# CSCI 2824: Discrete Structures

## Lecture 9: More Rules of Inference

# Reminders

Submissions:

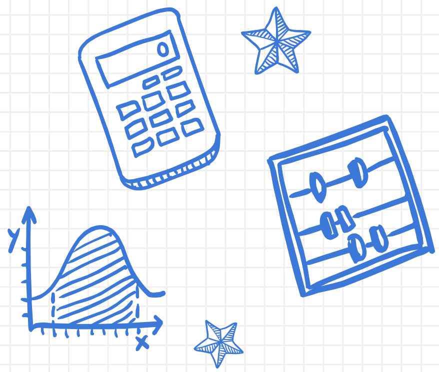
- Homework 3: Fri 9/20 at noon – 1 try
- Homework 4: Fri 9/13 at noon – Gradescope
- Quizlet 3 – due Wednesday @ 8pm

Readings:

- 1.6 – 1.8 this week
- Starting Ch. 2 next week

**Midterm – Tue October 1st at 6pm**

**<u>Any conflicts?</u> – email csci2824@colorado.edu**



# Rules of Inference

# What did we do last time?

- We have learned the rules of inference, and how to use them to construct **valid** arguments. (good arguments)
- We have learned how to identify a **sound** argument. (great arguments)

**Today**:

1. rules of inferences – cont.
2. identifying fallacious arguments
3. bring quantifiers into the mix

# Fallacies

Reminder: When the argument is valid AND the premises are true, we call the argument **sound**.

It is not uncommon to see invalid arguments out in the wild.
Usually people make invalid arguments when conditionals are involved.

**Example (from last time):** If it rains today, then my basement will flood.
My basement did not flood.
----------------------------------------------------------
∴ It did not rain today.

Was this argument valid?

# Fallacies

It is not uncommon to see invalid arguments out in the wild.

Usually people make invalid arguments when conditionals are involved.

**Example (from last time):** If it rains today, then my basement will flood.

My basement did not flood.

---------------------------------------------------

∴ It did not rain today.

1. $\neg q$
2. $p \rightarrow q$

------------

∴ $\neg p$

Was this argument valid? **Yes, by Modus Tollens**

What about this new argument?

**Example:** **<u>affirming the conclusion</u>**

If you study hard in this class, then you will get an A.

You got an A.

Therefore, you must have studied hard.

# Fallacies

**Example: affirming the conclusion**

If you study hard in this class, then you will get an A.

You got an A.

Therefore, you must have studied hard.

Do you see the problem?

You could have gotten an A for reasons aside from studying hard.

This has the form and is **not** a valid argument.

$$\begin{array}{l} p \rightarrow q \\ q \\ \hline \therefore\ p \end{array}$$

Note that if $p$=F and $q$=T, then both the premises are T **but the conclusion is F**.

# Fallacies

**Example:** **denying the hypothesis**

If you choose a strong password, then your email will not be hacked.

You did not choose a strong password.

Therefore, your email will be hacked.

# Fallacies

**Example:** __**denying the hypothesis**__

If you choose a strong password, then your email will not be hacked.
You did not choose a strong password.
Therefore, your email will be hacked.

Do you see the problem?
Your email might be hacked even if you have a strong password

This has the form

$$\begin{array}{rl} & p \to q \\ & \neg p \\ \hline \therefore & \neg q \end{array}$$

and is **not** a valid argument.

Note again that if *p*=F and *q*=T, then both the premises are T **but the conclusion is F**.

# Fallacies

**Example:** What rule of inference or logical fallacy is demonstrated by the following argument? Is the argument sound?

*If Earth is flat, then if you fly for a long time in one direction, you will fall off the edge.*

*The Earth is flat.*

*Therefore, if you fly for a long time in one direction for too long, you will fall off the Earth.*

# Fallacies

**Example:** What rule of inference or logical fallacy is demonstrated by the following argument? Is the argument sound?

*If Earth is flat, then if you fly for a long time in one direction, you will fall off the edge.*

*The Earth is flat.*

*Therefore, if you fly for a long time in one direction for too long, you will fall off the Earth.*

**Solution:** This argument is **valid**

It is **modus ponens**

… It is **NOT sound**.

