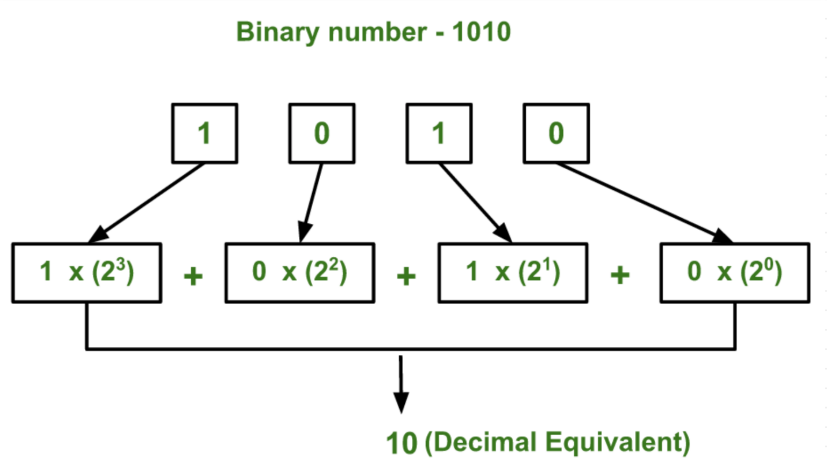


geeksforgeeks.org

CSCI 2824: Discrete Structures

Lecture 1: Intro & Binary Representation of Numbers

Hello world!

My name is IOANA FLEMING

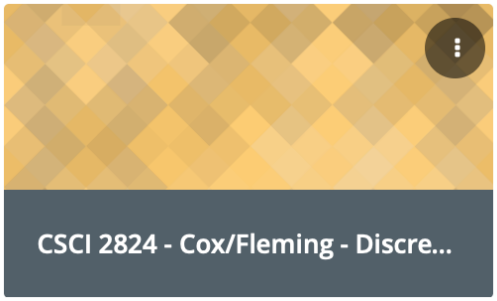
You can find me at:

