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CSCI 3104, Algorithms **Profs. Chen & Grochow**
Problem Set 10 – Due Wed April 22 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

Quicklinks: **?? ?? ?? ?? ??**

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1. Consider the following DP table for the Knapsack problem for the list

$$A = [(4,3), (1,2), (3,1), (5,4), (6,3)]$$

of (weight, value) pairs. The weight threshold $W = 10$.

- Fill in the values of the table.
- Draw the backward path consisting of backward edges and do not draw (or erase them) the edges that are not part of the optimal backward paths.

(a) Fill the table with the above requirements (You can also re-create this table in excel/sheet).

Weight	Value	i \ w	0	1	2	3	4	5	6	7	8	9	10
4	3	A[0..0]	0	0	0	0	3	3	3	3	3	3	3
1	2	A[0..1]	0	2	2	2	3	5	5	5	5	5	5
3	1	A[0..2]	0	2	2	2	3	5	5	5	6	6	6
5	4	A[0..3]	0	2	2	2	3	5	6	6	6	7	9
6	3	A[0..4]	0	2	2	2	3	5	6	6	6	7	9

(b) Which cell has the optimal value and what is the optimal value for the given problem?

The optimal value is the cell at row $A[0...3]$ and column 10, and the optimal value for the given problem is 9

(c) List out the optimal subset and provide it's weight and value.

$[(4,3), (1,2), (5,4)]$ is the optimal subset with the weight of 10 and the value of 9.

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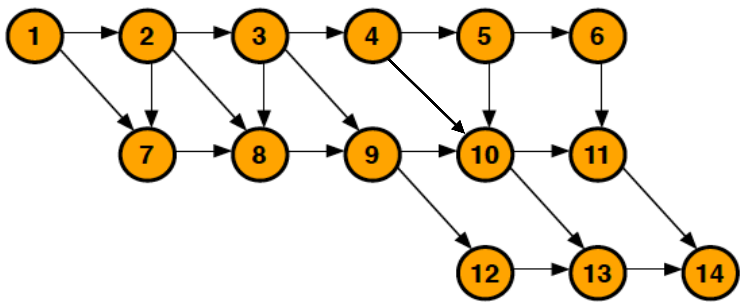
2. Given the following directed acyclic graph. Use dynamic programming to fill in a table that counts number of paths from each node j to 14, for $j \geq 1$. Note that a single vertex is considered a path of length 0.



After studying the graph, we can say a several things that relates to the problem. There are two ways/cases after the first node which are node 2 and node 7. In addition, the number of ways from node 2 to 14 and the number of ways from node 7 to node 14 together adds up to the number of ways from node 1 to node 14. In other words, the sum of the number of ways to node 14 from the nodes that node X is pointing to, is the same as the number of ways to get to node 14 from node X.

Let's say dp(n) is the the number of ways from node X to node 14 and here's a table that shows the number of ways from node 14 to node 1.

A	B	C
Node	**Number of ways to get to node 14**	**Explanation**
14	0	Base case
13	1	Base case
12	1	dp(12) = dp(13)
11	1	Base case
10	2	dp(11) + dp(13) = dp(10)
9	3	dp(10) + dp(12) = dp(9)
8	3	dp(9) = dp(8)
7	3	dp(8) = dp(7)
6	1	dp(11) = dp(6)
5	3	dp(6) + dp(10) = dp(5)
4	5	dp(5) + dp(10) = dp(4)
3	11	dp(4) + dp(8) + dp(9) = dp(3)
2	17	dp(3) + dp(7) + dp(8) = dp(2)
1	20	dp(2) + dp(7) = dp(1)

COLLABORATED WITH:
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