Example 18: Guessing 2/3rd of the average

Each of 3 people announces an integer 1 to K. If the 3 integers are different, the person whose integer is closest to $\frac{2}{3}$ of the average of the integers wins \$1. If 2 or more integers are the same, \$1 is split equally b/w the people whose integer is closest to $\frac{2}{3}$ of the average integer. Is there any integer k such that the action profile (k, k, k) in which every person announces the same integer k, is a Nash equilibrium?

If all 3 players announce same integer $k \geq 2$ then any one of them can deviate to $k-1$ and obtain \$1 rather than \$1/3. Thus no such action profile is a Nash equilibrium. If all 3 players announce 1, then no player can deviate and increase his payoff; thus (1, 1, 1) is a Nash equilibrium.

Now, consider an action profile in which not all 3 integers are the same; denote the highest by k^*.

- Suppose only one player names k^*, other integers are k_1 and k_2, with $k_1 \geq k_2$.

Average $= \dfrac{k_1 + k_2 + k^*}{3}$,

$$\left(\frac{2}{9}\right)(k_1 + k_2 + k^*) = \frac{2}{3} \text{ of Average}$$

If $k_1 \geq \frac{2}{9}(k^* + k_1 + k_2)$ then k^* is farther away, hence does not win.

If $k_1 < \frac{2}{9}(k^* + k_1 + k_2)$ then the difference b/w

k^* and $\frac{2}{9}(k^* + k_1 + k_2) = \frac{7}{9}k^* - \frac{2}{9}k_1 - \frac{2}{9}k_2$,

while the difference b/w k_1 and $\frac{2}{9}(k^* + k_1 + k_2) =$

$\frac{2}{9}k^* - \frac{7k_1}{9} + \frac{2k_2}{9}$.

Difference b/w former and latter is $= \frac{5}{9}k^* + \frac{5k_1}{9} - \frac{4}{9}k_2$

Which is greater than 0, so k_1 is closer to $\frac{2}{3}$ of ave. than is k^*. Hence the player who names k^* does not win, and is better off naming k_2, in which he obtains a share of the prize. Thus no such action profile is a Nash equilibrium.

- Suppose 2 player name k^*, and the third player $k < k^*$.

Average $= \frac{2k^* + k}{3}$

Now, $\frac{2}{3}\left(\frac{2k^* + k}{3}\right) = \frac{4}{9}k^* + \frac{2}{9}k < \frac{1}{2}(k^* + k)$,

so that the player who names k is the sole winner. Thus either of the other players can switch to naming k and obtain a share of prize. Thus no such action profile is Nash equilibrium.

Hence, Conclusion: only one Nash equilibrium. $(1,1,1)$

This game is studied experimentally by Nagel.