Application of Dynamic Programming to Solving Reservoir Operational Problems

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Practical Case Study

APPLICATION OF DYNAMIC PROGRAMMING TO SOLVING RESERVOIR OPERATIONAL PROBLEMS

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Abstract: Dynamic programming approach offers an exact solution to solving complex reservoir operational problems. In this research analysis, an attempt was made to evaluate the relevance of dynamic programming as an optimization tool. A brief overview including the characteristics, advantages and disadvantages of dynamic programming model was understudied. The focus was on the application of dynamic programming to handling the optimal allocation of the available water resources. Cases of large scale reservoir expansion problems were also considered and finally the optimal release policy for reservoir operations. The end result of the model formulation reveals the applicability of dynamic programming in resolving long time operational, water allocation and expansion problems in reservoir dynamics, operations and maintenance.

Keywords: Dynamic programming, optimal solution, reservoir operation modeling, water allocation

INTRODUCTION

A reservoir is an artificial lake used to store water. Reservoirs may be created in river valleys by the construction of a dam or may be built by excavation in the ground or by conventional construction techniques such a brickwork or cast concrete. The term reservoir may also be used to describe underground reservoirs such as an oil or water well [1].

Some of the applications of reservoir include; direct water supply, hydroelectricity, controlling water courses (Downstream water supply, Irrigation and flood control) and for recreational activities. Some of the known detrimental effects of reservoir include impoundment of free-flowing river habitat, blockage of fish migration and reduced water quality in downstream river reaches. Less-obvious effects include the interruption of geomorphologic processes that maintain aquatic habitat diversity required to sustain healthy riverine ecosystems [2].

The role of optimization in reservoir operation has increased substantially. Optimization models played a relatively minor role in the past, with reservoir releases operated mainly on the basis of predefined rules tested using simulation models. At present, most major river systems use optimization to identify the preferred release schedule, and refine this schedule using simulation. Even small river systems are now using optimization-based decision support systems. Because of this trend to rely on optimization, better methods are needed to incorporate ecological values as objectives. Among the approaches we reviewed, only real-time, minimum-flow-constrained methods using optimal control are actually in use. It is unlikely that any of the methods that incorporated population or water quality models is currently used to operate reservoirs [2].

Some of the optimization tool mostly employed in solving reservoir operation problems include; dynamic programming, Lagrange multiplier and linear programming approach. In terms of mathematical optimization, dynamic programming usually refers to a simplification of a decision by breaking it down into a sequence of decision steps over time. This is done by defining a sequence of value functions V_1, V_2, V_n with an argument (y) representing the state of the system at times i from 1 to n. The definition of $V_n(y)$ is the value obtained in state y at the last time n. The values V_i at earlier times I = n-1, n-2, 2, 1 can be found by working backwards, using a recursive relationship called the Bellman equation [3].

As a computer programming method, dynamic programming is mainly used to tackle problems that are solvable in polynomial terms. There are two key attributes that a problem must have in order for dynamic programming to be applicable: optimal substructure and overlapping subproblems [4]. Optimal substructure means that the solution to a given optimization problem can be obtained by the combination of optimal solutions to its sub problems. Consequently, the first step towards devising a dynamic programming solution is to check whether the problem exhibits such optimal substructure. Such optimal substructures are usually described by means of recursion. Overlapping subproblems means that the space of subproblems must be small, that is, any recursive algorithm solving the problem should solve the same subproblems over and over, rather than generating new subproblems [5].

CHARACTERISTICS OF DYNAMIC PROGRAMMING PROBLEMS

- A single n variable problem can be divided into n problems of single variable, provided the objective function of the optimization problem is separable with respect to stage, for example $R_1(x_1) + R_2(x_2)$ is a separable function
- The problem can be divided into stages with a policy required at each stage. In water allocation problems, each individual user constitute a stage and the amount of water allocated at a stage constitute a policy decision.
- With each stage, a number of states are associated. In water allocation problems, the quantity of water available at a stage for distribution defines the state at that stage.
- The policy decision transforms the present state into a state associated with the next stage.