ioana.fleming@colorado.edu

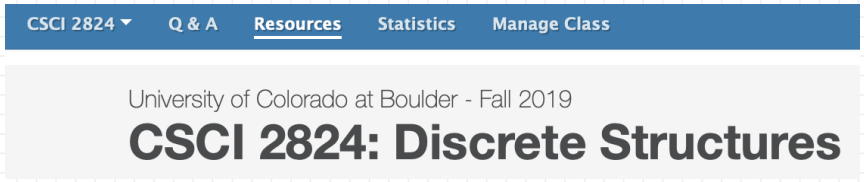
ECOT 735

Course Logistics - Platforms

1) Moodle – Online Homework, Online Quizlets, Grades



2) Piazza– Class discussion forum



3) Gradescope – Submission of written homework

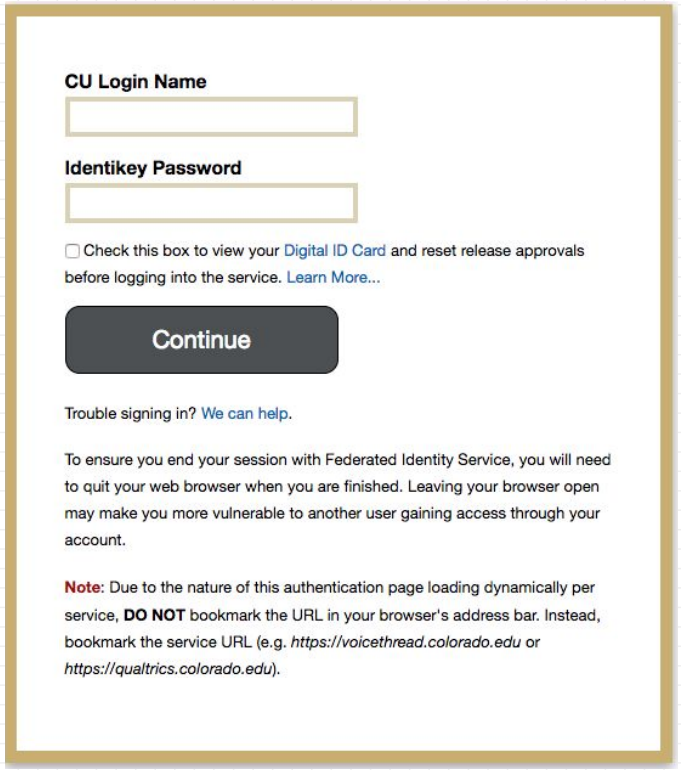
Course Logistics - Platforms

Moodle – Online Homework, Online Quizlets, Grades

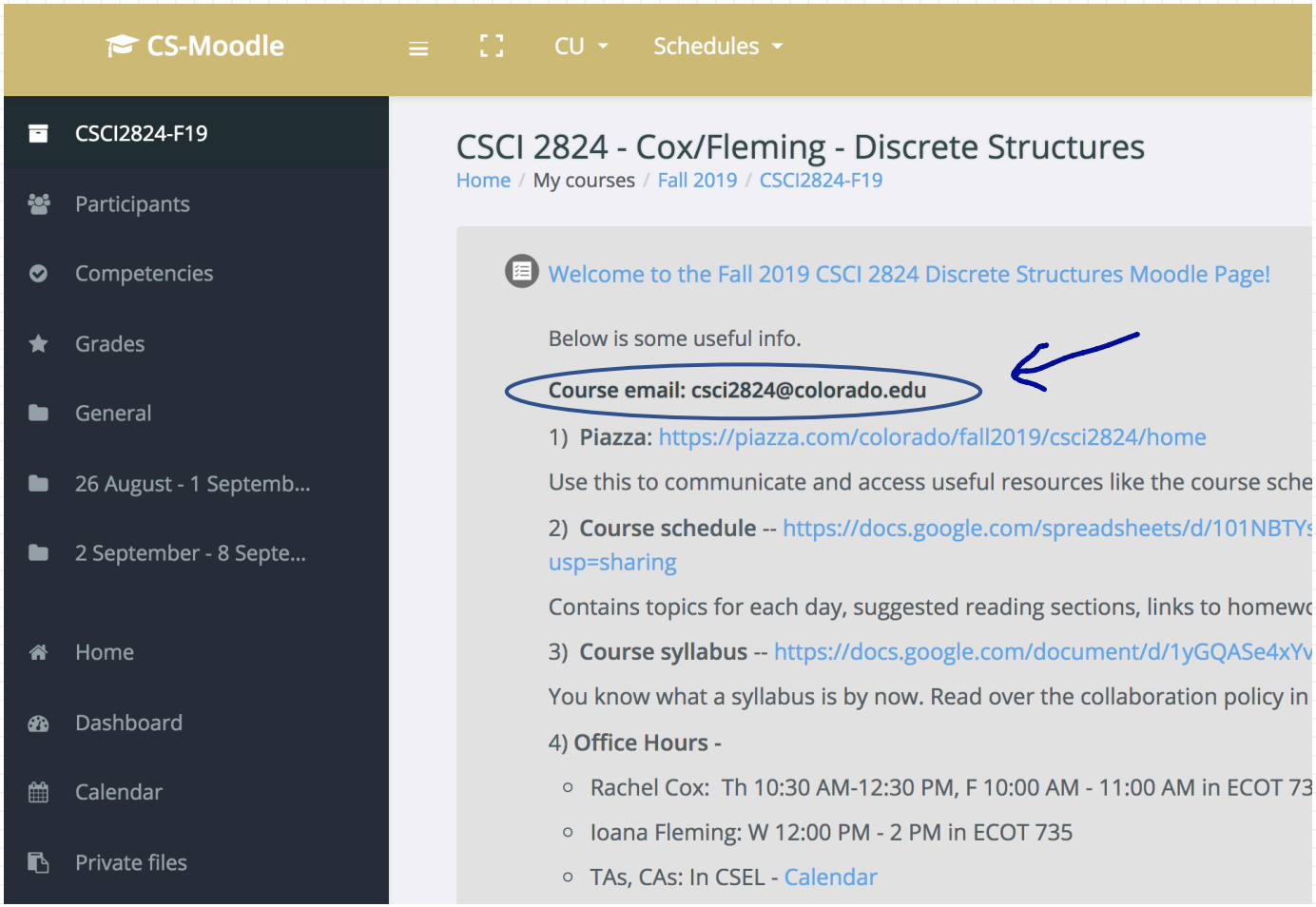
moodle.cs.colorado.edu

Enrollment key (case sensitive):

section 001 – 9am: CSCI2824-FALL2019
section 002 – 11am: CSCI2824-FALL2019



Course email



Piazza

https://piazza.com/colorado/fall2019/csci2824/home

- Homework assignments and solutions will be posted here
- Announcements, Videos
- Ask questions in Q & A forum (and answer other students' questions!)
  - There are hundreds of you and only a few of us -- get answers faster
- Discuss work, but **do not post solutions/vital code**
- Send **private** messages with your code/solution if you need feedback

Course Logistics - Platforms

Gradescope – I will enroll you. You'll receive an email once I've enrolled you. This is where you will turn in your written homework.

