1. (a). Here $n = 21 + 5 - 1 = 25$, $r = 5$

$$C(25,5) = \frac{25!}{(25-5)!\,5!}$$

$$= \frac{25!}{20!\,5!}$$

(b) Since now there are only $21 - 5 = 16$ cookies available to distribute, $n = 16$, $r = 5$.

$$C(16+5-1, 5)$$

$$= \frac{20!}{15!\,5!}$$

$$=$$

2. (a) The number of strings of Length $12 = 26^{12}$, Since there are 12 characters, and each character has 26 possibilities.

(b) If NINJA is at the start of the string. "NINJAXXXXXXX", since there are 7 available spaces, there are 26^7 possibilities.

Then, there are 8 possible starting positions for the word "NINJA",

there are $8 \cdot 26^8$ possibilities.

Lastly, check for repeated strings, there are 6.

Therefore, there are $8 \cdot 26^8 - 6$ strings.

(c). For the strings that contain "Turtles", there are $6 \cdot 26^5$ possibilities.
Since there are exactly two strings "NINJATURTLES" and "TURILESNINJA", we need to subtract 2.
Thus, the answer is

$$26^{12} - (8 \cdot 26^8 - 6 + 6 \cdot 26^5) + 2.$$

3. (a). The total outcomes is: $6^5 = 7776$

The total unique outcoms is: $6 \times 5 \times 4 \times 3 \times 2 = 720$

The probability $= \frac{720}{6^5} = \frac{5}{54}$

(b) case i : $\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{n}$

case ii : $\boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{n}$

case iii : $\boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{n}$

In each case, n has 6 possibilities.
However, we are double counting when
① $n = 5$ in case i & $n = 1$ in case ii
② $n = 6$ in case ii & $n = 2$ in case iii.
Hence, the number of total small straight is

$6 + 6 + 6 - 2 = 16$

The probability $= \frac{16}{6^5} = \frac{1}{486}$

(c).
case i : $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{n}$

case ii : $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{n}$

case iii : $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$ $\boxed{n}$

For the dices of above small stright in each case to be unique, n has two possibilities. Thus, $2 \times 3 = 6$.
The number of total small straight is 6,
and the number of total unique outcome is 720,
Thus, the probability of me have rolled a small straight is $\frac{6}{720} = \frac{1}{120}$.

4. (a). Thm: $(x+y)^{100} = \sum_{k=0}^{100} \binom{100}{0} x^{100-k} y^k$

The cofficient: $\binom{100}{43} x^{100-43} y^{43}$

$$= \binom{100}{43} x^{57} y^{43}$$

$$= \frac{100!}{57!\,43!}$$

(b). The cofficient: $\binom{100}{43} (-x)^{100-43} (-3y)^{43}$

$$= \binom{100}{43} (-x)^{57} (-3y)^{43}$$

$$= \frac{100!}{57!\,43!} \cdot (-1)^{57} (-3)^{43}$$

$$= \frac{100!}{57!\,43!} \cdot 3^{43}$$

(c). Thm: $(x+y)^{100} = \sum_{k=0}^{100} \binom{100}{0} x^{100-k} y^k$

From above, the exponent of x and y sum to 100,
However, in the given term $x^{58}y^{43}$, $58+43=101$.
Therefore, this term does not exist in the expansion of $(x+y)^{100}$.