- At a given stage for a given state, the recursive relationship identifies the optimal decision, given the optimal decision for each state at the previous stage
- The solution moves back ward or forward stage by stage, till optimal decision for the last stage is determined. From this solution the optimal decision for other stages are determined[6]

Advantages of Dynamic Programming

- In dynamic programming unlike linear programming, the formation for linear and non linear problems is the same. Thus no extra labour is required for non linear problems.
- In dynamic programming, the incorporation of constraints is easier than linear programming and non linear programming. The constraints serve a useful purpose. In this case, the constraints limit the feasible region and reduce the computational time.
- The stochastic nature of the problem can be easily considered in dynamic programming

Disadvantages of Dynamic Programming

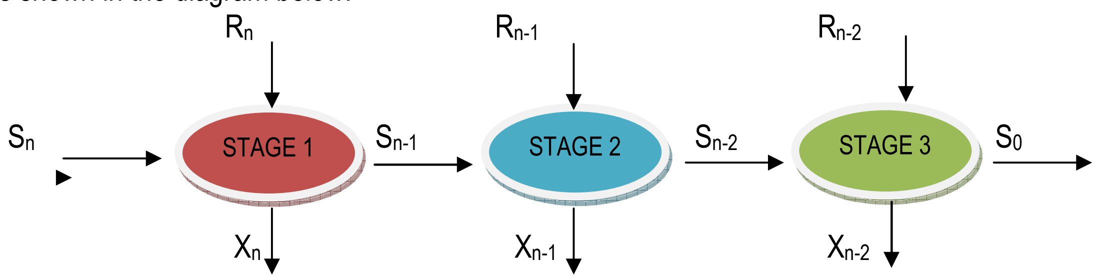
- In dynamic programming, there are no set procedures (algorithm) to solve any decision or allocation problem
- Dynamic programming cannot provide one time period (single stage) solution to problems unlike linear programming
- Design and formulation of recursive equations can be very complex and frustrating

RESERVOIR OPERATIONAL PROBLEMS

Operators of dams and reservoir are normally faced with multiples of problems ranging from; problems of water distribution, water allocation, release policies and reservoir expansion problems. These problems are natural and need to be appropriately addressed to allow for effective water distribution and allocation. One way to handle them is to model the problem into a dynamic programming format and then use the forward or backward dynamic programming approach to solve them. By this, it will be possible to determine the optimum release policy that will guarantee proper allocation of the available water resources. If water from a reservoir is to be distributed among n users for example, then each user should define one stage in the multistage decision problem. The water allocated to a particular user (i), should constitute the decision X_i at that stage. Using the following basic assumptions:

- Let S_n = Input to stage n
- X_n = Decision taken at stage n
- R_n = Return at stage n corresponding to the decision X_n for the input S_n

Consider a situation where (d) quantities of water are to be distributed; the input to the last stage n, i.e. (S_n) the water allocated to the user n, water available at stage (n+1) will be $S_n – X_n$. This relation defines the stage transformation. $S_n – 1$ forms the input to stage n – 1 and its equal to the amount of water to be allocated to all the remaining stages including the stage n – 1. This is shown in the diagram below:



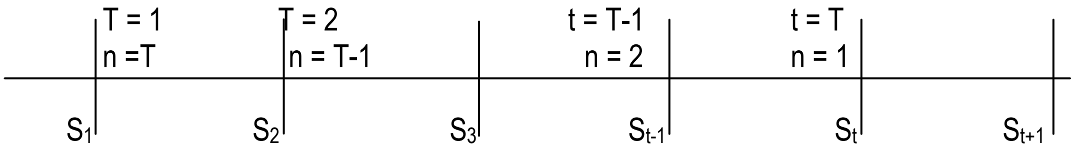
The solution of dynamic programming problems is based on Richard Bellman's principle of optimality; an optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal decision with respect to the state resulting from the initial decisions. The principle implies that, to arrive at an optimal decision at any stage, the previous stages must also be optimal. Given the state S_i of the system at stages i, one must proceed optimally till the last stage, irrespective of how one reached the state Si. The solution procedure using this principle involves the division of the original problem with n decision variables into n sub problems, each with one decision variable.

