POMDP PART 1
2019121010

X = 0.89
Y = 1
P(o = red | state = red) = 0.80
P(o = green | state = green) = 0.95

States = [‘R’ , ‘R’ , ‘G’ , ‘G’ , ‘R’]
Initial Belief state = [0.33, 0.33, 0, 0, 0.33]

$$b'(s_j) = \mathrm{P}(s_j \mid o, a, b) = \frac{P(o \mid s_j, a) \sum_{s_i \in S} P(s_j \mid s_i, a) b(s_i)}{\sum_{s_j \in S} P(o \mid s_j, a) \sum_{s_i \in S} P(s_j \mid s_i, a) b(s_i)}$$

The above formula is used to calculate the next belief state.

In the below calculations the meaning of the terms are as follows:

b = belief state probability for a state
obs = probability of observing the given observation in that state
trans = transition probability
answer = multiplication of above three

Now there are three actions that we take:-
1. Agent took action **Right** and observed **Red**

calculating denominator
From s1

To s1
b = 0.33000, obs = 0.80000, trans = 0.11000 answer=0.02904
To s2
b = 0.33000, obs = 0.80000, trans = 0.89000 answer=0.23496
To s3
b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s4
b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s5
b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000
Sum -> 0.264

From s2
To s1
b = 0.33000, obs = 0.80000, trans = 0.11000 answer=0.02904
To s2
b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s3
b = 0.33000, obs = 0.05000, trans = 0.89000 answer=0.01469
To s4
b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s5
b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000
Sum -> 0.043725

From s3
To s1
b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s2
b = 0.00000, obs = 0.80000, trans = 0.11000 answer=0.00000
To s3
b = 0.00000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s4

b = 0.00000, obs = 0.05000, trans = 0.89000 answer=0.00000
To s5
b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000
Sum -> 0.0

From s4
To s1
b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s2
b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s3
b = 0.00000, obs = 0.05000, trans = 0.11000 answer=0.00000
To s4
b = 0.00000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s5
b = 0.00000, obs = 0.80000, trans = 0.89000 answer=0.00000
Sum -> 0.0

From s5
To s1
b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s2
b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000
To s3
b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000
To s4
b = 0.33000, obs = 0.05000, trans = 0.11000 answer=0.00181
To s5
b = 0.33000, obs = 0.80000, trans = 0.89000 answer=0.23496
Sum -> 0.236775
Denominator -> 0.5445

