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CSCI 3104, Algorithms **Profs. Chen & Grochow**
Problem Set 11 – Due Wed April 29 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

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1. Indiana Jones is gathering n artifacts from a tomb, which is about to crumble and needs to fit them into 5 cases. Each case can carry up to W kilograms, where W is fixed. Suppose the weight of artifact i is the positive integer w_i. Indiana Jones needs to decide if he is able to pack all the artifacts. We formalize the Indiana Jones decision problem as follows.

   - Instance: The weights of our n items, $w_1, \ldots, w_n > 0$.
   - Decision: Is there a way to place the n items into different cases, such that each case is carrying weight at most W?

   Show that Indiana Jones $\in$ NP.

   The definition of NP is the set of decision problems such that if the answer is YES, then there exists a witness (succinctly: a proof) of that fact that can be checked in polynomial time. (From Week 14.pdf). To apply that with the given problem, in order to show that Indiana Jones $\in$ NP, we need to prove that the solution is in polynomial size and there's a possibility that a given solution can be checked in polynomial time.

   Let's say that Indiana Jones needs to fit the artifacts into 5 cases: I, J, K, L, M and each of them can fill up to i, j, k, l, m number of artifacts. In order to check that the solution is correct, we need to sum up the total weight of each cases, and then compare with W.

   To begin, let's calculate the total weight of the case I. Since adding two number(weight) of the artifacts takes just 1 step, adding i number of weights takes $(i-1)$ steps. Next, we need to compare the total number with the limit W which also takes 1 step. So we can conclude that it will take $(i-1)+1 = i$ steps to conclude the solution for the case I with i number of artifacts.

   For the cases J, K, L, and M, these also applies with the rest of the calculations with j, k, l, and m number of steps for the solution.

   In conclusion, the total numbers of all the steps takes $i+j+k+l+m = n$ which is $O(n)$.
   Furthermore, the solution $O(n)$ to the problem is in polynomial size due to the use of n items for different cases.

   Therefore, Indiana Jones $\in$ NP.

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2. A student has a decision problem L, which they know belongs to NP. This student wishes to show that L is NP-Complete. They attempt to do so by constructing a polynomial time reduction from L to SAT, a known NP-Complete problem. That is, the student attempts to show that $L \leq_p$ SAT. Determine if this student's approach is correct and justify your answer.

   After reading the problem, the student needs to present that $L \geq_p$ SAT instead of $L \leq_p$ SAT. Therefore, the student's approach is not correct

   To determine a decision problem L is NP-Complete, this has to mean that every problem in NP is able reduce to L in polynomial time, and L is in NP.

   From the given problem, we already know that the decision problem L belongs to NP. Now, in order to prove that L is NP-Complete, we need to show that every problem in NP is able to reduce to L in polynomial time. Furthermore, with the given SAT, a known NP-Complete problem, this gives us information that every problem in NP is able to reduce to SAT in polynomial time. So to prove that, we need to use the transitivity property of reduction to show that SAT can be reduced to L. However, the student attempts to prove that L can be reduced to SAT which is wrong.

   COLLABORATED WITH:
   RUIJIANG MA

   Thank you for the semester!