Dynamics of a mathematical model of cancer cells with chemotherapy

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Reference Paper

To build and analyze a mathematical model that describes the dynamic "battle" between the immune system, cancer cells, and chemotherapy treatment .

- **E(t):** Effector (Immune) Cells :The "Predator" or the body's "army".
- **T(t):** Tumor Cells :The "Prey," which grows logistically (like a population with limited resources)
- **M(t):** Chemotherapy Drug Concentration : An external intervention that is harmful to *both* cancer and immune cells and is cleared from the body over time

The entire system is defined by these three coupled equations:

1. $\frac{dE}{dt} = s + p\frac{ET}{h+T} - mET - \mu E - K_E ME$
   - (Immune source + recruitment) - (Tumor "fight-back" + natural death + chemo damage)
2. $\frac{dT}{dt} = rT(1 - bT) - a\frac{ET}{T+g} - K_T MT$
   - (Tumor logistic growth) - (Immune cleanup + chemo killing)
3. $\frac{dM}{dt} = -\gamma M + V_M(t)$
   - (Drug clearance) + (Drug administration)

The Improvisation (Stochastic & Network-Hybrid Model)

Core Idea: To evolve the model to a more biologically realistic model.

Step 1: Introducing Stochasticity (SDE Model)

- The deterministic ODEs were converted into Stochastic Differential Equations (SDEs).

$$dY = f(Y)dt + G(Y)dW(t)$$

$$dE = (s + p\frac{ET}{h+T} - mET - \mu E - K_E ME)dt + \sqrt{s + p\frac{ET}{h+T} + mET + \mu E + K_E ME}dW_1(t)$$
$$dT = (rT(1-bT) - a\frac{ET}{T+g} - K_T MT)dt + \sqrt{|rT(1-bT)| + a\frac{ET}{T+g} + K_T MT}dW_2(t)$$
$$dM = (-\gamma M + V_M(t))dt + \sqrt{\gamma M + V_M(t)}dW_3(t)$$

The Improvisation (Stochastic & Network-Hybrid Model)

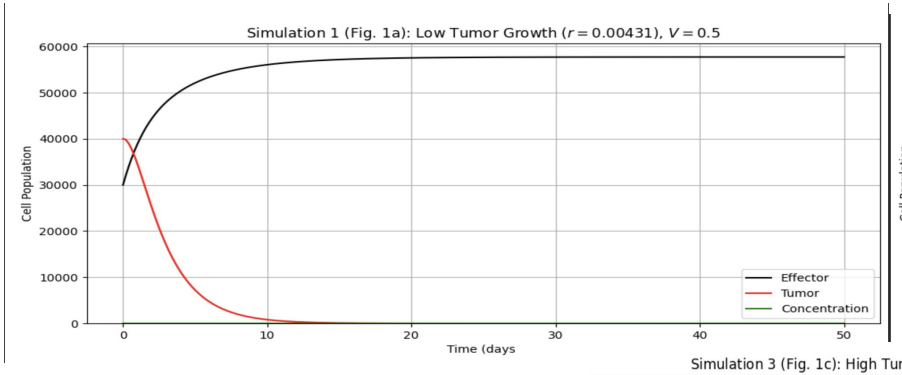
Integrating Network Science (Hybrid Model)

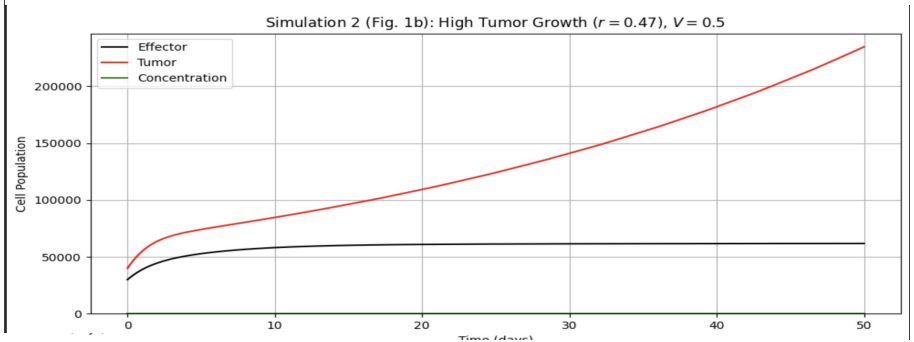
- The bulk tumor population T was replaced with an **agent-based model** using a networkx graph.
- **Nodes** = **Individual Tumor Cells**
- **Edges** = **Physical Adjacency** (cells within a certain radius).
- The global E (Immune) and M (Chemo) populations (from the SDE) now interact with this local network.