OPERATIONAL METHODOLOGY

Module One: Optimum Release Policy of Reservoir

The operational objective to maximize the total net benefits during a certain period in years can be formulated as: Maximize $\sum_{t=1}^{T} B_t(S_t R_t)$

Where B_t (S_t, R_t) is the net benefit during the period't' for given value of S_t and R_t, T is the number of record in the year. Using the backward recursive dynamic programming approach, one will be able to compute the actual release policy that will maximize the objective function.



From the diagram, n denotes the stage of dynamic programming, $f_t^{''}$ (S_t) is defined as the maximized net benefits up to and including the period't'.

In stage 1, n = 1 and t = T the total time period. The problem is solved only for one period and f_1^T (S_t) is obtained for all possible values of 'S_t' as

$$f_1^T (S_t) = \text{maximize } (B_t S_t, R_t)$$
$$0 \le R_T \le S_T + Q_T$$
$$S_T + Q_T - R_T \le K$$

Where

B_T = Benefits in the period T
R_T = Release of water during the period T
S_T = Storage of water during the period T
Q_T = Inflow of water during the period T
K = Live storage capacity of the reservoir

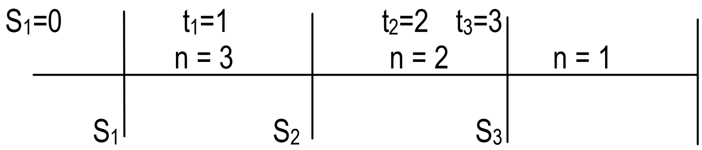
The two constraints $0 \le R_T \le S_T + Q_T$ and $S_T + Q_T - R_T \le K$, specify the feasible values for the release R_T over which the search is made.

Consider a reservoir of known capacity say 3 units; assume that inflows into the reservoir are given as 2, 1, and 3 units respectively. The release during a given season resulted in the net benefits as shown in the table below.

Table 1: Net benefit function for a certain release policy

S/No	Release of water	Benefits
1	0	-100
2	1	200
3	2	340
4	3	450
5	4	650
6	5	650
7	6	420

To determine the release policy, backward recursion has been adopted starting with the last stage.



STAGE 1

Here t = 3 and n = 1 and Q_3 = 3

$$f_3^1(S_3) = \text{Max.}\,[B_3(R_3)]$$
$$0 \le R_3 \le S_3 + Q_3$$
$$S_3 + Q_3 - R_3 \le 3$$

Table 2: Optimal Solution at Stage n =1

S_3	R_3	$B_3(R_3)$	$f_3^1(S_3)$	R_3^*
	0	-100		
0	1	200	450	3
	2	340		
	3	450		
	0	-100		
	1	200	650	4
1	2	340		
	3	450		
	4	650		
	1	200		
	2	340		
2	3	450	650	4, 5
	4	650		
	5	650		
	2	340		
	3	450		
3	4	650	650	4, 5
	5	650		
	6	420		

STAGE 2

Here t = 2 and n = 2 and $Q_2 = 1$

$$f_2^2 (S_2) = \text{Max.} (B_2 (R_2) + f_3^1 (S_2 + Q_2 - R_2)$$
$$0 \le R_2 \le S_2 + Q_2$$
$$S_2 + Q_2 - R_2 \le 3$$

Table 3: Optimal Solution at Stage n =2

S_2 (1)	R_2 (2)	$B_2 (R_2)$ (3)	$S_2 + Q_2 - R_2$ (4)	$f_3^1 (S_2 + Q_2 - R_2)$ (5)	Col. (3 + 4) (6)	$f_2^2 (S_2)$ (7)	R_2^* (8)
0	0	-100	1	650	550		
	1	200	0	450	650	650	1
1	0	-100	2	650	550		
	1	200	1	650	850	850	1
	2	340	0	450	790		
2	0	-100	3	650	550		
	1	200	2	650	850		
	2	340	1	650	990	990	2
	3	450	0	450	900		
3	1	-100	3	650	550		
	2	200	2	650	850		
	3	340	1	650	990	990	3
	4	450	0	450	900		