Course Logistics - Grading

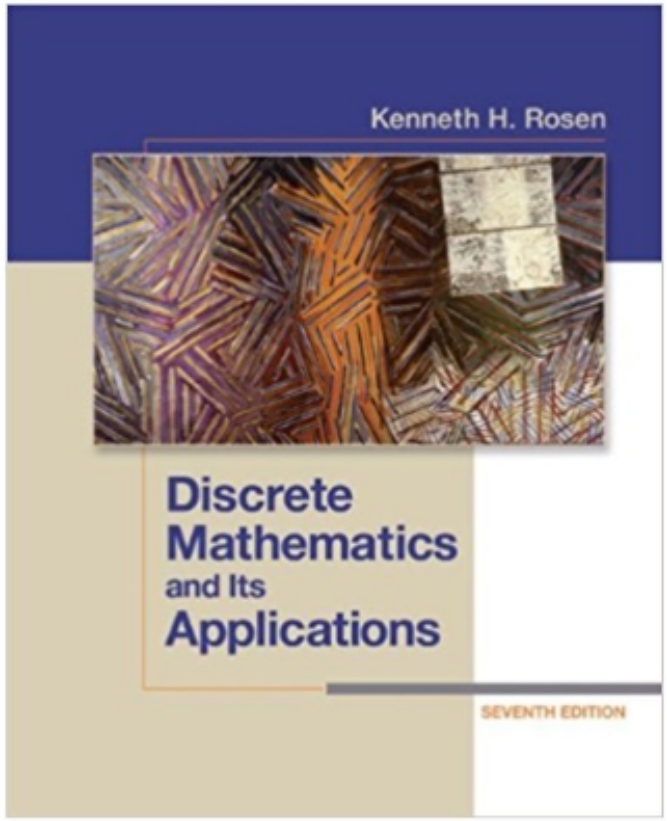
Weekly Homework (30%)
- Half Written, Half Online

Quizlets (10%)
- Online

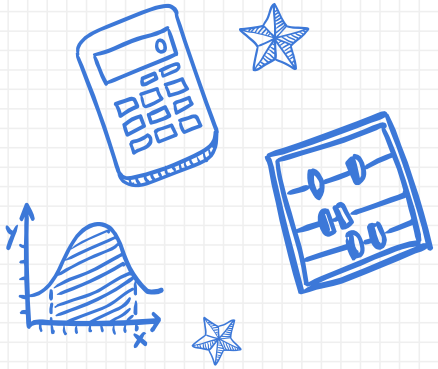
Two Midterms (20% each)
- October 1st , November 5th

Final Exam (20%)
- Section 001 – 9am: Wednesday December 18th , 1:30-4:00pm
- Section 002 – 11am: Sunday December 15th , 1:30-4:00pm

Course Logistics - Textbook



Discrete Mathematics and Its Applications, 7th Ed. by Kenneth H. Rosen



What is Discrete Structures?

What is Discrete Structures?

Discrete

- Logic
- Combinatorics
- Discrete Probability
- Recursion
- Sets
- Sequences
- Graph Theory

