Homework 1. | Daniel Kim

1. I believe that the mechanical degrees of freedom options are 6 or 7 degrees of freedom. Having more DoF means that you can have working space more possible and more reachable. Therefore, 7 degrees of freedom is advantage over the other because the robotic can move around more without moving the end factor. In addition, since it takes more coding and materials, 7 degrees of freedom should be more expensive.

2. The independent environmental degrees of freedom are X-axis, Y-axis, and Yaw-axis in two dimensional X-Y plane. To exemplify, the orientable robot can move along X-axis and Y-axis. There's Yaw-axis for rotation because the robot is orientable.

3. $\vec{V_1} = (0.966, 0.2588, 0)^T$ $\vec{V_2} = (-0.2588, 0.966, 0)^T$

a) $\cos\theta = \dfrac{\vec{V_1} \cdot \vec{V_2}}{|\vec{V_1}| \cdot |\vec{V_2}|}$

$$= \frac{0.966 \cdot (-0.2588) + 0.2588 \cdot 0.966 + 0 \cdot 0}{\sqrt{(0.966)^2 + (0.2588)^2} \times \sqrt{(-0.2588)^2 + (0.966)^2}}$$

$= 0$

$\therefore \ \theta = \pi/2.$

b) To calculate the third vector, we can matrix multiply $\vec{V_1}$ and $\vec{V_2}$

$$\vec{V_1} \cdot \vec{V_2} = \begin{vmatrix} x & Y & Z \\ 0.966 & 0.2588 & 0 \\ -0.2588 & 0.966 & 0 \end{vmatrix} = 0 \cdot x - 0 \cdot y + ((0.966)^2 + (0.2588)^2) Z$$

$= [0, 0, 1].$

4.

a) ${}^A_B R$ , $x_A \ Y_A \ Z_A$ , $x_B \ Y_B \ Z_B$.

$${}^A_B R = [{}^A x_B, \ {}^A Y_B, \ {}^A Z_B].$$

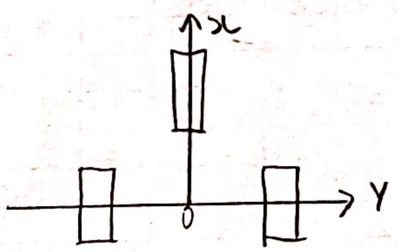
$$= \begin{bmatrix} x_B \cdot x_A & Y_B \cdot x_A & Z_B \cdot x_A \\ x_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ x_B \cdot Z_A & Y_B \cdot Z_A & Z_B \cdot Z_A \end{bmatrix}$$

b). $$^{A}x_{B} = \begin{bmatrix} x_B \cdot x_A & Y_B \cdot x_A & Z_B \cdot x_A \\ x_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ x_B \cdot Z_B & Y_B \cdot Z_A & Z_B \cdot Z_A \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} Y_B \cdot x_A \\ Y_B \cdot Y_A \\ Y_B \cdot Z_A \end{bmatrix}$$

c) $${}^{B}_{A}R = {}^{A}_{B}R^{T} = \begin{bmatrix} x_B \cdot x_A & x_B \cdot Y_A & x_B \cdot Z_A \\ Y_B \cdot x_A & Y_B \cdot Y_A & Y_B \cdot Z_A \\ Z_B \cdot x_A & Z_B \cdot Y_A & Z_B \cdot Z_A \end{bmatrix}$$

5. Coordinate system




Forward Kinematics equation

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} wr \\ 0 \\ \frac{wr}{l} \cdot \sin\phi \end{bmatrix}$$

6. ${}^{B}Q = (8, -4)$ , $(6, 2, 2.26893)$.

Homogenous Transform.

$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{{}^{A}_{B}R} & {}^{A}P \\ \hline 0 & 0 & 1 \end{array}\right] \begin{bmatrix} {}^{B}Q \\ 1 \end{bmatrix}$$

$$2.26893 \cdot \frac{180^\circ}{\pi} = 130^\circ$$

$$\begin{bmatrix} {}^{A}Q \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 6 \\ \sin\theta & \cos\theta & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.922 \\ 10.670 \\ 1 \end{bmatrix}$$