STAGE 3

Here t = 1 and n = 3 and $Q_2 = 2$

$$f_1^3 (S_1) = \text{Max.} (B_1 (R_1) + f_2^2 (S_1 + Q_1 - R_1)$$
$$0 \le R_1 \le S_1 + Q_1$$
$$S_1 + Q_1 - R_1 \le 3$$

Table 4: Optimal Solution at Stage n =3

S_1 (1)	R_1 (2)	$B_1 (R_1)$ (3)	$(S_1 + Q_1 - R_1)$ (4)	$f_2^2 (S_1 + Q_1 - R_1)$ (5)	Col. (3 + 4) (6)	$f_1^3 (S_1)$ (7)	R_1^* (8)
0	0	-100	2	990	850		
	1	200	1	850	1050	1050	1
	2	340	0	650	990		

From the calculation of stage 3, tracing back, we get

$R_1^* = 1$, from the last table of stage 3

$S_2' = S_1 + Q_1 - R_1^* = 0 + 2 - 1 = 1$

Corresponding $S_2' = 1$, $R_2^* = 1$ from stage 2 table

$S_3' = S_2' + Q_2 - R_2^* = 1+1 -1 = 1$

$R_3^* = 4$ from stage 1table. Thus the optimal release is 1, 1, and 4 during the three periods as shown in figure 1 below.

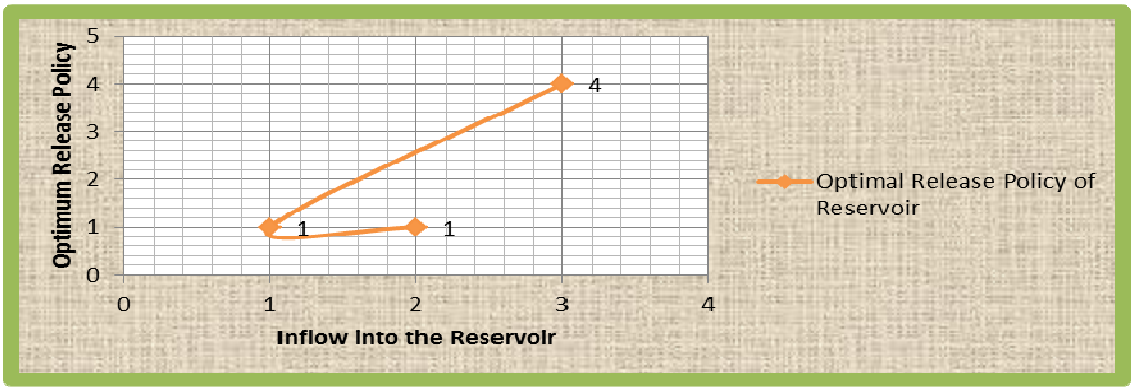


Fig 1: Optimum Release Policy of a Reservoir

From this release policy, the maximum net benefits resulted to 1050 units from the objective function therefore, f_1^3 (S_1) = 1050.

Module Two: Optimum Allocation Policy of Available Water

One most important problem associated with reservoir operation is how to allocate the available unit of water so as to maximize the objective function. Assumed that ten (10) units of water are to be allocated to three different faming communities whose benefit function equation are given as follows:

Community 1: $12X_1 – X_1^2$

Community 2: $8X_2 – X_2^2$

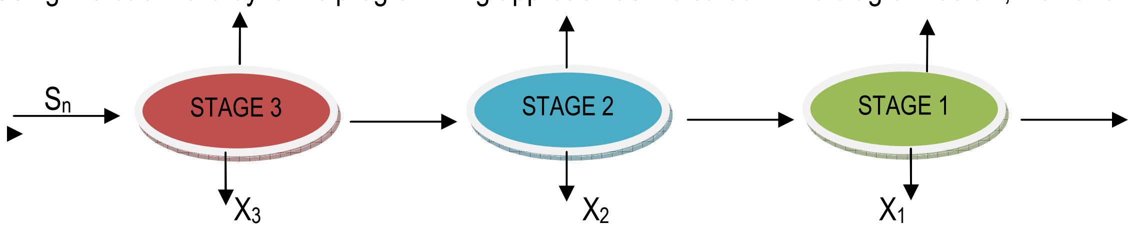
Community 3: $18X_3 – X_3^2$

Using discrete values: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and applying the benefits function, the table below was obtained

