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CSCI 2824: Discrete Structures

Lecture 13: More Set Operations

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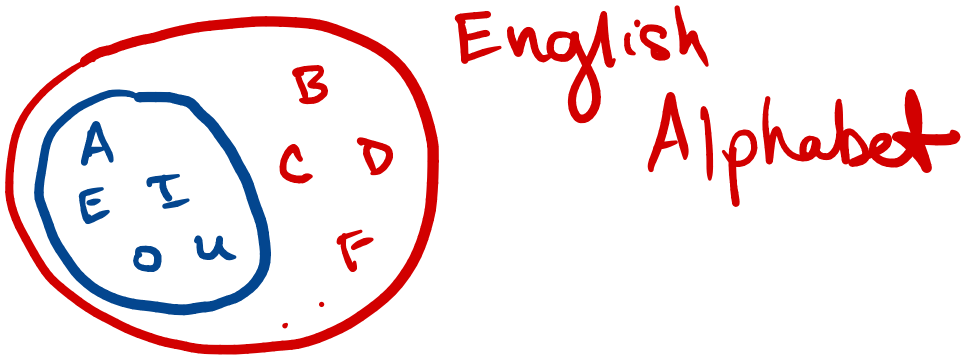
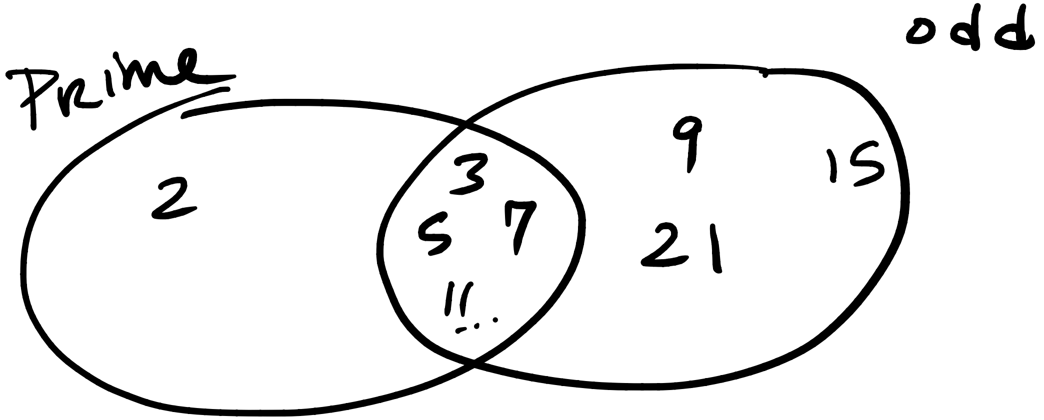
Set Operations

Example: Draw a Venn Diagram relating the set of all vowels to the set of all letters in the English alphabet.



Example: Draw a Venn diagram relating the set of all prime numbers and the set of odd numbers. (prime numbers are numbers that are only divisible by 1 and itself.



Set Operations

Example: Consider the sets $A = \{dumbo, jumbo\}$ and $B = \{a, b, c\}$. What is $A \times B$?

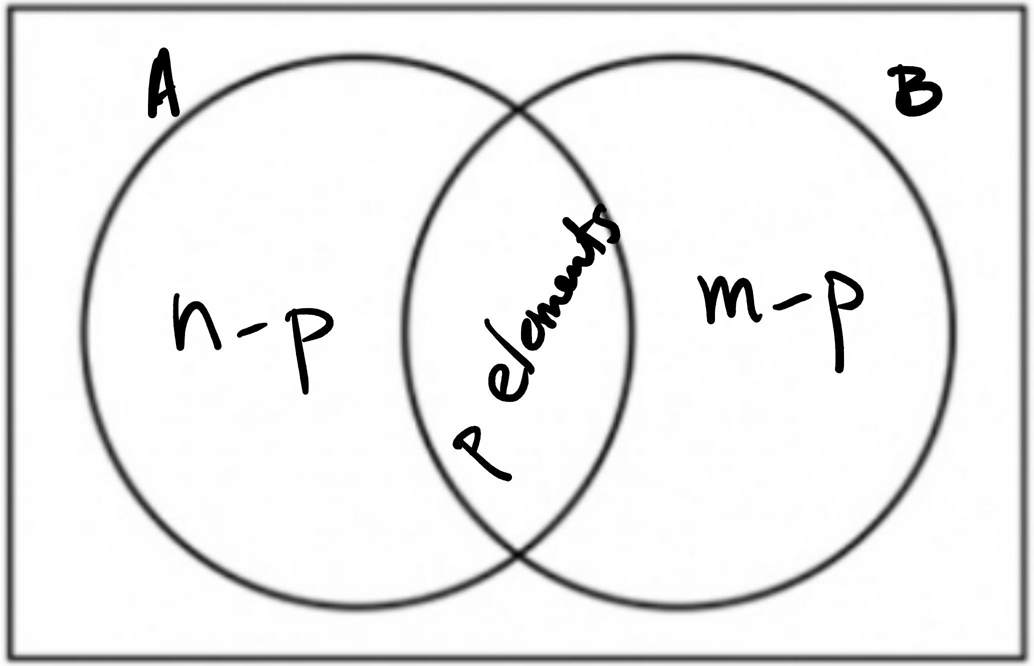
$$A \times B = \{ (dumbo, a), (dumbo, b), (dumbo, c), (jumbo, a), (jumbo, b), (jumbo, c) \}$$