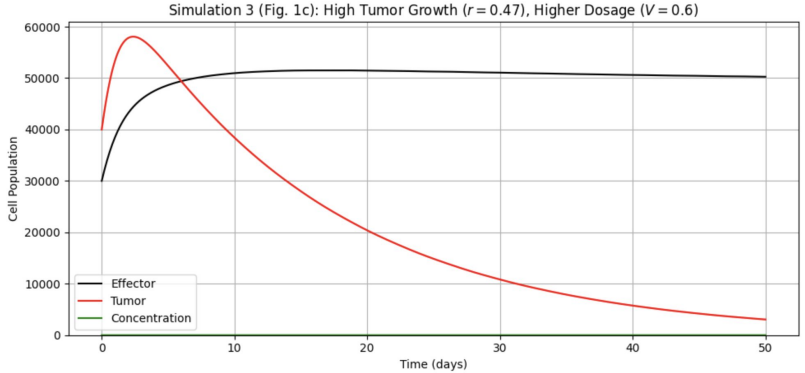
New Spatially-Aware Rules:

1. **Immune Attack (Surface vs. Core):** E cells no longer attack all T cells. They preferentially attack the tumor **"surface"** (nodes with a low network degree), leaving the **"core"** (high-degree nodes) shielded.
2. **Chemo Penetration (Gradient):** M concentration is not uniform. It is highest at a **"source node"** (representing a blood vessel) and it's killing effect decays exponentially with network distance (shortest_path_length).
3. **Tumor Growth:** Implemented agent-based proliferation using Lestari's r and b parameters, allowing living nodes to divide and add new nodes to the graph.