Table 5: Benefit function for a certain water allocation policy

S (unit of water)	Community 1	Community 2	Community 3
0	0	0	0
1	11	7	15
2	20	12	24
3	27	15	27
4	32	16	24
5	35	15	15
6	36	12	0
7	35	7	-21
8	32	0	-48
9	27	-9	-81
10	20	-20	-120

Using the backward dynamic programming approach as indicated in the diagram below, we have



Let the benefit function of community 3 be the optimum stage as shown in table three below.

Table 6: Benefit function for the optimal stage n = 3

S (unit of water)	Optimum Benefit (f^{3*})	Optimum Benefit (X_3*)
0	0	0
1	15	1
2	24	2
3	27	3
4	24	4
5	15	5
6	0	6
7	-21	7
8	-48	8
9	-81	9
10	-120	10

The optimum equation in stage two is given as
$f_2^*(S) = f_2(X_2) + f_3^*(S – X_2)$

Table 7: Benefit function for the optimal stage n = 2

X_2 S	0	1	2	3	4	5	6	7	8	9	10	f_2^*(s)	X_2^*
0	0	-	-	-	-	-	-	-	-	-	-	0	0
1	15	7	-	-	-	-	-	-	-	-	-	15	0
2	24	22	12	-	-	-	-	-	-	-	-	24	0
3	27	31	27	15	-	-	-	-	-	-	-	31	1
4	24	34	36	30	16	-	-	-	-	-	-	36	2
5	15	31	39	39	31	15	-	-	-	-	-	39	2 or 3
6	0	22	36	42	40	30	12	-	-	-	-	42	3
7	-21	7	27	39	43	39	27	7	-	-	-	43	4
8	-48	-14	12	30	40	42	36	22	0	-	-	42	5
9	-81	-41	-9	15	31	39	39	31	15	-9	-	39	5 or 6
10	-120	-74	-36	-6	16	30	36	34	24	6	-20	36	6

The optimum equation is given as: $f_1^*(S) = f_1(X_1) + f_2^*(S – X_1)$

Table 8: Benefit function for the optimal stage n = 1

X_2	0	1	2	3	4	5	6	7	8	9	10	f_1^*(s)	X_1^*
S	36	50	62	70	74	74	72	66	56	42	20	74	4 or 5

Optimum benefit = 74 corresponding to the optimal decision X_1 = 4 or 5
When X_1 = 4, there remain 10 – 4 unit of water i.e. S = 6
When X_1 = 5, there remain 10 – 5 unit of water i.e. S = 5
When S = 6 in stage two, optimal decision X_2 = 3
When S = 5 in stage two, optimal decision X_2 = 2 or 3

When $X_2 = 3$, there remain 6 – 3 unit of water i.e. S = 3
When $X_2 = 2$, there remain 5 – 2 unit of water i.e. S = 3
When $X_2 = 3$, there remain 5 – 3 unit of water i.e. S = 2
When S = 3 in stage one, optimal decision $X_3 = 3$
When S = 3 in stage one, optimal decision $X_3 = 3$
When S = 2 in stage one, optimal decision $X_3 = 2$
Thus the optimal allocation policy of the 10 units of water to the three different communities is given as shown below:

S/No	Community One		Community Two	Community Three
1	4 units		3 units	3 units
		Or		
2	5 units		2 units	3 units
		Or		
3	5 units		3 units	2 units