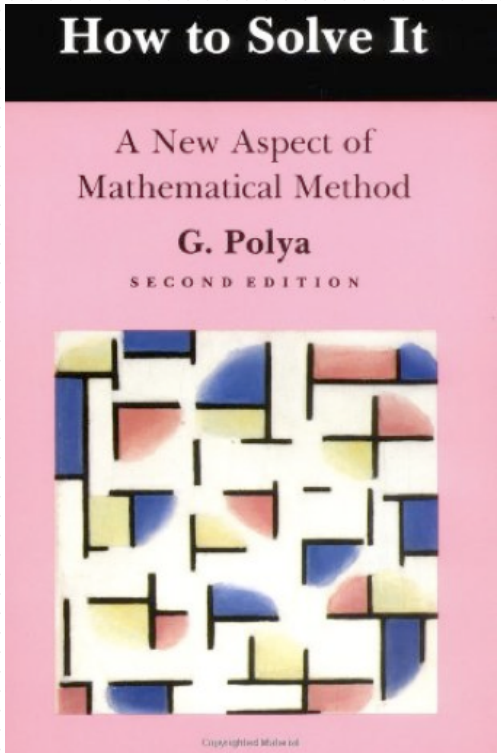
NOT Discrete

- Derivatives
- Integrals
- … things that involve infinitesimals

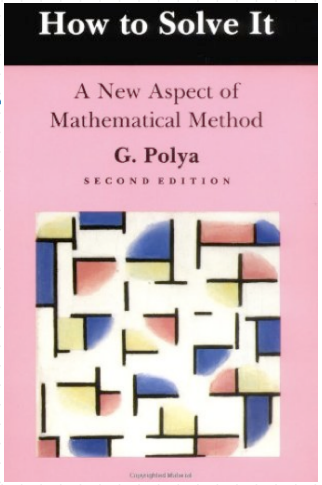
What is Discrete Structures?

The way I'd like to think about this course:

- How to solve problems:



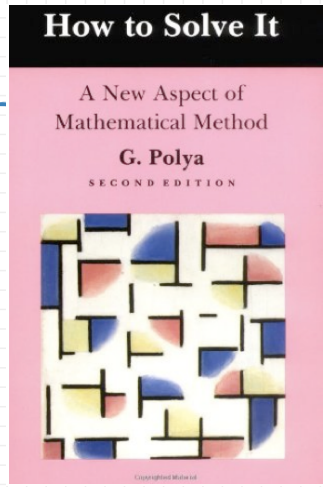
Polya's Strategies



Polya's First Principle: Understand the problem

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Polya's Strategies

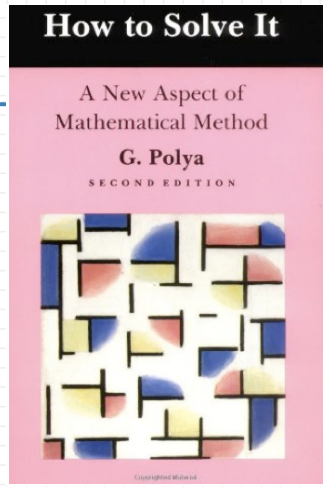


Polya's Second Principle: Devise a plan

Choose the best strategy. Solve lots of problems.

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious

Polya's Strategies



Polya's Third Principle: Carry out the plan

This step is usually easier than devising the plan

Polya's Forth Principle: Look back

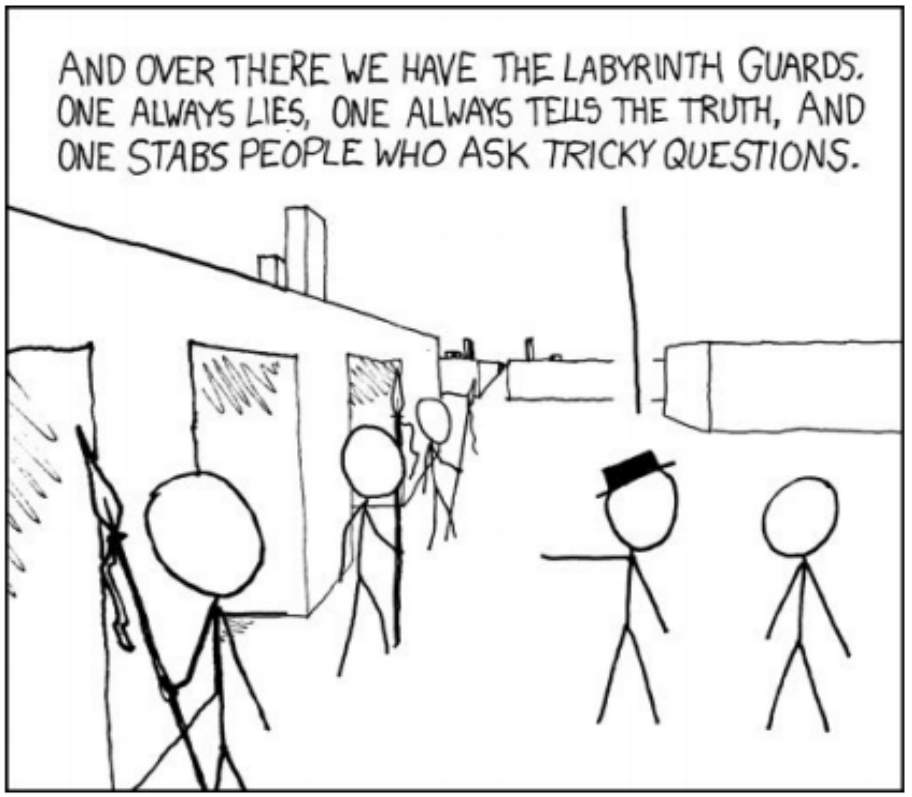
Much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems.

