

The first  $n^2$  is from line 2. For  $n * n$ , since the two loops on lines 3 and 4 are independent of one another, the number of times in line is the product of the number of times the two loops run which is  $n * n$ .

Next, since the three loops on lines 6, 7, and 8 are independent of one another, the number of times in line 9 run is the product of the three loops run which is  $((n^2) / 2) * \log_5(n)$ .

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**CSCI 3104, Algorithms**  
**Requiz Standard 3, Version A**

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**Instructions:** This quiz is open book and open note. You **may** post clarification questions to Piazza, with the understanding that you may not receive an answer in time and posting does count towards your time limit (30 min for 1x, 37.5 min for 1.5x, 45 min for 2x). Questions posted to Piazza **must be posted as PRIVATE QUESTIONS**. Other use of the internet, including searching for answers or posting to sites like Chegg, is strictly prohibited. Violations of these are grounds to receive a 0 on this quiz. Proofs should be written in **complete sentences**. **Show and justify all work to receive full credit.**

**Standard 3.** Analyze the worst-case runtime of the following algorithms. You should give your answer in big-Oh notation; try to give as tight a bound as possible (but you do not need to give an input which achieves your worst-case bound).

- In the column to the right of the code, indicate the cost of each line
- In the next column (all the way to the right), indicate the number of times each line is executed
- Below the code, justify your answers (show your work), and compute the total runtime in terms of big-Oh notation.
- In both columns, you don't have to put the exact values. For example, putting "c" for constant is fine, and the difference between  $n$  and  $n - 1$  is irrelevant here.

	Cost	# times run
// A is a square, 2D array; indexed starting from 1		
1 f(A[1, ..., n][1, ..., n]):		
2     let d be a copy of A	$n^2$	1
3     for i = 1 to n:	1	n
4         for j = 1 to n:	1	n
d[i][j] = i*j	1	n
5		
6     for i = 1 to n:	1	n
7         for (k = 1; k < len(A); k = k * 5):	1	$\log_5 n$
8             for j = 1 to n/2:	1	n/2
9                 print d[i][j] + d[i][k]	1	n

$$T(n) = n^2 + n \cdot n + \frac{n^2}{2} \cdot \log_5(n) = \Theta\left(\frac{n^2}{2} \cdot \log_5(n)\right)$$