Introduction to matrix multiplication

BrainScaleS-2 is capable of performing multiply-accumulate (MAC) operations within the analog network core. To multiply a vector with a matrix, the matrix is configured as synapse weights. The vector is encoded as pulse widths which stimulate synapse rows, therefore each synapse multiplies its weight with a vector entry. Neurons accumulate synaptic inputs from their column, thus accumulate the multiplication results from a matrix column.

Principles

No description has been provided for this image

The analog network core consists of a synapse matrix (green dots), a row of neurons (circles on bottom), and a column of synapse drivers (triangles on the left). The synapse drivers stimulate twin-rows of synapses for a variable time. By combining an excitatory and inhibitory synapse, we can allow for signed values in the weight matrix. The synapses send their charge output to the neurons at the bottom, which integrate the currents on the membrane. A columnar analog-to-digital converter (CADC) can digitize the membrane potentials of all neurons in parallel. The on-chip processor (PPU) will then read the result vector obtained by the CADC and could perform further operations with it.

No description has been provided for this image

In this example MAC operation, a vector (31, 5) is to be multiplied with a column of weights (20, -63).

To process the first vector entry, the top synapse driver activates the top twin-row of synapses for a long time as the vector value is 31 (= maximum of our 5-bit range). The positively signed synapse creates a low current as the weight is 20 (maximum would be 63 in our 6-bit range). The neuron receives a long but small current input and the membrane voltage rises.

For the next vector entry, the second synapse driver enables the synapses much shorter as the vector value is only 5. The current generated by the synapse is higher as its weight is maximum. The neuron receives a short but strong current input, but subtracts it from the membrane voltage as the synapse attaches to its inhibitory input.

References for further reading

The hardware perspective and a benchmark on MNIST handwritten digits is published in:

Weis J. et al. (2020) Inference with Artificial Neural Networks on Analog Neuromorphic Hardware. In: Gama J. et al. (eds) IoT Streams for Data-Driven Predictive Maintenance and IoT, Edge, and Mobile for Embedded Machine Learning. ITEM 2020, IoT Streams 2020. Communications in Computer and Information Science, vol 1325. Springer, Cham. https://doi.org/10.1007/978-3-030-66770-2_15

The integration into the PyTorch software frontend `hxtorch` and a benchmark on the human activity recognition dataset is published in:

Spilger P. et al. (2020) hxtorch: PyTorch for BrainScaleS-2. In: Gama J. et al. (eds) IoT Streams for Data-Driven Predictive Maintenance and IoT, Edge, and Mobile for Embedded Machine Learning. ITEM 2020, IoT Streams 2020. Communications in Computer and Information Science, vol 1325. Springer, Cham. https://doi.org/10.1007/978-3-030-66770-2_14

Example

First, we demonstrate how a neuron responds to synaptic currents in integration mode.

By altering the target parameters for calibration, we set the neurons to short synaptic time constants and long membrane time constants. When observing the membrane at the typical time ranges of many milliseconds biological time, we can observe step-like changes in the membrane potential for each synaptic stimulus. Since vector-matrix multiplication has no continuous time, we can send the vector entries much faster than biological spikes, at up to 125 MHz (hardware time, biological equivalent: 125 kHz).

In this example, we generate a few inputs and observe the neuron membrane during integration.

In order to use the microscheduler we have to set some environment variables first:

In [12]:
```
from _static.common.helpers import setup_hardware_client
setup_hardware_client()
```

```
INFO  09:33:54,546  demo_helpers Connection to hxcube7fpga0chip57_1 established
```

In [13]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import ipywidgets as widgets
```

