

Instructions: This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS**. Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences**. **Show and justify all work to receive full credit.**

Standard 19. We define the Multiplicative Rod Cutting problem as follows.

Input: A list of weights $w_1, \dots, w_n \geq 1$ for rods of length $1, \dots, n$, respectively.

Goal: Divide a rod of length n into pieces of lengths ℓ_1, \dots, ℓ_k (k can vary) to maximize the total value $\prod_{i=1}^k w_{\ell_i}$. You may assume that a rod of length 0 has weight 1.

Write down the recurrence for the optimal solution. Justify your answer.

In this particular problem, the sub problems are overlapping with each other due to the recursive algorithm revisiting the same problem multiple times. Let's say the length of the rod is 4, so there will be 8 (2^3) ways to cut the rod, and not cutting the rod at all is included. So, every inch has two options which are to cut it or not.

To add on, with the length 4, there will be 4 ways to cut it in half (4+0, 3+1, 2+2, 1+3). If the rod length is 3, there will be 3 ways to cut it in half. (3+0, 2+1, 1+2). If the rod length is 2, there will be 2 ways to cut it in half. (2 + 0, 1 + 1). If the rod length is 1, there will be 1 ways to cut it in half. (1 + 0) As you can see, when length is 3, this is also included from the sub problem when the length is 4 and continue on. Therefore, the recurrence for this optimal solution is 2^n