Logic:

“Two doors” riddle:

- Two doors, guarded by two guards.
- One door goes where you want to, but the other leads to certain death.
- One always lies, and one always tells the truth.

- **How can you ask**
  <u>only one of them</u>
  <u>only one question</u>
  to discover which door is which?



Discrete Probability:

e.g. The Monty Hall Problem

Three doors problem

- There are three doors.
- One has a nice prize behind it…
- … and the other two have goats.
- You get to pick a door and will be awarded the prize behind it.
- Then the host reveals a goat behind one of the other two doors.
- You now have the option to stick with your original door or switch.

- **Should you stick with your original door or switch? Or does it not matter?**

Graph Theory

Imagine that we have a gathering of six people, some of whom know each other and some of whom do not. Show that there must be a trio of people who are mutual friends or who are mutually unknown to each other.

Counting

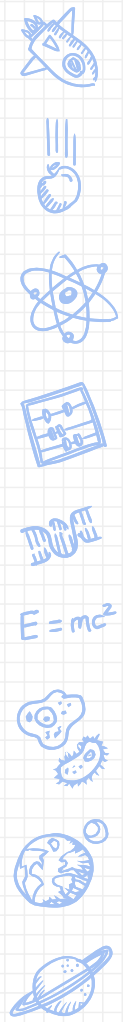
1. Suppose we have an N-bit word (a word consisting of N elements, each “0” or “1”). How many distinct such words are there?

2. How many distinct N-bit words are there which do not have two consecutive 0’s?

Counting

3. Lockers numbered 1 to 100 stand in a row at the school gym. When the first student arrives, she opens all the lockers. The second student then goes through and recloses all the even-numbered lockers; the third student changes the state of every locker whose number is a multiple of 3. This continues until 100 students have passed through. Which lockers are now open?

   – From P. Winkler, Mathematical Puzzles



One more

Mr. Smith and his wife invited four other couples for a party. When everyone arrived, some of the people in the room shook hands with some of the others. Of course, nobody shook hands with their spouse and nobody shook hands with the same person twice.

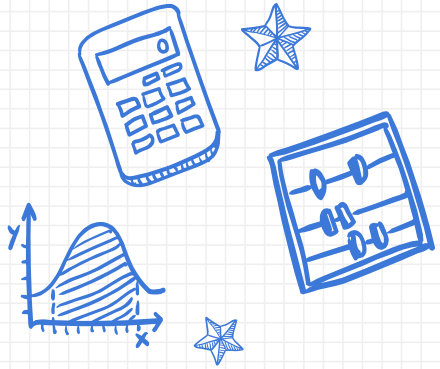
After that, Mr. Smith asked everyone how many times they shook someone's hand. He received different answers from everybody .

How many times did Mrs. Smith shake someone's hand?

Recursion:

e.g. The Tower of Hanoi

- Recursive solutions can be elegant and lead to more readable code
- Recursion may be frowned upon due to memory stack issues
- However, developing a recursive solution and then translating it to an iterative solution (loops) may be very helpful



Binary Representation of Numbers

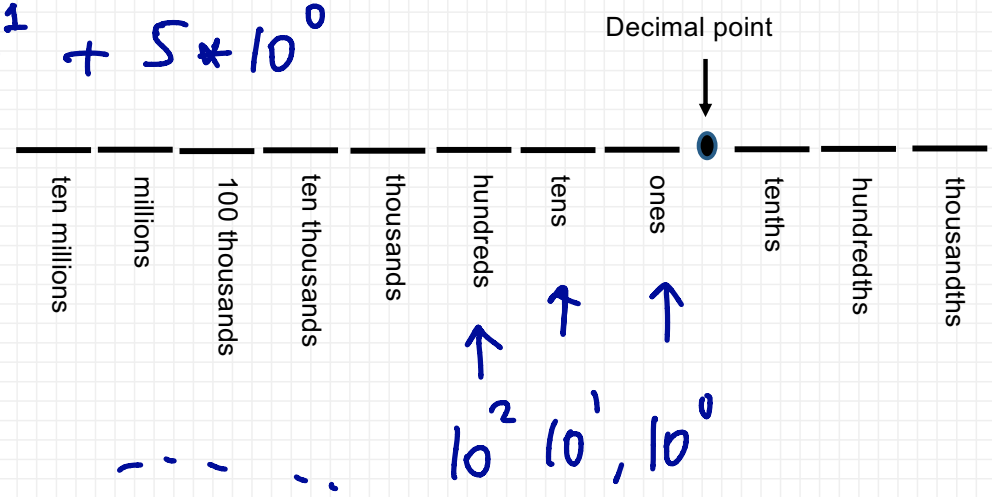
What's in a Number?