For s1 :-

calculating numerator

To s1

b = 0.33, obs = 0.8, trans = 0.11 answer=0.02904

To s2

b = 0.33, obs = 0.8, trans = 0.11 answer=0.02904

To s3

b = 0, obs = 0.8, trans = 0 answer=0.0

To s4

b = 0, obs = 0.8, trans = 0 answer=0.0

To s5

b = 0.33, obs = 0.8, trans = 0 answer=0.0

Numerator ->0.0290399 + 0.0290399 = 0.05808

Numerator / denominator = 0.10666

B'[s1'] = 0.10666

For s2:-

calculating numerator

To s1

b = 0.33000, obs = 0.80000, trans = 0.89000 answer=0.23496

To s2

b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000

To s3

b = 0.00000, obs = 0.80000, trans = 0.11000 answer=0.00000

To s4

b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000

To s5

b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000

numerator -> 0.23496

Numerator/Denominator = 0.4315151515151515

B'[s2] = 0.4315151515151515

For s3:-

calculating numerator

To s1

b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000

To s2

b = 0.33000, obs = 0.05000, trans = 0.89000 answer=0.01469

To s3

b = 0.00000, obs = 0.05000, trans = 0.00000 answer=0.00000

To s4

b = 0.00000, obs = 0.05000, trans = 0.11000 answer=0.00000

To s5

b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000

Numerator -> 0.014685

Numerator/Denominator = 0.02696969696969697

B'[s3] = 0.02696969696969697

For s4:-

calculating numerator

To s1

b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000

To s2

b = 0.33000, obs = 0.05000, trans = 0.00000 answer=0.00000

To s3

b = 0.00000, obs = 0.05000, trans = 0.89000 answer=0.00000

To s4

b = 0.00000, obs = 0.05000, trans = 0.00000 answer=0.00000

To s5

b = 0.33000, obs = 0.05000, trans = 0.11000 answer=0.00181

Numerator -> 0.0018149999999999998

Numerator/Denominator = 0.003333333333333333

B'[s4] = 0.003333333333333333

For s5:-

calculating numerator

To s1

b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000

To s2

b = 0.33000, obs = 0.80000, trans = 0.00000 answer=0.00000

To s3

b = 0.00000, obs = 0.80000, trans = 0.00000 answer=0.00000

To s4

b = 0.00000, obs = 0.80000, trans = 0.89000 answer=0.00000

To s5

b = 0.33000, obs = 0.80000, trans = 0.89000 answer=0.23496

Numerator -> 0.23496

Numerator/Denominator = 0.4315151515151515

B'[s5] = 0.4315151515151515

The Updated Belief state is as follows:-

[0.10666666666666666, 0.4315151515151515, 0.02696969696969697, 0.003333333333333333, 0.4315151515151515]

2. Agent took action **LEFT** and observed **GREEN**

Calculating denominator

From s1

To s1

b = 0.10667, obs = 0.20000, trans = 0.89000 answer=0.01899

To s2

b = 0.10667, obs = 0.20000, trans = 0.11000 answer=0.00235

To s3

b = 0.10667, obs = 0.95000, trans = 0.00000 answer=0.00000

To s4

b = 0.10667, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.10667, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.021333333333333333

From s2
To s1
b = 0.43152, obs = 0.20000, trans = 0.89000 answer=0.07681
To s2
b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000
To s3
b = 0.43152, obs = 0.95000, trans = 0.11000 answer=0.04509
To s4
b = 0.43152, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.12190303030303029

From s3
To s1
b = 0.02697, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.02697, obs = 0.20000, trans = 0.89000 answer=0.00480
To s3
b = 0.02697, obs = 0.95000, trans = 0.00000 answer=0.00000
To s4
b = 0.02697, obs = 0.95000, trans = 0.11000 answer=0.00282
To s5
b = 0.02697, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.007618939393939393

From s4
To s1

b = 0.00333, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.00333, obs = 0.20000, trans = 0.00000 answer=0.00000
To s3
b = 0.00333, obs = 0.95000, trans = 0.89000 answer=0.00282
To s4
b = 0.00333, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.00333, obs = 0.20000, trans = 0.11000 answer=0.00007
sum -> 0.002891666666666666

From s5
To s1
b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000
To s3
b = 0.43152, obs = 0.95000, trans = 0.00000 answer=0.00000
To s4
b = 0.43152, obs = 0.95000, trans = 0.89000 answer=0.36485
To s5
b = 0.43152, obs = 0.20000, trans = 0.11000 answer=0.00949
sum -> 0.37433939393939397
Denominator -> 0.5280863636363636

For s1:-

Calculating Numerator
To s1
b = 0.10667, obs = 0.20000, trans = 0.89000 answer=0.01899
To s2
b = 0.43152, obs = 0.20000, trans = 0.89000 answer=0.07681
To s3
b = 0.02697, obs = 0.20000, trans = 0.00000 answer=0.00000