*How do we know the Earth is not flat?*

# Rules of inference

**We've seen:** If Osiris can bleed, then Osiris is a mortal.
Osiris can bleed.

-----------------------------------------------------

Therefore, Osiris is a mortal.

**What about:** Everything that can bleed is mortal.
Osiris can bleed.

-----------------------------------------------------

Therefore, Osiris is a mortal.

# Rules of inference

Let's start with the basic rules of inference for quantifiers, and we will build up to Osiris.

**Universal Instantiation:**

$$\frac{\forall x\ P(x)}{\therefore \quad P(c)}$$

**Intuition:** If we know that *P(x)* is true for **all** *x* then we can insert **any** specific *x* (say, *x=c*) and know it's true.

**Example:** All owls are wise.
Therefore, Owl is wise.

So going from the universal case to the concrete one is pretty straightforward. What about the other way around?

# Rules of inference

It's okay to go from an **arbitrary** example to a universal statement.

**Universal Generalization:**

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x\, P(x)}$$

**This is subtle.** If you can prove something about an **arbitrary** element of your domain, then you can make a universal statement.

This does **not mean** that if you can prove something about a *specific* element and then make a *universal* statement.

# Rules of inference - Quantifiers

**Example**: Is the following argument valid?

All dogs go to heaven.

Laddie is a dog.

If Laddie goes to heaven, then he walks through the pearly gates.

---

Consequently, all dogs walk through the pearly gates.

**Example**: What about this one?

All dogs go to heaven.

"A" is an arbitrary dog.

If "A" goes to heaven, then s/he walks through the pearly gates.

---

Consequently, all dogs walk through the pearly gates.

# Rules of inference

But you can go from an **existential statement** to a **specific element**:

**Existential Instantiation:**

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some element } c}$$

**Example:**

- There exists an owl who wears a book on its head.
- We do not know what their name is, we just know this is true for *some* owl out there.

# Rules of inference

and you can also go from a **specific element** to an **existential statement**:

**Existential Generalization:**

$$\begin{array}{l} P(c) \text{ for some element } c \\ \hline \therefore \exists x\, P(x) \end{array}$$

**Example:**

- "Owl" wears a book on its head.
- Therefore, there exists an owl who wears a book on its head.

# Rules of inference

How do we prove it?

Everything that can bleed is mortal.

Osiris can bleed.

--------------------------------------------------

Therefore, Osiris is a mortal.

$$\forall x\,\big(B(x) \rightarrow M(x)\big)$$

$$B(Osiris)$$

---

$$\therefore M(Osiris)$$

# Rules of inference

Once we're able to instantiate variables from universal statements, we can use the propositional rules of inference to prove things from statements with quantifiers.

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(B(x) \rightarrow M(x))$ | premise |
| 2. | $B(Osiris)$ | premise |
| 3. | | |
| 4. | $\therefore M(Osiris)$ | modus ponens of (2) and (3) |

# Rules of inference

Once we're able to instantiate variables from universal statements, we can use the propositional rules of inference to prove things from statements with quantifiers.

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(B(x) \rightarrow M(x))$ | premise |
| 2. | $B(Osiris)$ | premise |
| 3. | $B(Osiris) \rightarrow M(Osiris)$ | Universal instantiation from (1) |
| 4. | $\therefore M(Osiris)$ | modus ponens of (2) and (3) |

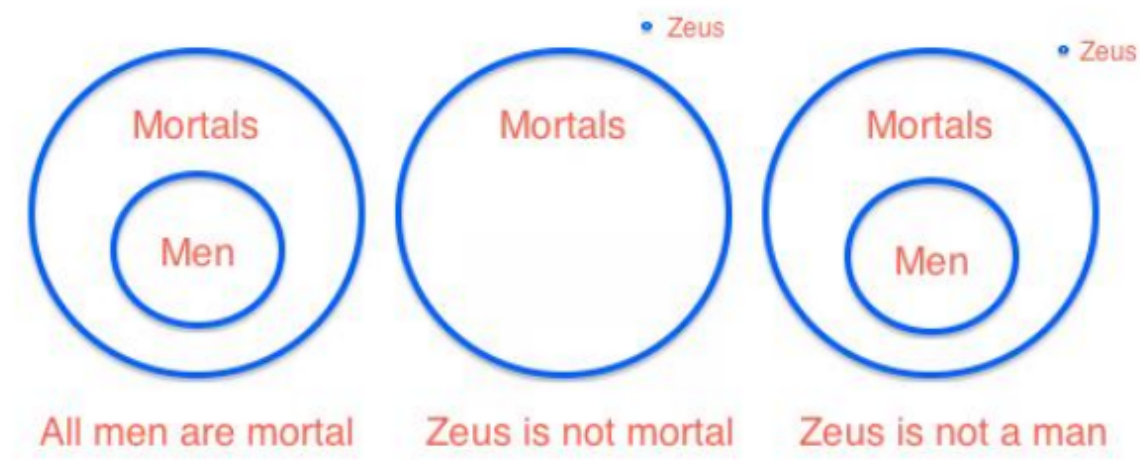
Sometimes this is called **Universal Modus Ponens**.