```python
import pynn_brainscales.brainscales2 as pynn
from pynn_brainscales.brainscales2 import Population
from pynn_brainscales.brainscales2.standardmodels.cells import SpikeSourceArray
from pynn_brainscales.brainscales2.standardmodels.synapses import StaticSynapse


def plot_membrane_dynamics(population: Population, segment_id=-1, ylim=None):
    """
    Plot the membrane potential of the neuron in a given population view. Only
    population views of size 1 are supported.
    :param population: Population, membrane traces and spikes are plotted for.
    :param segment_id: Index of the neo segment to be plotted. Defaults to
                       -1, encoding the last recorded segment.
    :param ylim: y-axis limits for the plot.
    """
    if len(population) != 1:
        raise ValueError("Plotting is supported for populations of size 1.")
    # Experimental results are given in the 'neo' data format
    mem_v = population.get_data("v").segments[segment_id].irregularlysampledsignals[0]

    plt.plot(mem_v.times, mem_v, alpha=0.5)
    print(f"Mean membrane potential: {np.mean(np.array(mem_v.base))}")
    plt.xlabel("Wall clock time [ms]")
    plt.ylabel("ADC readout [a.u.]")
    if ylim:
        plt.ylim(ylim)
```

Next we load a nightly calibration which is specifically tuned for the integration of synaptic inputs, for example it targets long membrane time constants. We save this calibration in two variables and use it later to define our neural network:

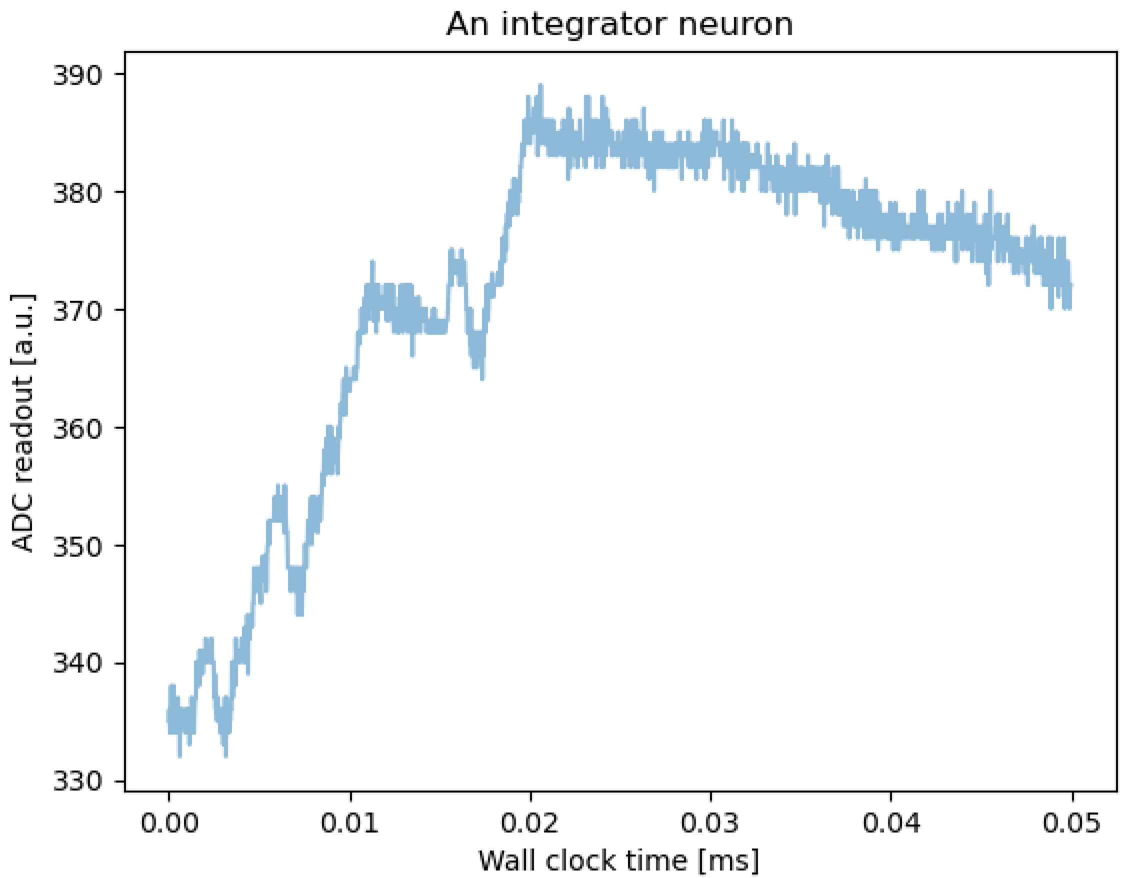
In [14]:
```python
from _static.common.helpers import get_nightly_calibration
calib = get_nightly_calibration("hagen_cocolist.pbin")
```

Now we define our experiment:

In [15]:
```python
def generate_external_inputs(stimulated_population):
    """
    Create off-chip populations serving as excitatory/inhibitory spike sources.
```