To s4

b = 0.00333, obs = 0.20000, trans = 0.00000 answer=0.00000

To s5

b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000

Numerator -> 0.09579636363636364

Numerator/Denominator = 0.181402835280042

B'[s1] = 0.181402835280042

For s2:-

calculating numerator

To s1

b = 0.10667, obs = 0.20000, trans = 0.11000 answer=0.00235

To s2

b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000

To s3

b = 0.02697, obs = 0.20000, trans = 0.89000 answer=0.00480

To s4

b = 0.00333, obs = 0.20000, trans = 0.00000 answer=0.00000

To s5

b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000

Numerator -> 0.007147272727272727

Numerator/Denominator = 0.013534287607915372

B'[s2] = 0.013534287607915372

For s3:-

calculating numerator

To s1

b = 0.10667, obs = 0.95000, trans = 0.00000 answer=0.00000

To s2

b = 0.43152, obs = 0.95000, trans = 0.11000 answer=0.04509

To s3

b = 0.02697, obs = 0.95000, trans = 0.00000 answer=0.00000

To s4

b = 0.00333, obs = 0.95000, trans = 0.89000 answer=0.00282

To s5

b = 0.43152, obs = 0.95000, trans = 0.00000 answer=0.00000

Numerator -> 0.04791166666666666

Numerator/Denominator = 0.09072695294904126

B'[s3] = 0.09072695294904126

For s4:-

calculating numerator

To s1

b = 0.10667, obs = 0.95000, trans = 0.00000 answer=0.00000

To s2

b = 0.43152, obs = 0.95000, trans = 0.00000 answer=0.00000

To s3

b = 0.02697, obs = 0.95000, trans = 0.11000 answer=0.00282

To s4

b = 0.00333, obs = 0.95000, trans = 0.00000 answer=0.00000

To s5

b = 0.43152, obs = 0.95000, trans = 0.89000 answer=0.36485

Numerator -> 0.367664393939393

Numerator/Denominator = 0.6962202004378302

B'[s4] = 0.6962202004378302

For s5:-

calculating numerator

To s1

b = 0.10667, obs = 0.20000, trans = 0.00000 answer=0.00000

To s2

b = 0.43152, obs = 0.20000, trans = 0.00000 answer=0.00000

To s3

b = 0.02697, obs = 0.20000, trans = 0.00000 answer=0.00000

To s4

b = 0.00333, obs = 0.20000, trans = 0.11000 answer=0.00007
To s5
b = 0.43152, obs = 0.20000, trans = 0.11000 answer=0.00949
Numerator -> 0.009566666666666666
Numerator/Denominator = 0.018115723725171215
B'[s5] = 0.018115723725171215

New Belief state
[0.181402835280042, 0.013534287607915372, 0.09072695294904126, 0.6962202004378302, 0.018115723725171215]

3. Agent took the action **Left** and observed **Green**
calculating denominator

From s1
To s1
b = 0.18140, obs = 0.20000, trans = 0.89000 answer=0.03229
To s2
b = 0.18140, obs = 0.20000, trans = 0.11000 answer=0.00399
To s3
b = 0.18140, obs = 0.95000, trans = 0.00000 answer=0.00000
To s4
b = 0.18140, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.18140, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.036280567056008390

From s2
To s1
b = 0.01353, obs = 0.20000, trans = 0.89000 answer=0.00241
To s2
b = 0.01353, obs = 0.20000, trans = 0.00000 answer=0.00000

To s3
b = 0.01353, obs = 0.95000, trans = 0.11000 answer=0.00141
To s4
b = 0.01353, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.01353, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.0038234362492360 92

From s3
To s1
b = 0.09073, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.09073, obs = 0.20000, trans = 0.89000 answer=0.01615
To s3
b = 0.09073, obs = 0.95000, trans = 0.00000 answer=0.00000
To s4
b = 0.09073, obs = 0.95000, trans = 0.11000 answer=0.00948
To s5
b = 0.09073, obs = 0.20000, trans = 0.00000 answer=0.00000
sum -> 0.025630364208104155

From s4
To s1
b = 0.69622, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.69622, obs = 0.20000, trans = 0.00000 answer=0.00000
To s3
b = 0.69622, obs = 0.95000, trans = 0.89000 answer=0.58865
To s4
b = 0.69622, obs = 0.95000, trans = 0.00000 answer=0.00000
To s5
b = 0.69622, obs = 0.20000, trans = 0.11000 answer=0.01532
sum -> 0.6039710238798176

