Daniel Kim
Section 002

1

a. $P * (Q \cap R) \subseteq (P * Q) \cap (P * R)$ ** First

Let's say $(x_1, y_1) \varepsilon P * (Q \cap R)$

$x_1 \varepsilon P,\ y_1 \varepsilon Q \cap R$ // Cartesian

$x_1 \varepsilon P,\ y_1 \varepsilon Q$ and $y_1 \varepsilon R$ // Intersection

$\{x_1 \varepsilon P,\ y \varepsilon Q\}$ and $\{x_1 \varepsilon P,\ y \varepsilon R\}$ // Idempotent

$(x_1, y_1) \varepsilon (P * Q)$ and $(x_1, y_1) \varepsilon (P * R)$ // Cartesian

$(x_1, y_1) \varepsilon (P * Q) \cap (P * R)$ // Intersection

$P * (Q \cap R) \subseteq (P * Q) \cap (P * R)$

To prove that $(P * Q) \cap (P * R) \subseteq P * (Q \cap R)$

Let $(x_2, y_2) \varepsilon (P * Q)$ and $(x_2, y_2)(P * R)$

$(x_2, y_2) \varepsilon (P * Q)$ and $(x_2, y_2)(P * R)$ // Cartesian Product

$x_2 \varepsilon P,\ y_2 \varepsilon Q$ and $x_2 \varepsilon P,\ y_2 \varepsilon R$ // Idempotent

$x_2 \varepsilon P,\ y_2 \varepsilon Q$ and $y_2 \varepsilon R$ // Intersection

$x_2 \varepsilon P,\ y_2 \varepsilon Q \cap R$ // Cartesian

$(x_2, y_2) \varepsilon P * (Q \cap R)$ // Cartesian Product

$(P * Q) \cap (P * R) \subseteq P * (Q \cap R)$

Therefore,

$P * (Q \cap R) = (P * Q) \cap (P * R)$

b. Prove that $P * (Q \cap R) = (P * Q) \cap (P * R)$ - set builder notation

$= P * (Q \cap R) = \{ (x, y) \mid (x\ \varepsilon P)\ and\ (y\ \varepsilon (Q \cap R)) \}$

// Cartesian Product

$= \{ (x, y) \mid (x\ \varepsilon P)\ and\ (y\ \varepsilon\ Q)\ and\ (y\ \varepsilon R) \}$

// Intersection

$= \{ (x, y) \mid (x\ \varepsilon P)\ and\ (y\ \varepsilon\ Q)\ and\ (x\ \varepsilon P)\ and\ (y\ \varepsilon R) \}$

// Idempotent

$= \{ (x, y) \mid (x, y)\ \varepsilon (P * Q)\ and\ (x, y)\ \varepsilon (P * R) \}$

// Cartesian Product

$= \{ (x, y) \mid (P * Q) \cap (P * R) \}$

// Intersection

$= (P * Q) \cap (P * R)$

2

a. ***A*** - ***B***

i. Finite:

Let's say that both ***A*** and ***B*** are real numbers and both be the exact same set. If we say ***A*** - ***B***, it will be empty of size zero which is finite

ii. Countably infinite

Let's say ***A*** is a set of real numbers and ***B*** is real number minus negative integers

(***B*** = R - $\mathbb{Z}$) which is a set of all numbers but no negative integers. So ***A*** - ***B*** = $\mathbb{Z}^-$ is countably infinite

iii. Uncountable infinite

Let's say A is a set of positive real numbers and ***B*** be the negative real numbers. So ***A*** - ***B*** = ***A*** is uncountable

a. Let's say the real numbers in [3,4] are countable
Then we go for an enumeration of the form
$A_1 = 3 . a_{11}\ a_{12}\ a_{13...}$ and $A_2 = 3 . a_{21}\ a_{22}\ a_{23}....$

and so on of all the real numbers in [3,4] where $a_{ij} \varepsilon \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
We say $\bar{a}_{kk} = 9 - a_{kk}$ and $A_3 = 3.\bar{a}_{11}\bar{a}_{12}\bar{a}_{13}....$
$A_3 = 3.\bar{a}_{11}\bar{a}_{12}\bar{a}_{13}....$ is different from $A_1 = 3.a_{11}a_{12}a_{13...}$ in a_{11} because
a_{kk} does not equal to $\bar{a}_{kk} = 9 - \bar{a}_{kk}$ otherwise a_{kk} is 9/2
We can even say that the general form $A_k = 3.a_{k1}a_{k2}a_{k3}.....$ differs from A_2
Because of these reasons we can say the real numbers in [3,4] does not include in the enumeration
Therefore, the real numbers in [3,4] are uncountable

b. Let's say that the set of irrationals in [3,4] are countable. We already know that the rationals are countable and because of irrationals and rationals are both countable, the union of those two sets are countable also. In addition, the union of two countable sets is countable which does not prove that the real numbers are countable. Therefore, we can say that the set of irrationals in [3,4] are uncountable