Example: Consider the numbers **235** and **1130**. Assume they are both "decimal".

How can we expand them as powers of 10?

$$235 = 2*100 + 3*10 + 5$$

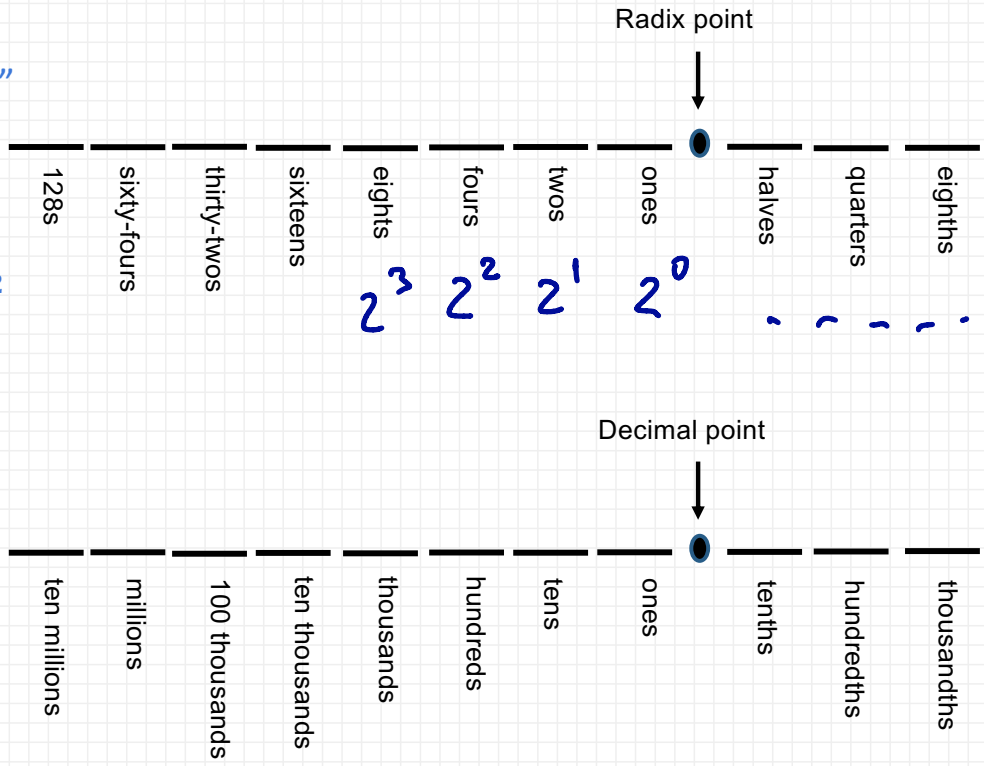
$$= 2*10^2 + 3*10^1 + 5*10^0$$



Binary Number System

0 1

- Representing numbers in "base 2"
- Denote a binary number by $(binary\ number)_2$



Powers of 2

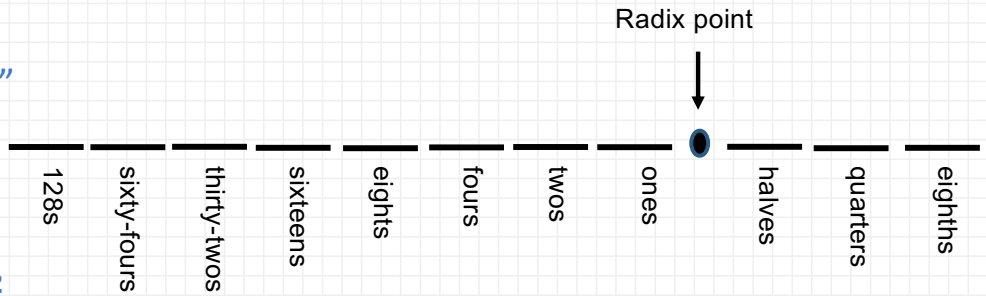
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$

