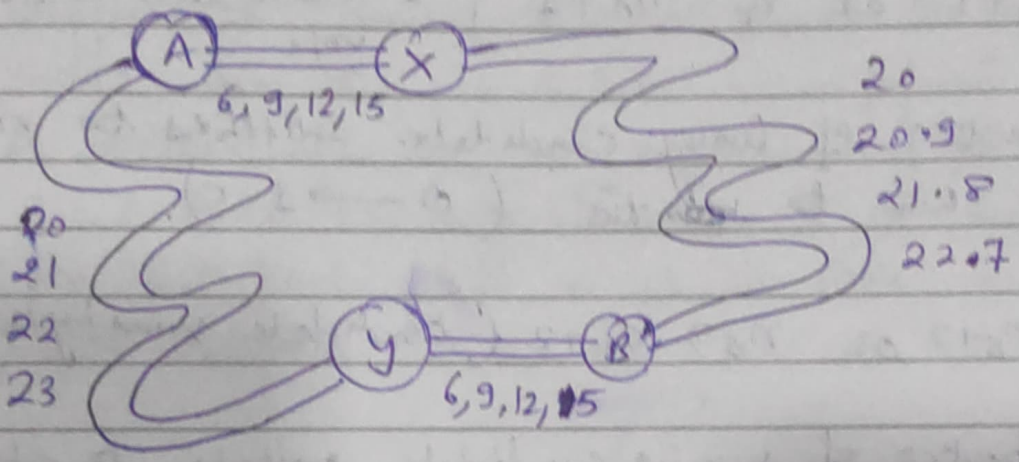
Example 20: Choosing a route

Four people must drive from A to B at the same time. 2 routes are available, one via X and one via Y.



The roads from A to X and Y to B are both short and narrow; one car takes 6 minutes, each additional car increases the travel time per car by 3 minutes. (Eg: If 2 cars drive from A to X, each car takes 9 minutes).

The roads from A to Y and X to B are long and wide, on A to Y one car takes 20 min, and each additional car increases time per car by 1 minute; on X to B one car takes 20 min. and each additional car increases travel time per car by 0.9 min.

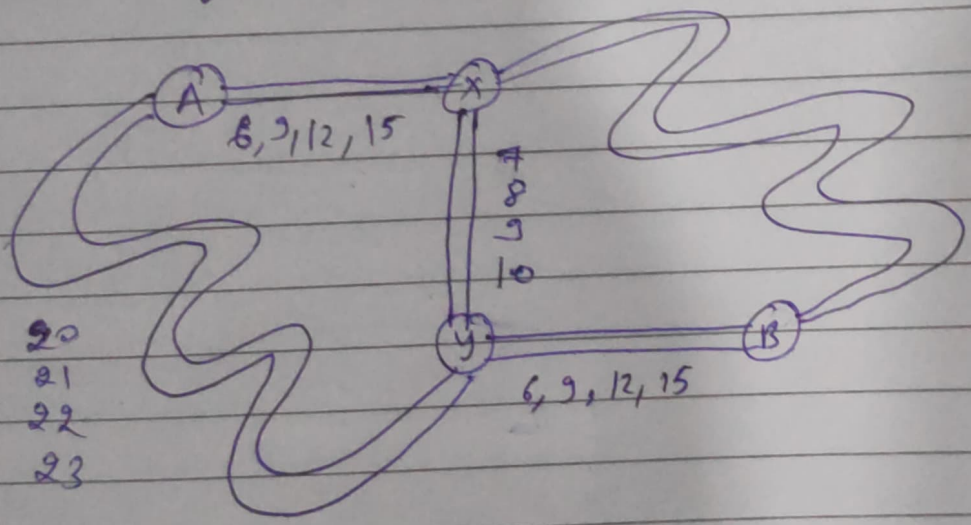
Strategic Game:

Players: 4 people
Actions: {X, Y}
Preferences: Each players' pay off is the negative of travel time.

Nash equilibrium: 2 people take each route. (For any other case, a person taking popular route is better off switching to other route).

Each person's travel time is 29.9 or 30 minutes.
(X) (Y)

Now suppose that a relatively short, wide road is built from X to Y, giving each person 4 options for travel from A to B: A-Y-B, A-X-B, A-X-Y-B, A-Y-X-B. Assume that a person who takes A-X-Y-B travels the A-X portion at the same time as someone who takes A-X-B, and Y-B portion at the same time as A-Y-B. On the road b/w X and Y, one car takes 7 minutes and each additional car increases time for car by 1 minute. Find the Nash equilibria in the new situation. Compare 2 situations.



Earlier, 2 people take route A-X-B and 2, A-Y-B, resulting in total time for each = 29.9 or 30 min.
However, if a person taking A-X-B switches to A-X-Y-B, then his total time = 9 + 7 + 12 = 28 min.
Hence, there is no equilibrium in which new road is not used.

Nash equilibrium: one person takes A-X-B, 2 people takes A-X-Y-B, and one person takes A-Y-B.
Each person's travel time = 32 minutes.
Thus, in the equilibrium with new road every person's travel time increases, from either 29.9 or 30 to 32 minutes.