# Rules of inference - Quantifiers

Universal Modus Tollens

$\forall x\ (P(x) \rightarrow Q(x))$
$\neg Q(a)$, where $a$ is a particular element in the domain

$\therefore \neg P(a)$



# Rules of inference - Quantifiers

**Example:** All kittens are cute

Some kittens do not like milk

---

Consequently, some cute creatures do not like milk

Let *K(x)* mean *x* is a kitten,
*C(x)* mean *x* is a cute creature and
*M(x)* mean *x* likes milk.

$\therefore$

Let's prove this formally!

# Rules of inference - Quantifiers

**Example:** All kittens are cute

Some kittens do not like milk

---

Consequently, some cute creatures do not like milk

Let *K(x)* mean *x* is a kitten,
*C(x)* mean *x* is a cute creature and
*M(x)* mean *x* likes milk.

$$\forall x\,(K(x) \rightarrow C(x))$$

$$\exists x\,(K(x) \land \neg M(x))$$

---

$$\therefore \quad \exists x\,(C(x) \land \neg M(x))$$

Let's prove this formally!

# Rules of inference - Quantifiers

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(K(x) \rightarrow C(x))$ | premise |
| 2. | $\exists x\,(K(x) \wedge \neg M(x))$ | premise |
| 3. | | |
| 4. | | |
| 5. | | |
| 6. | | |
| 7. | | |
| 8. | | |
| 9. | $\therefore \exists x\,(C(x) \wedge \neg M(x))$ | |

$$\begin{array}{ll} & \forall x\,(K(x) \rightarrow C(x)) \\ & \exists x\,(K(x) \wedge \neg M(x)) \\ \hline \therefore & \exists x\,(C(x) \wedge \neg M(x)) \end{array}$$

# Rules of inference - Quantifiers

$$\forall x\,(K(x) \rightarrow C(x))$$
$$\exists x\,(K(x) \wedge \neg M(x))$$
$$\therefore \quad \exists x\,(C(x) \wedge \neg M(x))$$

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(K(x) \rightarrow C(x))$ | premise |
| 2. | $\exists x\,(K(x) \wedge \neg M(x))$ | premise |
| 3. | $K(a) \wedge \neg M(a)$ | existential instantiation (2) (for some $a$) |
| 4. | $K(a) \rightarrow C(a)$ | universal instantiation (1) |
| 5. | $K(a)$ | simplification (3) |
| 6. | $C(a)$ | modus ponens (4), (5) |
| 7. | $\neg M(a)$ | simplification (3) |
| 8. | $C(a) \wedge \neg M(a)$ | conjunction (6), (7) |
| 9. | $\therefore \exists x\,(C(x) \wedge \neg M(x))$ | existential generalization (8) |

# Rules of inference - Quantifiers

**Example:** Translate and show the argument is valid.

All elves are very clever

No tall people eat snozzberries

People that do not eat snozzberries are stupid

---

Consequently, elves are short

Let *E(x)* mean *x* is an elf,

*C(x)* mean *x* is clever,

*T(x)* mean *x* is tall, and

*S(x)* mean *x* eats snozzberries.