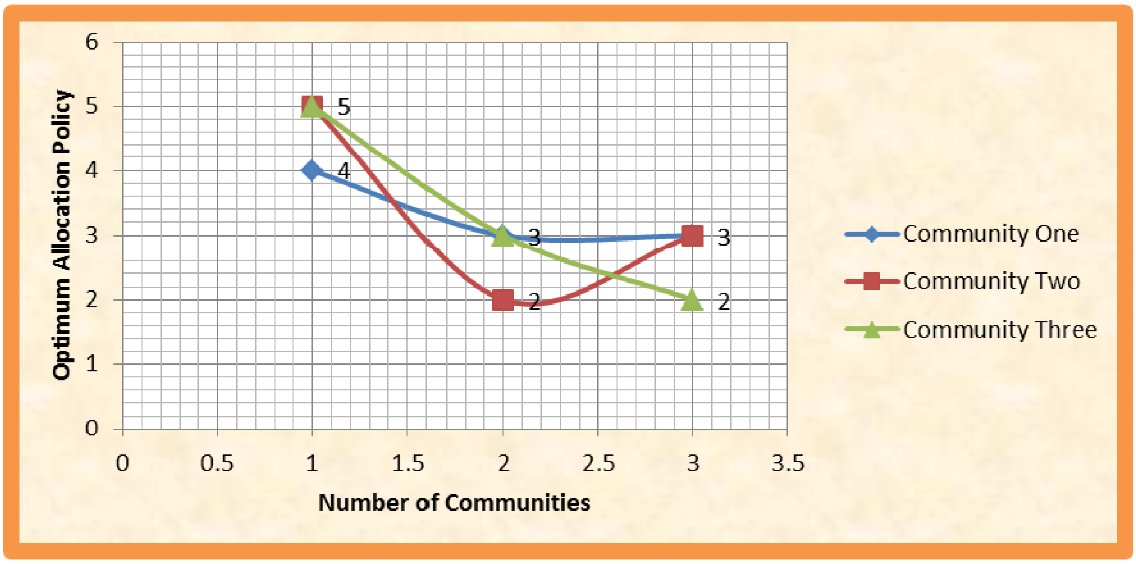


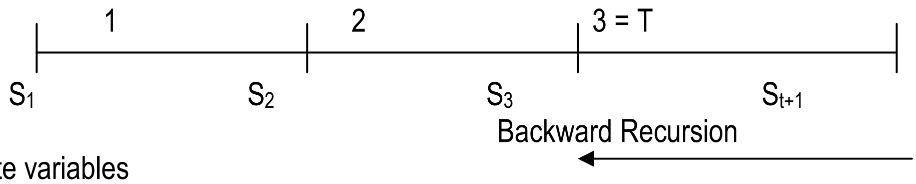
Fig 2: Optimum Allocation Policy for Available Water Resources.

Module Three: Optimum Policy for Reservoir Expansion

A lot of uncertainties accompany investement involving the expansion of an already existing project. The decision have to be taken of investments at different times starting with the present. Usually such decisions are taken on 5 years duration for investment and the position is reviewed and further decision is taken and so on. As the decisions are to be taken on the investments that are actually needed to be incurred in future, actually a great deal of uncertainty exists about the economics and the actual capacity needed at different times with such investment decisions. For solving capacity expansion problem with dynamic programming, terms used in the program must be defined first. Consider a case where the capacity of a reservoir is to be increased from the initial 15 units to 18 units, 24 units and 30 units at the end of 6, 12 and 18 years after commissioning so that increased demand are always met, the discounted present worth of cost are given in the table below.

Table 9: Discount present water of cost for additional capacity

STAGE		Additional Capacity					
		0	3	6	9	12	15
	Period in years						
1	1 – 6	0	22	39	87	131	283
2	7 – 12	0	41	116	95	422	-
3	13 – 18	0	156	344	-	-	-



Let S = State variables

X = Decision variables

The problem can then be solved starting from the last period (T): [13 – 18] using backward dynamic programming approach.