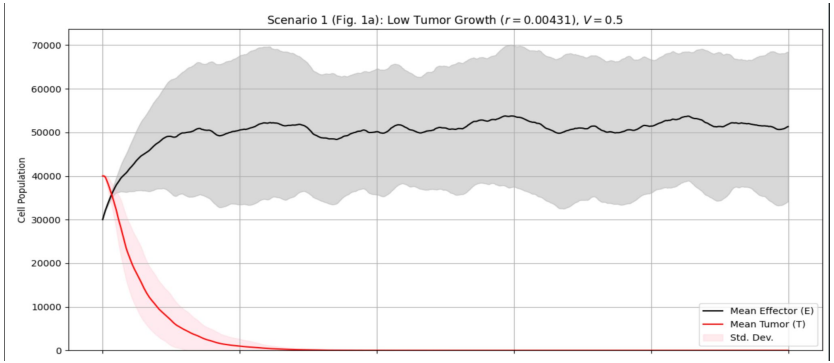
Cancer Chemotherapy model simulation

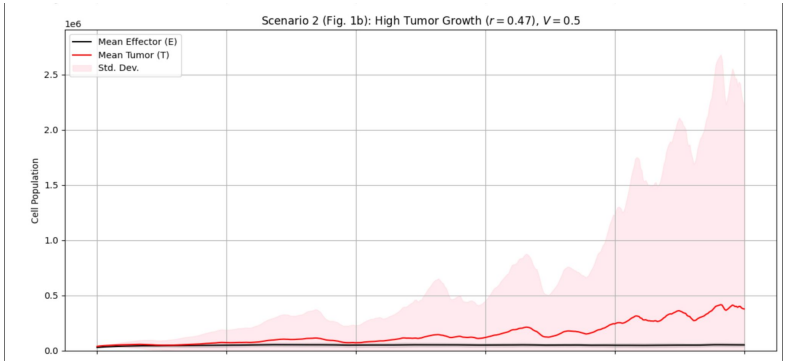


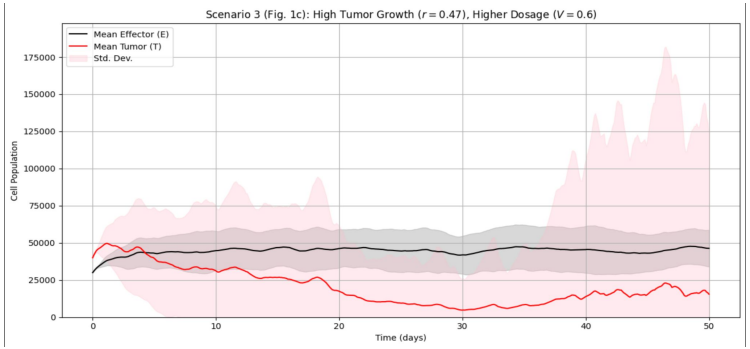


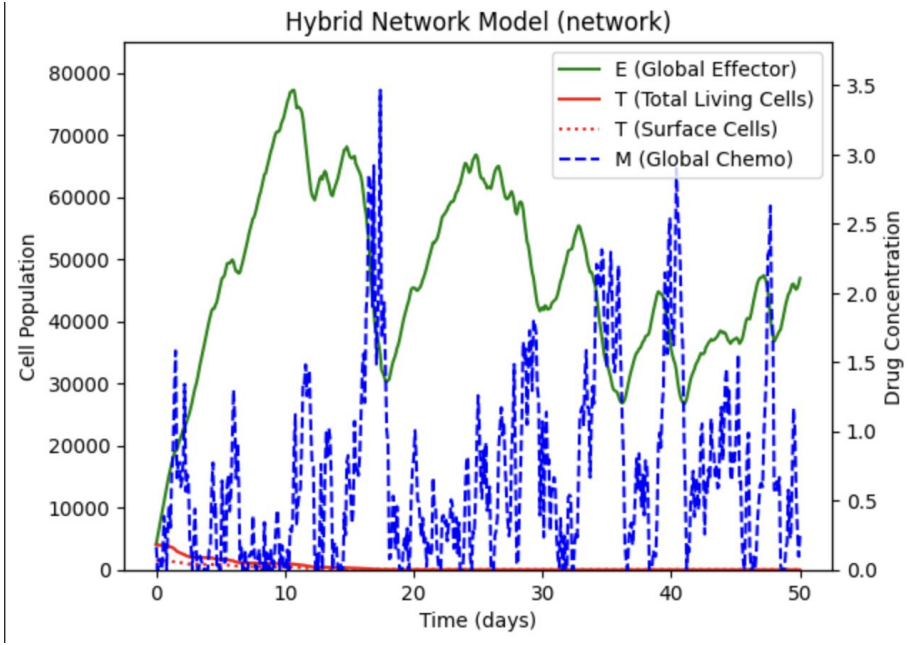


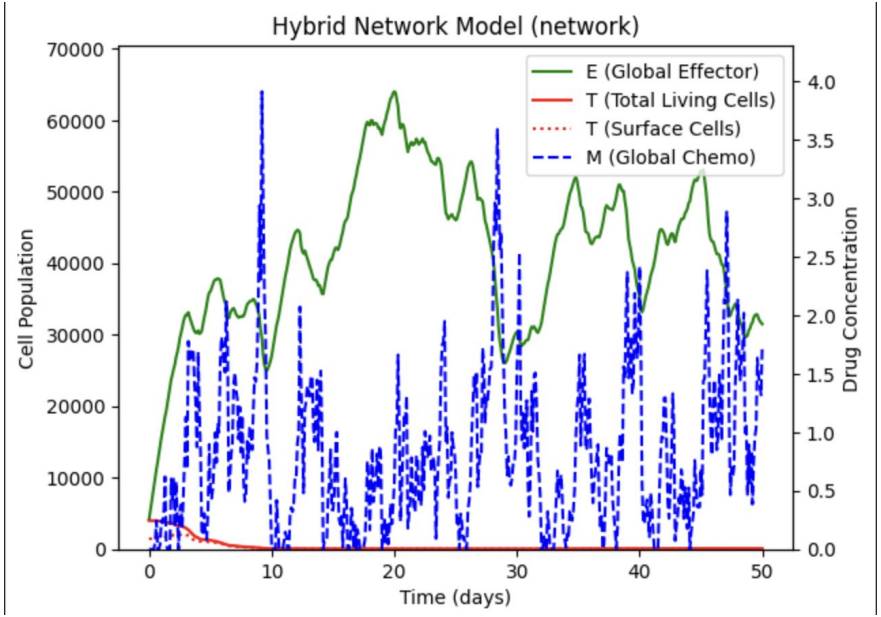
SDE simulation of Cancer Chemotherapy model








Hybrid Network Model (network)
E (Global Effector)
T (Total Living Cells)
T (Surface Cells)
M (Global Chemo)
Cell Population
Drug Concentration
Time (days)
0
10000
20000
30000
40000
50000
60000
70000
80000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
10
20
30
40
50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2.5
3.0
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20
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40
50


Thank
you!