$\therefore$

# Rules of inference - Quantifiers

**Example:** Translate and show the argument is valid.

All elves are very clever

No tall people eat snozzberries

People that do not eat snozzberries are stupid

---

Consequently, elves are short

Let *E(x)* mean *x* is an elf,
*C(x)* mean *x* is clever,
*T(x)* mean *x* is tall, and
*S(x)* mean *x* eats snozzberries.

$$\forall x\,(E(x) \rightarrow C(x))$$

$$\neg\exists x\,(T(x) \wedge S(x))$$

$$\forall x\,(\neg S(x) \rightarrow \neg C(x))$$

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$$\therefore \quad \forall x\,(E(x) \rightarrow \neg T(x))$$

# Rules of inference - Quantifiers

$\forall x\,(E(x) \rightarrow C(x))$

$\neg \exists x\,(T(x) \wedge S(x))$

$\forall x\,(\neg S(x) \rightarrow \neg C(x))$

$\therefore \quad \forall x\,(E(x) \rightarrow \neg T(x))$

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(E(x) \rightarrow C(x))$ | premise |
| 2. | $\neg \exists x\,(T(x) \wedge S(x))$ | premise |
| 3. | $\forall x\,(\neg S(x) \rightarrow \neg C(x))$ | premise |
| | $\therefore \forall x\,(E(x) \rightarrow \neg T(x))$ | |

| Step | Justification |
| --- | --- |
| $\therefore \forall x\,(E(x) \rightarrow \neg T(x))$ | |

$$\forall x\,(E(x) \rightarrow C(x))$$

$$\neg \exists x\,(T(x) \wedge S(x))$$

$$\forall x\,(\neg S(x) \rightarrow \neg C(x))$$

$$\therefore \quad \forall x\,(E(x) \rightarrow \neg T(x))$$

| | Step | Justification |
|---|---|---|
| 1. | $\forall x\,(E(x) \to C(x))$ | premise |
| 2. | $\neg\exists x\,(T(x) \land S(x))$ | premise |
| 3. | $\forall x\,(\neg S(x) \to \neg C(x))$ | premise |
| 4. | $E(a) \to C(a)$ | universal instantiation (1) **(*a* is arbitrary)** |
| 5. | $\forall x\,(\neg T(x) \lor \neg S(x))$ | DeMorgan (2) |
| 6. | $\neg T(a) \lor \neg S(a)$ | universal instantiation (5) **(*a* is arbitrary)** |
| 7. | $\neg S(a) \to \neg C(a)$ | universal instantiation (3) **(*a* is arbitrary)** |
| 8. | $C(a) \to S(a)$ | contraposition (7) |
| 9. | $E(a) \to S(a)$ | hypothetical syllogism (4), (8) |
| 10. | $\neg S(a) \lor \neg T(a)$ | commutation (6) |
| 11. | $S(a) \to \neg T(a)$ | that one rule (10) |
| 12. | $E(a) \to \neg T(a)$ | hypothetical syllogism (9), (11) |
| 13. | $\therefore \forall x\,(E(x) \to \neg T(x))$ | universal generalization (12) |

$$\forall x\,(E(x) \to C(x))$$
$$\neg\exists x\,(T(x) \land S(x))$$
$$\forall x\,(\neg S(x) \to \neg C(x))$$
$$\therefore \quad \forall x\,(E(x) \to \neg T(x))$$

**Note:** We can jump from (12) to (13) because *a* was arbitrary.

That is, steps (4), (6) and (7) work for every *a* in our domain.

# Rules of inference - Quantifiers

**Example**: For the following argument, explain which rules of inference are used for each step.

Somebody in this class enjoys whale watching.
Every person who enjoys whale watching cares about ocean pollution.
Therefore, there is a person in this class who cares about ocean pollution.

## Rules of inference - Quantifiers

**FYOG:** Translate the following argument using quantifiers. Then show that the conclusion follows from the premises by logical inference.