$f_3(S_3) = \min [C_3X_3]$

$24 \le S_3 \le 30$

$S_3 + X_3 = 30$

Table 10: Backward Recursive Analysis for stage n = 3

S_3	X_3	C_3X_3	$f_3(S_3)$	X_3^*
24	6	344	344	6
27	3	156	156	3
30	0	0	0	0

$f_2(S_2) = \min [C_2(X_2) + f_3(S_2 + X_2)]$

$18 \le S_2 \le 30$

$24 \le S_2+X_2 \le 30$

Table 11: Backward Recursive Analysis for stage n = 2

S_2	X_2	$C_2(X_2)$	$S_2 + X_2$	$f_3(C_2+X_2)$	$C_2(X_2) + f_3(C_2+X_2)$	$f_2(S_2)$	X_2^*
18	3	41	21	-	-		
	6	116	24	344	460	251	9
	9	95	27	256	251		
	12	422	30	0	422		
21	0	0	21	-	-		
	3	41	24	344	385	95	9
	6	116	27	156	272		
	9	95	80	0	95		
24	0	0	24	344	344		
	3	41	27	156	197	116	6
	6	116	30	0	116		
27	0	0	27	156	156		
	3	41	30	0	41	41	3
30	0	0	30	0	0	0	0

$f_1(S_1) = \min[C_1(X_1) + f_2(S_1+X_1)]$
$S_1 = 15$
$18 \leq S_1 + X_1 \leq 30$

Table 12: Backward Recursive Analysis for stage n = 1

S_1	X_1	$C_1(X_1)$	$S_1 + X_1$	$f_2(C_1+X_1)$	$C_1(X_1) + f_2(C_1+X_1)$	$f_1(S_1)$	X_1^*
15	3	22	18	422	444		
	6	39	21	95	134		
	9	87	24	116	203	134	6
	12	131	27	41	172		
	15	283	30	0	283		

Tracing back for minimum value of column (6) we obtain the value of X_1^*, X_2^*, and X_3^* and the capacities. For minimum due to column (6), the value of additional capacities is 6 units. Taking additional capacity as 6, the values of X_1^*, X_2^*, and X_3^* are shown below.

	X_1^*	X_2^*	X_3^*
	6	9	0
Capacity	15	21	30

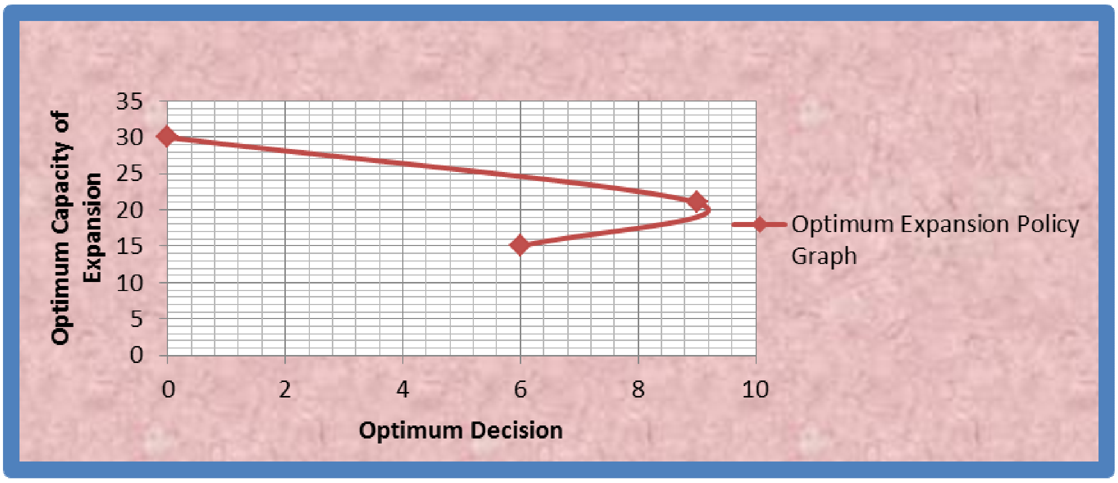


Fig 3: Optimum Expansion Policy Graph

The optimum investment is 134 units.

CONCLUSION

The applicability of dynamic programming to resolving complex reservoir operational problems cannot be over emphasized. Dynamic programming offers a unified approach to solving multi stage reservoir problems. Central to the methodology is the ability to model the problem into a stage wise format, thereafter the optimal solution is obtained via solving the Bellman's optimality equation. The domain of the model equation is the state space of the system to be controlled or optimized. The ability to simulate unified constraint to the problem under study justifies the workability of the dynamic programming model. In this research model, suitable constraint including the cost of reservoir expansion in present worth and water availability has been fully evaluated.

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