From s5
To s1
b = 0.01812, obs = 0.20000, trans = 0.00000 answer=0.00000
To s2
b = 0.01812, obs = 0.20000, trans = 0.00000 answer=0.00000
To s3
b = 0.01812, obs = 0.95000, trans = 0.00000 answer=0.00000
To s4
b = 0.01812, obs = 0.95000, trans = 0.89000 answer=0.01532
To s5
b = 0.01812, obs = 0.20000, trans = 0.11000 answer=0.00040
sum -> 0.0157153903315860 25

Denominator -> 0.6854207817247522

For s1:-

calculating numerator
To s1
b = 0.18140, obs = 0.20000, trans = 0.89000 answer=0.03229
To s2
b = 0.01353, obs = 0.20000, trans = 0.89000 answer=0.00241
To s3
b = 0.09073, obs = 0.20000, trans = 0.00000 answer=0.00000
To s4
b = 0.69622, obs = 0.20000, trans = 0.00000 answer=0.00000
To s5
b = 0.01812, obs = 0.20000, trans = 0.00000 answer=0.00000
Numerator -> 0.03469880787405641

Numerator/Denominator = 0.05062409661221883
B'[s1] = 0.05062409661221883

For s2:-

calculating numerator

To s1

b = 0.18140, obs = 0.20000, trans = 0.11000 answer=0.00399

To s2

b = 0.01353, obs = 0.20000, trans = 0.00000 answer=0.00000

To s3

b = 0.09073, obs = 0.20000, trans = 0.89000 answer=0.01615

To s4

b = 0.69622, obs = 0.20000, trans = 0.00000 answer=0.00000

To s5

b = 0.01812, obs = 0.20000, trans = 0.00000 answer=0.00000

Numerator -> 0.020140260001090267

Numerator/Denominator = 0.02938378954663515

B'[s2] = 0.02938378954663515

For s3:-

calculating numerator

To s1

b = 0.18140, obs = 0.95000, trans = 0.00000 answer=0.00000

To s2

b = 0.01353, obs = 0.95000, trans = 0.11000 answer=0.00141

To s3

b = 0.09073, obs = 0.95000, trans = 0.00000 answer=0.00000

To s4

b = 0.69622, obs = 0.95000, trans = 0.89000 answer=0.58865

To s5

b = 0.01812, obs = 0.95000, trans = 0.00000 answer=0.00000

Numerator -> 0.5900685125252125

Numerator/Denominator = 0.8608850625164868

B'[s3] = 0.8608850625164868

For s4:-

calculating numerator

To s1

b = 0.18140, obs = 0.95000, trans = 0.00000 answer=0.00000

To s2

b = 0.01353, obs = 0.95000, trans = 0.00000 answer=0.00000

To s3

b = 0.09073, obs = 0.95000, trans = 0.11000 answer=0.00948

To s4

b = 0.69622, obs = 0.95000, trans = 0.00000 answer=0.00000

To s5

b = 0.01812, obs = 0.95000, trans = 0.89000 answer=0.01532

Numerator -> 0.02479781099280707

Numerator/Denominator = 0.03617895992357764

B'[s4] = 0.03617895992357764

For s5:-

calculating numerator

To s1

b = 0.18140, obs = 0.20000, trans = 0.00000 answer=0.00000

To s2

b = 0.01353, obs = 0.20000, trans = 0.00000 answer=0.00000

To s3

b = 0.09073, obs = 0.20000, trans = 0.00000 answer=0.00000

To s4

b = 0.69622, obs = 0.20000, trans = 0.11000 answer=0.01532

To s5

b = 0.01812, obs = 0.20000, trans = 0.11000 answer=0.00040

Numerator -> 0.01571539033158603

Numerator/Denominator = 0.02292809140108176

New Belief state is:-

[0.05062409661221883, 0.02938378954663515, 0.8608850625164868, 0.03617895992357764, 0.02292809140108176]