed as the sum of two perfect squares statement is false.

c. Let a and b be integers. If a and b are expressible in the form 4n +1 where n is an integer, then a*b is also expressible in that form.

Direct proof
$a = 4n_1 + 1$
$b = 4n_2 + 1$
$a \cdot b = (4n_1 + 1)(4n_2 + 1)$
$= 4^2 n_1 n_2 + 4n_1 + 4n_2 + 1$
$= 4\,(4n_1 n_2 + n_1 + n_2) + 1$
$a \cdot b = 4\,(4n_1 n_2 + n_1 + n_2) + 1$
$4n + 1 = 4\,(4n_1 n_2 + n_1 + n_2) + 1$
$n = 4n_1 n_2 + n_1 + n_2$

n_1 and n_2 are integers and when $n = 4n_1 n_2 + n_1 + n_2$, n is also an integer. Therefore, $a \cdot b$ can be expressed in some kind of form of $4n + 1$.

3.

a. Let a and b integers and define $c = a \cdot b + a + b$. Prove that c is even and only if a and b are both even.

Proof by case
$c = a \cdot b + a + b$
Even = 2x, Odd = 2x + 1

Case 1:
Let's say a is even and b is odd
a = 2x, b = 2y + 1 for some x and y

c = (2x)(2y + 1) + 2x + (2y + 1)
c = 2 [x(2y + 1) + x + y] + 1
∴ c = odd

Case 2:
Let's say a is odd and b is even.
a = 2x + 1, b = 2y for some x and y

c = (2x + 1)2y + (2x + 1) + 2y
c = 2 [y(2x + 1) + x + y] + 1
∴ c = odd

Case 3:

Let's say a and b are both odd
$a = 2x + 1$, $b = 2y + 1$ for some x and y

$c = (2x + 1)(2y + 1) + 2x + 1 + 2y + 1$
$c = 4xy + 2x + 2y + 1 + 2x + 2y + 2$
$c = 2 [(2xy + x + y + x + y + 1)] + 1$
$\therefore c = \text{odd}$

Case 4:
Let's say a and b are both even
$a = 2x$, $b = 2y$ for some x and y

$c = 2x(2y) + 2x + 2y$
$c = 2 [2xy + x + y]$
$\therefore c = \text{even}$

Because c turns out even only when both a and b are even, c is even and only if a and b are both even statement is true

b. Suppose n is a positive integer, 5 divides 4n, then 5 divides n
5 divides 4n
    $4n = 5m, m \in n$
    We need to prove that if n is even, m is also even
    $2(2n) = 5m,$
    If m is even, let's say $m = 2k$
    $2(2n) = 5(2k)$
    $2n = 5k$

    We say $k = 2u$ and run the process again
    $2n = 5(2u)$
    $n = 5u$
    Because $n = 5u$, n is a multiple of 5
    $\therefore$ 5 divides n

4.

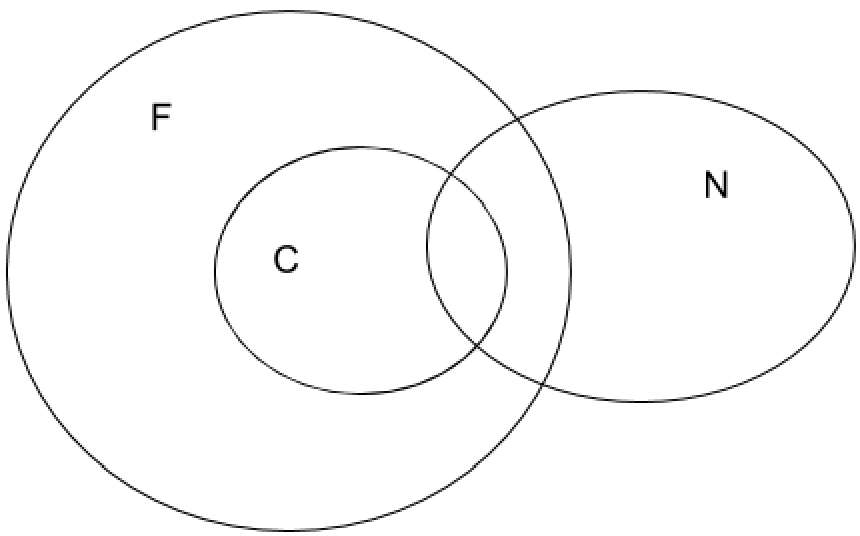
a. $P(a)$, $a = \{ \emptyset, 1, \{ \text{cat}, \text{dog}, 7\}\}$

Find the set of all subsets of set a

P(a) = { ∅, {∅}, {1} {cat, dog, 7}, { ∅, 1}} ,{{ ∅, {cat, dog, 7}}, {1, {cat, dog, 7}}, { ∅, 1, {cat, dog, 7}}

∴ P(a) = { ∅, 1, {cat, dog, 7}}

b. C = set of All CU faculty
   N = set of all people contains the letter "o"
   F = set of all people who works at any University.



C ⊆ F Because C cannot be F but F can be C

c. 1 ∈ {1, {1}} because 1 is an element of {1, {1}}
   1 ⊆ {1, {1}} because 1 is not a set itself so cannot be a subset

5

a. $| A \cup B \cup C | = | A | + | B | + | C | - | A \cap B | - | A \cap C | - | B \cap C | + | A \cap B \cap C |$

Let $D = B \cup C$

$| A \cup B \cup C | = | A \cup D |$

$= | A | + | D | - | A \cap D |$

$= | A | + | B \cup C | - | A \cap (B \cup C) |$

Using Distributive law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= | A | + | B \cup C | - | (A \cap B) \cup (A \cap C) |$

$= | A | + (| B | + | C | - | B \cup C |) - (| A \cap B | + | A \cap C | - | (A \cap B) \cup (A \cap C) |)$

Using Idempotent law: $(A \cap B) \cup (A \cap C) = (A \cap B \cap C)$
$= |A| + |B| + |C| - |B \cup C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

b. $|B| = 13$, $|E| = 11$, $|N| = 22$, $|B \cap E| = 5$, $|B \cap N| = 10$, $|E \cap N| = 8$, $|B \cap E \cap N| = 4$

Number of surfers wiped
$|B \cup E \cup N| = 13 + 11 + 22 - 5 - 10 - 8 + 4 = 27$

Number of surfers not wiped (Answer)
$40 - 27 = 13$

Therefore, 13 people did not get wiped out