3

a. Given $a_n = 2 * a_{n-1} - 2$ and $a_0 = -1$
= $2 (2 * a_{n-2} - 2) - 2$
= $2^2 * a_{-2} - 2^2 - 2$ * Keep adding $2 * a_{n-1} - 2$
= $2^2 (2 * a_{n-3} - 2) - 2^2 - 2$
= $2^3 * a_{n-3} - 2^3 - 2^2 - 2$
= $2^3 (2 * a_{n-4} - 2) - 2^3 - 2^2 - 2$
…..
= $2^n * a_0 - 2^n - 2^{n-1} - 2^{n-2} - ..... - 2^2 - 2$
So when $a_0 = -1$,
= $2^n * - 1 - 2^n - 2^{n-1} - ...... 2^3 - 2^2 - 2$
= $-2 (1 + 2 + 2^2 + 2^3 + 2^{n-2}) - 2^{n+1}$
= $-2 * -(1-2)^{n-1} - 2^{n+1}$
= $-2^{n+1} - 2^n + 2$

b) $a_n = (n + 2) a_{n-1}$ and $a_0 = 3$

$\frac{a_n}{a_{n-1}} = n + 2$

$\frac{a_n}{a_{n-1}} * \frac{a_{n-1}}{a_{n-2}} * \ldots\ldots \frac{a_2}{a_1} * \frac{a_1}{a_0}$

$= (n + 2) * (n + 1) * n * \ldots\ldots * 3 = \frac{(n+2)!}{2}$

Therefore,

$a_n = \frac{3}{2}(n + 2)!$

c) $a_n = 5(-1)^n = n + 2$

Calculate a_{n-1} and a_{n-2} and substitute the values in

$a_n = a_{n-1} + 2a_{n-2} + 2a_n - 9$

a_{n-1} becomes $5(-1)^{n-1} - (n - 1) + 2$

a_{n-2} becomes $5(-1)^{n-2} - (n - 2) + 2$

Substituting the values in a_n

$5(-1)^{n-1} - (n - 1) + 2 + 2(5(-1)^{n-2} - (n - 2) + 2) + 2n - 9$

$= 5(-1)^{n-1} - n + 3 + 10(-1)^{n-2} - 2n + 8 + 2n - 9$

$= 5(-1)^{n-1} + 10(-1)^{n-2} - n + 2$

$= 5(-1)^{n-1} - n + 2$

$= a_n$

4

a. $f : Z * Z \rightarrow Z,\ f(m, n) = 2m - n$

Let , $K \varepsilon Z$

$K = 2(K + 9) - (K - 2P)\ \ P \varepsilon Z$

$m = K + P,\ n = K + 2P$

Therefore, f is onto because we can say any $K \varepsilon Z$ in the form of $2m - n$

b. $f : Z * Z \rightarrow Z, f(m, n) = m^2 - n^2$

Let $K \varepsilon Z$, then,

$K = [\frac{K+2}{2}]^2 - [\frac{K-1}{2}]^2 = m^2 - n^2$

Therefore, f is onto because we can say any $K \varepsilon Z$ in the form of $m^2 - n^2$

c. C = the set of all residents in colorado

$f : C \rightarrow Z$

$f(x) = x \quad \forall x \varepsilon C$

Z = association with each person

Because it is one to one as each and every person is associated to every specific number so it is an identity map. Also, this map is not onto because there might be some number of set Z not involved to any person for having set Z infinite and set C finite

d. C = the set of all residents in colorado

$f : C \rightarrow Z$

$f(x) = x^2 \quad \forall x \varepsilon C$

Each person (with a number) is associated with the square of the number of person. This is one to one because every person is associated with one specific number. For example, 2nd person is involved with the unique number 4