```
    Feel free to modify the `{exc,inh}_spiketimes` and the `weight` of the stimulation.

    :param simulated_population: Population to map inputs to.
    """
    exc_spiketimes = [1, 3, 4, 5, 7, 8, 9, 10, 15, 17, 18, 19]  # us
    exc_spiketimes = np.array(exc_spiketimes) / 1e3
    exc_stim_pop = pynn.Population(1, SpikeSourceArray(spike_times=exc_spiketimes))
    pynn.Projection(exc_stim_pop, stimulated_p,
                    pynn.AllToAllConnector(),
                    synapse_type=StaticSynapse(weight=63),
                    receptor_type="excitatory")

    inh_spiketimes = [2, 6, 16]  # us (bio: ms)
    inh_spiketimes = np.array(inh_spiketimes) / 1e3
    inh_stim_pop = pynn.Population(1, SpikeSourceArray(spike_times=inh_spiketimes))
    pynn.Projection(inh_stim_pop, stimulated_p,
                    pynn.AllToAllConnector(),
                    synapse_type=StaticSynapse(weight=-63),
                    receptor_type="inhibitory")

plt.figure()
plt.title("An integrator neuron")

# reset membrane potential before beginning of experiment (it floats otherwise)
pynn.setup(initial_config=calib)

# use calibrated parameters for neuron
silent_p = pynn.Population(2, pynn.cells.HXNeuron())
stimulated_p = pynn.Population(1, pynn.cells.HXNeuron())
generate_external_inputs(stimulated_p)
stimulated_p.record(["v", "spikes"])

pynn.run(50e-3)  # run for 50 us
plot_membrane_dynamics(stimulated_p)
plt.show()
```

```
Mean membrane potential: 371.4507141113281
```




```
WARN  09:44:19,145  lib-rcf.OnDemandUpload.loop_upload Error while uploading: Remote call timeout exceeded. No respon
se from peer.
WARN  09:54:24,977  lib-rcf.OnDemandUpload.loop_upload Error while uploading: Remote call timeout exceeded. No respon
se from peer.
WARN  10:04:30,813  lib-rcf.OnDemandUpload.loop_upload Error while uploading: Remote call timeout exceeded. No respon
se from peer.
WARN  10:14:36,645  lib-rcf.OnDemandUpload.loop_upload Error while uploading: Remote call timeout exceeded. No respon
se from peer.
WARN  10:24:42,477  lib-rcf.OnDemandUpload.loop_upload Error while uploading: Remote call timeout exceeded. No respon
se from peer.
```

In the plot, you can see the integration phase in the beginning and a random drift after all inputs are received. Since the leakage is disabled, we do not decay to a controlled leak potential.

For practial usage, the vector entries will be sent much faster, at a rate of up to 125 MHz, so the membrane has much less time to drift than here. In the beginning, the membrane potential is reset to a known starting voltage and in the end, the potential is digitized immediately after all inputs are received.

We will now use the `hxtorch` software frontend which provides a vector-matrix multiplication on chip and supports the standard layers used in deep neural networks. First, we investigate the characteristics of the analog MAC operation.

In [ ]:

```

```