

### Example 19: Voter participation

Two candidates, A and B, compete in an election of the  $n$  citizens,  $k$  support A and  $m(n-k)$  support B. Each citizen decide to vote, at a cost  $c$ , for the candidate he supports, or to abstain. A citizen who abstains receives payoff of 2 if the candidate he supports wins, 1 if this candidate ties for first place, 0 if he loses. A citizen who votes receives  $2-c$ ,  $1-c$ ,  $-c$  in 3 cases. ( $0 < c < 1$ )

(a) for  $k=m=1$

2 Citizens:

Citizen 1 \ Citizen 2		
	Vote	Abstain
Vote	$1-c, 1-c$	$2-c, 0$
Abstain	$0, 2-c$	$1, 1$

This game is same as Prisoner's Dilemma  
(Abstain  $\equiv$  Quiet, Vote  $\equiv$  Fight)

(b)  $k=m$ , find Nash equilibria.

(let  $n_A \Rightarrow$  votes for A,  $n_B \Rightarrow$  votes for B.)

The cases in which  $n_A \leq n_B$  are symmetric with those in which  $n_A \geq n_B$ .

- $n_A = n_B = k = m$  (all citizens vote): A citizen who switches from voting to abstaining causes candidate he supports to lose. So it is Nash equilibrium.

- $n_A = n_B \leq k$  (not all citizens vote; the candidate tie):  
A citizen who switches from abstaining to voting, causes candidate he supports to win. ( $1 \rightarrow 2-c$ ).

- $n_A = n_B + 1$  or  $n_B = n_A + 1$  (a candidate wins by one vote):

A supporter of losing candidate switches to voting causing him to ~~lose~~ tie. ( $0 \rightarrow 1-c$ ).

- $n_A \geq n_B + 2$  or  $n_B \geq n_A + 2$  (candidate wins by 2 or more votes):

A supporter of winning candidate switches to abstaining, does not affect the outcome. So this is not a strict Nash equilibrium. Also ( $2-c \rightarrow 2$ ).

1. Nash equilibrium: all ~~candidate~~ citizens vote

(c)  $k < m$

There is no Nash equilibrium.

- $n_A = n_B \leq k$ : A supporter of B switches to voting causing B to win rather than tie. ( $1 \rightarrow 2-c$ ).
- $n_A = n_B + 1$  or  $n_B = n_A + 1$ : A supporter of losing candidate switches to voting causes tie. ( $0 \rightarrow 1-c$ ).
- $n_A \geq n_B + 2$  or  $n_B \geq n_A + 2$ : A supporter of winning candidate switches to abstaining ( $2-c \rightarrow 2$ ) does not affect outcome.