Babies are illogical

Nobody is despised who can wrangle a crocodile

Illogical people are despised

---

Therefore, babies cannot wrangle crocodiles

# Rules of inference - Quantifiers

**Recap:**

- We have learned the rules of inference, and how to use them to construct **valid** arguments. (good arguments)
- We have learned how to identify a **sound** argument. (great arguments)
- We have learned how to recognize common **fallacious** arguments. (awful arguments)

**Next time:**

- Proofs:  strategies and pitfalls
- Handling Crocodiles

# Extra Practice

**Example**: Translate and show the argument is valid

All hummingbirds are richly colored

No large birds live on honey

Birds that do not live on honey are dull in color

---

Therefore, hummingbirds are small

**Example:** Translate the following argument using quantifiers. Then show that the conclusion follows from the premises by logical inference

Babies are illogical

Nobody is despised who can manage a crocodile

Illogical people are despised

---

Therefore, babies cannot manage crocodiles

Let $B(x)$ mean "$x$ is a baby", $L(x)$ mean "$x$ is logical", $C(x)$ mean "$x$ can handle a crocodile", and $D(x)$ mean "$x$ is despised"

# Solutions

**Example**: Translate and show the argument is valid

All hummingbirds are richly colored
No large birds live on honey
Birds that do not live on honey are dull in color

---

Therefore, hummingbirds are small

This can be symbolized as

$$\begin{array}{ll}
 & \forall x\,(B(x) \rightarrow C(x)) \\
 & \neg\exists x\,(L(x) \wedge H(x)) \\
 & \forall x\,(\neg H(x) \rightarrow \neg C(x)) \\
\hline
\therefore & \forall x\,(B(x) \rightarrow \neg L(x))
\end{array}$$

| | step | justification |
|---|---|---|
| 1. | $\forall x\,(B(x) \rightarrow C(x))$ | premise |
| 2. | $\neg\exists x\,(L(x) \wedge H(x))$ | premise |
| 3. | $\forall x\,(\neg H(x) \rightarrow \neg C(x))$ | premise |
| 4. | $B(a) \rightarrow C(a)$ | universal inst. (1) |
| 5. | $\forall x\,(\neg L(x) \vee \neg H(x))$ | DeMorgan (2) |
| 6. | $\neg L(a) \vee \neg H(a)$ | universal inst. (5) |
| 7. | $\neg H(a) \rightarrow \neg C(a)$ | universal inst. (3) |
| 8. | $C(a) \rightarrow H(a)$ | contraposition (6) |
| 9. | $B(a) \rightarrow H(a)$ | hypothetical syll. (4), (9) |
| 10. | $\neg H(a) \vee \neg L(a)$ | commutation (6) |
| 11. | $H(a) \rightarrow \neg L(a)$ | that one rule (10) |
| 12. | $B(a) \rightarrow \neg L(a)$ | hypothetical syll. (9), (11) |
| 13. | $\therefore \forall x\,(B(x) \rightarrow \neg L(x))$ | universal gen. (12) |

**Note**: Step 13 works b/c all instances of $a$ were arbitrary

**Example:** Translate the following argument using quantifiers. Then show that the conclusion follows from the premises by logical inference

Babies are illogical

Nobody is despised who can manage a crocodile

Illogical people are despised

---

Therefore, babies cannot manage crocodiles

Let $B(x)$ mean "$x$ is a baby", $L(x)$ mean "$x$ is logical", $C(x)$ mean "$x$ can handle a crocodile", and $D(x)$ mean "$x$ is despised"

Then the argument we want to derive is as follows

$$\begin{array}{ll} & \forall x\,(B(x) \to \neg L(x)) \\ & \forall x\,(C(x) \to \neg D(x)) \\ & \forall x\,(\neg L(x) \to D(x)) \\ \hline \therefore & \forall x\,(B(x) \to \neg C(x)) \end{array}$$

| | step | justification |
|---|---|---|
| 1. | $\forall x\,(B(x) \to \neg L(x))$ | premise |
| 2. | $\forall x\,(C(x) \to \neg D(x))$ | premise |
| 3. | $\forall x\,(\neg L(x) \to D(x))$ | premise |
| 4. | $B(a) \to \neg L(a)$ | universal inst. (1) |
| 5. | $\neg L(a) \to D(a)$ | universal inst. (3) |
| 6. | $B(a) \to D(a)$ | hypothetical syll.(4), (5) |
| 7. | $C(a) \to \neg D(a)$ | universal inst. (2) |
| 8. | $D(a) \to \neg C(a)$ | contraposition of (7) |
| 9. | $B(a) \to \neg C(a)$ | hypothetical syll. (6), (8) |
| 10. | $\therefore\ \forall x\,(B(x) \to \neg C(x))$ | universal gen. (9) |