Name: Daniel Kim
Section: 002

1.
    a.
        i. $\forall x(\exists x (K(x)) \land \exists x(\neg K(x)))$
        ii. So K(x) mean “x is a knight” and $\neg K(x)$ = “x is not a knight”. Every inhabitant told me “some of us are knights and some of us are knaves.” Since those two statements are the same as ($\exists x (K(x)) \land \exists x(\neg K(x))$), and after speaking to every inhabitant on the island, the final result will be $\forall x(\exists x (K(x)) \land \exists x(\neg K(x)))$.
    b.
        i. $\exists x \forall x (\neg K(x))$
        ii. Since that one inhabitant said “All of us are knaves”, it becomes $\forall x (K(x))$, and after we only spoke to that one inhabitant, the final result is $\exists x \forall x (\neg K(x))$.
2.

A	B	C	A ∨ ¬B	A ∧ ¬C	B ⊕ C
T	T	T	T	F	F
T	F	T	T	F	T
T	T	F	T	T	T
F	T	T	F	F	F
T	F	F	T	T	F
F	F	T	T	F	T
F	T	F	F	F	T
F	F	F	T	F	F

After these statements from the three inhabitants, we can conclude with A: “A is a knight”, B: “B is a knight”, and C: “C is a night”. After looking at the A’s claim, the statement would be A V ¬B since A is “A is a knight” and ¬B is “B is a knave”. After looking at the B’s claim, the statement would be A ∧ ¬C for B claiming A is a knight and C is a knave. After looking at C’s claim, it would be B ⊕ C because they both cannot be knights and knaves, just one of each. Using these statements, we can use the truth table and find where all 3 claims are true. Therefore, A and B are knights and C is a knave.

3.

a.

p	q	¬p	¬q	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	F

Since $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow q)$ is a contradiction, the statement is unsatisfiable. Let's say, p: Daniel works diligently and q : Daniel gets paid lots of money. $(p \leftrightarrow q)$ means Daniel gets paid lots of money if and only if Daniel works diligently. $(\neg p \leftrightarrow q)$ explains that Daniel gets paid lots of money if Daniel does not work diligently. So the statement is not satisfiable because Daniel earns lots of money if Daniel works diligently if and only if Daniel earns lots of money and Daniel does not work diligently $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow q)$ is a contradiction.

b. Show that $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

p	q	r	$p \to r$	$q \to r$	$p \land q$	$(p \to r) \lor (q \to r)$	$(p \to q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

$(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$	
$(\neg p \lor r) \lor (q \rightarrow r)$	RBI
$(\neg p \lor r) \lor (\neg q \lor r)$	RBI
$\neg p \lor r \lor \neg q \lor r$	Simplification
$(\neg p \lor \neg q) \lor r$	Idempotent law
$\neg (p \land q) \lor r$	De Morgan's law
$\neg (\neg (p \land q)) \rightarrow r$	RBI

4.

Number of students	Number of courses/buildings
1	1
2	2
3	2
4	3
5	3
6	3
7	3
8	4

	1 C/B	2 C/B	3 C/B	4 C/B
1 Student	1	0	1	0
2 Students	0	1	0	1
3 Students	1	1	0	0
4 Students	1	1	1	0
5 Students	0	0	1	0

6 Students	0	1	1	0
7 Students	1	0	0	0
8 Students	1	1	1	1

a. From the tables, I have found the pattern for maximum number of students depending on the number of courses which is $2^n - 1 = x$. The reason why we subtracted by 1 is because all students need to take at least one course and at least one student needs to be in one specific course. N represents the number of courses/buildings and x represents the max number of students. Since x is 500, n becomes 8.9 and needs to be rounded up which is 9 buildings.

b. For *n* number of buildings, the maximum number of students is x from $2^n - 1 = x$

5.

a.

i. Z (l) ∧ ¬Z (p)

ii. ¬Z(s)

iii. Z (p) → ¬Z(l)

iv. Z(l) ↔ Z(s) ∨ Z(g)

b. The group's pizza toppings requirements are satisfiable. The set of pizza toppings that satisfies the requirements are licorice and granola. Rafael must have licorice and cannot have the peanut butter. In addition, they cannot have salami because of Michelangelo. Donatello's requirement is already fulfilled because of Rafael's request for not adding peanut butter, so licorice will be in the requirement for Donatello. For Leonardo, we already know that we will have licorice, so salami or granola will be the option. Since Michelangelo does not want salami, granola will be the requirement which there won't be any problems for everyone. To conclude, adding licorice and granola satisfies the requirement.