Powers of 10

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
$10^4 = 10{,}000$
$10^5 = 100{,}000$
$10^6 = 1{,}000{,}000$
$10^7 = 10{,}000{,}000$
$10^8 = 100{,}000{,}000$

Binary Number System

- Representing numbers in "base 2"
- Denote a binary number by $(binary\ number)_2$



Powers of 2

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$

Binary – why do we care?

- Computers are good are knowing whether something is "on" (1) or "off" (0)
- Two options → base-2 system

Example: holding down Ctrl (0/1)? Alt (0/1)? Del (0/1)?

⇒ Abort only if (111)

0 or 1

$235 = 1*2^7 + 1*2^6 + 1*2^5 + 0*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0$

Example:

Convert 235 from decimal to binary

$128 \rightarrow 2^7$

$235 - 128 = 107$

$64 \rightarrow 2^6$

$107 - 64 = 43$

$32 \rightarrow 2^5$

$43 - 32 = 11$

$8 \rightarrow 2^3$

$11 - 8 = 3$

2

1

Powers of 2:

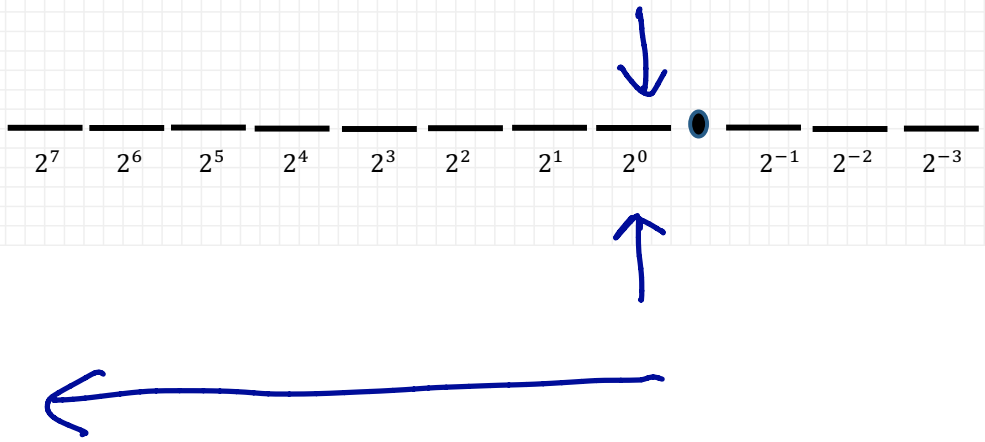
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
1	1	1	0	1	0	1	1	.			

235

Example:

Convert 235 from decimal to binary in a more systematic way



An Algorithm for Converting Decimal Integers to Binary

Let N be a nonnegative integer. Move from right to left.

Is N even? or odd?

If N is even, set bit to 0, reset $N = \frac{N}{2}$

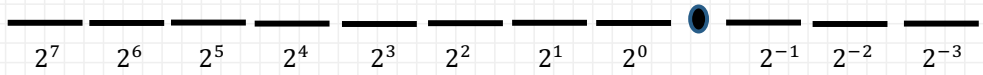
If N is odd, set bit to 1, reset $N = \frac{N-1}{2}$

Move left to the next bit

Repeat until N = 0

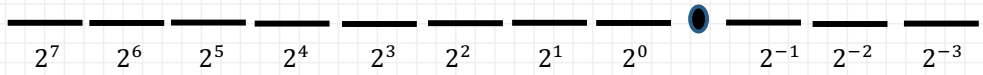
Example:

Convert 235 from decimal to binary using the algorithm we just defined



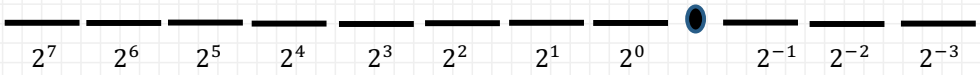
Example:

Convert 160 and 161 from binary to decimal using the algorithm we just defined



Example:

Convert 1100101 from binary to decimal



Example:

What's the largest number you can store as a 32-bit signed int?