$$B \times A = \{ (a, dumbo), (b, dumbo), (c, dumbo), (a, jumbo), (b, jumbo), (c, jumbo) \}$$

$(a, dumbo) \in B \times A$ But $\{(a, dumbo)\} \subseteq B \times A$

Set Operations

Example: How many elements are in the set $A \cup B$?



$|A| = n$

$|B| = m$

$|A \cap B| = p$

note: $n-p + p + m-p = n+m-p$

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= n + m - p$$

Set Operations

- **Python** has some nice functionality that can help you convert lists of elements into sets, and perform some operations on them.

```
In [8]: mylist = [1,2,3,1,4]

In [9]: myset = set(mylist)

In [10]: print(myset)
{1, 2, 3, 4}
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In [15]: A = set([1,2,3,4])

In [16]: B = set([3,4,5,6])

In [17]: print(set.intersection(A,B))
{3, 4}

In [18]: print(set.union(A,B))
{1, 2, 3, 4, 5, 6}

In [19]: print(set.difference(A,B))
{1, 2}

In [20]: print(set.difference(B,A))
{5, 6}
```

- **If/when** the time comes, you should feel free to explore these functions for manipulating sets…
  … he said with a knowing grin.

Set Operations

Example: Use set identities to prove $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$ ★

$$\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B \cap C})$$ DeMorgan's Law for Sets.

$$= \overline{A} \cap (\overline{B} \cup \overline{C})$$ DeMorgan's Law for sets

$$= (\overline{B} \cup \overline{C}) \cap \overline{A}$$ Commutativity

$$= (\overline{C} \cup \overline{B}) \cap \overline{A}$$ Commutativity

Set Operations

Example: If P is the set of prime numbers, then what is $\overline{P}$?

$\overline{P}$ would be the set of composite numbers.

$P = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

$\overline{P} = \{1, 4, 6, 8, 9, 10, 12, 14, 15, \ldots\} = \mathbb{Z}^+ - P$

Set Operations

practice

e.g. $\mathcal{P}(A-B)$ compare to (g)

Example: Suppose $A=\{b,c,d\}$ and $B=\{a,b\}$. Find:

(a) $(A\times B)\cap(B\times B)$ (d) $(A\cap B)\times A$ (g) $\mathscr{P}(A)-\mathscr{P}(B)$

(b) $(A\times B)\cup(B\times B)$ (e) $(A\times B)\cap B$ (h) $\mathscr{P}(A\cap B)$

(c) $(A\times B)-(B\times B)$ (f) $\mathscr{P}(A)\cap\mathscr{P}(B)$ (i) $\mathscr{P}(A)\times\mathscr{P}(B)$

(a) $A\times B=\{(b,a),(b,b),(c,a),(c,b),(d,a),(d,b)\}$

$B\times B=\{(a,a),(a,b),(b,a),(b,b)\}$

$(A\times B)\cap(B\times B)=\{(b,a),(b,b)\}$

(g) $\mathcal{P}(A)=\{\cancel{\emptyset},\cancel{\{b\}},\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$

$-\ \mathcal{P}(B)=\{\emptyset,\{a\},\{b\},\{a,b\}\}$

$\mathcal{P}(A)-\mathcal{P}(B)=\{\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$